Das anomale magnetische Moment des Myons - neue Physik jenseits des Standardmodells ?

F. Jegerlehner, DESY Zeuthen

Physikalisches Kolloquium,

Fakultät für Physik und Astronomie, Universität Bochum

News Release

BROOKHAVEN NATIONAL LABORATORY

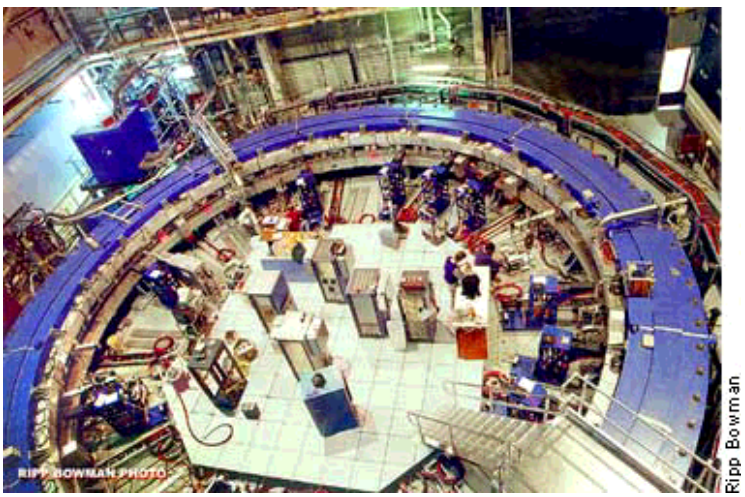
01-12
February 8, 2001

For more information, contact:
Karen McNulty Walsh, (631)344-8350, kmcnulty@bnl.gov,
or Mona S. Rowe, (631)344-5056, mrowe@bnl.gov

Physicists Announce Possible Violation of Standard Model of Particle Physics

UPTON, NY -- Scientists at the U.S. Department of Energy's Brookhaven National Laboratory, in collaboration with researchers from 11 institutions in the U.S., Russia, Japan, and Germany, today announced an experimental result that directly confronts the so-called Standard Model of particle physics. "This work could open up a whole new world of exploration for physicists interested in new theories, such as supersymmetry, which extend the Standard Model," says Boston University physicist Lee Roberts, co-spokesperson for the experiment.

"We are now 99 percent sure that the present Standard Model calculations cannot describe our data"



The g-2 muon storage ring at Brookhaven National Lab. Hi-Res

2.6 σ !

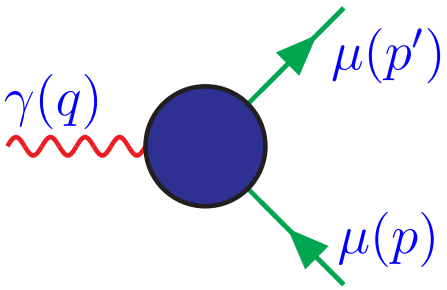
$$\left|a_\mu^{\rm exp} - a_\mu^{\rm the}\right| = 426(165) \times 10^{-11}$$

Lepton magnetic moments: basics

Magnetic moment:

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\vec{s} \;\; ; \quad g_\mu = 2\,(1+a_\mu)$$

Dirac: $g_\mu = 2$ **,** a_μ **muon anomaly**



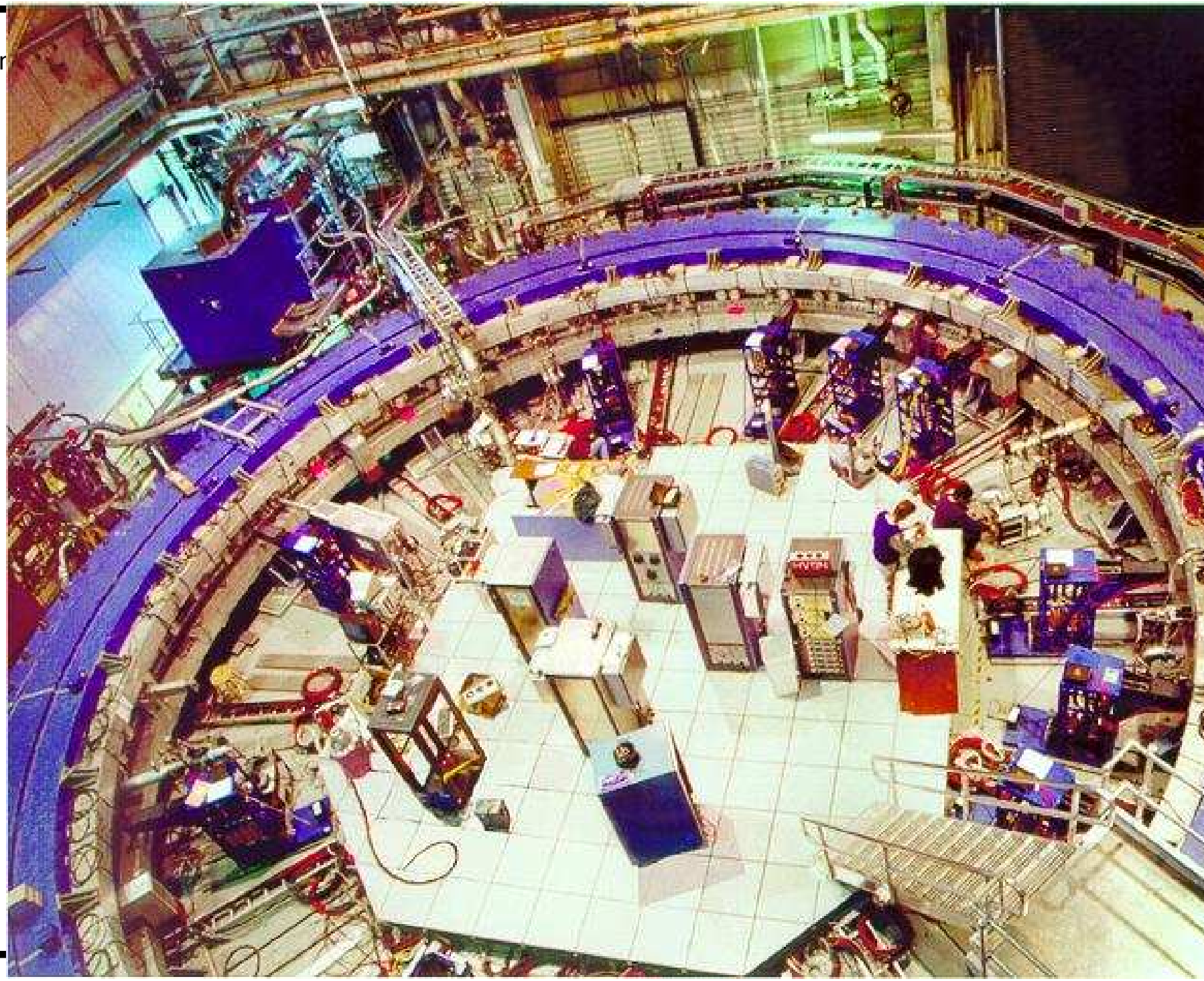
$$= (-ie)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu}F_2(q^2)\right]u(p)$$

$$F_1(0) = 1 \;\; ; \quad F_2(0) = a_\mu$$

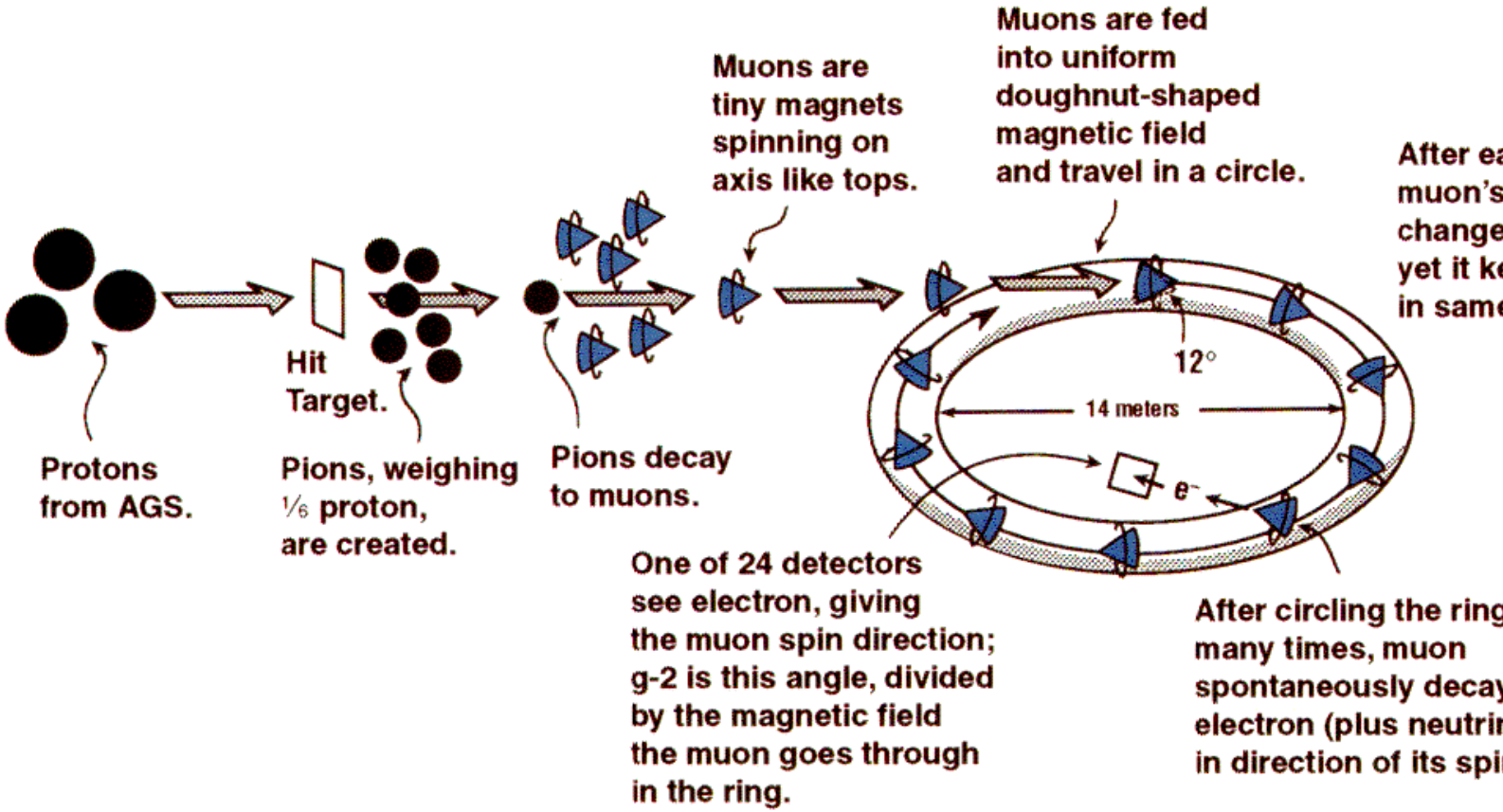
a_μ **responsible for the Larmor precession**

directly proportional at magic energy $\sim$ 3.1 GeV

BNL (g-2) Experir



LIFE OF A MUON: THE g-2 EXPERIMENT



$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right]^{E\sim 3.1\text{GeV}}_{\text{at "magic } \gamma\text{"}} \simeq \frac{e}{m}\left[a_\mu \vec{B}\right]$$

❶ $\alpha_{\rm em}$ **from** $a_e \equiv (g_e - 2)/2$

❷ **The anomalous magnetic moment of the muon**

❸ **Standard model prediction for** a_μ

❹ a_μ **status**

❺ **New physics**

❻ $\alpha_{\rm em}(s)$ **in precision physics**

❼ **Comments on recent progress**

❽ **Status and outlook**

Abstract: The muon anomalous magnetic moment is one of the most precisely measured quantities in particle physics. New high precision measurements at Brookhaven indicate a 2.6 sigma deviation from the electroweak Standard Model and thus might signal a contribution from yet unknown new physics. I review the status of the theoretical predictions and in particular discuss the role of hadronic vacuum polarization effects and the light--by--light scattering contribution. Possible explanations for the deviation are presented.

❶ Most precise $\alpha_{\rm em}$ from $a_e \equiv (g_e - 2)/2$

Compare extraordinary precise measurements of electron anomalous magnetic moment

$$\boxed{a_e \equiv (g_e - 2)/2}$$

$$\begin{aligned} a_{e^-}^{\rm exp} &= 0.001\,159\,652\,188\,4(43), \\ a_{e^+}^{\rm exp} &= 0.001\,159\,652\,187\,9(43), \end{aligned}$$

Dehmelt et al. 1987

with the prediction

$$\begin{aligned} a_e^{\rm SM} = & \frac{\alpha}{2\pi} - 0.328\,478\,444\,00\left(\frac{\alpha}{\pi}\right)^2 \\ & + 1.181\,234\,017\left(\frac{\alpha}{\pi}\right)^3 \\ & - 1.5098(384)\left(\frac{\alpha}{\pi}\right)^4 \\ & + 1.66(3)\times 10^{-12} \quad \textbf{(hadronic \& electroweak loops)} \end{aligned}$$

currently provides the best determination of the fine structure constant

$$\boxed{\alpha^{-1}(a_e) = 137.035\,999\,58(52)\ .}$$

To test the Standard Model requires comparison with an alternative measurement of α with comparable accuracy. Unfortunately, the next best determination of α, from the quantum Hall effect,

$$\alpha^{-1}(qH) = 137.036\,003\,00(270),$$

has a considerably larger error. If one assumes that

$$\left|\Delta a_e^{\text{New Physics}}\right| \simeq m_e^2/\Lambda^2$$

where Λ approximates the scale of "New Physics", then the agreement between $\alpha^{-1}(a_e)$ and $\alpha^{-1}(qH)$ currently probes

$$\Lambda \lesssim \mathcal{O}\,(\textbf{100 GeV}).$$

To access the much more interesting

$$\Lambda \sim \mathcal{O}\,(\textbf{TeV})$$

region would require an order of magnitude improvement in a_e^{exp} (technically feasible), an improved calculation of the 4-loop QED contribution to a_e^{SM} and a much better independent measurement of α^{-1} by almost two orders of magnitude.

❷ The Anomalous Magnetic Moment of the Muon

a_μ most precise test of quantum field theory model (QED/SM)

Experimental results: from muon storage rings CERN 1977:

$$a_\mu^{\rm exp} = 116\,592\,\underline{300}(840) \times 10^{-11}$$

(CERN 1977).

Ongoing experiment (E821) at BNL:

BNL 1998 with μ^+:

$$a_\mu^{\rm exp} = 116\,592\,\underline{050}(460) \times 10^{-11}$$

(CERN'77 + BNL'98).

BNL 1999 with μ^+:

$$a_\mu^{\rm exp} = 116\,592\,\underline{020}(160) \times 10^{-11}$$

(BNL'99).

Averaged together:

$$a_\mu^{\rm exp}({\rm Average}) = 116\,592\,\underline{023}(151) \times 10^{-11}$$

(CERN'77 + BNL'98 & '99).

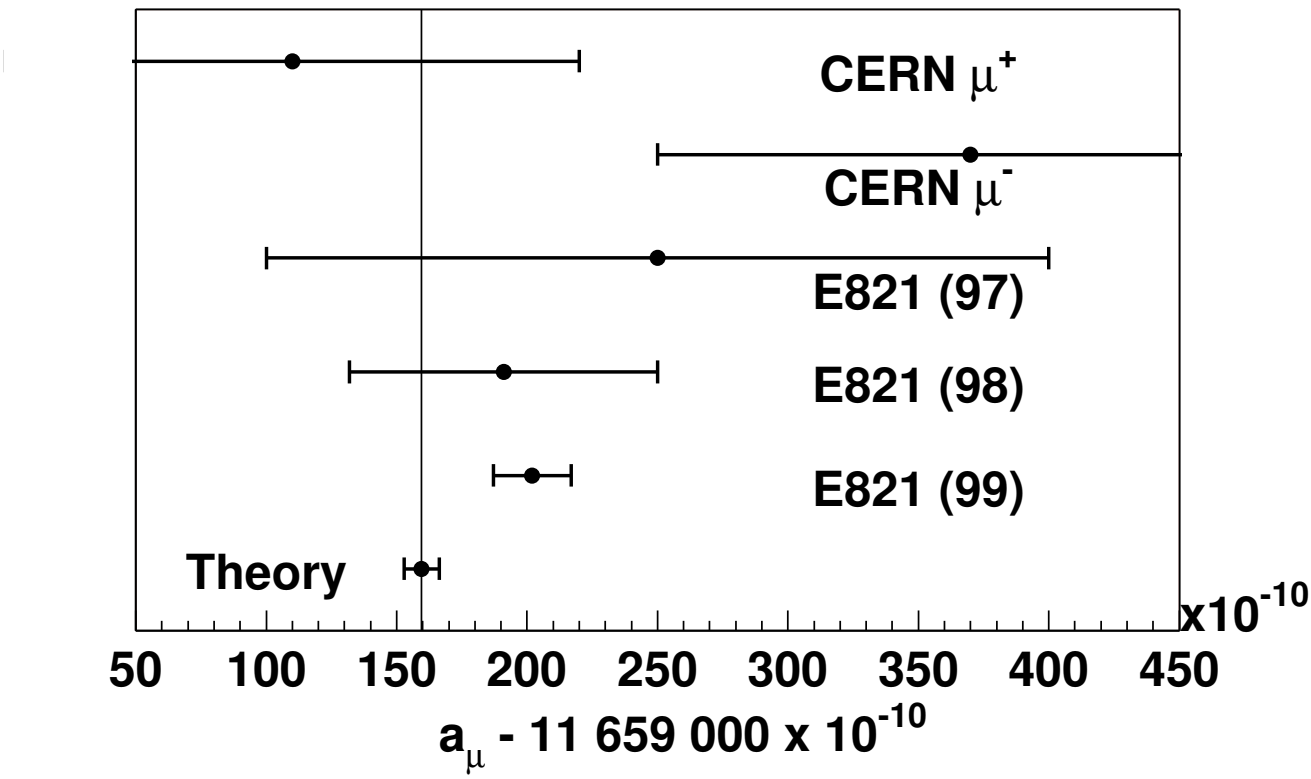
Future: 2000 data with μ^+, 2001 data with μ^-

goal $\pm 40 \times 10^{-11}$ final precision

$\downarrow$

current deviation $\longrightarrow$ 5.4 σ ! limited by theory

Experimental Summary 2001



Muon (g-2) Collaboration

nucl-ex/0102017

Uncertainties:

expt	1.30ppm
theo	0.66ppm
BNL design	0.35ppm

Meaning for precision tests and "New Physics":

- $a_\mu^{\rm exp}$ **is 350 times less precise than** $a_e^{\rm exp}$
- **much more sensitive to hadronic and electroweak quantum loops as well as "New Physics" effects, since such contributions are generally proportional to** m_l^2**.**

The $m_\mu^2/m_e^2 \simeq 40\,000$ **enhancement more than compensates for the reduced experimental precision and makes** $a_\mu^{\rm exp}$ **a much better probe of short-distance phenomena**

more than a factor 100

in sensitivity !

❸ Standard Model Prediction for a_μ

① QED Contribution

The QED contribution to a_μ has been computed (or estimated) through 5 loops

$$\begin{aligned} a_\mu^{\rm QED} &= \frac{\alpha}{2\pi} + 0.765\,857\,376(27)\left(\frac{\alpha}{\pi}\right)^2 \\ &\quad +24.050\,508\,98(44)\left(\frac{\alpha}{\pi}\right)^3 \\ &\quad +126.07(41)\left(\frac{\alpha}{\pi}\right)^4 \\ &\quad +930(170)\left(\frac{\alpha}{\pi}\right)^5 . \end{aligned}$$

Growing coefficients in the α/π expansion reflect the presence of large $\ln\frac{m_\mu}{m_e} \simeq 5.3$ terms coming from electron loops. Employing the value of α from a_e leads to

$$\boxed{a_\mu^{\rm QED} = 116\,584\,705.7(2.9)\times 10^{-11}\ .}$$

The current uncertainty is well below the $\pm 40\times 10^{-11}$ ultimate experimental error anticipated from E821 and should, therefore, play no essential role in the confrontation between theory and experiment.

# of loops	$a_\mu^{\rm QED} \times 10^{11}$
1	116140972.87 (0.44)
2	413217.60 (0.02)
3	30141.90 (0.00)
4	367.01 (1.19)
5	6.29 (1.15)
tot	116584705.66 (2.80)

❶ 1 diagram — Schwinger 1948

❷ 7 diagrams — Peterman 1957, Sommerfield 1957

❸ 72 diagrams — Lautrup, Peterman, de Rafael 1974; Laporta, Remiddi 1993

❹ few 100 diagrams — Kinoshita 1999

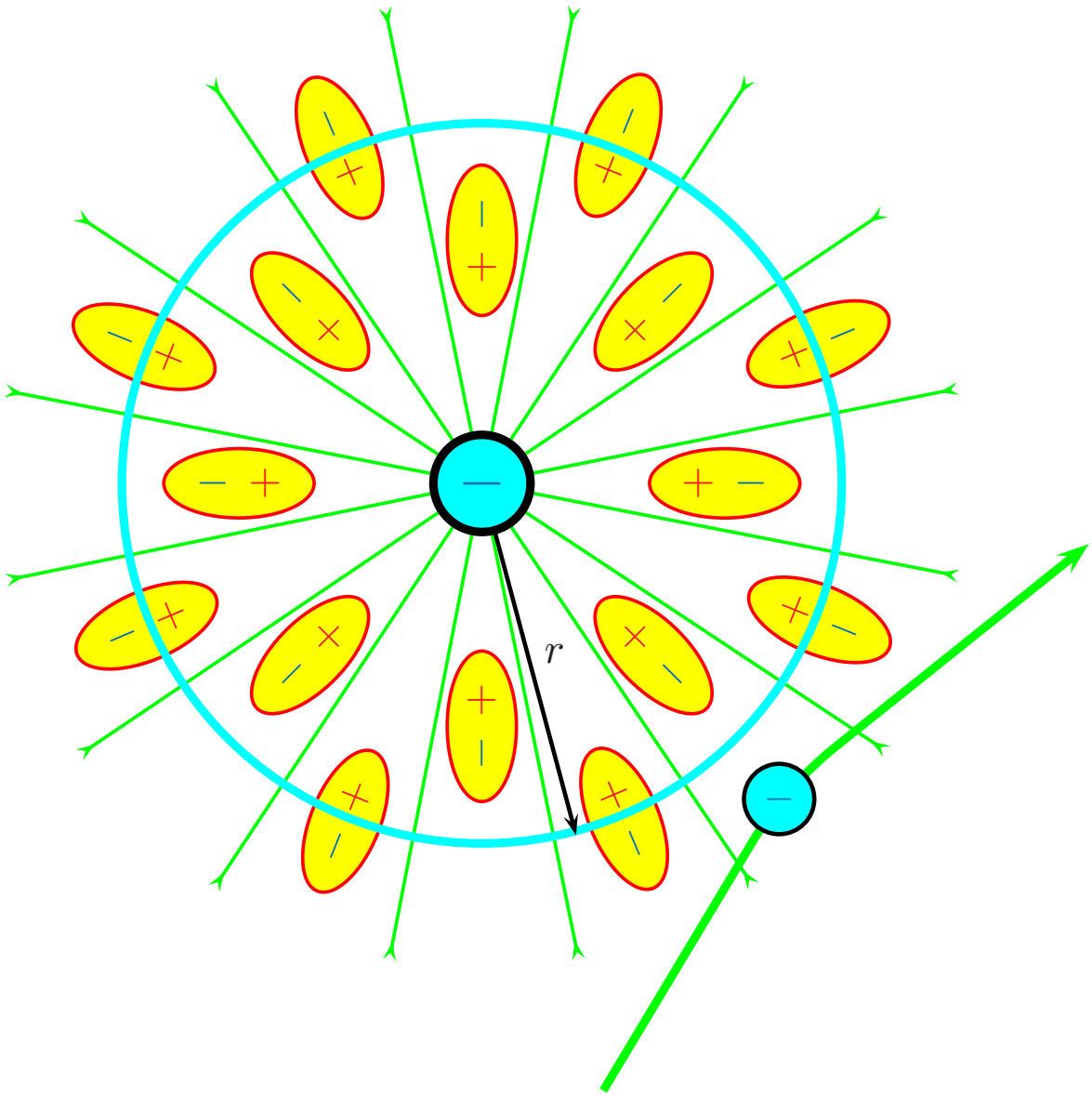
② Hadronic Contributions

General problem in electroweak precision physics: contributions from hadrons (quark loops) at low energy scales

Digression

Most important example: photon vacuum polarization $\Rightarrow$ electro-magnetic charge screening (effective or running charge)

Vacuum polarization and charge screening

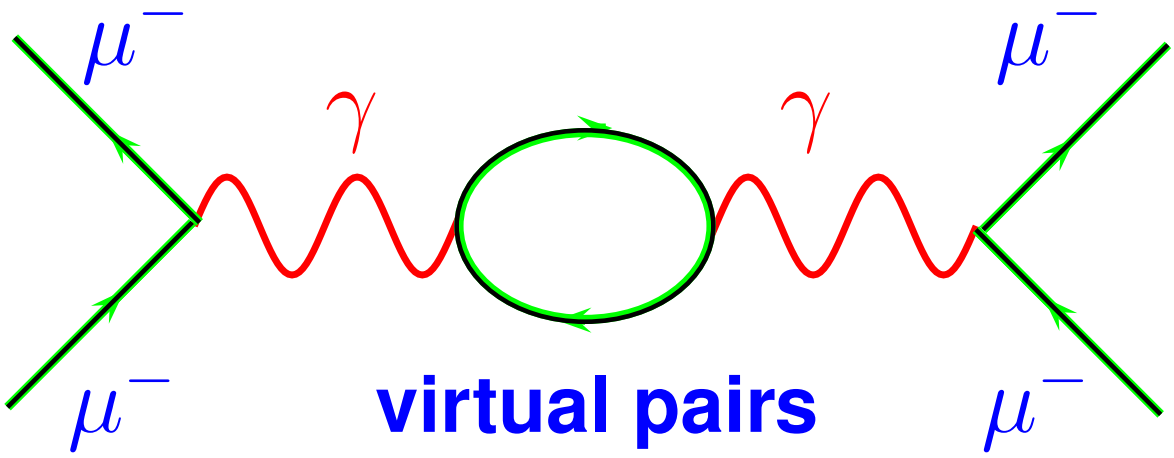


In a collision of impact energy E the effective charge is the charge contained in the sphere of radius $r \simeq 1/E$, which due to vacuum polarization is larger than the classical charge seen in a large shere ($r \to \infty$)

$\Downarrow$

charge screening

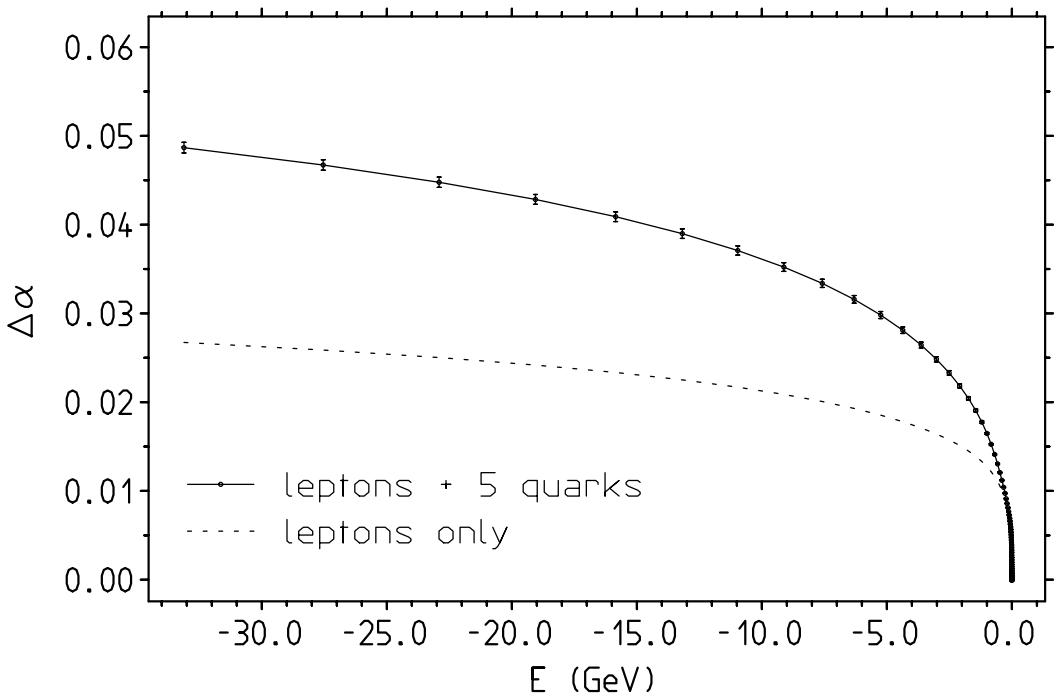
(charge renormalization).



$$\gamma^* \longrightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-, u\bar{u}, d\bar{d}, \cdots \longrightarrow \gamma^*$$

Effective finestructure "constant" $\alpha(E)$

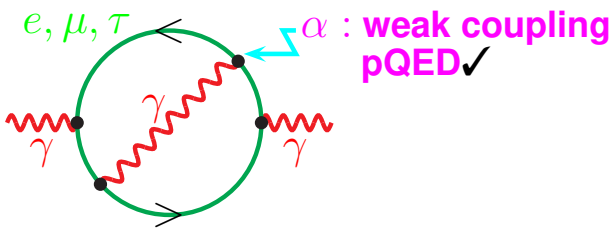
appears in many places in physics! **large EWRC !**



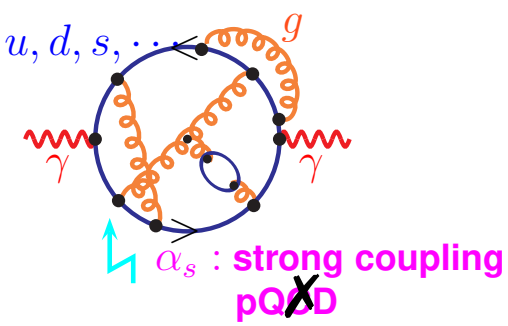
❑ **The fine structure constant α of Quantum Electrodynamics and its extension to the electroweak Standard Model is one of the fundamental constant of nature.**

❑ **Due to charge screening effects (vacuum polarization) it is in fact a function $\alpha(E)$ of the energy scale E typical for a given process ($\alpha(0) = \alpha$).**

❑ **The contribution of the leptons to the vacuum polarization can be reliably calculated in perturbation theory, since $\alpha(0) = 1/137.036..$ is small.**



❑ **In contrast the quark/hadron contributions cannot be reliably calculated in perturbation theory because they exhibit strong interaction and the strong coupling $\alpha_s(E)$ increases at low energies (due to the asymptotic freedom=anti-screening) such that the perturbation expansion fails to "converge".**



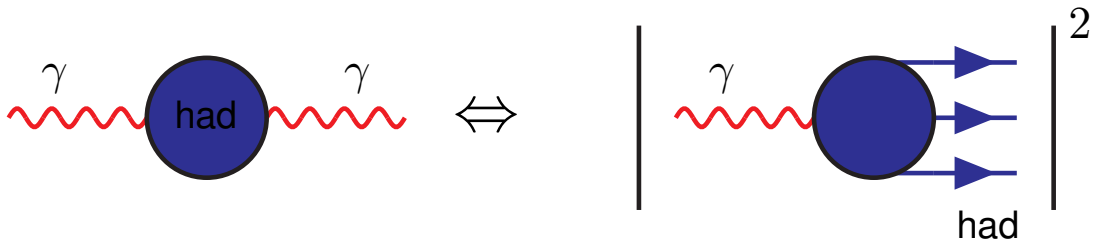
❑ **The way out:**

non–perturbative approach by dispersion relation

The total cross section

$$\sigma(e^+e^- \to hadrons)$$

for hadro-production in e^+e^-– annihilation directly determines the hadronic contribution to the photon vacuum polarization function $\Pi'_\gamma(q^2)$



$\Pi'_\gamma(q^2)$ $\sim \sigma^{\rm had}_{\rm tot}(q^2)$

causality (analyticity) + unitarity (optical theorem)

$$\Delta\alpha(s) \equiv -4\pi\alpha \mathrm{Re}\left[\Pi'_\gamma(s) - \Pi'_\gamma(0)\right]$$

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\; \mathrm{P}\!\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \mathrm{P}\!\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm QCD}_\gamma(s')}{s'(s'-s)} \right)$$

where

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi \mathrm{Im}\Pi'_\gamma(s)$$

Compilation: Davier, Höcker 97

Theory = pQCD: Groshny et al. 91, Chetyrkin et al. 97

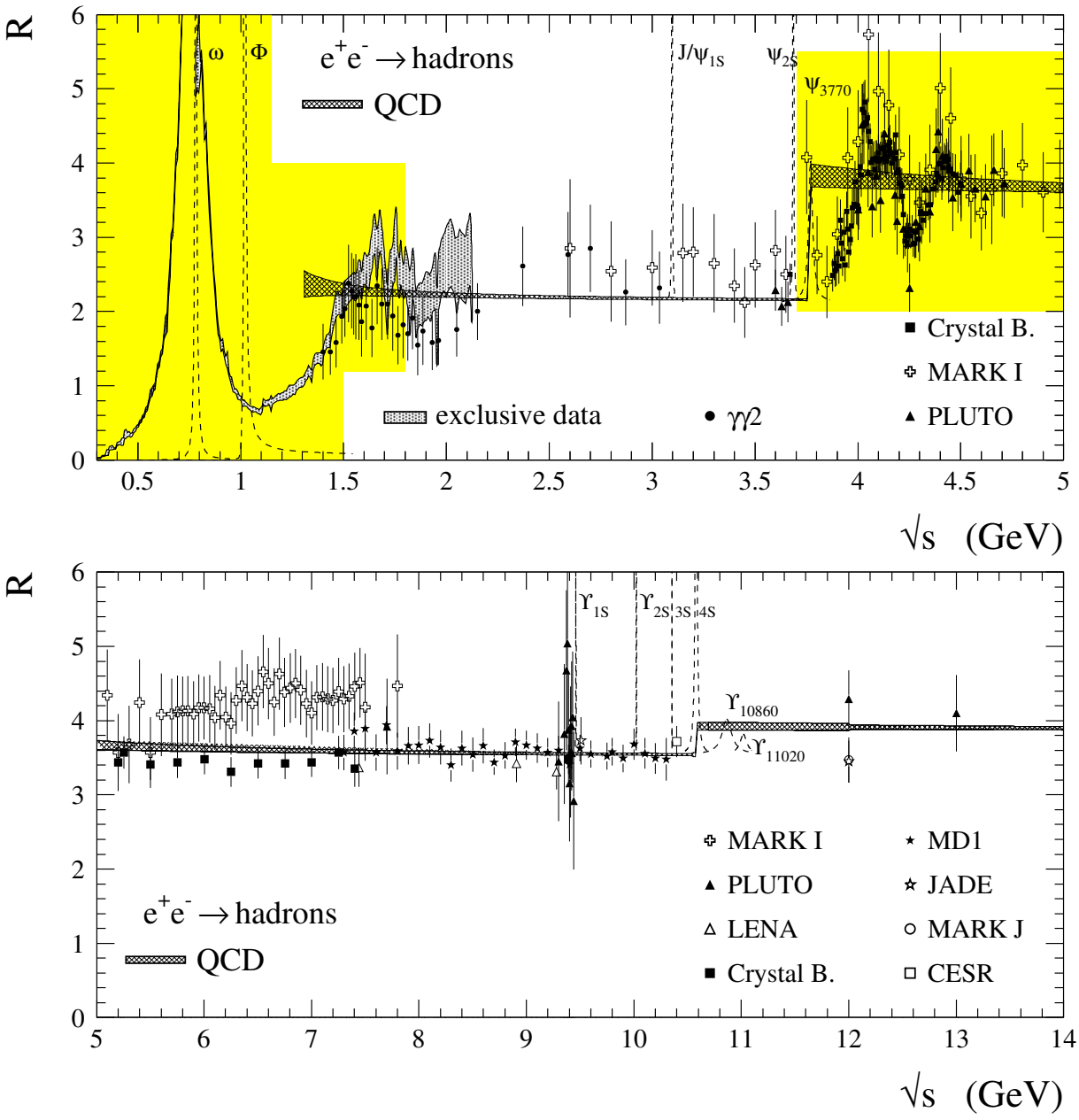
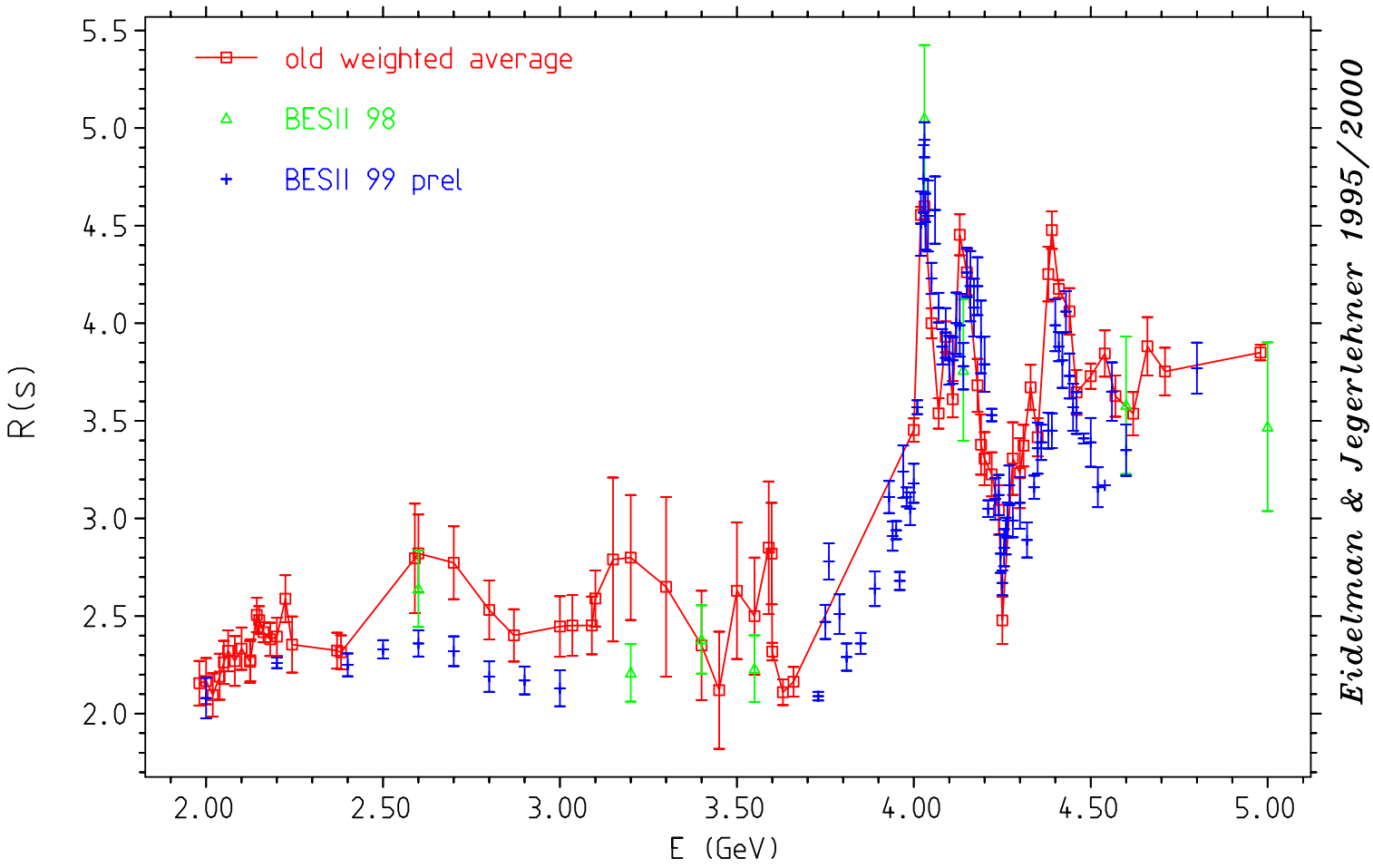


Figure 1: *Inclusive hadronic cross section ratio in e^+e^- annihilation versus the c.m. energy $\sqrt{s}$. Additionally shown is the QCD prediction of the continuum contribution as explained in the text. The shaded areas depict regions were experimental data are used for the evaluation of $\Delta\alpha_{\rm had}(M_{\rm Z}^2)$ and $a_\mu^{\rm had}$ in addition to the measured narrow resonance parameters. The exclusive e^+e^- cross section measurements at low c.m. energies are taken from DM1,DM2,M2N,M3N,OLYA,CMD,ND and τ data from ALEPH (see Ref. [1] for detailed information).*

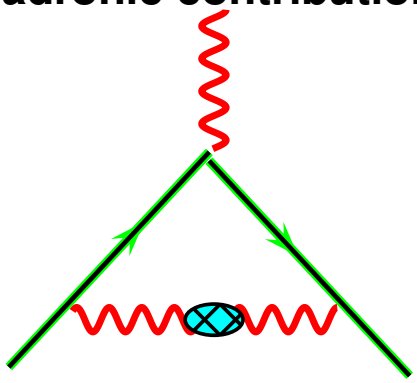
New BESII data (Feb. 2001)



data 20% syst error $\longrightarrow$ 7% syst error !

Hadronic contribution to $(g-2)$ of the muon

❑ **Leading order hadronic contributions**



$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{4m_\pi^2}^{E_{\rm cut}^2} ds\, \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int\limits_{E_{\rm cut}^2}^{\infty} ds\, \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right)$$

- **Note the extra $1/s$–enhancement of contributions from low energies in a_μ as compared to $\Delta\alpha$.**

$$a_\mu^{\rm had} = (697.40 \pm 10.47) \times 10^{-10} \qquad \textbf{(FJ 01)}^*$$

$$a_\mu^{\rm had} = (692.4 \pm 6.2) \times 10^{-10} \qquad \textbf{(Davier, Höcker 98b)}$$

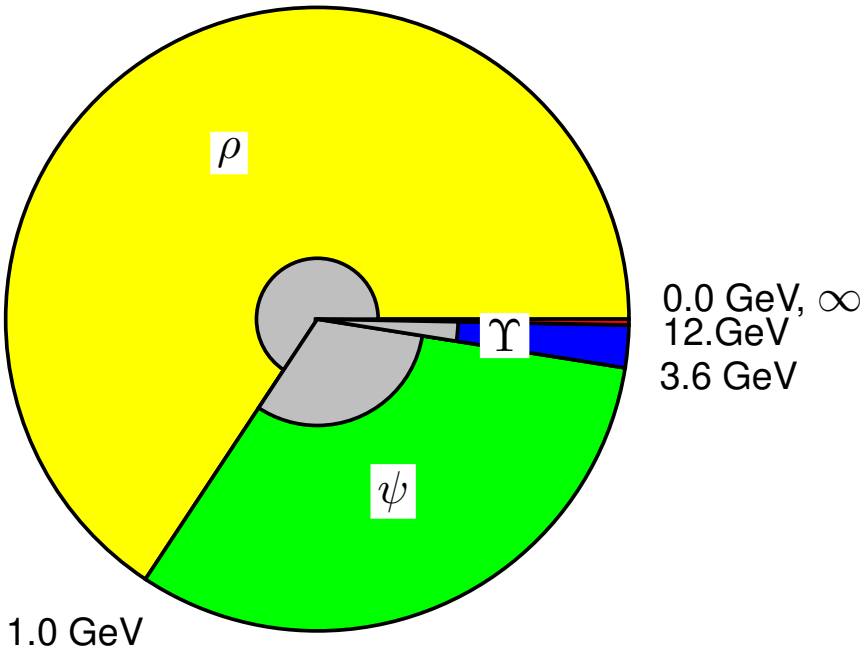
$$a_\mu^{\rm exp} = (\cdots \pm 4.0) \times 10^{-10} \qquad \textbf{(BNL E821 (0?))}$$

*[CDM-2 99, BESII 99]

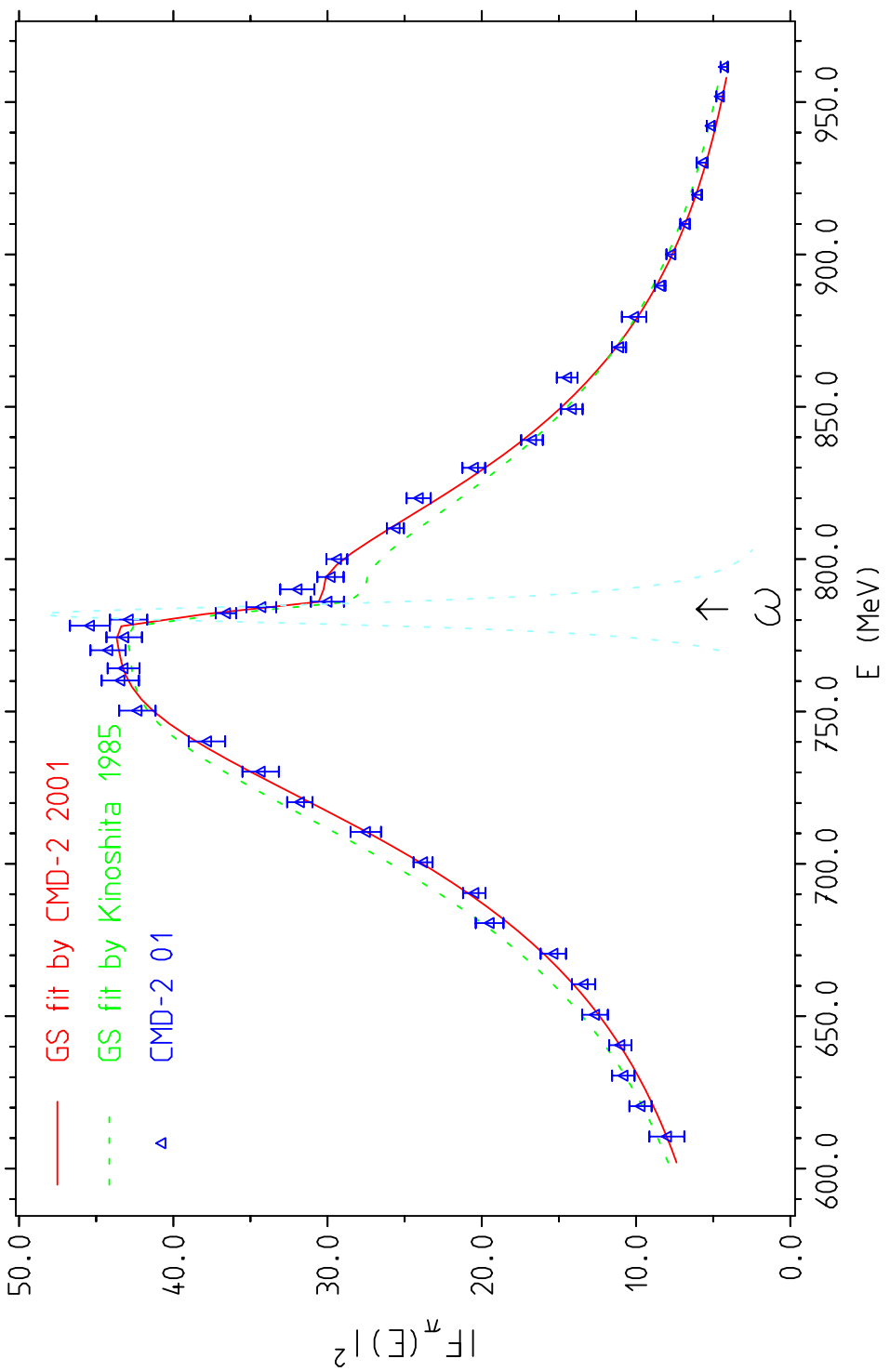
Distribution of hadronic contributions to $a_\mu^{\rm had}$

e^+e^- data based approach:

Based on available data up to 12 GeV (FJ 00)

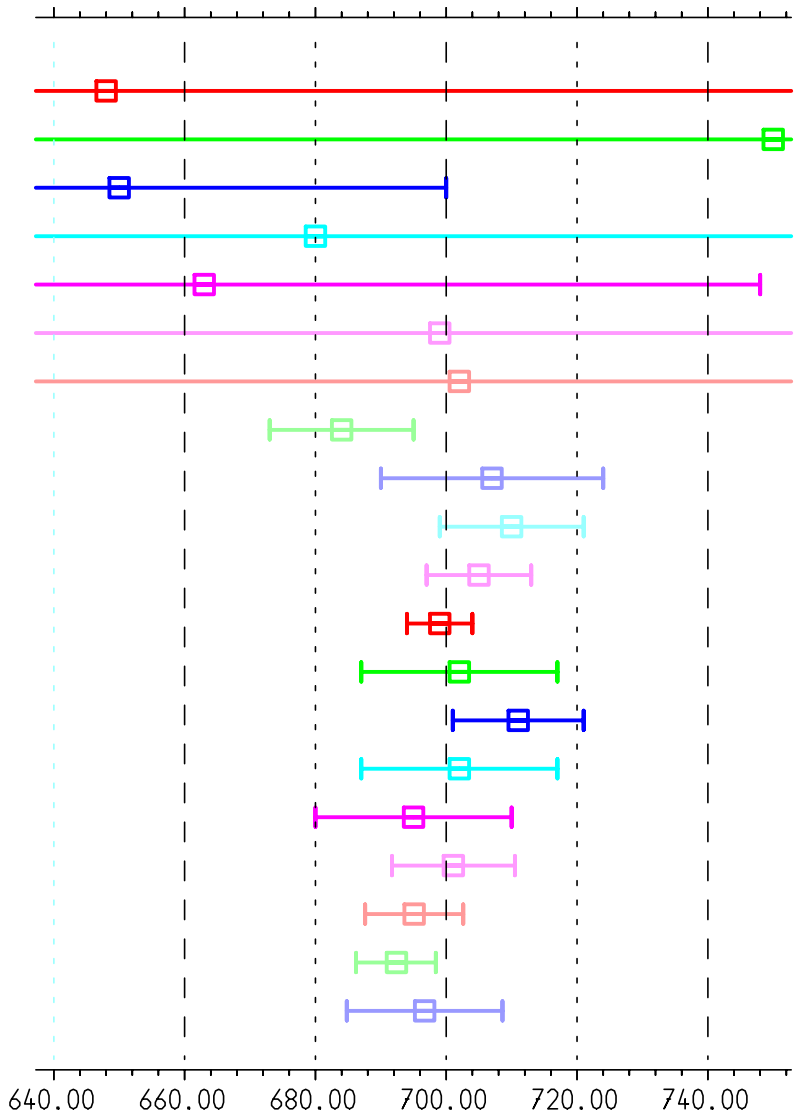


Shaded area: 10 $\times$ error (scaled up)

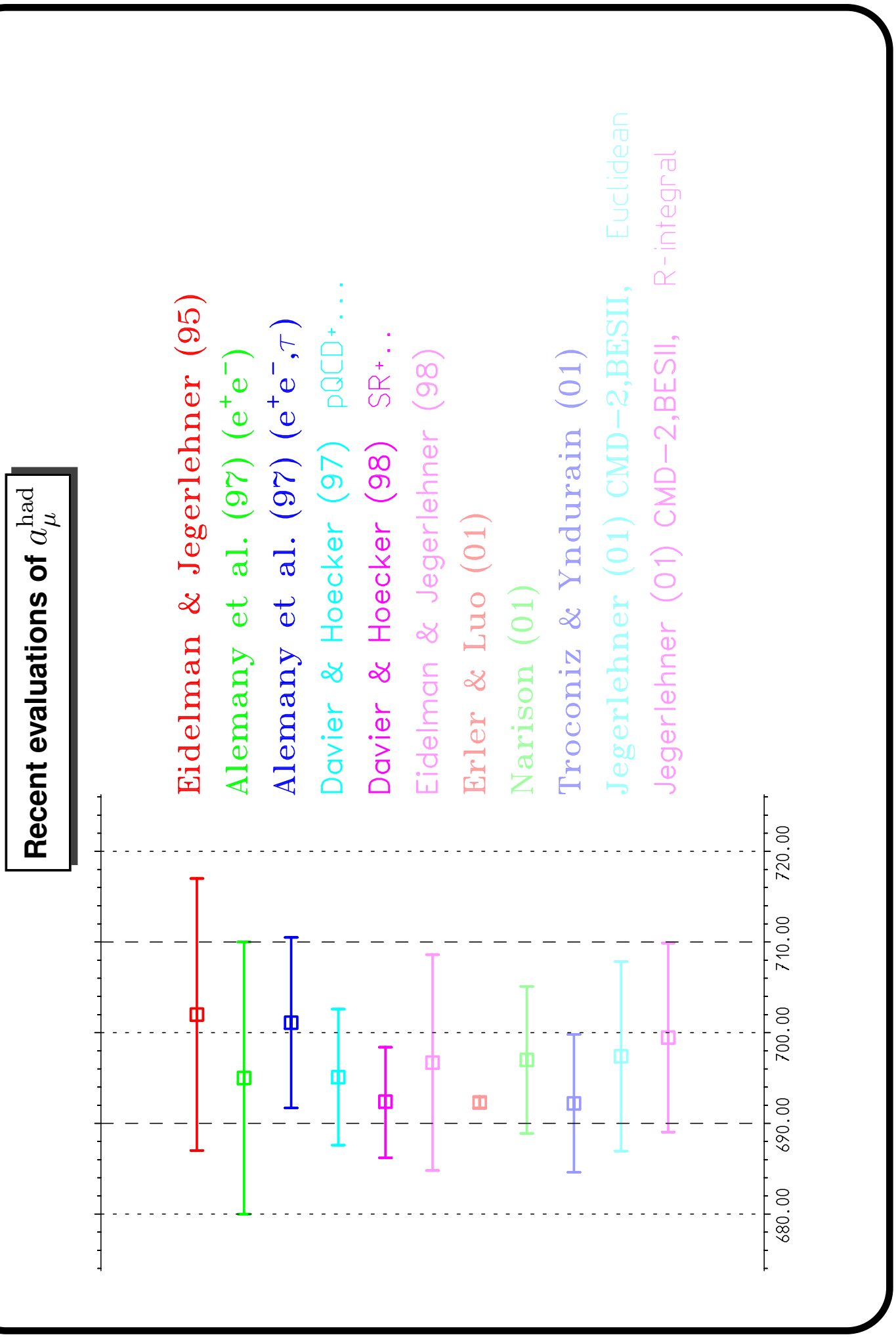

GS fit by CMD-2 2001
GS fit by Kinoshita 1985
CMD-2 01
ω
E (MeV)
$|F_\pi(E)|^2$
600.0
650.0
700.0
750.0
800.0
850.0
900.0
950.0
0.0
10.0
20.0
30.0
40.0
50.0

Evaluations of $a_\mu^{\rm hadrons} \times 10^{10}$ by different authors (before 2000)

Only the leading hadronic vacuum polarization diagram is included.

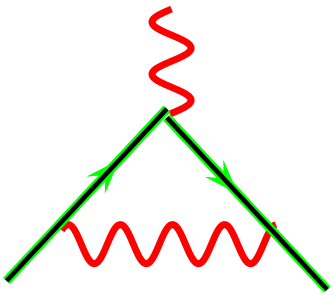


Bouchiat & Michel (61)
Kinoshita & Oakes (67)
Gourdin & de Rafael (69)
Bramon, Etim & Greco (72)
Barger, Long & Olsen (75)
Calmet et al. (76)
Narison (78)
Barkov et al. (85)
Kinoshita et al. (85)
Casas et al. (85)
Martinovic & Dubnicka (90)
Dubnickova et al. (92)
Eidelman & Jegerlehner (95)
Adel & Yndurain (95)
Worstell & Brown (95)
Alemany et al. (97) (e^+e^-)
Alemany et al. (97) (e^+e^-,τ)
Davier & Hoecker (97) pQCD+...
Davier & Hoecker (98) SR+..
Eidelman & Jegerlehner (98)


Recent evaluations of $a_\mu^{\rm had}$
Eidelman & Jegerlehner (95)
Alemany et al. (97) (e^+e^-)
Alemany et al. (97) (e^+e^-,τ)
Davier & Hoecker (97) pQCD+...
Davier & Hoecker (98) SR+...
Eidelman & Jegerlehner (98)
Erler & Luo (01)
Narison (01)
Troconiz & Yndurain (01)
Jegerlehner (01) CMD−2,BESII, Euclidean
Jegerlehner (01) CMD−2,BESII, R-integral
680.00
690.00
700.00
710.00
720.00

❹ a_μ Status:

- QED



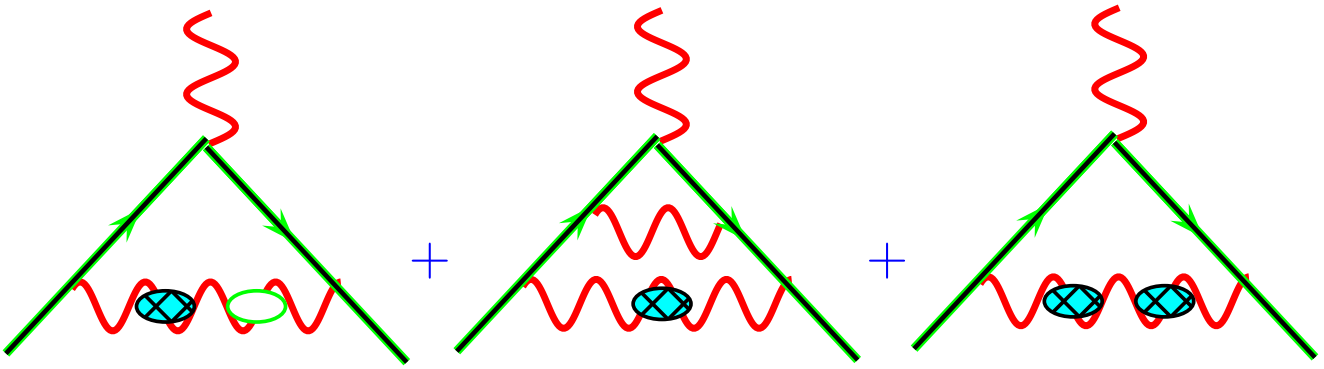
$$a_\mu^{\mathrm{QED}(1)} = (116140981 \pm 6) \times 10^{-11}$$

$$\vdots$$

$$a_\mu^{\mathrm{QED}(5)} = (116584706 \pm 3) \times 10^{-11}$$

(Schwinger 48, $\cdots$, Kinoshita 93, $\cdots$, Laporta, Remiddi 96)

- Higher order hadronic contributions

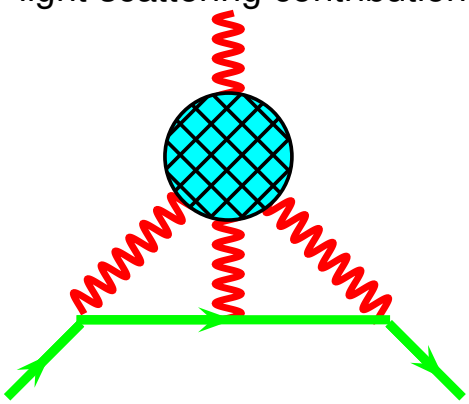


$$a_\mu^{\mathrm{had}(2)} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds\, K^{(2)}(s)\, \sigma_{\mathrm{had}}(s)$$

$$a_\mu^{\mathrm{had}(2)} = -(100 \pm 6) \times 10^{-11}$$

(Krause 96)

- Hadronic light–by–light scattering contribution



QCD inspired models used: ENJL and HGS models supplemented by VMD

$$a_\mu^{\rm light-by-light} = \begin{cases} -(92 \pm 32) & BPP96 \\ -(79 \pm 15) \times 10^{-11} & HK98 \\ -(85 \pm 25) & CM98 \end{cases}$$

model ambiguity? (Bijnens, Pallante, Prades 96, Hayakawa, Kinoshita 98)

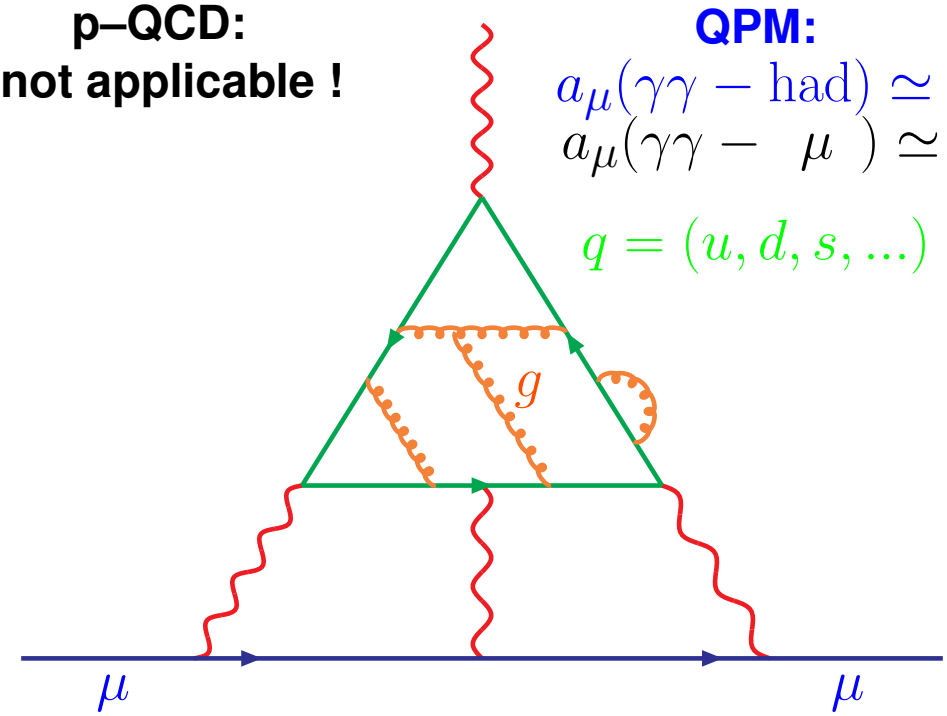
Hadronic light–by–light scattering diagrams:

p–QCD:
not applicable !

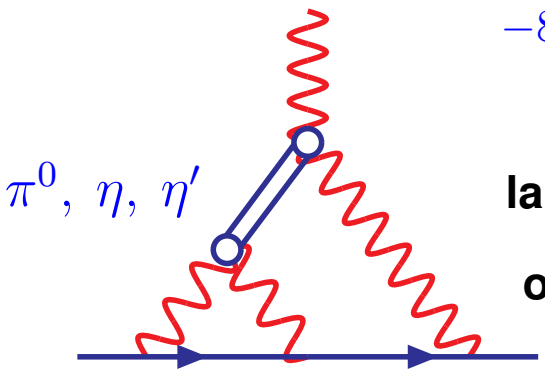
QPM:

$$a_\mu(\gamma\gamma - \text{had}) \simeq 150 \times 10^{-11}$$
$$a_\mu(\gamma\gamma - \ \mu\ ) \simeq 464 \times 10^{-11}$$

$q = (u, d, s, ...)$



Low energy effective theory: e.g. ENJL



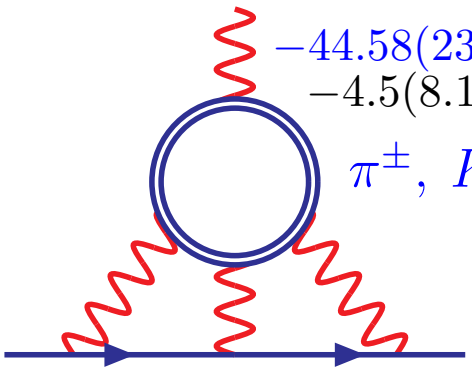
$-82.7(6.4) \times 10^{-11}$

yields

large contribution

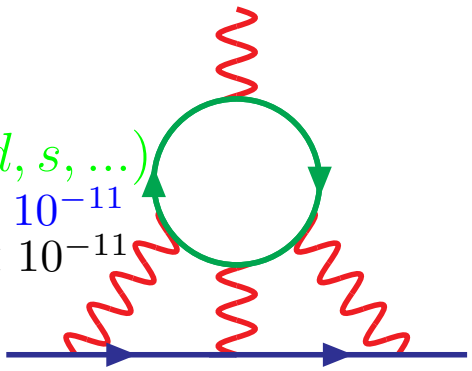
of opposite sign

L.D.



$-44.58(23) \times 10^{-11}$
$-4.5(8.1) \times 10^{-11}$ VDM

L.D.



$+62(3) \times 10^{-11}$
VDM $+9.7(11.1) \times 10^{-11}$

S.D.

NEW: **reconsideration of π^0 – exchange contribution**

Knecht, Nyffeler and Knecht, Nyffeler, de Rafael 2001

i) $\mathcal{O}(N_c)$ **and** χ**PT at** p^6**:** π^0, η, η'

ii) $\mathcal{O}(N_c)$ **and** χ**PT at** p^8**: four meson exchange etc.**

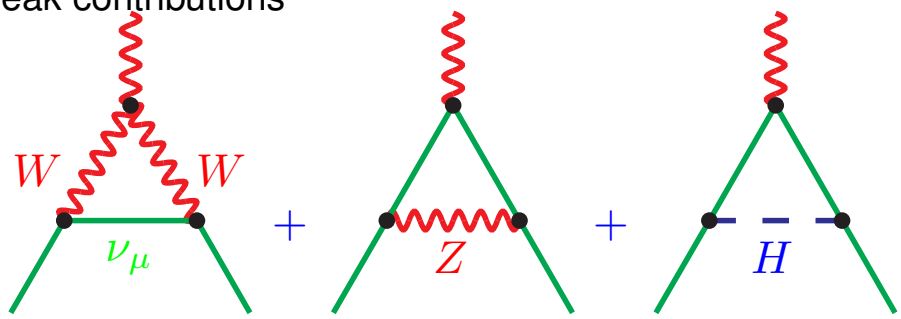
iii) $\mathcal{O}(1)$ **and** χ**PT at** p^4**:** $\pi^\pm, K^\pm$ **loops**

$10^{11} \times a_\mu$	BPP	HK(S)	KN
i)	$-85(13)$	$-83(06)$	$+83(12)$
ii)	$+12(06)$	$+08(11)$	-
iii)	$-19(13)$	$-4.5(8.1)$	-
sum	$-92(32)$	$-79(15)$	$-$
ave	$-(85 \pm 25)$		$+(82 \pm 25)$

changes results in central value by

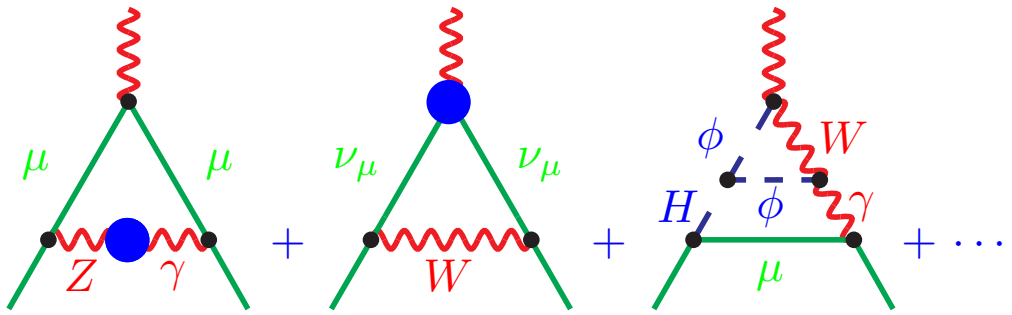
$$+167 \times 10^{-11}$$

- Electroweak contributions



$$a_\mu^{\rm weak(1)} = (195 \pm 0) \times 10^{-11}$$

Higgs contribution tiny!



$$a_\mu^{\rm weak(2)} = -(44 \pm 4) \times 10^{-11}$$

(Czarnecki, Krause, Marciano 98)

a_μ Summary:

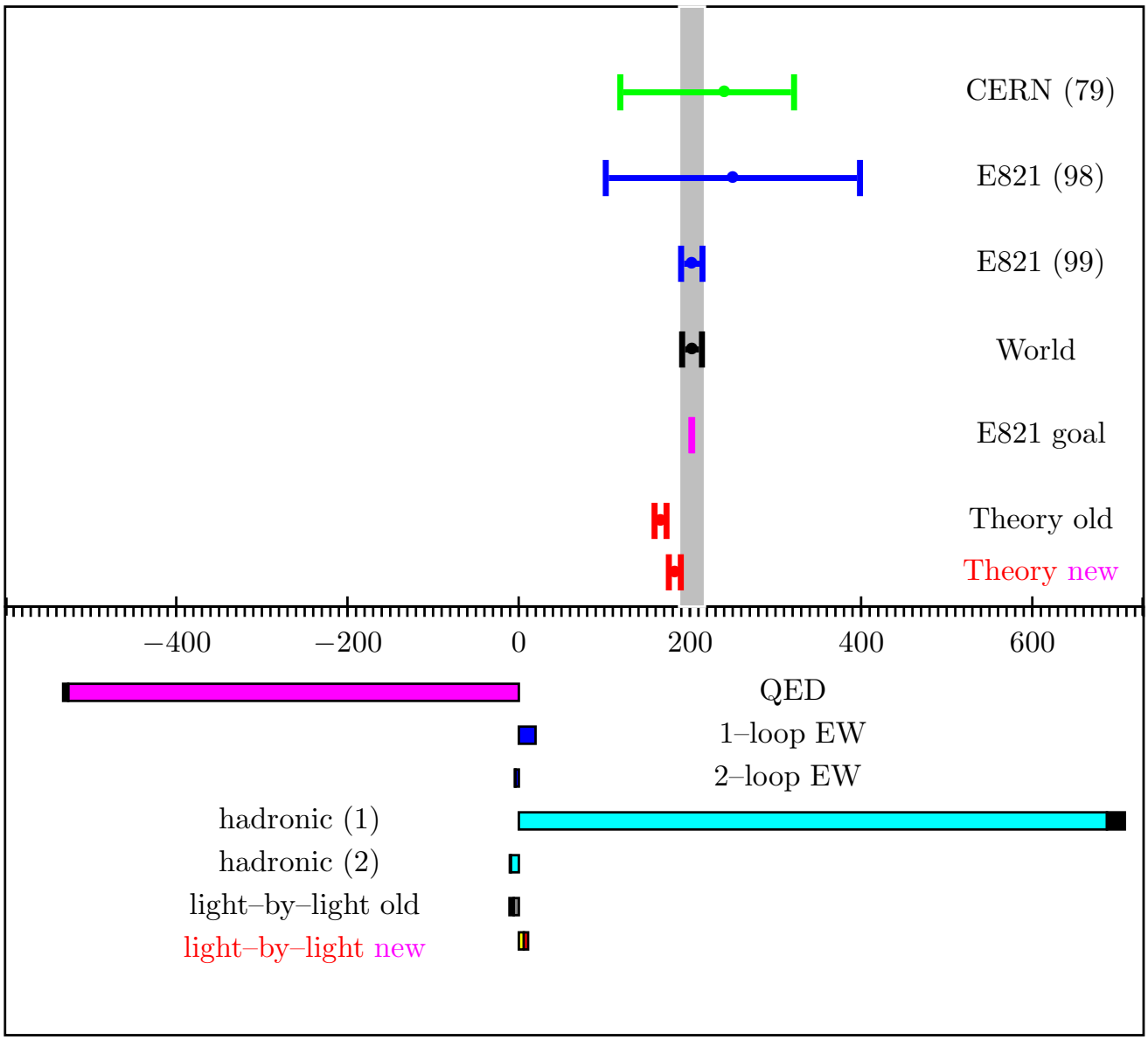
	$a_\mu \times 10^{11}$		
CERN experiment (79)	116592400	$\pm$	850
BNL E821 exp. (98)	116592500	$\pm$	1520
BNL E821 exp. (99)	116592020	$\pm$	160
Average	116592023	$\pm$	151
BNL E821 goal $\longrightarrow$	$\cdots$	$\pm$	40
Theory [DH98]	116591595	$\pm$	67
Theory [FJ00]	116591658	$\pm$	116
Theory [KN01]	116591825	$\pm$	116[c]
QED	116584706	$\pm$	3
1–loop EW	195		
2–loop EW	$-$ 44	$\pm$	4
hadronic (1)	6988	$\pm$	111
$\longrightarrow$	6925	$\pm$	62[a]
hadronic (2)	$-$ 100	$\pm$	6
light–by–light ?	$-$ 92	$\pm$	32
$\longrightarrow$	$-$ 79	$\pm$	15[b]
?	$+$ 83	$\pm$	25[c]

[a] Davier, Höcker 98

[b] Hayakawa, Kinoshita 98

[c] Knecht, Nyffeler 01

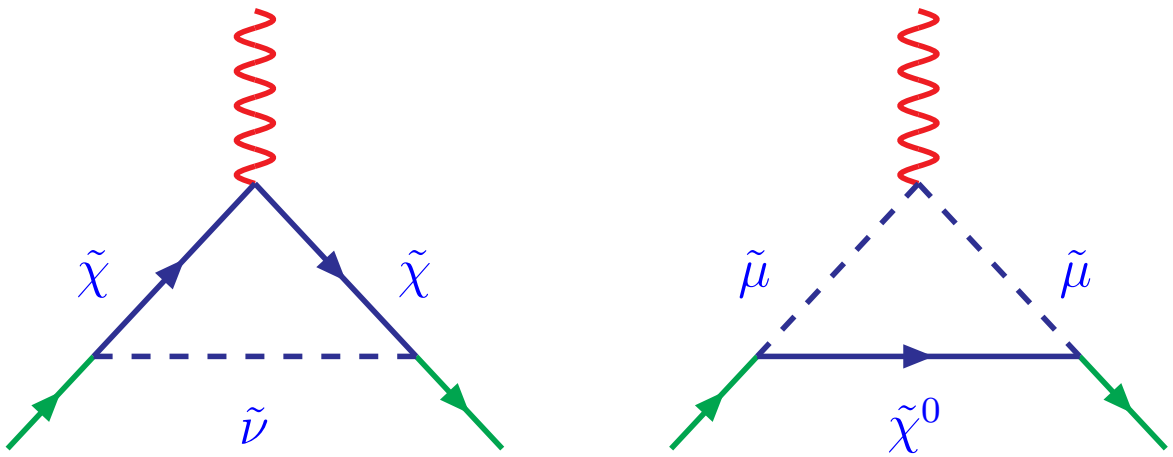
a_μ-11659000 $\times 10^{-10}$: theory vs. experiment in the year 2001 for (g-2) of the muon. The new E821 expriment at Brookhaven revield a 2.6 σ deviation from the theory. The various types of SM contributions are shown in the lower part of the figure.



❺ New Physics

① SUSY

The supersymmetric contributions to a_μ stem from smuon–neutralino and sneutrino-chargino loops:



Some contributions are enhanced by $\tan\beta \equiv \frac{v_2}{v_1}$ which may be large (in some cases of order $m_t/m_b \approx 40$). Then

$$\left|a_\mu^{\rm SUSY}\right| \simeq \frac{\alpha(M_Z)}{8\pi \sin^2\theta_W}\,\frac{m_\mu^2}{\widetilde{m}^2}\,\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{\widetilde{m}}{m_\mu}\right)$$

$\widetilde{m} = m_{\rm SUSY}$ **typical SUSY loop mass**

In large $\tan\beta$ regime

$$\left|a_\mu^{\rm SUSY}\right| \simeq 130\times 10^{-11}\left(\frac{100\ {\rm GeV}}{\widetilde{m}}\right)^2\tan\beta$$

$a_\mu^{\rm SUSY}$ **generally has the same sign as the** μ**-parameter in SUSY models. For positive sgn**(μ)**:**

$$\tan\beta \left(\frac{100\ \textbf{GeV}}{\widetilde{m}}\right)^2 \simeq 3.3 \pm 1.3$$

or

$$\widetilde{m} \simeq (55\ \textbf{GeV})\sqrt{\tan\beta}$$

Negative μ **models give the opposite sign contribution to** a_μ **and are strongly disfavored.**

For $\tan\beta$ **in the range** $4 \sim 40$**, assuming** $\widetilde{m} > 100$ **GeV:**

$$\widetilde{m} \simeq 100 - 450\ \textbf{GeV}$$

precisely the range where SUSY particles are often expected.

Note: Ferrara, Remiddi 1974

$$a_\mu^{\rm tot} = a_\mu^{\rm SM} + a_\mu^{\rm SUSY} = 0$$

in SUSY limit

unbroken SUSY: $a_\mu^{\rm SUSY} < 0$

broken SUSY: either sign

$(g-2)$ **requires:** $a_\mu^{\rm SUSY} > 0$ **in addition to** $a_\mu^{\rm SM} > 0$

② Anomalous W couplings

Anomalous W boson couplings:

Magnetic dipole moment

$$\mu_W = \frac{e}{2m_W}(1+\kappa+\lambda)$$

Electric quadrupole moment

$$Q_W = -\frac{e}{2m_W}(\kappa-\lambda)$$

$\kappa = 1$ **and** $\lambda = 0$ **in the SM Contribution to** a_μ**:**

$$a_\mu(\kappa,\lambda) \simeq \frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2}\left[(\kappa-1)\ln\frac{\Lambda^2}{m_W^2} - \frac{1}{3}\lambda\right]$$

Λ **high momentum cutoff required to give a finite result.**

For $\Lambda \simeq 1$ **TeV the BNL constraint yields**

$$\kappa - 1 = 0.37 \pm 0.14 \qquad \textbf{(BNL 01)}$$

Is already ruled out by $e^+e^- \to W^+W^-$ **data at LEP II**

$$\kappa - 1 = 0.04 \pm 0.08 \qquad \textbf{(LEP II)}$$

③ New Physics: general structure

For a variety of models typically

$$|a_\mu(\textbf{New Physics})| \simeq m_\mu^2/M^2 \quad \cdot \underbrace{\frac{\alpha}{\pi}}_{\text{if loop induced}}$$

Current constraint suggests (very roughly)

$$\boxed{M \simeq 1-2\ \textbf{TeV}}$$

Of course, for a specific model, one must check that the sign of the induced $a_\mu^{\rm NP}$ is in accord with experiment (i.e. it should be positive).

Since Feb. 8, 2001 (BNL):

226 citations

mostly

new physics explanations

❻ $\alpha_{\rm em}(s)$ in Precision Physics

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

Precision physics limitations:

$\alpha\ ,\ G_\mu, M_Z$ **most precise input parameters**

partially non-perturbative relationship

$\Updownarrow$

precision predictions

$\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$$\begin{array}{lcll} \frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times\ 10^{-9} \\ \frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times\ 10^{-6} \\ \frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times\ 10^{-5} \\ \frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 1.6 \div 6.8 & \times\ 10^{-4} \end{array}$$

$\Rightarrow$ **now a limiting factor for precision physics**

Formal definition:

Effective QED coupling constant at scale $\sqrt{s}$:

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)}$$

with

$$\Delta\alpha(s) = -4\pi\alpha \mathrm{Re}\left[\Pi'_\gamma(s) - \Pi'_\gamma(0)\right]$$

where $\Pi'_\gamma(s)$ is the photon vacuum polarization function

$$i\int d^4x\, e^{iq\cdot x}\langle 0|\mathrm{T} j^\mu_{em}(x) j^\nu_{em}(0)|0\rangle = -(q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi'_\gamma(q^2)$$

and $j^\mu_{em}(x)$ is the electro-magnetic current.

Contributions:

❑ Leptons

Calculable in perturbation theory: Free lepton loop affected by small electro-magnetic corrections.

$$\begin{aligned}
\Delta\alpha_{\mathrm{leptons}}(s) = & \\
&= \sum_{\ell=e,\mu,\tau} \frac{\alpha}{3\pi}\left[-\frac{8}{3} + \beta_\ell^2 - \frac{1}{2}\beta_\ell(3-\beta_\ell^2)\ln\left(\frac{1-\beta_\ell}{1+\beta_\ell}\right)\right] \\
&= \sum_{\ell=e,\mu,\tau} \frac{\alpha}{3\pi}\left[\ln\left(s/m_\ell^2\right) - \frac{5}{3} + O\left(m_\ell^2/s\right)\right] \\
&= 0.03142 \text{ for } s = M_Z^2
\end{aligned}$$

where $\beta_\ell = \sqrt{1 - 4m_\ell^2/s}$.

❑ Quarks

The contribution of the five light quarks (u, d, s, c, b) is not calculable in perturbation theory: Free quark loop strongly modified by strong interactions at low energy. Way out: unitarity and the analyticity of $\Pi'_\gamma(s)$ allows us to write

(Cabibbo, Gatto 1961)

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\int\!\!\!\!\!\!\diagup^{E^2_{\rm cut}}_{4m^2_\pi} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \int\!\!\!\!\!\!\diagup^{\infty}_{E^2_{\rm cut}} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

where

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi {\rm Im}\Pi'_\gamma(s)$$

has been measured in e^+e^-–annihilation experiments up to about $E_{\rm cut} \sim 40$ GeV. By virtue of the asymptotic freedom of QCD the high energy tail can be safely calculated using pQCD (Gorishny, Kataev, Larin 91).

$E_{\rm cut} \gtrsim 12.GeV$

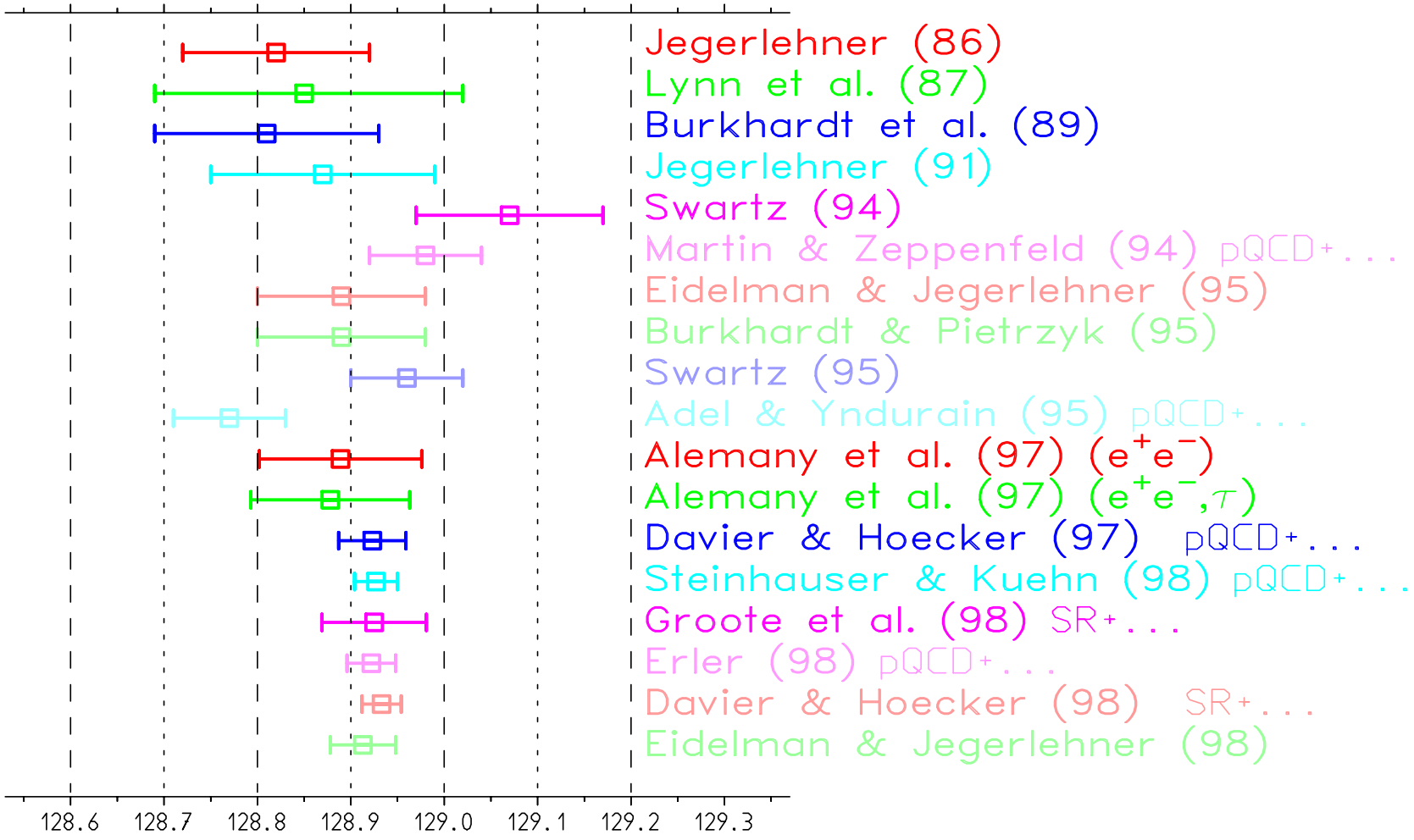
$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.0280 \pm 0.0007 \quad \mathbf{E} \quad (EJ95)$$
$$\to \quad 0.02763 \pm 0.00016 \quad \mathbf{T} \quad (DH98)$$

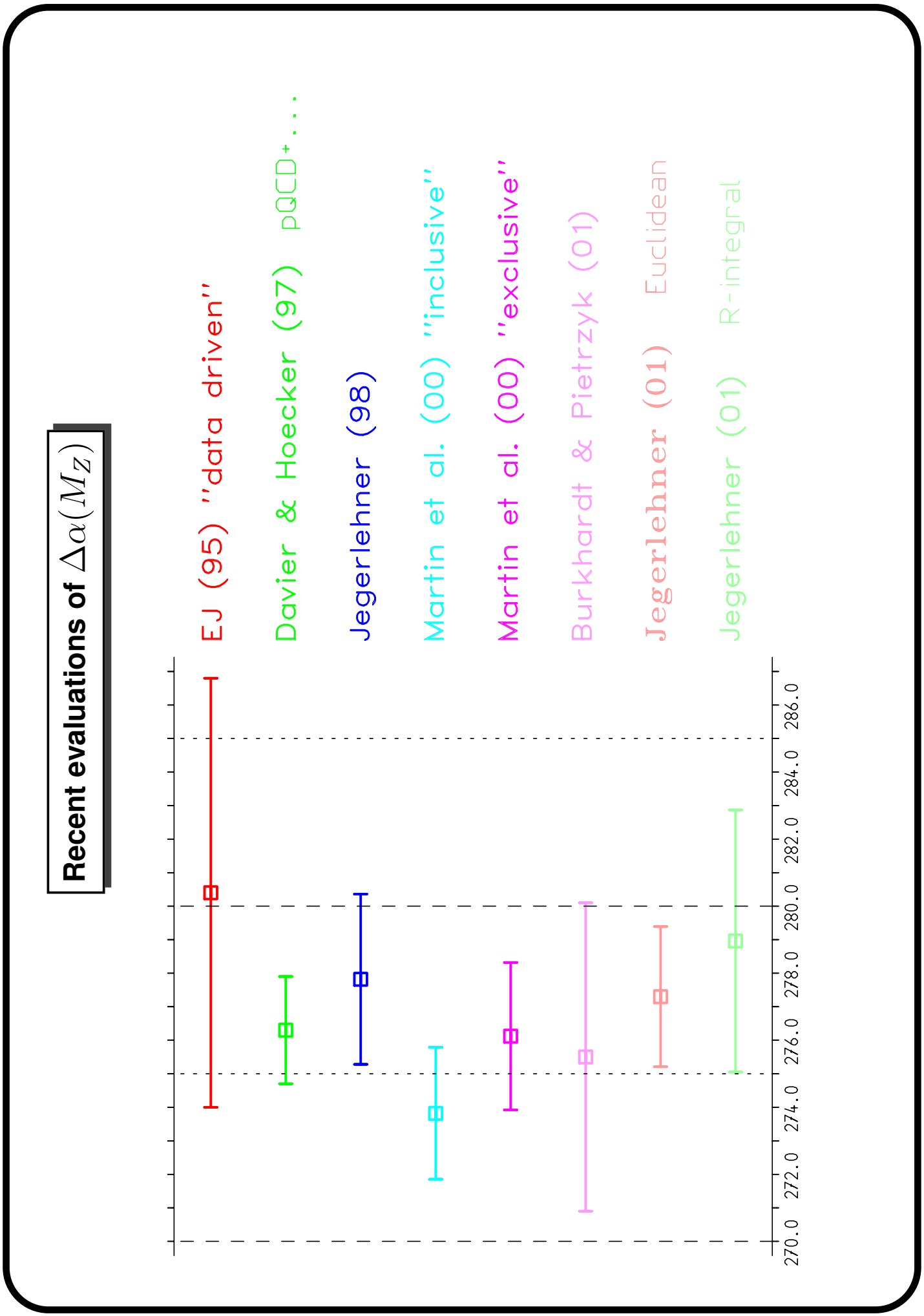
$E_{\rm cut} \sim 1.7GeV$

$$\alpha^{-1}(M_Z) = 128.89 \pm 0.09 \quad \mathbf{E} \quad (EJ95)$$
$$\to \quad 128.933 \pm 0.021 \quad \mathbf{T} \quad (DH98)$$

Evaluations of $\alpha(M_Z)$ by different authors (before 2000)

All values have been rescaled to $M_Z = 91.1887$ GeV. The small top quark and the W contributions are not included.




Recent evaluations of $\Delta\alpha(M_Z)$
EJ (95) "data driven"
Davier & Hoecker (97) pQCD+...
Jegerlehner (98)
Martin et al. (00) "inclusive"
Martin et al. (00) "exclusive"
Burkhardt & Pietrzyk (01)
Jegerlehner (01) Euclidean
Jegerlehner (01) R-integral
270.0
272.0
274.0
276.0
278.0
280.0
282.0
284.0
286.0

$\Delta\alpha$ in electroweak precision physics

Calculate $\sin^2\Theta_i$ from α, G_μ and M_Z :

$$\sqrt{2}\,G_\mu\,M_Z^2\,\sin^2\Theta_i\,\cos^2\Theta_i = \pi\alpha\,\frac{1}{1-\Delta r_i}$$

Sirlin 1980

Various definitions of $\sin^2\Theta_i$, which coincide at tree level: From weak gauge boson masses, from electroweak gauge couplings and from the neutral current coupling of the charged fermions:

$$\begin{aligned}
\sin^2\Theta_W &= 1-\frac{M_W^2}{M_Z^2}\\
\sin^2\Theta_g &= e^2/g^2\\
\sin^2\Theta_f &= \frac{1}{4|Q_f|}\left(1-\frac{v_f}{a_f}\right)\ ,\quad f\neq\nu
\end{aligned}$$

$$\begin{aligned}
\Delta r_i &= \Delta r_i(\alpha,\ G_\mu,\ M_Z, m_H,\ m_{f\neq t},\ m_t)\\
\Delta r_i &= \Delta\alpha - f_i(\sin^2\Theta_i)\,\Delta\rho + \Delta r_{i\,\mathrm{rem}}
\end{aligned}$$

$\uparrow$

universal

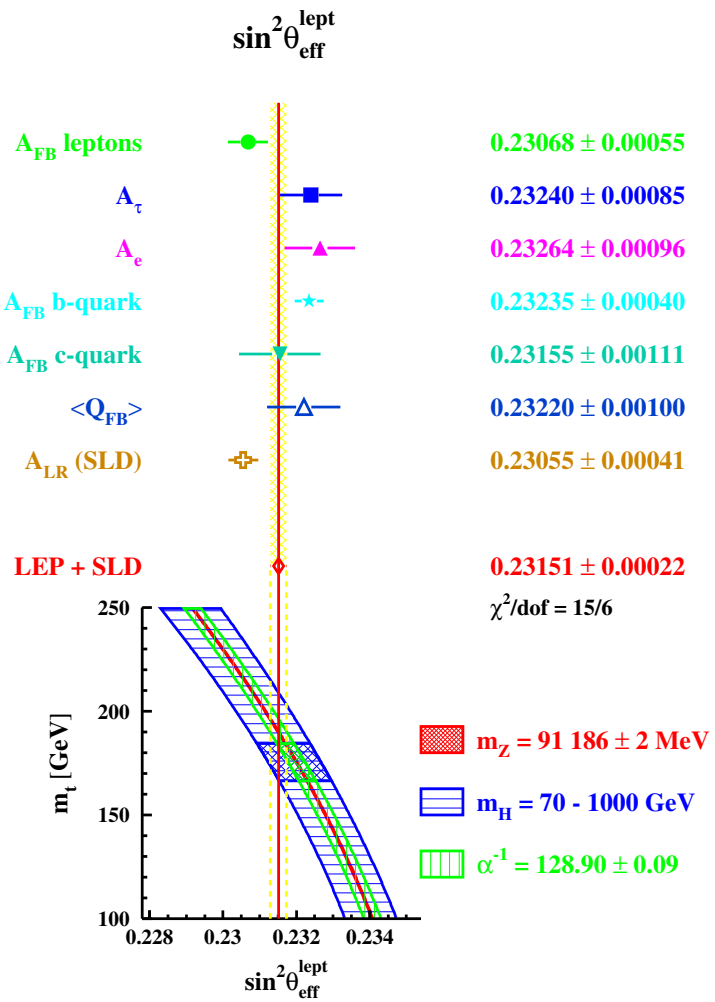
$\rightarrow$ **predictions for M_W, A_{LR}, A_{FB}^f, Γ_f, $\cdots$**

$$\delta\Delta\alpha \rightarrow \delta M_W\ ,\quad \delta\sin^2\Theta_i\ ,\ \cdots$$

TOP mass bound

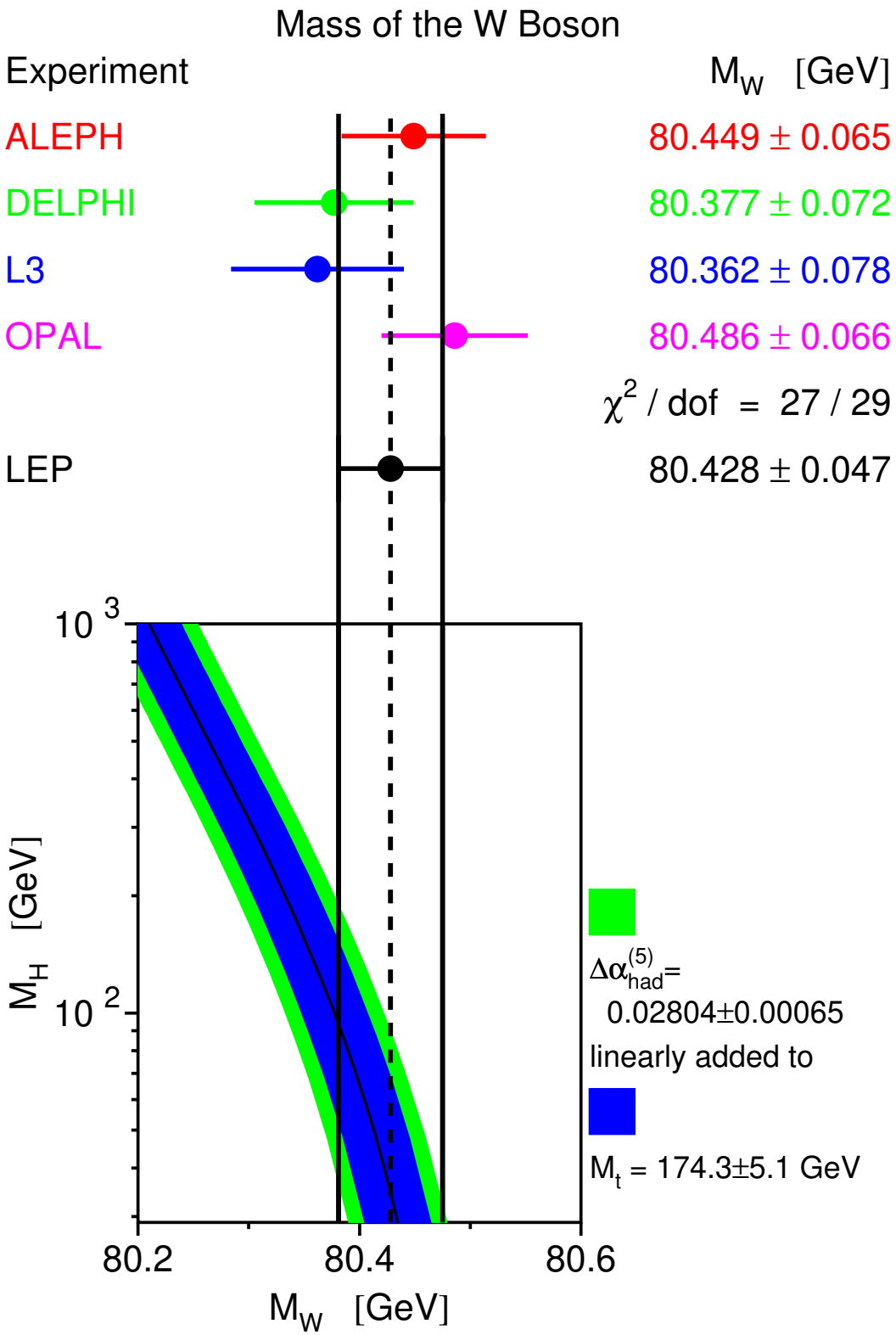
$m_t = 170 \pm 10^{+17}_{-19}$ GeV LEP 1995

where 60 GeV $< m_H <$ 1000 GeV assumed

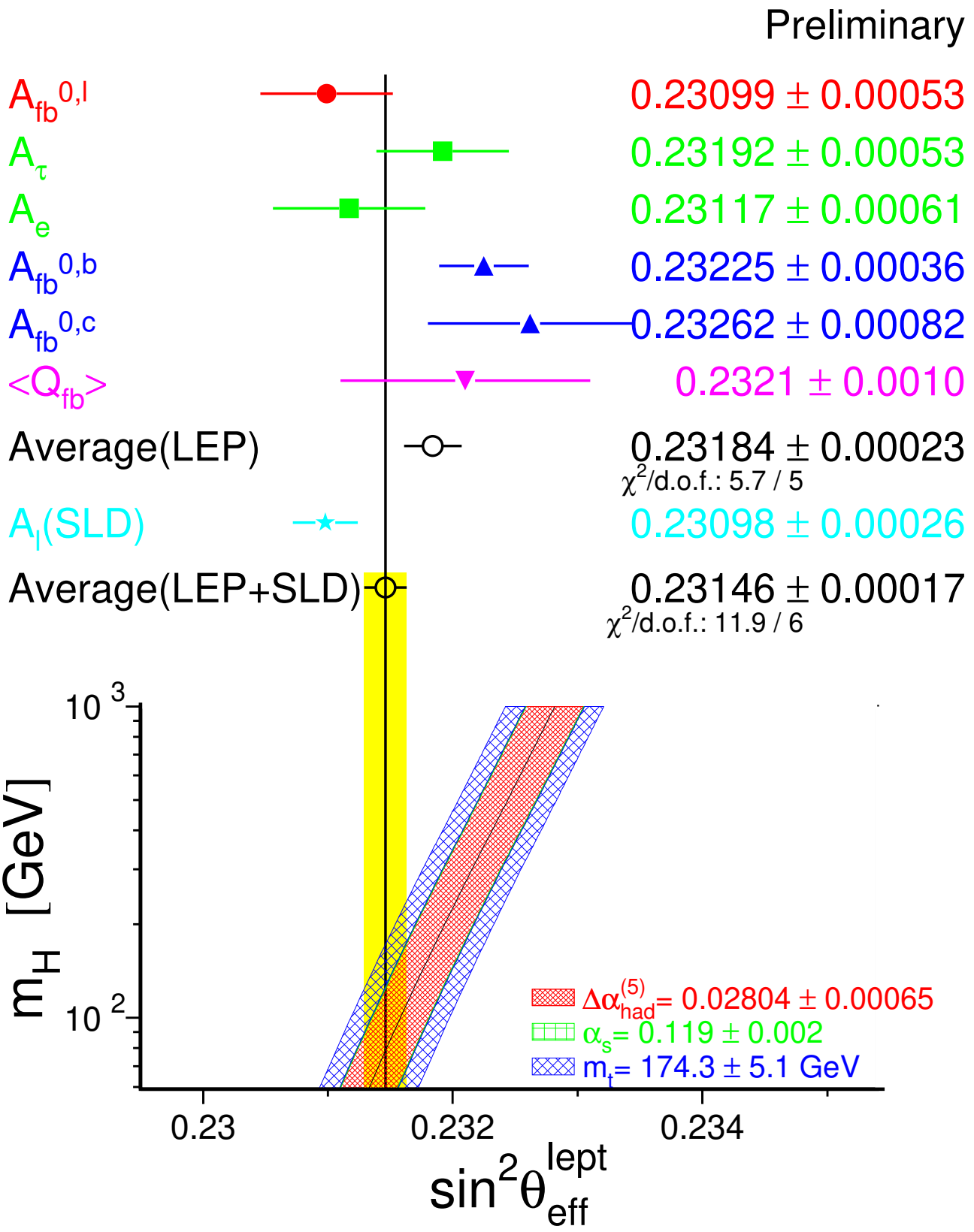


(LEP Electroweak Working Group: J. Alcaraz et al. 97)

$\Rightarrow$ **TOP discovery: CDF, D0 (95/96)**



(LEP Electroweak Working Group: D. Abbaneo et al. 00)



(LEP Electroweak Working Group: D. Abbaneo et al. 00)

❼ Comments on Recent Progress

① Additional data: τ–data + CVC

ALEPH–Coll., (OPAL, CLEO), Alemany, Davier, Höcker 96

$$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+ e \to X^0$$

where X^- and X^0 are hadronic states related by isospin rotation. The e^+e^- cross–section is then given by

$$\sigma^{I=1}_{e^+e^-\to X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \le M_\tau$$

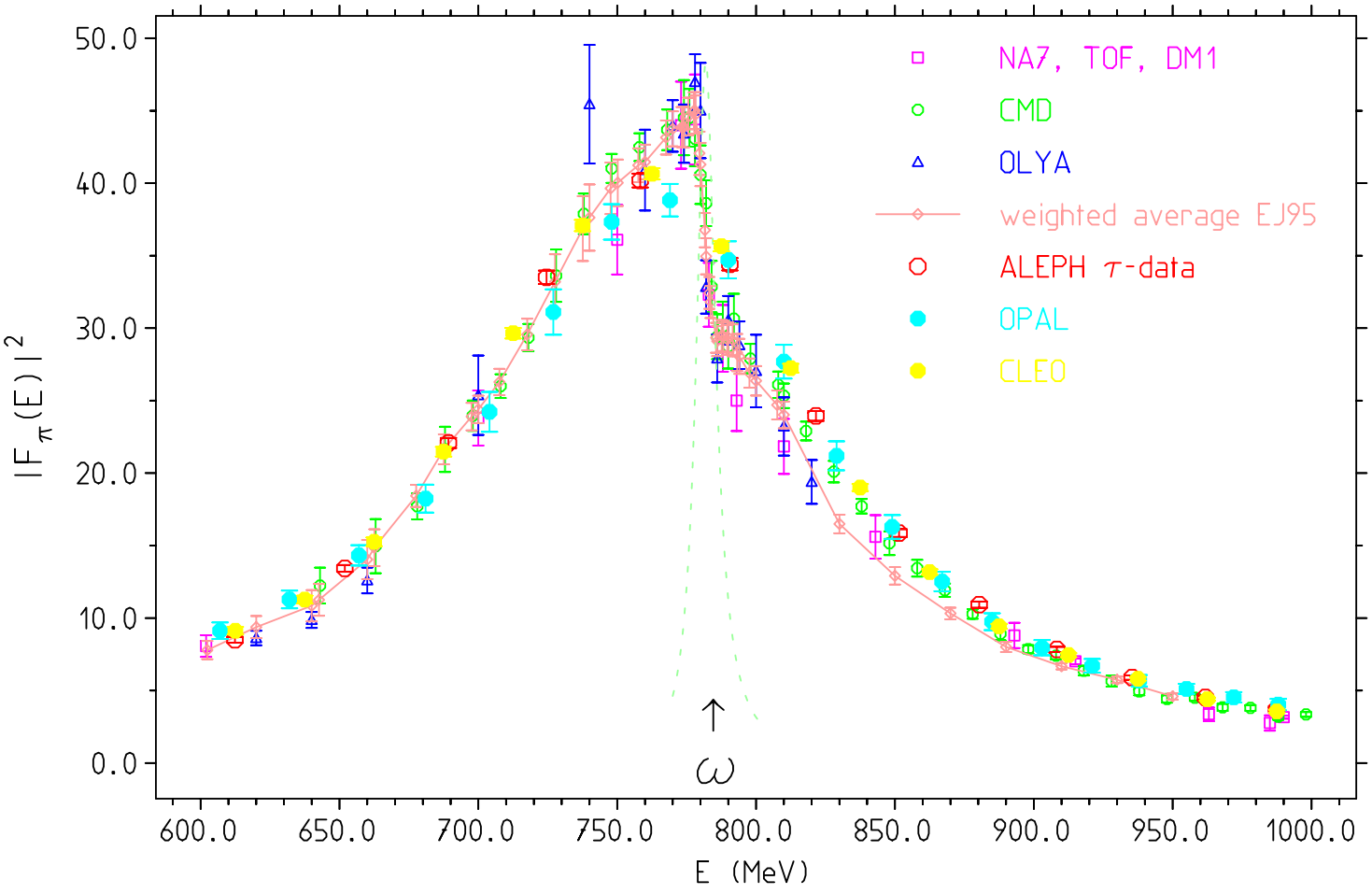
in terms of the τ spectral function v_1.

- mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ–resonance contribution)
- which is dominating in $a_\mu^{\rm had}$ (72%)

$$I = 1 \sim 75\% ; \; I = 0 \sim 25\%$$

τ–data cannot replace e^+e^-–data

$$\begin{aligned} \delta a_\mu &: \; 15.6 \times 10^{-10} \; \to \; 10.2 \times 10^{-10} \\ \delta\Delta\alpha &: \; 0.00067 \; \to \; 0.00065 \end{aligned}$$



systematic deviations largely can be understood!

e^+e^-–data*= data corrected for isospin violations:

$$|F(s)|^2 \text{ = (}|F(s)|^2\text{–data) / } \left|\left(1+\frac{\epsilon s}{(s_\omega - s)}\right)\right|^2$$

with

$$s_\omega = (M_\omega - \tfrac{i}{2}\Gamma_\omega)^2$$

ϵ **determined by fit to the data:** $\epsilon = 0.00172$

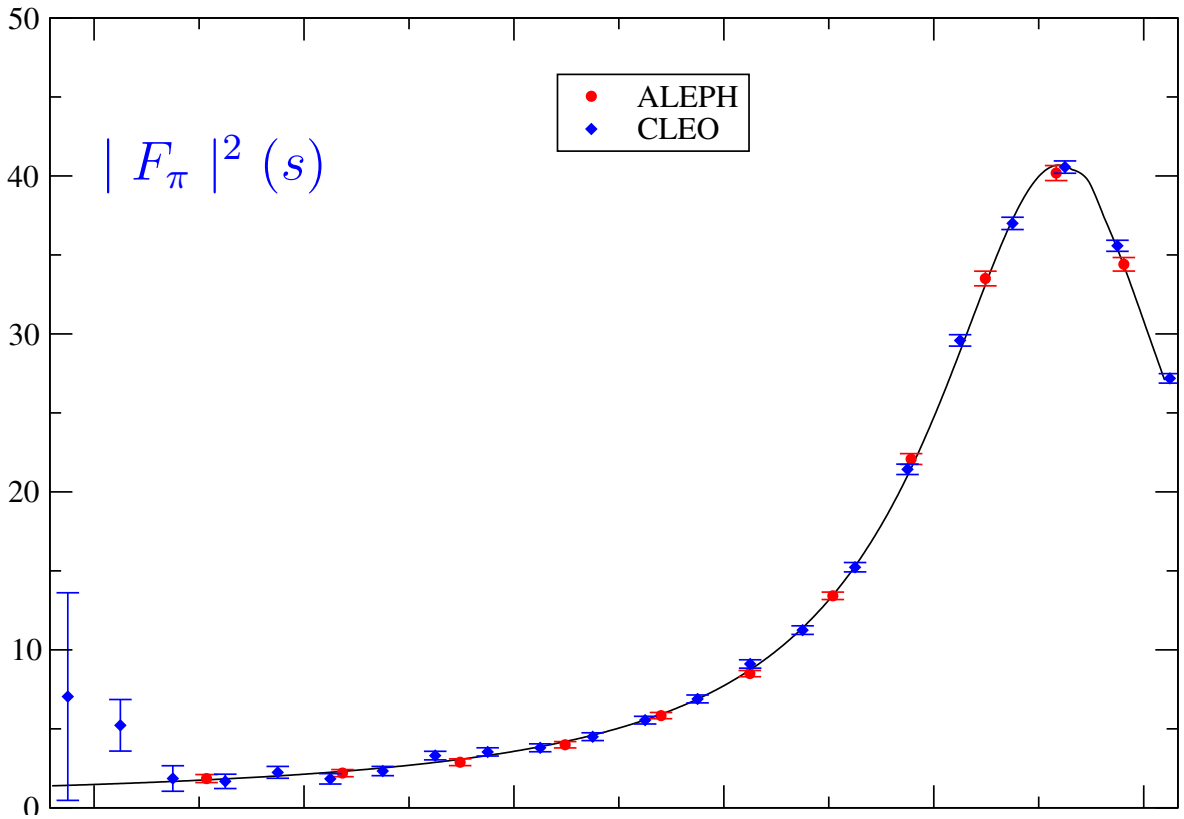
(Leutwyler 00)

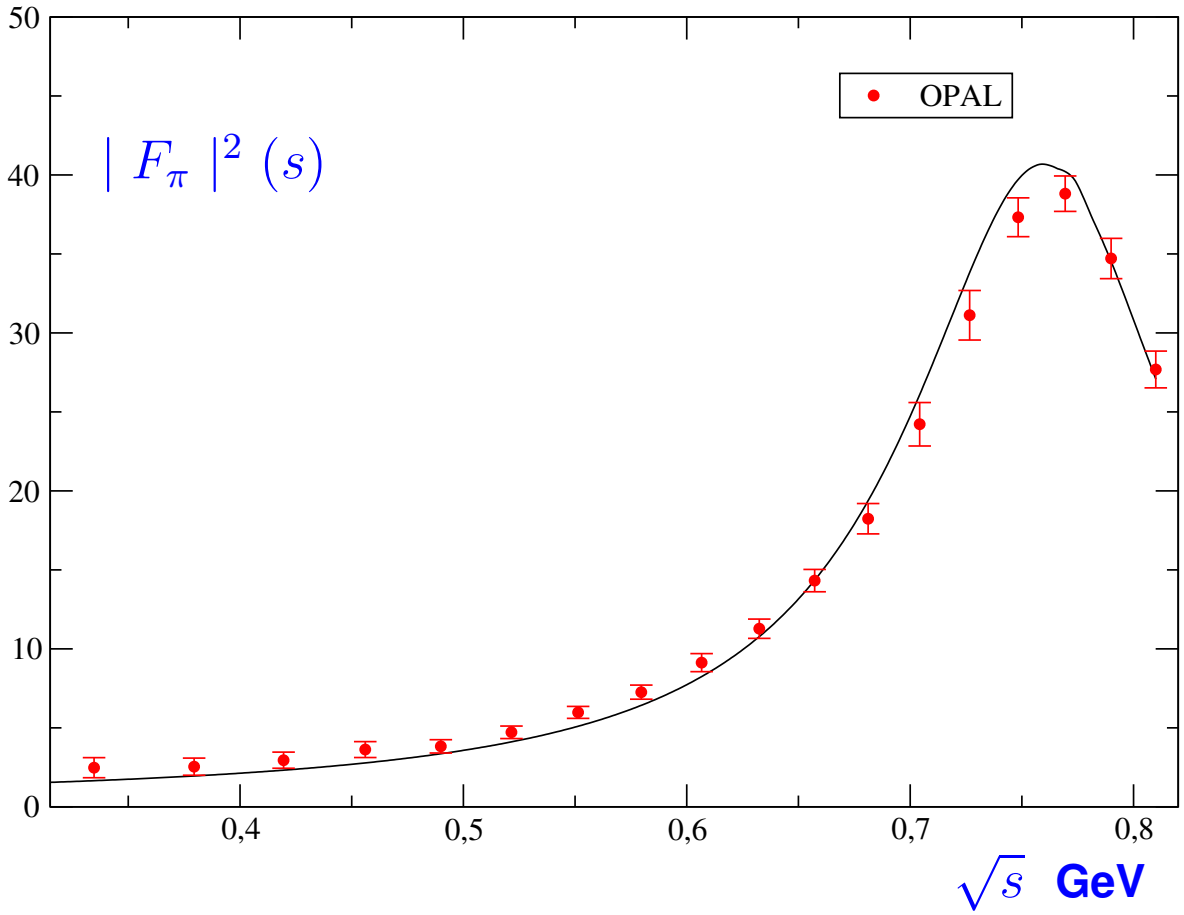
After correction no systematic deviations between CLEO/ALEPH and DM1/OLYA/CDM1/CDM2 data beyond fluctuations of e^+e^- data, while OPAL data show substantial deviations.

work in progress

G. Colangelo, J. Gasser, F. J.,H. Leutwyler

NA7+CMD-2 e^+e^-–data analytized+unitarized+chiral limit: solid line vs. τ –data





② Progress in pQCD calculations

Groshny, Kataev, Larin 91, Chetyrkin, Kühn et al. 97, Vermaseren, Larin, Ritbergen, Rodrigo, Pich, Santamaria, Chetyrkin, Kniehl, Steinhauser 97, ...

Assume pQCD to be valid down to M_τ

$$\sigma_{\rm tot}(e^+e^- \to \text{hadrons}) \sim \sigma_{\rm tot}(e^+e^- \to \text{quarks and gluons})$$

The assumption seems to be supported by

- **the apparent applicability of pQCD to τ physics. In fact the running of $\alpha_s(M_\tau) \to \alpha_s(M_Z)$ from the τ mass up to LEP energies agrees well with the LEP value. The estimated uncertainty may be debated, however.**
- **non–perturbative (NP) effects if parametrized as prescribed by the operator product expansion (OPE) of the electro-magnetic current correlator are seemingly small.**

For a more conservative approach see below (scheme dependence, mass effects, Euclidean approach)

③ Non perturbative effects?

Parametrize NP effects at sufficiently large energies and away from resonances as prescribed by the OPE. Non–vanishing gluon and light quark condensates (Shifman, Vainshtein, Zakharov 79) imply the leading power corrections

$$\begin{aligned}\Pi_\gamma^{\prime \rm NP}(Q^2) &= \frac{4\pi\alpha}{3}\sum_{q=u,d,s} Q_q^2 N_{cq}\cdot\Big[\frac{1}{12}\left(1-\frac{11}{18}a\right)\frac{<\frac{\alpha_s}{\pi}GG>}{Q^4}\\ &+ 2\left(1+\frac{a}{3}+\left(\frac{11}{2}-\frac{3}{4}l_{q\mu}\right)a^2\right)\frac{<m_q\bar{q}q>}{Q^4}\\ &+\left(\frac{4}{27}a+\left(\frac{4}{3}\zeta_3-\frac{257}{486}-\frac{1}{3}l_{q\mu}\right)a^2\right)\sum_{q'=u,d,s}\frac{<m_{q'}\bar{q'}q'>}{Q^4}\Big]\\ &+ \cdots\end{aligned}$$

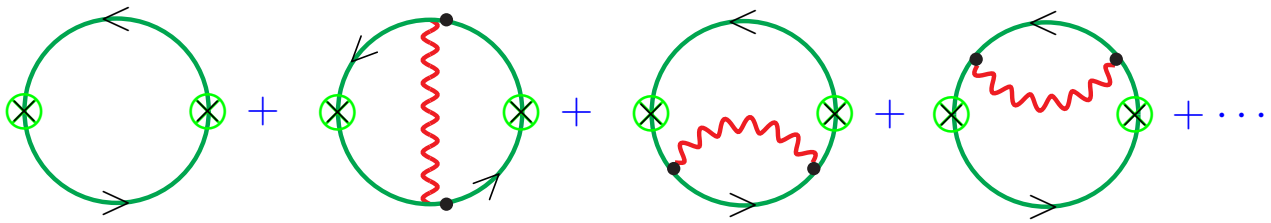
where $a\equiv\alpha_s(\mu^2)/\pi$ **and** $l_{q\mu}\equiv\ln(Q^2/\mu^2)$.
$<\frac{\alpha_s}{\pi}GG>$ **and** $<m_q\bar{q}q>$ **are the scale-invariantly defined condensates. Sum rule estimates of the condensates yield typically (large uncertainties)**

$$\begin{aligned}<\tfrac{\alpha_s}{\pi}GG> &\sim (0.389\ \textbf{GeV})^4\\ <m_q\bar{q}q> &\sim -(0.098\ \textbf{GeV})^4 \ \text{ for } \ q=u,d\\ <m_q\bar{q}q> &\sim -(0.218\ \textbf{GeV})^4 \ \text{ for } \ q=s\end{aligned}$$

Note that the above expansion is just a parameterization of the high energy tail of NP effects associated with the existence of non–vanishing condensates. There are other kind of NP phenomena like bound states, resonances, instantons and what else. The dilemma with is that it works only for Q^2 large enough and it has been successfully applied in heavy quark physics. It fails do describe NP physics at lower Q^2, once it starts to be numerically relevant pQCD starts to fail because of the growth of the strong coupling constant.

④ Controlling pQCD via the Adler function

❶ pQCD calculations of vacuum polarization amplitudes



up to 4–loops massless — Groshny, Kataev, Larin 91

up to 3–loops massive — Chetyrkin, Kühn et al. 97

up to 2–loops massive BF–MOM RG — F. J., Tarasov 98

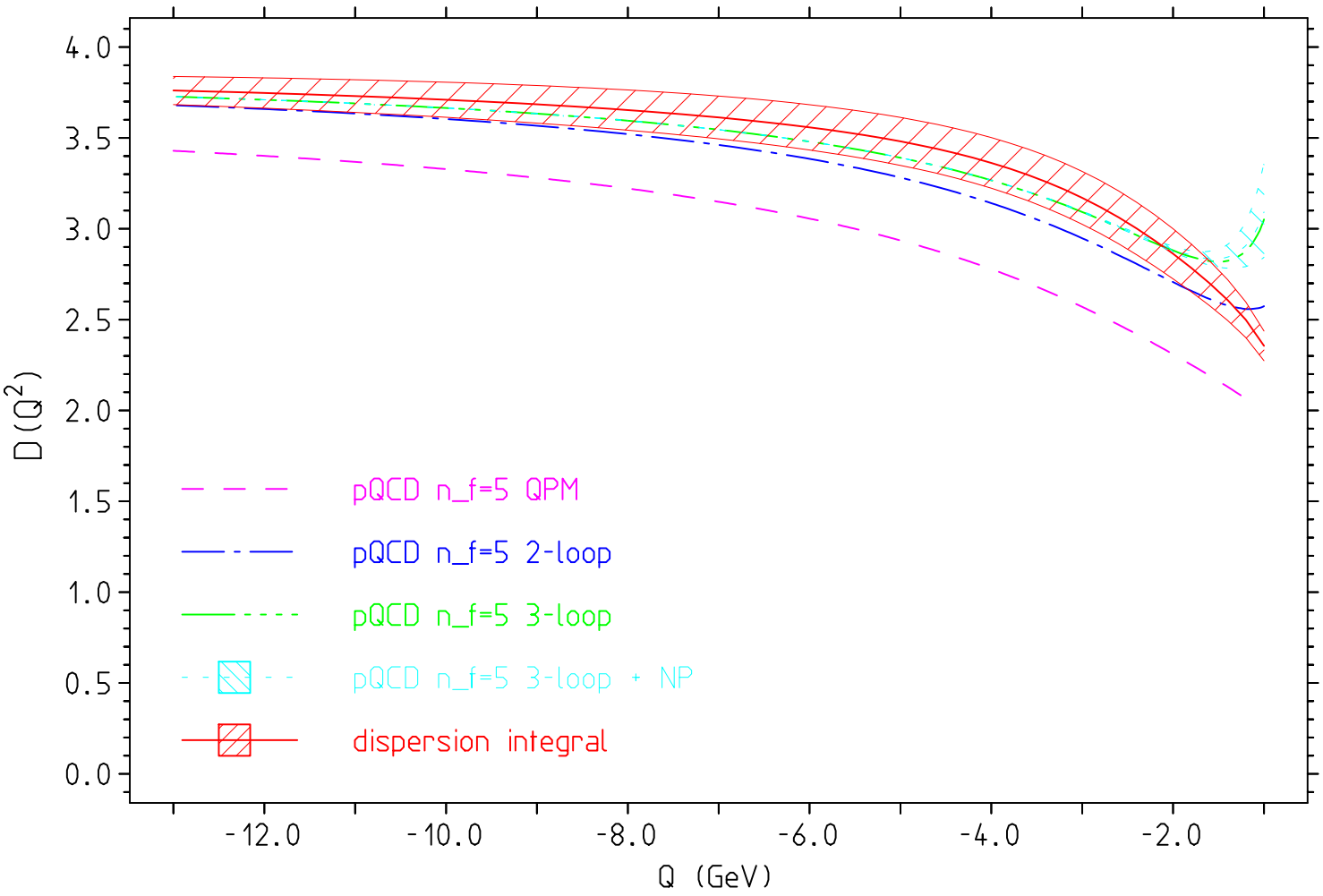
❷ use old idea: testing non–perturbative effects with help of the Adler function

Eidelman, F. J., Kataev, Veretin 98

$$
\begin{aligned}
D(-s) &\doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) \\
&= -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds} \\
\Rightarrow \qquad D(Q^2) &= Q^2 \int\limits_{4m_\pi^2}^{\infty} ds\, \frac{R(s)}{(s+Q^2)^2}
\end{aligned}
$$

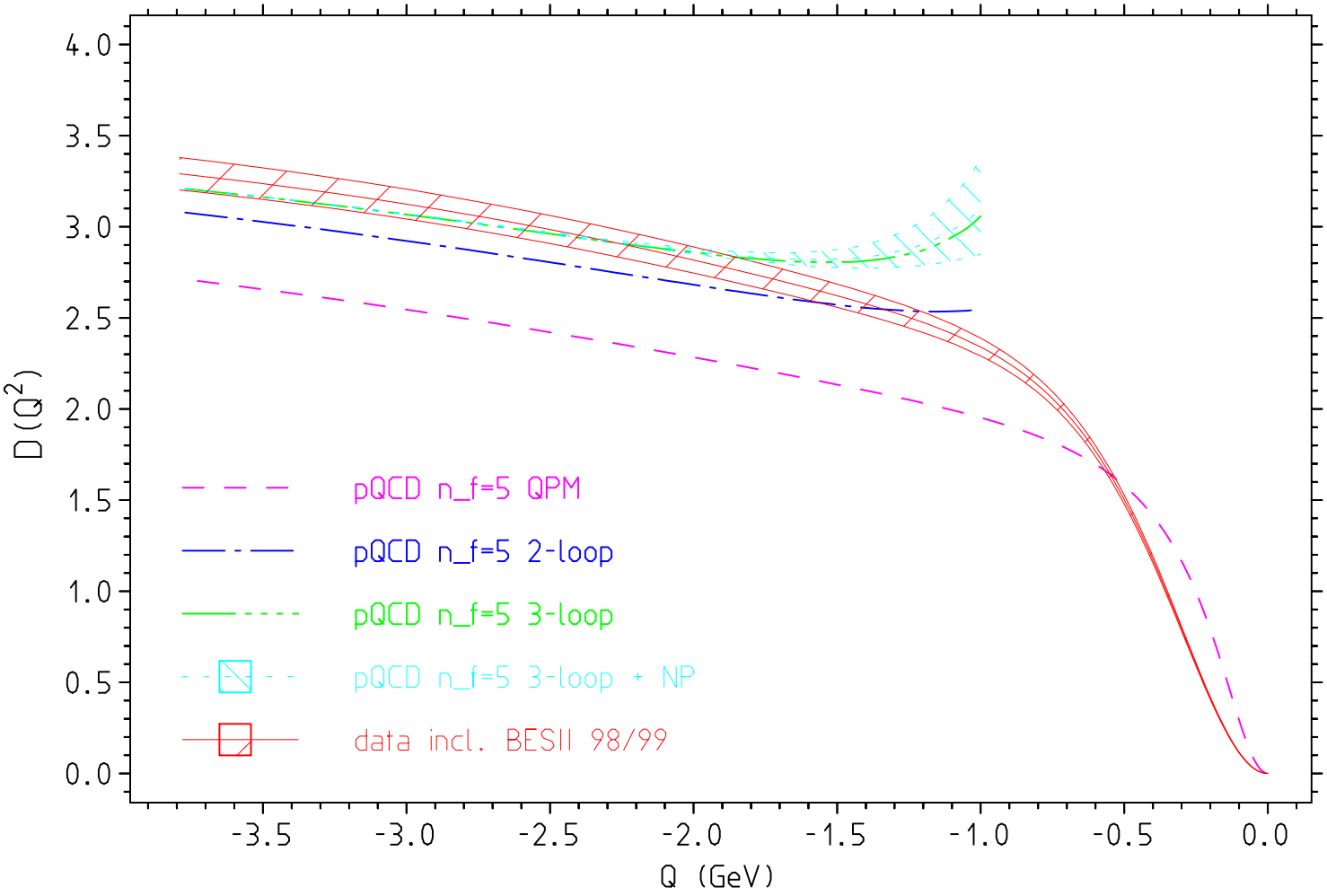
pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

"Experimental" Adler–function versus theory (pQCD + NP)



(Eidelman, F.J., Kataev, Veretin 98)

“Experimental” Adler–function versus theory (pQCD + NP)



(Update 2000: EJKV98 + BESII 98/99 prel.)

$\Rightarrow$ pQCD works well to predict $D(Q^2)$ down to $s_0 = (2.5\,\text{GeV})^2$ (not down to m_τ ! however); use this to calculate

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int dQ'^2 \frac{D(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0 = (2.5\,\text{GeV})^2$: **(FJ 98)**

$$\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} = 0.007541 \pm 0.000254$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.027737 \pm 0.000254$$

$$\alpha^{-1}(-M_Z^2) = 128.917 \pm 0.035$$

The virtues of this analysis are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.5 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!**

Remaining problems are the following:

a) contributions to the Adler function up to three–loops all have the same sign and are substantial. Four– and higher–orders could still add up to non-negligible contribution. An error for missing higher order terms is not included.

b) the link between space–like and time-like region is the difference

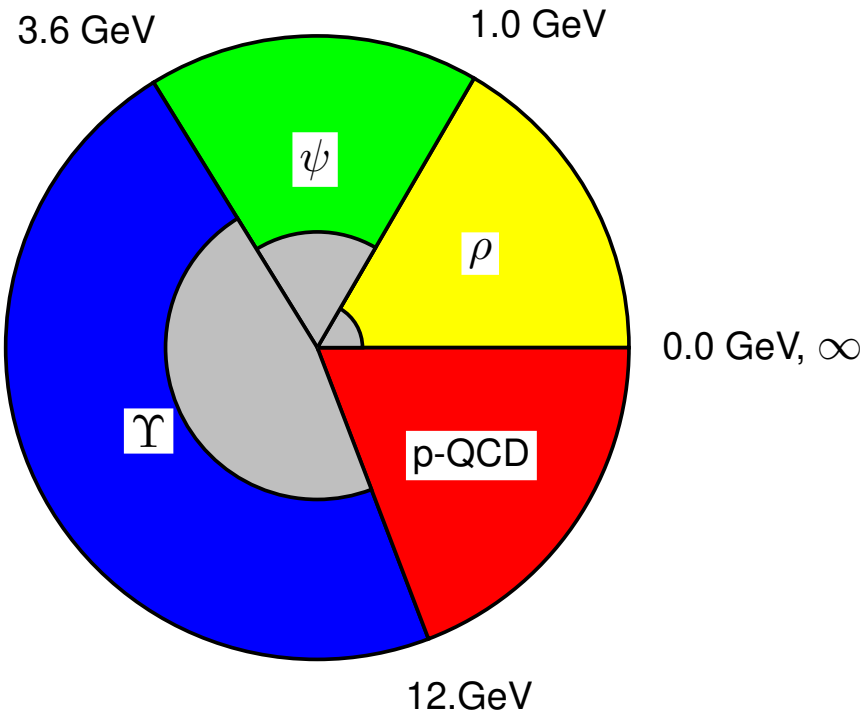
$$\Delta = \Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.000045 \pm 0.000002$$

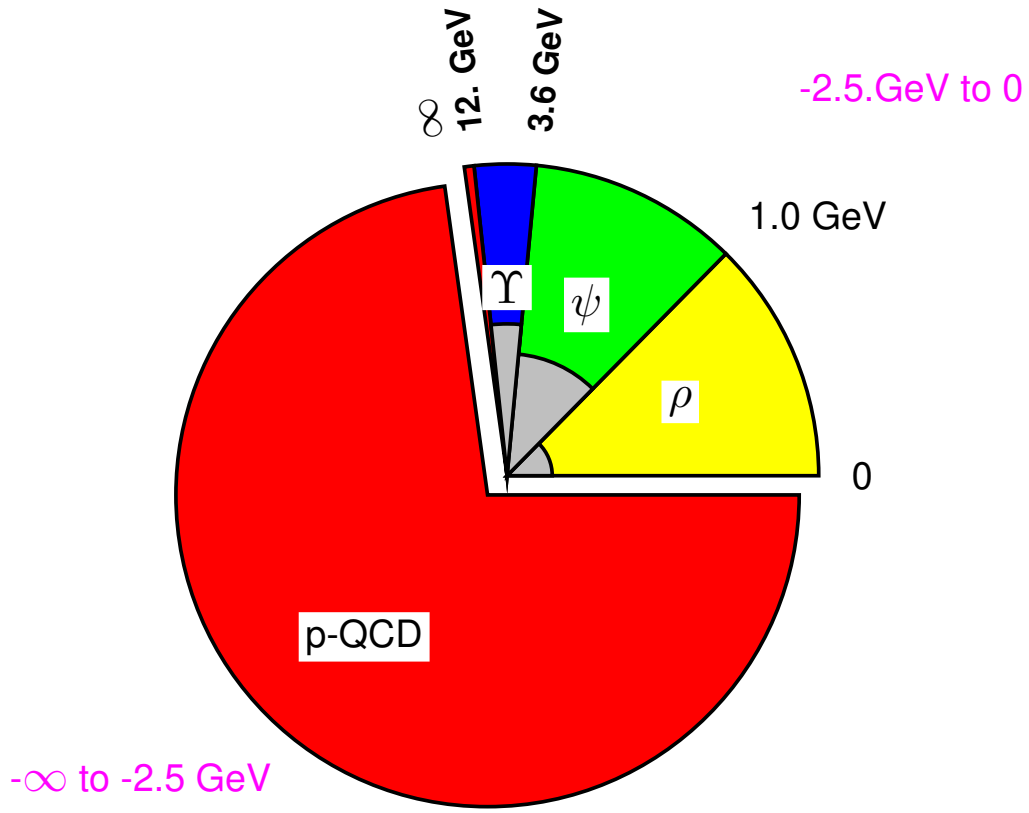
which can be calculated in pQCD. It accounts for the $i\pi$–terms from the logs $\ln(-q^2/\mu^2) = \ln(|q^2/\mu^2|) + i\pi$: Including this term yields

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027782 \pm 0.000254$$

$$\alpha^{-1}(M_Z^2) = 128.913 \pm 0.035$$

One may ask the question whether terms like Δ should be resummed at all, i.e., included in the running coupling. Usually such terms tend to cancel against constant rational terms which are not included in the renormalization group (RG) evolution. Note that the Dyson resummation in general is not a systematic resummation of leading, sub-leading etc. terms as the RG resummation is.





Shaded area: 10 $\times$ error (scaled up)

$a_\mu^{\rm had}$ via the Adler function

t–channel or Euclidean field theory approach

non–perturbative regime directly accessible by lattice QCD !

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int_0^\infty \frac{ds}{s}\int_0^1 dx\,\frac{x^2\,(1-x)}{x^2+(1-x)s/m_\mu^2}\,e^2\frac{1}{\pi}{\rm Im}\Pi'_{\gamma\,{\rm had}}$$

interchange of integrations yields:

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha^{\rm had}\left(-Q^2(x)\right) \qquad (1)$$

where $Q^2(x)\equiv\frac{x^2}{1-x}m_\mu^2$ is the space like square momentum transfer or

$$x = \frac{Q^2}{2m_\mu^2}\left(\sqrt{1+\frac{4m_\mu^2}{Q^2}}-1\right)$$

1) calculate the Adler function from the e^+e^---data and pQCD tail above 11 GeV

2) calculate the corresponding shift $\Delta\alpha^{\rm had}$ with or without t--channel cut at 2.5 GeV

3) calculate $a_\mu^{\rm had}$

By writing $(1-x) = -\frac{1}{2}\frac{d}{dx}(1-x)^2$ and performing a partial integration we obtain

$$a_\mu^{\rm had} = \frac{\alpha}{\pi} m_\mu^2 \int\limits_0^1 dx\, x\,(2-x)\,\Big(D(Q^2(x))/Q^2(x)\Big) \qquad (2)$$

the number of integrations is reduced by one.

The evaluation in both forms provides a good stability test of the numerical integrations involved.

Utilizing the most recent compilation of the e^+e^-–data we obtain the following results:

$a_\mu^{\rm had}$	method
6994.7 (104.3)	e^+e^--data ,time-like, pQCD above 11 GeV
6974.0 (104.4)	Eq. (1)
6974.0 (104.7)	Eq. (1) pQCD above 2.5 GeV
6974.0 (104.7)	Eq. (2) pQCD above 2.5 GeV

$$\boxed{6974 \pm 105}$$

τ–data: central value $\nearrow$, uncertainty $\searrow$

Status as I see it:

$$\begin{array}{lcll} & \boxed{212} & & \boxed{1\sigma} \\ \left|a_\mu^{\rm exp} - a_\mu^{\rm the}\right| & = (379 \pm 190) \times 10^{-11} & \Rightarrow & 2\sigma \quad \text{present} \\ & (??? \pm 123) \times 10^{-11} & \Rightarrow & 3\sigma \quad \text{final} \\ & & & \boxed{2\sigma} \end{array}$$

Knecht, Nyffeler Nov. 2001 -167 in central value

Status and Outlook

$a_\mu^{\rm had}$: **uncertainty from $\pi^+\pi^-$, ρ^0 dominates**

$a_\mu^{\rm had}$ **and uncertainties in units 10^{-11}.**

$a_\mu^{\rm had}$	δa_μ	input
6967	156	e^+e^- data $<$ 12. GeV + pQCD (EJ95, ADH98)
7011	94	$e^+e^- + \tau$ data $<$ 12. GeV + pQCD (ADH98)
6951	75	data $<$ 1.8 GeV and 3.3-5.0 GeV + pQCD (DH98a)
6924	62	in addition OPE (sum rules) (DH98b)
6987	111	EJ95 update incl. CMD-2 99, BESII 99 (FJ01)
6974	105	"Adler based" (EJKV00, FJ01)
—	151	BNL a_μ–experiment (BNL 01)
—	40	BNL final goal

$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$:

$$\frac{\delta M_W}{M_W} \sim \frac{1}{2}\frac{\sin^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 0.23\,\delta\Delta\alpha$$

$\delta\Delta\alpha$	δM_W (MeV)	Ref.	Method
0.00065	12.0	[EJ95]	data $< 12.$ GeV
0.00016	3.0	[DH98]	data < 1.7 GeV
0.00025	4.6	[FJ98]	Euclidean > 2.5 GeV
0.000197	3.6	[MOR00]	rescaled data, pQCD 2.8-3.7, 5-∞
0.000220	4.0	[MOR00]	same but "exclusive"
0.000460	8.5	[BP00]	data
0.000391	7.2	[FJ00]	as EJ95 + new data (BESII)
0.00007	1.3	$\delta\sigma \lesssim 1\%$ up to J/ψ	
0.00005	0.9	$\delta\sigma \lesssim 1\%$ up to Υ	
WA	22.0	Osaka 2000	

$$\frac{\delta \sin^2\theta}{\sin^2\theta} \sim \frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 1.54\,\delta\Delta\alpha$$

$\delta\Delta\alpha$	$\delta \sin^2\theta$	Ref.	Method
0.00065	0.000232	[EJ95]	data $< 12.$ GeV
0.00016	0.000057	[DH98]	data < 1.7 GeV
0.00025	0.000089	[FJ98]	Euclidean > 2.5 GeV
0.000197	0.000070	[MOR00]	rescaled data, pQCD 2.8-3.7, 5-∞
0.000220	0.000078	[MOR00]	same but "exclusive"
0.000460	0.000164	[BP00]	data
0.000391	0.000139	[FJ00]	as EJ95 + new data
0.00007	0.000025	$\delta\sigma \lesssim 1\%$ up to J/ψ	
0.00005	0.000018	$\delta\sigma \lesssim 1\%$ up to Υ	
WA	0.000160	LEP+SLD	

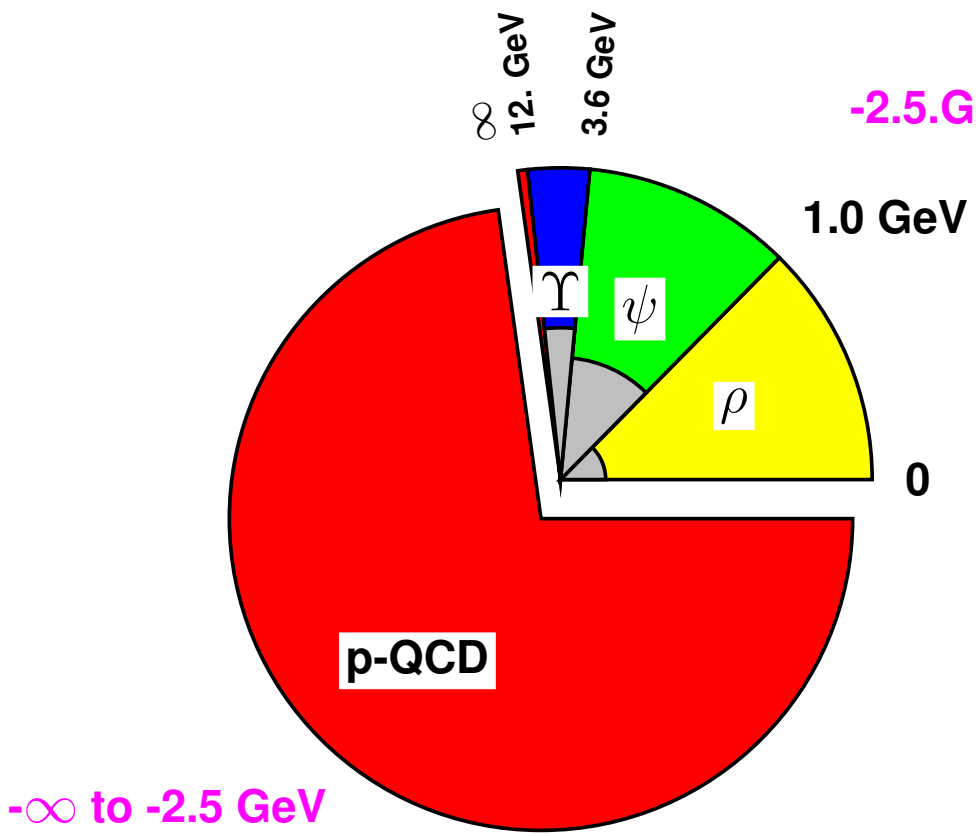
Future:

- **KLOE (DAΦNE Frascati): expect a 1% measurement for $\sigma(e^+e^- \to hadrons)$ at $\sqrt{s} \lesssim$ 1 GeV**
  - **photon tagging method** **theory: $\sim$ 2%**
  - **$\pi^+\pi^-$ inv. mass spectrum** **theory: $\sim$ 0.5%**
  - ✗ **very important new masurement with different systematics (cross check of Novosibirsk measurements)**
  - ✗ **still a lot of theory efforts necessary !**
    - **large angle Bhabha ?**
    - **RC to photon tagging ?**
    - **photon radiation from hadrons ?**
  - ✗ **eagerly awaiting for fist release of KLOE data big attention for new input for $g-2$ now !**
- **CMD-2, SND (VEPP-2M Novosibirsk): CMD-2 final data analysis 0.6% systematics ? upgrade up to 2 GeV**
- **BES (BEPC Beijing): upgrade, higher luminosity, improved detector 2 to 5 GeV region**
- **SLAC PEP-N project: $R(s)$ measurement in range $M_\Phi < E_{\rm cm} < M_{J/\psi}$**
  Also: radiative return $\pi\pi$ inv. mass spectrum at B factories (BABAR/SLAC, BELLE/KEK)

Long term: need $\sigma(e^+e^- \to hadrons) \lesssim$ 1% up to 3.6 GeV both for $a_\mu^{\rm had}$ and for $\Delta\alpha^{\rm had}$:

Distribution of hadronic contributions to $\Delta\alpha^{\rm had}$

"Adler function based" approach (EJKV99,FJ 00):



Shaded area: 10 $\times$ error (scaled up)

- **τ–charm factory**

  able to perform an energy scan between 2 and 3.6 GeV

  would satisfy requirements of future precision experiments

  g-2, GigaZ,...

- **Last but not least: need further progress in theory**
  - ✗ **more careful study of isospin breaking in τ–data vs. e^+e^-–data**
  - ✗ **constraints to $F_\pi(s)$ from χPT, unitarity and analyticity below the ρ**
  - ✗ **the real theoretical challenge: the hadronic light–by–light scattering contribution !**

In view of the more precise measurement to come from the BNL g-2 Collaboration the subject remains a big challenge for all participants. It remains an exciting possibility that we are on the way to establish "new physics" beyond the SM