PHE 86-13

PROCEEDINGS

OF THE

XX INTERNATIONAL SYMPOSIUM AHRENSHOOP

on the Theory of Elementary Particles

from 13 to 17 October 1986

INSTITUT FÜR HOCHENERGIEPHYSIK
AKADEMIE DER WISSENSCHAFTEN DER DDR
BERLIN-ZEUTHEN · DDR

December 1986

NON-ABELIAN GAUGE THEORY IN A CONSTANT HOMOGENEOUS BACKGROUND

Properties of Radiative Corrections and the Two-Loop Effective Potential

H. J. Kaiser
Institut für Hochenergiephysik der AdW, Berlin-Zeuthen, DDR

K. Scharnhorst
Sektion Physik der Humboldt-Universität Berlin, DDR

E. Wieczorek
Institut für Hochenergiephysik der AdW, Berlin-Zeuthen, DDR

1. Introduction

Effective Lagrangians or potentials can be considered as the basic tools to determine the ground state of a QFT. Especially constant homogeneous fields have been discussed as candidates for ground state configurations in non-abelian gauge theories [1]. In 1-loop approximation the effective potential V_{eff} of the gauge field sector has an imaginary part for colour electric as well as for colour magnetic fields, which is usually understood as a signal for the instability of such configurations[1,2]. By studying radiative corrections in constant background fields beyond the 1-loop approximation we hope to obtain a better understanding of V_{eff} and especially of the physical meaning of $\mathrm{Im}\, V_{eff}$.

What is the problem with $\mathrm{Im}\, V_{eff}$? To clarify this point let us turn to QED. Since the classical Schwinger paper[3] it is well-known that the 1-loop expression for V_{eff} has an

imaginary part for constant electric background fields (but not for magnetic ones), which is simply connected with the pair creation rate in an external electric field. The same applies to the quark loop contribution to V_{eff} in QCD[4]. Recently also the 2-loop contributions to V_{eff} for QED have been obtained[5,6]. The imaginary part (in case of electric fields) turned out to be finite and independent of the choice of renormalization conditions[6]. This is fully in accordance with the understanding that $\mathrm{Im}\, V_{eff}$ is connected with an observable quantity which is uniquely determined provided that the usual renormalizations of QED (including the electron mass renormalization) have been performed.

To determine radiative corrections in the non-abelian case we derive in Section 2 the necessary propagators in the framework of a modified real proper-time representation which separately treats the contribution of the negative modes to the gluon propagator. In comparison with formal generalizations of the Schwinger propagator to the non-abelian case[2] the propagator obtained here is better suited to evaluate radiative corrections and to disentangle real and imaginary parts in V_{eff}. In Section 3 radiative corrections and their almost (i.e. up to phase factors) translation invariant structure will be discussed. For this purpose choosing the background field as $B_\mu = -\frac{1}{2} F_{\mu\nu} x_\nu$ (Fock-Schwinger gauge) is of crucial importance. In the resulting expression for $V^{(2)}_{eff}$ all the phase factors drop out and $\mathrm{Im}\, V^{(2)}_{eff}$ can be easily expressed in terms of the renormalized gluon tensor $\Pi_{\mu\nu}$. This finally allows to discuss the physical meaning of Im V_{eff} (gauge field sector) from a new point of view. A discussion of the results will be given in Section 4.

2. Ghost and Gluon Propagators in the Background Field

In a Euclidean SU(2)-symmetric Yang-Mills theory we introduce a constant colour-magnetic background field

$$B^c_\mu = \frac{1}{2} F^c_{\mu\nu} x_\nu = -\frac{1}{2}\delta^{c3} B\, \varepsilon^{\perp}_{\mu\nu} x_\nu , \quad \varepsilon^{\perp}_{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{2.1}$$

The kernel for the ghost field is

$$\tilde{K}^{ab} = -D^{ac}_{\varrho} D^{cb}_{\varrho} \tag{2.2}$$

and for the gluon field

$$K^{ab}_{\mu\nu} = \delta_{\mu\nu} \tilde{K}^{ab} + 2g\, \varepsilon_{abc} F^c_{\mu\nu} \tag{2.3}$$

where

$$D^{ab}_\mu = \delta^{ab}\partial_\mu + g\, \varepsilon^{abc} B^c_\mu . \tag{2.4}$$

Here the background gauge fixing term with gauge parameter $\alpha=1$ has been chosen as usual. For simplicity of notation we take $gB>0$. It is straightforward to diagonalize $\tilde{K}^{ab}$ and $K^{ab}_{\mu\nu}$ in colour indices

$$\tilde{K} = U \begin{pmatrix} k^+ & & \\ & k^- & \\ & & \Delta \end{pmatrix} U^{-1}$$

$$K_{\mu\nu} = U \begin{pmatrix} \delta_{\mu\nu} k^+ + 2igB\,\varepsilon^{\perp}_{\mu\nu} & & \\ & \delta_{\mu\nu} k^- - 2igB\,\varepsilon^{\perp}_{\mu\nu} & \\ & & \delta_{\mu\nu}\Delta \end{pmatrix} U^{-1} \tag{2.5}$$

$$\Delta = \partial_\varrho^2$$

and to diagonalize $K_{\mu\nu}$ further in space-time indices

$$\delta_{\mu\nu} k^+ + 2igB\,\varepsilon^{\perp}_{\mu\nu} = R \begin{pmatrix} k^{+-} & & & \\ & k^{++} & & \\ & & k^+ & \\ & & & k^+ \end{pmatrix} R^{-1}$$

$$\delta_{\mu\nu} k^- - 2igB\,\varepsilon^{\perp}_{\mu\nu} = R \begin{pmatrix} k^{-+} & & & \\ & k^{--} & & \\ & & k^- & \\ & & & k^- \end{pmatrix} R^{-1} \tag{2.6}$$

where

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 & 0 \\ -1 & -i & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}, \quad R = \frac{1}{\sqrt{2}} \begin{pmatrix} i & 1 & 0 & 0 \\ -1 & -i & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{pmatrix}$$

$$k^{+\pm} = k^+ \pm 2gB, \qquad k^{-\pm} = k^- \pm 2gB. \tag{2.7}$$

The Green function of an operator K with the eigenvalue equation

$$K u_n = \lambda_n u_n \tag{2.8}$$

is given by

$$G(x,x') = \sum_n \frac{u_n(x)\, u_n^*(x')}{\lambda_n}. \tag{2.9}$$

To compute the inverses of the operators

$$k^{\pm} = -\left[\partial_\varrho^2 - \frac{g^2B^2}{4}(x_1^2+x_2^2) \pm igB(x_1\partial_2 - x_2\partial_1)\right] \tag{2.10}$$

we substitute $z_{1,2} = \sqrt{gB/2}\; x_{1,2}$ and introduce the creation and annihilation operators

$$\begin{aligned} d^\dagger &= \frac{1}{2}\left[-\partial_1 + z_1 + i(-\partial_2 + z_2)\right] \\ d &= \frac{1}{2}\left[\partial_1 + z_1 - i(\partial_2 + z_2)\right] \\ \tilde{d}^\dagger &= \frac{1}{2}\left[-\partial_1 + z_1 - i(-\partial_2 + z_2)\right] \\ \tilde{d} &= \frac{1}{2}\left[\partial_1 + z_1 + i(\partial_2 + z_2)\right] \end{aligned} \tag{2.11}$$

with the commutators

$$[a, a^+] = [\tilde{a}, \tilde{a}^+] = 1$$

$$[a, \tilde{a}] = [a^+, \tilde{a}^+] = [a, \tilde{a}^+] = [\tilde{a}, a^+] = 0 \tag{2.12}$$

and arrive after a Fourier transform for the x_3, x_4 -dependence at the expressions

$$h^+(x_1, x_2, k_{\parallel}) = k_{\parallel}^2 + gB\,(2a^+a + 1)$$

$$h^-(x_1, x_2, k_{\parallel}) = k_{\parallel}^2 + gB\,(2\tilde{a}^+\tilde{a} + 1). \tag{2.13}$$

The operators h^+ and h^- are 2-dimensional harmonic oscillator Hamiltonians. Their eigenfunctions are

$$u_{mn}(z_\perp) = \frac{\text{const}}{\sqrt{\pi\, m!\, n!}} (a^+)^m (\tilde{a}^+)^n e^{-z_\perp^2/2}$$

$$= \frac{\text{const.}}{2^{m+n}} \frac{e^{-z_\perp^2/2}}{\sqrt{\pi\, m!\, n!}} \sum_{\mu,\nu} \binom{m}{\mu}\binom{n}{\nu} i^{m-n+\mu-\nu} H_{\mu+\nu}(z_1)\, H_{m+n-\mu-\nu}(z_2) \tag{2.14}$$

and their (infinitely degenerate) eigenvalues

$$\lambda_{mn}^+ = k_{\parallel}^2 + (2m+1)\,gB, \qquad \lambda_{mn}^- = k_{\parallel}^2 + (2n+1)\,gB \tag{2.15}$$

for h^+ and h^- respectively. We emphasize that the complete set of eigenfunctions for the 2-dimensional oscillator problem is given by the u_{mn} of eq. (2.14) and not by u_{mo} or u_{on} alone.

We use the integral representation of the Hermite polynomials

$$H_n(x) = \frac{(-1)^n}{2\sqrt{\pi}} e^{x^2} \int_{-\infty}^{\infty} dk\, (ik)^n e^{-k^2/4 + ikx} \tag{2.16}$$

to evaluate (2.14) and obtain

$$u_{mn}(z_\perp) = \sqrt{\frac{gB}{2}} \frac{e^{z_\perp^2/2}}{4\pi^{3/2}\sqrt{m!\,n!}} \int d^2k_\perp\, e^{-k_\perp^2/4 + ik_\perp z_\perp} \times$$

$$\times \left(\frac{-ik_1 + k_2}{2}\right)^m \left(\frac{-ik_1 - k_2}{2}\right)^n \tag{2.17}$$

properly normalized in accordance with

$$\sum_{m,n} u_{mn}(x_\perp)\, u_{mn}^*(x_\perp') = \delta(x_\perp - x_\perp').$$

Now we find for the inverses of our operators h^+ and h^-

$$[h^\pm(x,x')]^{-1} = \int \frac{d^2k_{\parallel}}{(2\pi)^2} e^{ik_{\parallel}(x-x')_{\parallel}} \sum_{m,n} \frac{u_{mn}(x_\perp)\, u_{mn}^*(x_\perp')}{\lambda_{mn}^\pm}$$

$$= \int \frac{d^2k_{\parallel}}{(2\pi)^2} \frac{e^{ik_{\parallel}(x-x')_{\parallel}}}{4\pi} e^{\pm i \frac{gB}{2}(x_1x_2' - x_2x_1')} \times$$

$$\times \int_0^1 \frac{d\alpha}{1-\alpha}\, \alpha^{\left(\frac{k_{\parallel}^2}{2gB} - \frac{1}{2}\right)} e^{-\frac{gB}{4}\frac{1+\alpha}{1-\alpha}(x_\perp - x_\perp')^2} \tag{2.18}$$

By means of a Fourier transform of the factor $\exp(..(x_\perp - x_\perp')^2)$ and with the substitution $\alpha = e^{-s}$ we can rewrite the result in the form of the propagator for scalar QED

$$[h^\pm(x,x')]^{-1} = \frac{1}{2gB} e^{\pm i\varphi(x,x')} \int \frac{d^4k}{(2\pi)^4} e^{ikx} \times$$

$$\times \int_0^\infty \frac{ds}{\cosh s/2}\, e^{-\frac{k_{\parallel}^2 s}{2gB} - \frac{k_\perp^2}{gB}\tanh s/2} \tag{2.19}$$

where the phase

$$\varphi(x,x') = \frac{gB}{2}(x_1x_2' - x_2x_1') \tag{2.20}$$

can be represented as an integral along the straight line connecting x and x'

$$\varphi(x,x') = \int_x^{x'} dy_\mu\, B_\mu. \tag{2.21}$$

Apart from the phase the expression (2.19) is translation-invariant.

The inverses of the operators $h^\pm + 2gB$ are simply obtained by substituting $\exp(-s(1 + k_{\parallel}^2/2gB))$ for $\exp(-sk_{\parallel}^2/2gB)$ in (2.19).

In the case of the inverses of $h^\pm - 2gB$ we have a negative mode at m=0 resp. n=0. The formal substitution $\exp(-s(-1 + k_{\parallel}^2/2gB))$ in (2.19) would lead to a divergent integral over s. Therefore we separate the negative mode.

We take u_{on} from (2.17) and perform the sum

$$\sum_{n=0}^{\infty} u_{on}(z)\, u_{on}^{*}(z') = \frac{1}{\pi}\, e^{i\varphi(z,z')}\, e^{-(z_\perp - z_\perp')^2/2} \tag{2.22}$$

This gives the m=0 contribution

$$\begin{aligned}\left[h^{+}(x,x') - 2gB\right]^{-1} &= \frac{gB}{2} \int \frac{d^2 k_\parallel}{(2\pi)^2}\, \frac{e^{ik_\parallel (x-x')_\parallel}}{k_\parallel^2 - gB}\, e^{i\varphi(x,x')}\, e^{-\frac{gB}{4}(x_\perp - x_\perp')^2} \\ &= 2\, e^{i\varphi(x,x')} \int \frac{d^4 k}{(2\pi)^4}\, \frac{e^{ik(x-x')}}{k_\parallel^2 - gB}\, e^{-\frac{k_\perp^2}{gB}}\end{aligned} \tag{2.23}$$

The remaining sum

$$\sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \frac{u_{mn}(z_\perp)\, u_{mn}^{*}(z_\perp')}{\lambda_{mn}^{+} - 2gB} \tag{2.24}$$

contributing to $\left[h^{+} - 2gB\right]^{-1}$ can be done without difficulties.

The result for $\left[h^{-} - 2gB\right]^{-1}$ is obtained from $\left[h^{+} - 2gB\right]^{-1}$ by changing the sign of the phase.

Collecting all contributions we present the final results:

The ghost propagator is

$$G^{ab}(x,x') = \phi^{ab}(x,x')\, D^{0}(x-x') + \frac{\delta^{a3}\delta^{b3}}{4\pi^2 (x-x')^2} \tag{2.25}$$

and the gluon propagator

$$\begin{aligned}G^{ab}_{\mu\nu}(x,x') &= \phi^{ab}(x,x')\left[\frac{D^{+}(x-x') + D^{-}(x-x')}{2}\, \delta^{\perp}_{\mu\nu} + D^{0}(x-x')\, \delta^{\parallel}_{\mu\nu}\right] \\ &+ \frac{i}{B}\left(F_{\mu\nu}\, \phi(x,x')\right)^{ab} \frac{D^{+}(x-x') - D^{-}(x-x')}{2} + \frac{\delta^{a3}\delta^{b3}\delta_{\mu\nu}}{4\pi^2 (x-x')^2}\end{aligned} \tag{2.26}$$

where the phase factor is given by

$$\phi^{ab}(x,x') = \left(e^{ig\int dy_\mu B_\mu}\right)^{ab} - \delta^{a3}\delta^{b3} \tag{2.27}$$

involving the line integral as in eq. (2.21). Furthermore,

$$\left(F_{\mu\nu}\right)^{ab} = -i\, B\, \varepsilon^{\perp}_{\mu\nu}\, \varepsilon^{ab3} \tag{2.28}$$

is the field strength in the adjoint representation.

The D-functions read explicitely

$$\begin{aligned}D^{0}(x) &= \frac{1}{2gB}\int \frac{d^4 k}{(2\pi)^4}\, e^{ikx} \int_0^\infty \frac{ds}{\cosh s/2}\, e^{-\frac{k_\parallel^2 s}{2gB} - \frac{k_\perp^2}{gB}\tanh s/2} \\ D^{+}(x) &= \frac{1}{2gB}\int \frac{d^4 k}{(2\pi)^4}\, e^{ikx}\left[\frac{k_\perp^2}{gB}\int_0^\infty \frac{ds}{\cosh s/2}\int_{\tanh s/2}^{1} d\tau\, e^{\left(1 - \frac{k_\parallel^2}{2gB}\right)s - \frac{k_\perp^2}{gB}\tau}\right. \\ &\left. - \int_0^\infty \frac{ds}{\cosh s/2}\, e^{-\frac{k_\parallel^2 s}{2gB} - \frac{k_\perp^2}{gB}} + 4\, \frac{e^{-\frac{k_\perp^2}{gB}}}{\frac{k_\parallel^2}{gB} - 1}\right] \\ D^{-}(x) &= \frac{1}{2gB}\int \frac{d^4 k}{(2\pi)^4}\, e^{ikx}\int_0^\infty \frac{ds}{\cosh s/2}\, e^{-\left(\frac{k_\parallel^2}{2gB} + 1\right)s - \frac{k_\perp^2}{gB}\tanh s/2} .\end{aligned} \tag{2.29}$$

Let us note finally that choosing the background field in a different gauge (e.g. $B_1 = -Bx_1$, $B_2 = B_3 = B_4 = 0$) one would not arrive at structures like (2.25) or (2.26)

3. Two-Loop Corrections to Im V_{eff} —

Evaluating 2-loop corrections to V_{eff} amounts essentially to perform perturbation calculations where the usual propagators are replaced by those in a background field. Of course, all the interactions with the background field are summed up already. Let us also mention that - due to the negative modes - an intermediate regularization of the underlying functional integral has to be supposed. Note, however, that it is one and the same procedure which on the one hand justifies functional integration and the appearance of the gluon determinant in the 1-loop expression for V_{eff} and on the other hand the perturbation theory around the background field configuration.

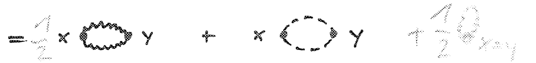
Such a perturbation theory and the resulting radiative corrections will differ considerably from the structures known in the zero-field case. The reason is that the phase factors in the expressions (2.25), (2.26) for the propagators are neither diagonal in colour space nor translational invariant.

Let us first note the couplings between the gauge field fluctuations Q . The case of the ghost-gluon vertex is obvious. From the gauge fixing term (in background gauge) we get the Faddeev-Popov Lagrangian $\bar{c}_a D_\mu^{ab}(B) D_\mu^{bd}(B+Q) c_d$ which leads to the vertex $g\bar{c}_a D_\mu^{ab}(B) f_{bde} Q_\mu^e c_d$. Expanding the 3- and 4-point vertices of the gauge field around B one obtains, besides the well-known Q^4 term, the cubic term

$$\frac{g}{2} f_{abc} Q_\mu^b Q_\nu^c \left(D_\mu^{ad} Q_\nu^d - D_\nu^{ad} Q_\mu^d \right). \qquad (3.1)$$

As a rule, the couplings in the background case are the usual ones up to the modification that all derivatives must be replaced by covariant ones. This fact, together with the form (2.25), (2.26) of the propagators, allows to simplify the expressions for radiative corrections to a large extent.

Since we are primarily interested in the imaginary part of the 2-loop effective potential $V_{eff}^{(2)}$ it is sufficient to restrict the consideration to the general structure of the polarization tensor

$\Pi_{\mu\nu}(x,y) =$ 

(the lines denote gluon resp. ghost propagators in the background field). These diagrams lead to the following form for $\Pi_{\mu\nu}$

$$\Pi_{\mu\nu}^{ab}(x,y) = \delta^{a3}\delta^{b3}\,\Pi_{\mu\nu}^{(1)}(x-y) + \phi^{ab}(x,y)\,\Pi_{\mu\nu}^{(2)}(x-y)$$
$$+\, i\left\{ (F_{\mu\varrho}\phi)^{ab}(x-y)_\varrho (x-y)_\nu - (\mu \leftrightarrow \nu) \right\} \Pi^{(3)}(x-y) \qquad (3.2)$$

(compare (2.27) and (2.28)). Here $\Pi_{\mu\nu}^{(1)}$, $\Pi_{\mu\nu}^{(2)}$, and $\Pi^{(3)}$ are functions of x-y only, i.e. they are translation invariant. The factors which are not invariant under translation are present in (2.25) and (2.26) already. Therefore the expression (3.2) may be called almost translation invariant. In deriving (3.2) we have used

$$D_{x_\nu}^{ab}\left\{ \phi^{bc}(x,y) f(x-y) \right\} = \phi^{ac}(x,y)\frac{\partial}{\partial x_\nu} f(x-y) + \frac{ig}{2}(F_{\nu\varrho}\phi)^{ac}(x-y)_\varrho f(x-y)$$
$$\varepsilon^{bce}\varepsilon^{b'c'e'}\phi^{cb'}(x,y)\phi^{c'b}(y,x) = -2\,\delta^{e3}\delta^{e'3}$$
$$\varepsilon^{bce}\varepsilon^{b'c'e'}\left(F_{\mu\nu}\phi(x,y)\right)^{cb'}\phi^{c'b}(x,y) = 0 \qquad (3.3)$$
$$\varepsilon^{bce}\varepsilon^{b'c'e'}\left(F_{\mu\nu}\phi(x,y)\right)^{cb'}\left(F_{\varrho\sigma}\phi(y,x)\right)^{c'b} = -2\,\delta^{e3}\delta^{e'3}B^2\,\varepsilon_{\mu\nu}^{\perp}\,\varepsilon_{\varrho\sigma}^{\perp}$$

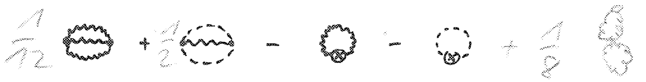
together with the obvious relations

$$\phi^{ab}(x,y) = \phi^{ba}(y,x),$$
$$\phi^{ab}(x,y)\,\phi^{bc}(y,x) = \delta^{ac}\left(\delta^{a1}+\delta^{a2}\right),$$
$$G_{\mu\nu}^{ab}(x,y) = G_{\nu\mu}^{ba}(y,x), \qquad G^{ab}(x,y) = G^{ba}(y,x).$$

Knowledge of the explicit expressions for $\Pi_{\mu\nu}^{(1)}, \Pi_{\mu\nu}^{(2)}, \Pi^{(3)}$ in terms of D^+, D^-, and D^0 is not necessary for the subsequent considerations. What we have to study, however, is the structure of the conterterms which are required to compensate the ultraviolet singularities in (3.2). Let us at first look at the short-distance behaviour of the propagators. The leading singularity of D^+, D^-, and D^0 is that of a scalar zero-field propagator $D^+ \approx D^- \approx D^0 \sim (x-y)^{-2}$. With $\phi^{ab}(x,y) \approx \delta^{ab}$ we obtain $G^{ab} \sim \delta^{ab}(x-y)^{-2}$ and $G_{\mu\nu}^{ab} \sim \delta^{ab}\,\delta_{\mu\nu}\,(x-y)^{-2}$. Therefore the short-distance singularity of $\Pi_{\mu\nu}^{ab}(x,y)$ and correspondingly the counterterm has the structure

$\delta^{ab}(\delta_{\mu\nu}\Delta - \partial_\mu \partial_\nu)\,\delta(x-y)$ as in the B=0 case.

Let us turn now to V_{eff}. In the 2-loop approximation one has to take into account the following diagrams (including counterterms):



This can be written in terms of the polarization tensor

$$V^{(2)}_{eff} = \frac{1}{V_4} \int dx\, dy \left\{ G^{ab}_{\mu\nu}(x,y)\, \Pi^{ab}_{\mu\nu}(x,y) - c.t. \right\} \tag{3.4}$$

and applying (3.2), (3.3), and (2.26) we get

$$V^{(2)}_{eff} = \frac{1}{V_4} \int dx\, dy \left\{ \left[\frac{D^+ + D^-}{2}\, \delta^{\perp}_{\mu\nu} + D^0\, \delta^{\parallel}_{\mu\nu} \right] \Pi^{(2)}_{\mu\nu} + B\,(D^+ - D^-)\,(x-y)^2_\perp\, \Pi^{(3)} + \frac{\delta_{\mu\nu}\, \Pi^{(4)}_{\mu\nu}}{4\pi^2 (x-y)^2} - c.t. \right\}. \tag{3.5}$$

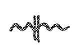
The phase factors have completely disappeared from the final expression (3.5). Thus $V^{(2)}_{eff}$ is given in terms of translation invariant functions only, which has already been anticipated in dividing out the (infinite) 4-volume V_4. The representation (3.5) allows to separate real and imaginary parts immediately.

The imaginary part of $V^{(2)}_{eff}$ originates from the negative mode contribution to D^+ namely

$$\frac{e^{-k_\perp^2/gB}}{k_\parallel^2 - gB} = e^{-k_\perp^2/gB} \left\{ P \frac{1}{k_\parallel^2 - gB} \pm i\pi\, \delta(k_\parallel^2 - gB) \right\}. \tag{3.6}$$

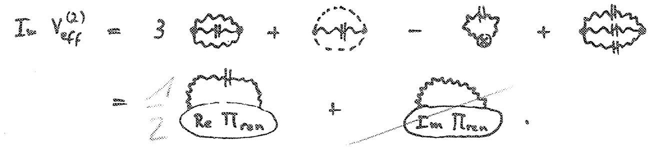
In Euclidean theory it is not obvious how to justify an a-priori choice of the sign. (Compare the analogous situation in the 1-loop case.) Let us denote the imaginary contribution

to the gluon propagator graphically:



$$= \pm \frac{i\pi}{gB}\, \delta(k_\parallel^2/gB - 1)\, e^{-k_\perp^2/gB}. \tag{3.7}$$

Then we obtain trivially that $\mathrm{Im}\, V^{(2)}_{eff}$ is determined by the following diagrams (now uncrossed lines are understood to represent the gluon propagator from which the imaginary contribution (3.7) is removed):



(3.8)

Here a warning is in order: The upper diagrams represent a simple book-keeping of imaginary parts and have nothing to do with usual unitarity cutting rules! In (3.8) Re Π_{ren} denotes the polarization tensor evaluated with two real and two imaginary parts of the gluon propagator, whereas in Im Π_{ren} (beeing in fact uv finite from the beginning) one of the gluon propagators is reduced to the cut contribution:

$\mathrm{Im}\, \Pi_{ren} = 2$  (3.9)

Leaving aside the question whether the second diagram in (3.8) is finite or not, we observe that $\mathrm{Im}\, V^{(2)}_{eff}$ contains Re Π_{ren} which is undetermined up to an arbitrary term const. $\delta^{ab}(\Delta\, \delta_{\mu\nu} - \partial_\mu \partial_\nu)\,\delta(x-y)$.

Therefore Im $V^{(2)}_{eff}$ remains undetermined in any case.

4. Discussion

We regard three of the results presented here as useful new achievements.

That is at first the representation (2.26), (2.29) of the gluon propagator in constant homogeneous background fields given in the form of a real Euclidean proper-time representation. It has turned out to be necessary, however, to separate the contribution of the negative modes to the propagator, which later on becomes the origin of imaginary parts of the radiative corrections. The explicit expressions for the propagators allow to study radiative corrections in perturbation theory. The justification for doing this is the same regularizing procedure (tacitly agreed upon) which makes the gluon determinant to appear in the one-loop effective potential in spite of the fact that the negative modes spoil the Gaussian behaviour of the functional integral. We would like to emphasize that whenever we discuss $V_{eff}^{(1)}$ the two-(and higher-)loop contributions should be kept on equal rights.

The structure of $\Pi_{\mu\nu}$ and $V_{eff}^{(2)}$ given in (3.2) and (3.5) respectively should be considered as the second interesting insight. This shows especially that the phase factors in (2.26) and (3.2) have been dropped out and $\mathrm{Im}\ V_{eff}^{(2)}$ can be obtained immediately from the imaginary part (3.6) related to the negative modes.

The final result concerns $\mathrm{Im}\ V_{eff}^{(2)}$ which turns out to be subject to renormalization arbitrariness. Another line of arguments leading to the same result can be found in /7/.
In other words, $\mathrm{Im}\ V_{eff}^{(2)}$ can be chosen arbitrarily and e.g. such as to compensate the one-loop expression for $\mathrm{Im}\ V_{eff}$.
This leads us to our main conclusion: The imaginary part of the effective potential for constant colour magnetic fields cannot have a quantitative physical meaning like $\mathrm{Im}\ V_{eff}(E)$ in QED. Whereas this is the statement of a direct result from diagram analysis, physical questions concerning the stability of colour magnetic external fields remain open.
To study whether there is a non-zero probability for gluon pair production would be equivalent to discuss the time evolution of an in-vacuum state characterized by such an external field. The peculiar role of a colour magnetic field shows up also in broken gauge theories (Yang-Mills-Higgs theories).
It is known that $\mathrm{Im}\ V_{eff}^{(1)}$ is non-zero for $gB > M_g^2$ but zero otherwise. This could naively be interpreted as if weak fields were unable to produce massive pairs. This is in sharp contradistinction to unstable QED situations: Arbitrarily weak constant electric fields can produce e^+e^- pairs in principle.

The investigation of propagators and effective potentials has been performed in the Euclidean variant of the gauge theory. If one would like to study the effect of colour electric fields one had to perform an analytic continuation of all expressions together with the replacement $E_{Eucl} \to -iE$.

This would additionally introduce those imaginary contributions to V_{eff} which are known from the QED case. Our problem, however, is mainly the imaginary part which originates from the negative modes of the gluon kernel.

The authors gratefully acknowledge interesting discussions with J. Ambjorn.

References

/1/ I.A.Batalin, S.G.Matinjan, G.K.Savvidi, Jad.Fiz.26(1977)4o7
N.K.Nielsen, P.Olesen, Nucl.Phys. B144(1978)376

/2/ J.Ambjorn, R.J.Hughes, Ann.Phys.(NY) 145(1983)34o

/3/ J.Schwinger, Phys.Rev. 82(1951)664

/4/ P.H.Cox, A.Yildiz, Phys.Rev. D32(1985)819

/5/ W.Dittrich, M.Reuter: Effective Lagrangians in Quantum Electrodynamics, Springer-Verlag, Berlin 1985

/6/ S.L.Lebedev, V.I.Ritus, JETF 86(1984)4o8

/7/ K.Scharnhorst, Thesis

SUPERSYMMETRIC QUANTUM MECHANICS IN CURVED MANIFOLD

V. de Alfaro
Dept. of Theoretical Physics
University of Torino, Italy
C. D'Azeglio 46, 10125 Torino Italy

1. Introduction

The problem of quantizing a particle in a curved metric is not a new one. The requirement of general coordinate invariance for the quantum observables has been extensively investigated (1,2,3,4) both in the canonical and in the functional frames. In the canonical approach a main question is the ordering of non commuting operators in quantum observables. From the quantum functional point of view the same problem appears through the prescription for the passage to the continuum in the functional integral: different prescriptions correspond to different orderings.

The problem is of interest again, both for the relativistic point and the relativistic string in curved metric, particularly for the supersymmetric versions of these models. It is indeed expected that SuSy helps in problems of ordering and a number of interesting papers have appeared in this connection (5,6,7,8).

In this ordering problem the relative role of SuSy and General Coordinate Transformation (GCT) invariance is not always evident; recently S. Fubini, G. Furlan, M. Roncadelli and myself have dedicated attention to the problem (9) and this review is based on our work on the subject. I shall just give here a sketch of our discussion. We shall use the canonical approach, using operators. In sect. 2 I shall introduce the model by the superfield formalism; sect. 3 discusses the canonical formalism and the quantum form for the Hamiltonian and