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The vectorboson and graviton propagators in the presence of multipole forces

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Abstract. For a statistical mechanical system with short-range exchange and long-range multipole interactions we investigate the effect of the multipole forces on the large distance behaviour. The euclidean field theory corresponding to the large distance asymptote of the system exhibits the spin-1 and spin-2 particle propagators in a new class of non-local gauges. Some interesting properties of these cases are discussed.

1. Introduction

It is well known from the theory of critical phenomena that the long-range asymptote of the correlation functions of a statistical mechanical system represent a renormalizable euclidean field theory near the critical point [1, 2]. Provided the Osterwalder–Schrader conditions are fulfilled the euclidean field theory is equivalent to a *bona fide* local quantum field theory in Minkowski space [3]. The generating functional of the statistical correlation functions in the long-range effective form constitutes the generating functional path-integral of a euclidean field theory. Thus it intrinsically defines a quantum field theory.

Elementary particle interactions may thus be considered as the long-range effective interactions of an appropriate statistical mechanical system having a fundamental cut-off and exhibiting short-range exchange interactions. As a cut-off the inverse of the Planck length $1_0 \simeq 10^{-33}$ cm presumably sets a fundamental scale [4]. The idea that such a cut-off has a physical meaning is rather attractive for several reasons. It amounts to a new view of elementary particle interactions where most of its simplicity and regularity may be attributed to universality of statistical mechanical systems near a critical point.

Within this scheme the renormalizability of the long-range effective theory acquires a natural explanation. The non-renormalizable couplings turn out to be irrelevant since they are strongly scaled down at large distances. Even more exciting is the possibility that symmetries may be generated dynamically at large distances [5].

We also know from the renormalization group (RG) analysis of statistical mechanical systems that in a space of dimension $d > 4$ the Gaussian (free field) fixed point describes correctly the long-range behaviour of such systems if we require them to have a stable equilibrium state (ground state). The borderline case $d = 4$ where the

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interaction sets in, as $d \to 4$ from above, happens to coincide with the dimension of physical space-time. Thus use of perturbation theory to a large extent finds a physical justification.

From universality we expect that the long-range effective interactions depend in a significant manner neither on the way the cut-off enters nor on most of the details of the interactions of the original system. The cut-off serves only to set a scale. The interactions are considered to be restricted only by such general principles as positivity, existence of the infinite volume limit, and, to some extent, translational and rotational invariance.

We have to expect that apart from the short-range exchange interactions, the long-range forces are also present. Since the effect of such forces, which we assume to have a multipole expansion, already shows some interesting features in the Gaussian approximation, we present its investigation as a first step. As we shall see, the physical parameters related to the multipole terms formally enter as gauge parameters in the long distance asymptote. Gauge parameters thus can be considered as physical parameters of the cut-off system which turn out to be irrelevant in the long-range regime because of universality.

In Section 2 we specify the model under consideration. The corresponding RG-transformations are written down in Section 3. Afterwards the dipole (spin-1) and quadrupole (spin-2) propagators are discussed in, respectively, Sections 4 and 5.

We shall use the term 'fixed point' here modulo the canonically marginal variables (gauge parameters?). In order to investigate the fixed-point properties of these variables, we have to go beyond the Gaussian case. This consideration will be the subject of forthcoming publications.

2. Multipole interactions (see [5] for dipole interactions)

We consider a statistical mechanical system with the Gaussian fluctuation variables $S_{x,\alpha} \in \{q(x), D_i(x), Q_{ij}(x), \ldots\}$, where $q(x)$ is a scalar, $D_i(x)$ a vector, $Q_{ij}(x)$ a symmetric traceless tensor under spatial rotations. The associated interaction energy is

$$
\begin{aligned}
H_q &= -\sum_{x,y} K_{x-y} q_x q_y && \text{(monopole)} \\
H_D &= -\sum_{x,y} K_{x-y,ik} D_{xi} D_{yk} && \text{(dipole)} \\
H_Q &= -\sum_{x,y} K_{x-y,ijkl} Q_{xij} Q_{ykl} && \text{(quadrupole)}
\end{aligned}
\tag{1}
$$

etc.,

where x and y are points on a regular lattice.

Mixed interaction terms are also present in general. The kernels in (1) consist of an arbitrary rotation covariant short-range exchange interaction term K^{sr} and a long-range interaction term K^{lr} having a multipole expansion.

The multipole potentials in d-dimensional space are the usual derivatives of the monopole potential restricted to the lattice.

$$K_x^{\mathrm{lr}} = \Phi = -\frac{1}{r^{d-2}}$$

$$K_{x,ik}^{\mathrm{lr}} = \partial_i \partial_k \Phi = -(d-2)\left\{d\frac{x_i x_k}{r^{d+2}} - \frac{\delta_{ik}}{r^d}\right\} \tag{2}$$

$$K_{x,ijkl}^{\mathrm{lr}} = \partial_i \partial_j \partial_k \partial_l \Phi =$$
$$= -d(d-2)\left\{(d+4)(d+2)\frac{x_i x_j x_k x_l}{r^{d+6}} - (d+2)\frac{(\delta_{ij} x_k x_l + \mathrm{perm})}{r^{d+4}} + \right.$$
$$\left. + \frac{(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})}{r^{d+2}}\right\}$$

etc.

The partition function (ground state expectation value) is of the form

$$Z = \int \prod dS_{x,\alpha}\, e^{-\overline{H}(S)} \tag{3a}$$

where the Gaussian distributions

$$\prod_{x,\alpha} \rho(S_{x,\alpha}) = e^{-(1/2)b\Sigma_{x,\alpha} S_{x\alpha} S_{x\alpha}}$$

are included in a Hamiltonian $\overline{H}(S)$. The interaction Hamiltonians $\overline{H}(S)$ correspond in field theory language to the euclidean action.

The generating functional for the correlation functions is written as

$$Z\{J\} = Z^{-1} \int \prod dS_{x,\alpha}\, e^{-\overline{H}(S)}\, e^{(J,S)}. \tag{3b}$$

The long-range correlations are essentially unaffected if we replace the lattice by a rotation invariant cut-off $\Lambda = a^{-1}$ in momentum space [1] in the thermodynamic limit. The leading terms of the kernels (2) at large distances are then easily evaluated in momentum space from the integral

$$\mathbf{K}_{\cdot}^{\mathrm{lr}}(q) = \int_{|x|>a} d^d x\, K_{x,\cdot}^{\mathrm{lr}}\, e^{iqx} \tag{4}$$

(see Appendix A). Together with the exchange interaction and site spin distribution terms [1] we have to order $0(q^3)$ for the kernels of $\overline{H}(S)$

$$\mathbf{K}(q) = a_0\{q^{-2} + a_1 + a_2 q^2\} + (m_0^2 + \bar{c}q^2)$$

$$\mathbf{K}_{ik}(q) = b_0\left\{(1 + b_1 q^2)\left(d\frac{q_i q_k}{q^2} - \delta_{ik}\right)\right\} + (m_0^2 + \bar{c}q^2)\,\delta_{ik} + \overline{h}q_i q_k$$

$$\mathbf{K}_{ijkl}(q) = c_0\left\{(d+4)(d+2)\frac{q_i q_j q_k q_l}{q^2} - (d+2)(\delta_{ij} q_k q_l + \mathrm{perm}) + \right. \tag{5}$$
$$\left. + (\delta_{ij}\,\delta_{kl} + \delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{ik})q^2\right\} +$$
$$+ (m_0^2 + \bar{c}q^2)\tfrac{1}{2}(\delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{jk}) + \overline{h}(\delta_{ik} q_j q_l + \cdots)$$

etc.

In each expression the first term is the multipole term. The remaining terms are up to the Gaussian contribution to m_0^2 rotation-covariant exchange interaction terms. We notice that higher multipole contributions are $0(q^3)$ or higher. Thus they never contribute to the leading behaviour which then is dominated by the short-range exchange interactions. Up to the quadrupole–quadrupole interactions, however, the multipole forces contribute non-analytical (non-local) terms.

3. The renormalization group

The long-range behaviour of a statistical mechanical system can be investigated systematically by the renormalization group (RG) method of Wilson [1]. In the Gaussian (free field) case the RG has the trivial form

$$\mathbf{K}'(q', s) = \xi^2(s)s^{-d}\mathbf{K}\left(\frac{q'}{s}\right). \tag{6}$$

s is the scale parameter $s > 1$ and $\xi(s) = s^x$ is the renormalization factor of the fluctuation variable S. The dimension of S is defined by $d_s = d - x$.

The scale exponent x is fixed by the condition that the leading q-dependent exchange force term has coefficient $0(1)$ (wave function renormalization of the kinetic term $\partial S\, \partial S$). In the Gaussian case we have thus $x = (d + 2/2)$ and $d_s = (d - 2/2)$ is canonical.

The parameters of the kernels (5) transform as follows

$$\left.\begin{aligned} m_0'^2 &= s^2 m_0^2 \\ \bar{c}' &= \bar{c} \\ \overline{h'} &= \overline{h} \end{aligned}\right\} \quad \text{(short range)}$$

$$\left.\begin{aligned} a_0' &= s^4 a_0 \\ a_0' a_1' &= s^2 a_0 a_1 \\ a_0' a_2' &= a_0 a_2 \end{aligned}\right\} \quad \text{(monopole)} \tag{7}$$

$$\left.\begin{aligned} b_0' &= s^2 b_0 \\ b_0' b_1' &= b_0 b_1 \end{aligned}\right\} \quad \text{(dipole)}$$

$$c_0' = c_0 \quad \text{(quadrupole).}$$

Below we shall discuss the infrared fixed point ($s \to \infty$) of these transformations and the behaviour of the propagators. We require the thermodynamic limit to exist for the system under consideration. Thus the monopole term has to vanish $a_0 = 0$. $q(x)$ then describes a scalar boson, which will not be considered further.

4. Behaviour of the dipole propagator

The propagators are given by the inverses of the kernels (5). The dipole propagator has the form

$$\mathbf{G}_{ik}(q) = \frac{1}{m^2 + cq^2}\left\{\delta_{ik} - \frac{g + hq^2}{g + hq^2 + m^2 + cq^2}\frac{q_i q_k}{q^2}\right\} \tag{8}$$

with $g = b_0 d$, $h = \bar{h} + b_0 b_1 d$, $c = \bar{c} - b_0 b_1$ and $m^2 = m_0^2 - b_0$. g is a relevant parameter of dimension 2, which relative to the Gaussian fixed point behaves like the mass m^2 whereas c and h are marginal and dimensionless. We may choose $c = 1$. It is remarkable that there are *two* different fixed points for which $\mathbf{G}_{ik}(q)$ stays regular

$$\begin{aligned} &\text{(i)} \quad m^* = 0 \quad \text{and} \quad g^* = 0 \\ &\text{(ii)} \quad m^* = 0 \quad \text{and} \quad g^* = \infty. \end{aligned} \tag{9}$$

We notice that except for $h = -1$, $\mathbf{G}_{ik} = 0(q^{-2})$ as $q^2 \to \infty$ and for $m^2 = 0$, $\mathbf{G}_{ik}$ becomes independent of g in both regimes $q^2 \to \infty$ and $q^2 \to 0$. At the dipolar fixed point (ii) we have

$$\mathbf{G}_{ik} = \frac{1}{q^2}\left\{\delta_{ik} - \frac{q_i q_k}{q^2}\right\}$$

which is the massless spin-1 (photon) propagator in a particular non-local gauge. If $m^2 = 0(a^2)$ and $g = fm^2$ (8) can be considered as a massive vector boson propagator in a class of renormalizable covariant gauges h and f being the gauge parameters. Near the propagator pole we have

$$\mathbf{G}_{ik} \propto \frac{1}{m^2 + q^2}\left\{\delta_{ik} + \frac{q_i q_k}{m^2}\right\}$$

for arbitrary h and f. Thus $\mathbf{G}_{ik}$ has the behaviour of a unitary vector boson propagator as $q^2 \to -m^2$.

If dipole interactions are absent, we are at the isotropic fixed point (i) and $\mathbf{G}_{ik}$ *has the form of a massive vectorboson propagator in the 't Hooft gauge* [6].

$$\mathbf{G}_{ik} = \frac{1}{m^2 + q^2}\left\{\delta_{ik} - \frac{h}{m^2 + (1+h)q^2} q_i q_k\right\}. \tag{9a}$$

The gauge $h = -1$ corresponds to the non-renormalizable unitary gauge, i.e. $\mathbf{G}_{ik} = 0(1)$ as $q^2 \to \infty$. For $m^2 = 0$ we have the photon propagator in the standard class of covariant gauges. It is remarkable that in the absence of dipole forces the covariant forms of the propagators of a vectorial fluctuation variable are obtained in a natural way in the 't Hooft gauge. It also should be noticed that it is the potential and not the field strength that enters in a direct way in this approach to quantization. This fact has no natural explanation in standard approaches [7].

In the presence of interaction terms S^4 it is known that the dipole interactions lead to a new infrared stable non-trivial dipolar fixed point in $d = 4 - \varepsilon$ ($\varepsilon > 0$) dimensions [5]. The usual non-trivial isotropic fixed point is infrared unstable relative to dipole perturbations. The much more interesting dynamical fixed point appearing in the competition of S^4 and $S^2\,\partial S$ interactions (Yang-Mills fixed point) will be discussed in a separate paper.

5. Behaviour of the quadrupole propagator

The quadrupole propagator may be chosen to have the form (see Appendix B)

$$\mathbf{G}_{ijkl} = \frac{1}{m^2 + cq^2} Y_{ijkl}(q) \tag{10}$$

with

$$Y_{ijkl} = \frac{1}{2}\left[\left(\delta_{ik} - \alpha\frac{q_i q_k}{q^2}\right)\left(\delta_{jl} - \alpha\frac{q_j q_l}{q^2}\right) + \text{perm}\right] -$$
$$- \gamma\left(\delta_{ij} - \alpha\frac{q_i q_j}{q^2}\right)\left(\delta_{kl} - \alpha\frac{q_k q_l}{q^2}\right).$$

This requires the addition of two further terms

$$a_0\,\delta_{ij}\,\delta_{kl} \quad \text{and} \quad a_2\left(\delta_{ij}\frac{q_k q_l}{q^2} + \delta_{kl}\frac{q_i q_j}{q^2}\right) \tag{11}$$

to the quadrupole kernel. These terms do not couple to Q_{ij} because of the tracelessness and, therefore, may be chosen so that (10) has a convenient form.

With the parameters

$$g = c_0(d+4)(d+2), \qquad h = \bar{h} - c_0(d+2), \qquad c = \bar{c} + c_0 \quad \text{and} \quad m^2 = m_0^2$$

we have

$$\begin{aligned} \alpha &= \frac{hq^2}{m^2 + cq^2 + hq^2} \\ \gamma &= \frac{g(m^2 + cq^2) - h^2q^2}{d[g(m^2 + cq^2) - h^2q^2] + h^2q^2} = \frac{\rho m^2 + (c\rho - 2)q^2}{d\rho m^2 + [d(c\rho - 2) + 2]q^2} \end{aligned} \tag{12}$$

and

$$\begin{aligned} a_0 &= (m^2 + cq^2)\frac{(m^2 + cq^2)g - h^2q^2}{h^2q^2} = -\tfrac{1}{2}(m^2 + cq^2)[2q^2 - \rho(m^2 + cq^2)]q^{-2} \\ a_2 &= \frac{(m^2 + cq^2)g - h^2q^2}{h} = -\tfrac{1}{2}h[2q^2 - \rho(m^2 + cq^2)]. \end{aligned} \tag{13}$$

The canonical quadrupole fixed point is $m^* = 0$.

The parameter ρ is defined by $g = (\rho/2)h^2 \cdot h$ and g (or ρ) are canonically marginal (dimensionless). Obviously G_{ijkl} can be considered as the propagator of a spin-2 particle in a class of covariant renormalizable gauges g and h being the gauge parameters. We are free to choose $c = 1$. The coefficients a_0, a_2 and γ are non-analytic functions of g and h. a_0 and a_2 are analytic however in h and $\rho = (2g/h^2)$. When $m^2 \neq 0$ a_0 is non-analytic in q^2. At $m^2 = 0$ the expression $\gamma^{-1} = d - (2/2 - \rho)$ can attain any value by the appropriate choice of ρ. Actually for $m^2 = 0$ α *and* γ are q-independent and thus can be considered as the gauge parameters.

Special choices of ρ are listed in Table 1. All coefficients are bounded for $q^2 \to \infty$, which is identical with $m^2 = 0$.

For $h = -1$ we have $\alpha = -(q^2/m^2)$ and hence a non-renormalizable gauge. $h = -1$ and $\rho = 0$ for $m^2 \neq 0$ is the unitary gauge of the massive spin-2 particle and $\rho = 0$, $\alpha = (hq^2/m^2 + (1+h)q^2)$ is the corresponding 't Hooft gauge. On the other hand, for $m^2 = 0$, $\alpha = (h/1+h)$ we obtain the local and covariant Veltman gauge for the graviton with $\alpha = 0$ and $\rho = 1$ [8]. One must remember that the gauge parameters

Table 1
Some special gauges of the spin 2 propagator

ρ	m^2	γ	a_0	a_2
0	m^2	$\frac{1}{d-1}$	$-a_1$	$-a_3$
1	m^2	$\frac{q^2-m^2}{(d-2)q^2-dm^2}$	$-\frac{1}{2}a_1\left(2-\frac{a_1}{q^2}\right)$	$-\frac{1}{2}a_3\left(2-\frac{a_1}{q^2}\right)$
1	0	$\frac{1}{d-2}$	$-\frac{1}{2}a_1$	$-\frac{1}{2}a_3$
∞	m^2	$\frac{1}{d}$	∞	$\infty(h \neq 0)$ $0(h=0)$
ρ	0	$\left(d-\frac{2}{2-\rho}\right)^{-1}$	$-\frac{1}{2}a_1(2-\rho)$	$-\frac{1}{2}a_3(2-\rho)$

(14)

originate from physical couplings on the lattice at short distances. They get their physical irrelevance (if so) in the long-range regime. *The important thing to notice is that within our class of gauges* the *massive and the massless spin-2 propagators with the correct number of physical polarizations are related by a continuous change of the gauge parameters.* Thus this class of gauges may be important for the study of the renormalization of gravitation, particularly in connection with the infrared problem. This situation leads to the possibility of giving the graviton a regularization mass. Physically the gauge $g = 0$ corresponds to the absence of quadrupole interactions. The gauge $h^2 = 2g$ involves quadrupole interactions except in the limit $h \to 0$ (Veltman gauge). Whether h and g are true gauge parameters has to be answered, of course, from dynamical considerations. This question is under investigation.

6. Conclusion

By looking at spin-1 and spin-2 particles as long-range modes of a statistical mechanical system involving some fundamental cut-off we find, in a natural way, certain classes of gauges for the propagators. The multipole interactions lead to new classes of non-local gauges. In the spin-1 case the new gauge includes the 't Hooft gauge. For the spin-2 field we obtain a gauge which connects in a continuous way the massive and the massless propagators in the standard covariant gauges. The new gauge, therefore, may be important for the study of the quantization and the renormalization of gravitational interactions.

Two further comments are in order. First, in our approach to quantization, the potentials, not the field strengths, enter in a natural way. Second, if the short-range exchange forces are present, the leading modes at large distances have a spin $\leqslant 2$. This statement means that the higher spin modes do not appear in the long-range asymptote.

Appendix A

We have to evaluate the integrals

$$\int_{|x|\geqslant a} d^d x\, x_{i_1}\cdots x_{i_n} \frac{1}{|x|^{d+\sigma}} e^{iqx} = (-i)^n \frac{\partial}{\partial q_{i_1}}\cdots\frac{\partial}{\partial q_{i_n}} \mathbf{K}_{q,\sigma} \tag{A1}$$

with

$$\mathbf{K}_{q,\sigma} = \int_{|x|\geqslant a} d^d x\, |x|^{-(d+\sigma)} e^{iqx}.$$

We get

$$\mathbf{K}_{q,\sigma} = (2\pi)^{d/2} a^{-\sigma} f_{\sigma,0}(a|q|) \tag{A2}$$

where

$$f_{\sigma,0}(z) = z^{\sigma} \int_z^{\infty} dr\, r^{-[(d/2)+\sigma]} J_{(d-2/2)}(r).$$

$J_\nu(r)$ is the Bessel function of index ν, and $z = a|q|$. Using then

$$\frac{\partial}{\partial q_i} f(z) = a^2 q_i \frac{1}{z}\frac{d}{dz} f(z)$$

$$f_{\sigma,k}(z) = \left(\frac{1}{z}\frac{d}{dz}\right)^k f_{\sigma,0}(z) = z^{\sigma-2k} \int_z^{\infty} dr\, r^{-[(d/2)+\sigma-k]} J_{(d-2/2)+k}(r)$$

and the relation

$$J_{\nu-1}(r) + J_{\nu+1}(r) = \frac{2\nu}{r} J_\nu(r)$$

we obtain the multipole kernels

$$\mathbf{K} = -(2\pi)^{d/2} \frac{1}{q^2} \int_{a|q|}^{\infty} dr\, r^{-[(d/2)-2]} J_{(d/2)-1}(r)$$

$$\mathbf{K}_{ik} = (2\pi)^{d/2}(d-2)\left\{ d\frac{q_i q_k}{q^2} - \delta_{ik}\right\} \int_{a|q|}^{\infty} dr\, r^{-d/2} J_{(d/2)+1}(r)$$

$$\begin{aligned} \mathbf{K}_{ijkl} = {} & -(2\pi)^{d/2}(d-2)\, d\Big\{(d+4)(d+2)\frac{q_i q_j q_k q_l}{q^2} - \\ & - (d+2)(\delta_{ij} q_k q_l + \cdots) + \\ & + (\delta_{ij}\,\delta_{kl} + \cdots) q^2\Big\} \int_{a|q|}^{\infty} dr\, r^{-[(d/2)+2]} J_{(d/2)+3}(r) \end{aligned} \tag{A3}$$

etc.

The integrals in (A3) have a Taylor expansion in $z = a|q|$ at $z = 0$ given by

$$\int_z^{\infty} dr\, r^{-(\nu-1)} J_\nu(r) = \int_0^{\infty} dr\, r^{-(\nu-1)} J_\nu(r) - \sum_{n=1}^{\infty} \frac{z^{2n}}{2n!} h_{(0)}^{(2n-1)} \tag{A4}$$

with $h(z) = z^{-(\nu-1)} J_\nu(z)$. The multipole terms thus have the form given in Equation (5).

Each multipole kernel is multiplied by a coupling constant which is included in the coefficients a_0, b_0 and c_0. The coefficients a_1, b_1 and a_2 follow from (A4).

Appendix B

The quadrupole kernel has the form

$$\mathbf{K} = G^{-1} = \sum a_i X_i$$

where the X_i denote the following basis of operators:

$$X_0 = \delta_{ij}\,\delta_{kl}$$

$$X_1 = \tfrac{1}{2}(\delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{jk})$$

$$X_2 = \left(\delta_{ij}\frac{q_k q_l}{q^2} + \delta_{kl}\frac{q_i q_j}{q_2}\right) = Z_1 + Z_2$$

$$X_3 = \frac{1}{2}\left(\delta_{ik}\frac{q_j q_l}{q^2} + \delta_{il}\frac{q_j q_k}{q^2} + \delta_{jk}\frac{q_i q_l}{q^2} + \delta_{jl}\frac{q_i q_k}{q^2}\right) - 2\frac{q_i q_j q_k q_l}{q^4}$$

$$X_4 = \frac{q_i q_j q_k q_l}{q^4}.$$

Their multiplication table is listed below in Table 2.

Table 2
Multiplication table of the matrices X_i.

	X_0	X_1	X_3	X_4	Z_1	Z_2
X_0	dX_0	X_0	0	Z_1	dZ_1	X_0
X_1	X_0	X_1	X_3	X_4	Z_1	Z_2
X_3	0	X_3	X_3	0	0	0
X_4	X_4	X_4	0	X_4	X_4	Z_2
Z_1	X_0	Z_1	0	Z_1	Z_1	X_0
Z_2	dZ_2	Z_2	0	X_4	dX_4	Z_2

X_1 is the unit operator and

$$GG^{-1} = G^{-1}G = X_1.$$

For the propagator we find

$$G = \Sigma b_i X_i$$

with

$$b_1 = \frac{1}{a_1}; \qquad b_3 = -b_1\frac{a_3}{a_1 + a_3}$$

$$b_0 = -b_1\frac{a_0 a_1 + \Delta}{\Sigma a_1 + (d-1)\,\Delta}$$

$$b_2 = -b_1\frac{a_2 a_1 - \Delta}{\Sigma a_1 + (d-1)\,\Delta}$$

$$b_4 = -b_1\frac{a_4 a_1 + d\Delta}{\Sigma a_1 + (d-1)\,\Delta}$$

where

$$\Delta = a_0 a_4 - a_2 a_2$$
$$\Sigma = d a_0 + a_1 + 2a_2 + a_4.$$

Since G^{-1} couples to a symmetric traceless tensor, the parameters a_0 and a_2 may be chosen freely. G may then be chosen to have the following standard form:

$$G = b_1 Y$$
$$Y = \frac{1}{2}\left[\left(\delta_{ik} - \alpha\frac{q_i q_k}{q^2}\right)\left(\delta_{jl} - \alpha\frac{q_j q_l}{q^2}\right) + \cdots\right] -$$
$$- \gamma\left[\left(\delta_{ij} - \alpha\frac{q_i q_j}{q^2}\right)\left(\delta_{kl} - \alpha\frac{q_k q_l}{q^2}\right)\right].$$

This determines a_0 and a_2. We find

$$a_0 = \frac{a_1}{a_3}\frac{\Xi}{a_3} \qquad a_2 = \frac{\Xi}{a_3}$$

and

$$\alpha = \frac{a_3}{a_1 + a_3} \qquad \gamma = \frac{\Xi}{d\Xi + a_3 a_3}$$

with $\Xi = a_1(a_4 - 2a_3) - a_3 a_3$.

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