DESY 03-155
SFB/CPP-03-47
October 15, 2003

Iso–spin violating effects in e^+e^- vs. τ measurements of the pion form factor $|F_\pi|^2(s)$.*

Stephane Ghozzi* and Fred Jegerlehner†

Deutsches Elektronen Synchrotron
Platanenallee 6, D-15738 Zeuthen, Germany

Abstract

We study possible so far unaccounted iso-spin breaking effects in the relation between the pion form factor as determined in e^+e^- experiments and the corresponding quantity obtained after accounting for known iso-spin breaking effects by an iso-spin rotation from the τ–decay spectra. In fact the observed 10% discrepancy in the respective pion form factors may be explained by the iso-spin breaking which is due to the difference between masses and widths of the charged and neutral ρ mesons. Since the hadronic contribution to the muon anomalous magnetic moment can be calculated directly in terms of the e^+e^-–data [our estimate is $a_\mu^{\rm had(1)} = (694.8 \pm 8.6) \times 10^{-10}$] the corresponding evaluation seems to be more reliable. The τ–data are usefull at the presently aimed level of accuracy only after appropriate input from theory.

1 Introduction

The most precise measurement of the low energy pion form factor in e^+e^-–annihilation experiments is from the CMD-2 collaboration. The updated results for the processes $e^+e^- \to \rho \to \pi^+\pi^-$ has just been published [1]. The update appeared necessary due to an overestimate of the integrated luminosity in previous analyzes. The latter was published in 2002 [2]. A more progressive error estimate (improving on radiative corrections, in particular) allowed a reduction of the systematic error from 1.4% to 0.6 % .

Since 1997 precise τ–spectral functions became available [3, 4, 5] which, to the extent that flavor $SU(2)_{\rm f}$ in the light hadron sector is a symmetry, allows to obtain the iso–vector part of the e^+e^-–cross section [6]. In this way τ data may help to substantially improve our knowledge of $|F_\pi|^2(s)$, which is an important input for the evaluations of the hadronic vacuum polarization contributions to the anomalous magnetic moment of the muon a_μ and of the effective fine structure constant $\alpha_{\rm em}(M_Z)$ an important input for

*Work supported in part by TMR, EC-Contract No. HPRN-CT-2002-00311 (EURIDICE), and by DFG under Contract SFB/TR 9-03.

*On leave from École Normale Supérieure, Departement de Physique, Paris
†e-mails: stephane.ghozzi@ens.fr, fred.jegerlehner@desy.de

LEP/SLC precision physics (see e.g. [7]). The idea to use the τ spectral data to improve the evaluation of the hadronic contributions a_μ^{had} and $\Delta\alpha^{\rm had}$ was pioneered in [8].

With increasing precision of the low energy data it more and more turned out that we are confronted with a serious obstacle to further progress: in the region just above the ω–resonance, the iso-spin rotated τ–data, corrected for the known iso-spin violating effects [9], do not agree with the e^+e^-–data at the 10% level [10]. Before the origin of this discrepancy is found it will be hard to make progress in pinning further down theoretical uncertainties in the predictions for a_μ and $\alpha_{\rm em}(M_Z)$.

In this context iso-spin breaking effects in the relationship between the τ– and the e^+e^-–data have been extensively investigated in [9]. One point which in our opinion has not been satisfactorily clarified is the role of the the iso–spin breaking effects in the charged vs. neutral ρ line–shape, which must manifest themselves in $m_{\rho^\pm} - m_{\rho^0}$ and $\Gamma_{\rho^\pm} - \Gamma_{\rho^0}$. Looking at the particle data tables [11], there is no established non-zero result as yet. Earlier statements about the problem in [8, 9, 10] adopted essentially the PDG estimate $m_{\rho^\pm} - m_{\rho^0} = 0 \pm 1$ MeV[1] while a more recent analysis of the CMD-2 and the ALEPH and CLOE data yields $m_{\rho^\pm} - m_{\rho^0} = 2.6 \pm 0.8$ MeV and $\Gamma_{\rho^\pm} - \Gamma_{\rho^0} = 3.1 \pm 1.7$ MeV [21] where the uncertainty is our estimate. The corresponding iso–spin corrections may still look to small to account fully for the observed discrepancy in the spectral functions but they clearly point towards a substantial reduction of the problem. Our strategy therefore here is a different one. Our hypothesis is that as a leading effect the discrepancy very likely is due to the iso–spin breaking by the charged vs. neutral ρ–meson parameters. A similar but subleading contribution is expected to come from possible iso-spin violations in the respective backgrounds (encoded usually by the ρ', ρ'' contributions). Since the fit formulae adopted, like the Gounaris-Sakurai formula [13], are far from being based on first principles we should not trust to much in the fitting procedures based on them. E.g., usually just a set of resonances $\rho, \omega, \rho', \rho''$ is included but we have no idea about the background (continuum) which also should be included somehow. We also would like to stress that, at the level of accuracy the present discussion advances, in future it will make no sense to compile ρ masses and width for mixed charges, as done in the particle data tables.

Whether the observed discrepancy is an experimental problem, or just a so far underestimated iso–spin breaking effect will also be settled, hopefully, by new results for hadronic e^+e^- cross–sections which are under way from KLOE, BABAR and BELLE. These experiments, running at fixed energies, are able to perform measurements via the radiative return method [14, 15, 16]. Results presented recently by KLOE seem to agree very well with the final CMD-2 e^+e^-–data.

2 The τ vs. e^+e^- problem

The iso-vector part of $\sigma(e^+e^- \to \text{hadrons})$ may be calculated by an iso-spin rotation from τ–decay spectra, to the extend that the so–called conserved vector current (CVC) would be really conserved (which it is not, see below). The relation may be derived by comparing

[1] For theoretical estimates see [12] and references therein.

the relevant lowest order diagrams which for the e^+e^- case translates into

$$\sigma_{\pi\pi}^{(0)} \equiv \sigma_0(e^+e^- \to \pi^+\pi^-) = \frac{4\pi\alpha^2}{s}\, v_0(s) \tag{1}$$

and for the τ case into

$$\frac{1}{\Gamma}\frac{d\Gamma}{ds}(\tau^- \to \pi^-\pi^0\nu_\tau) = \frac{6|V_{ud}|^2 S_{EW}}{m_\tau^2}\frac{B(\tau^- \to \nu_\tau\, e^-\,\bar{\nu}_e)}{B(\tau^- \to \nu_\tau\, \pi^-\pi^0)}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\, v_-(s) \tag{2}$$

where $|V_{ud}| = 0.9752 \pm 0.0007$ [11] denotes the CKM weak mixing matrix element and $S_{\mathrm{EW(new)}} = 1.0233 \pm 0.0006$ [$S_{\mathrm{EW(old)}} = 1.0194$] accounts for electroweak radiative corrections [17, 18, 19, 9]. The spectral functions are obtained from the corresponding invariant mass distributions. The $B(i)$'s are branching ratios. SU(2) symmetry (CVC) would imply

$$v_-(s) = v_0(s)\ \ . \tag{3}$$

The spectral functions $v_i(s)$ are related to the pion form factors $F_\pi^i(s)$ by

$$v_i(s) = \frac{\beta_i^3(s)}{12}|F_\pi^i(s)|^2\ \ ;\quad (i=0,-) \tag{4}$$

where $\beta_i^3(s)$ is the pion velocity. The difference in phase space of the pion pairs gives raise to the relative factor $\beta^3_{\pi^-\pi^0}/\beta^3_{\pi^-\pi^+}$ [8, 20].

It is important to check what precisely the experimental data in each case represent. In CMD-2 e^+e^- measurements final state radiation (FSR) as well as vacuum polarization effects are not included. They have been subtracted together with the initial state photon radiation. Hard FSR photons were rejected to a large extent. Thus $F_\pi(s)$ from CMD-2 is the undressed (from VP and FSR) pion form–factor (see e.g. [22]). In contrast τ decay spectra include photon radiation, which hence has to be subtracted a posteriori (the correction factor G_{EM} below), while photon vacuum polarization effects play no role. This is because the τ decay as a charged current (CC) process proceeds by the heavy W exchange, which makes it an effective four–fermion interaction with Fermi constant G_F as a coupling in place of $\alpha(s)$. In contrast to α the Fermi coupling G_F is not running up to LEP energy scales. Electroweak short distance corrections (hadronic relative to leptonic channel) give raise to the correction factor $S_{\mathrm{EW}} = 1 + \delta_{\mathrm{EW}}$, which is dominated by a leading large logarithm $(1+(\alpha/\pi)\ \ln(M_Z/m_\tau))$ which should be resummed using the renormalization group [17]. Note that the overall coupling drops out from the ratios in (2). This also makes it evident that the subtraction of the large and strongly energy dependent vacuum polarization effects (see e.g. Fig. 1 in [23]) necessary for the e^+e^-–data, which seems to worsen the problem, was properly treated in the analyzes.

Before a precise comparison via (3) is possible all kind of iso-spin breaking effects have to be taken into account. As mentioned earlier, this has been investigated carefully in [9] for the most relevant $\pi\pi$ channel. Accordingly, we may write the corrected version of (3) (see [9] for details) in the form

$$\sigma_{\pi\pi}^{(0)} = \left[\frac{K_\sigma(s)}{K_\Gamma(s)}\right]\ \frac{d\Gamma_{\pi\pi[\gamma]}}{ds} \times \frac{R_{\mathrm{IB}}(s)}{S_{\mathrm{EW}}} \tag{5}$$

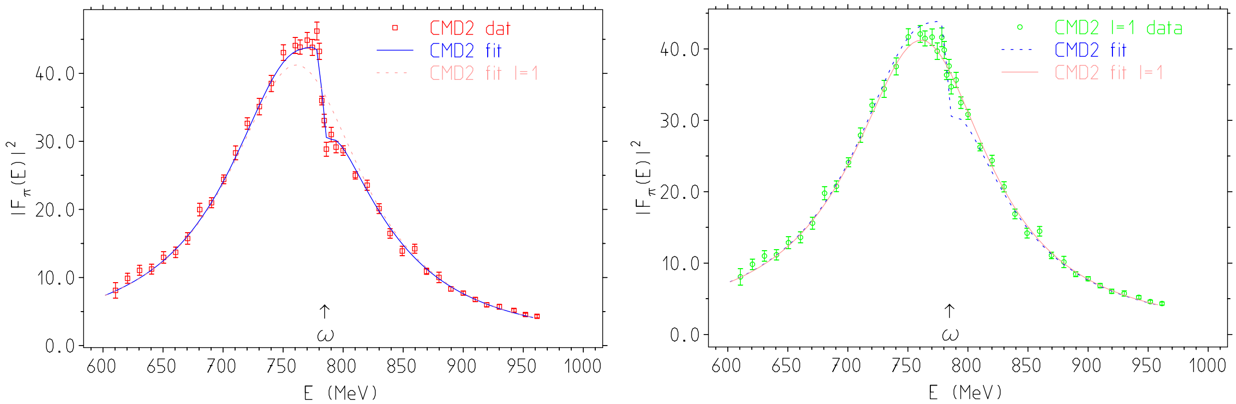

Figure 1: CMD-2 data for $|F_\pi|^2$ in $\rho-\omega$ region together with Gounaris-Sakurai fit. Left: before subtraction, right: after subtraction of the ω.

with

$$K_\Gamma(s)=\frac{G_F^2\,|V_{ud}|^2\,m_\tau^3}{384\pi^3}\,\left(1-\frac{s}{m_\tau^2}\right)^2\,\left(1+2\,\frac{s}{m_\tau^2}\right)\quad;\quad K_\sigma(s)=\frac{\pi\alpha^2}{3s}\,, \tag{6}$$

and the iso-spin breaking correction

$$R_{\rm IB}(s)=\frac{1}{G_{\rm EM}(s)}\,\frac{\beta^3_{\pi^-\pi^+}}{\beta^3_{\pi^-\pi^0}}\,\left|\frac{F_V(s)}{f_+(s)}\right|^2\quad. \tag{7}$$

The latter includes the photonic (long range) corrections (based on scalar QED), phase space corrections due to the $\pi^\pm-\pi^0$ mass difference and the form–factor corrections which are dominated by the SU(2) breaking $\rho-\omega$ mixing effect and the electromagnetic shifts in the masses and the widths of the ρ's. These corrections were applied in [10] and were not able to resolve the puzzle of the observed discrepancy between $v_-(s)$ and $v_0(s)$ (see [10] for details and Fig. 3 below).

One possible iso–spin breaking effect which has been mentioned or very briefly discussed only in [8, 9, 10] is the possibility that the electromagnetic corrections which accounts for the difference in masses and widths of the neutral vs. charged ρ–mesons (remember that for similar bound states the π's we have $m_{\pi^\pm}-m_{\pi^0}=4.5935\pm0.0005$ MeV) might have been underestimated so far. These iso–spin violating differences are not established experimentally, not even the sign. We therefore ask the question whether applying an iso–spin breaking correction which accounts for that could resolve the puzzle of the above mentioned discrepancy.

What we do is the following: we take the CMD-2 data and subtract off the ω–contribution. The latter $I=0$ part enters via $\rho-\omega$ mixing, which is a consequence of iso–spin violation due to the mass difference m_u-m_d of the light quarks. To this end we may take the Gounaris–Sakurai kind parameterization (see [13, 2] for details) (FSR not included; BW=Breit-Wigner; $\mathrm{BW^{GS}}$=Gounaris-Sakurai modified Breit-Wigner)

$$F_\pi(s)=\frac{\mathrm{BW}^{\rm GS}_{\rho(770)}(s)\cdot\left(1+\delta\frac{s}{M_\omega^2}\mathrm{BW}_\omega(s)\right)+\beta\,\mathrm{BW}^{\rm GS}_{\rho(1450)}(s)+\gamma\,\mathrm{BW}^{\rm GS}_{\rho(1700)}(s)}{1+\beta+\gamma} \tag{8}$$

of the CMD-2 data and set the mixing parameter $\delta=0$. In this way we obtain the iso–vector part of the square of the pion form factor $|F_\pi|^{2\,I=1}_{(e^+e^-)}(s)$ displayed in Fig. 1. To

	τ ALEPH	τ CLEO	τ OPAL	e^+e^- CMD-2
m_{ρ^0}	-	-	-	$773.14 \pm 0.56 \pm 0.12$
Γ_{ρ^0}	-	-	-	$146.43 \pm 0.68^{+0.12}_{-0.13}$
m_{ρ^-}	$775.83 \pm 0.50^{+0.34}_{-0.25}$	$775.26 \pm 0.35^{+0.30}_{-0.29}$	$778.50 \pm 1.19^{+0.29}_{-0.23}$	-
Γ_{ρ^-}	$147.76 \pm 0.67^{+0.10}_{-0.09}$	$147.35 \pm 0.49^{+0.12}_{-0.21}$	$152.50 \pm 1.64^{+1.15}_{-1.50}$	-
S	1.43	1.67	0.85	1.11

Table 1: Results of fits to the iso–spin breaking corrected pion form factors squared for τ (ALEPH and CLEO) and e^+e^- (CMD-2) data. The Gounaris-Sakurai parameterization of the ρ line shape is utilized. Masses and widths in MeV. $S = \sqrt{\chi^2/(n-1)}$

the τ version of the pion form factor, following from (2) and (4), we perform the iso–spin breaking corrections

$$r_{\rm IB}(s) = \frac{1}{G_{\rm EM}(s)} \frac{\beta^3_{\pi^-\pi^+}}{\beta^3_{\pi^-\pi^0}} \frac{S_{\rm EW(old)}}{S_{\rm EW(new)}} \tag{9}$$

with $G_{\rm EM}(s)$ from [9]. The such obtained corrected[2] pion form factor $|F_\pi|^{2\,I=1}_{(\tau)}(s)$ is to be compared with $|F_\pi|^{2\,I=1}_{(e^+e^-)}(s)$. The ratio shows the unexpected large deviations from unity (see Fig. 3). While the ALEPH and CLEO data clearly exhibit the structure as expected from an increase of mass and width of the ρ the OPAL data show a different form of the spectrum. The reason for the problem with the OPAL data is due to the fact that at the ρ apparently the cross section is too low. Since the distribution is normalized to the very precisely known total branching fraction, the tails of the resonance get enhanced, which leads the the structure actually seen (the apparent widths gets enhanced). Of course the data points of the spectrum have imposed strong error correlations via the normalization to the integral rate. In the figure only the diagonal elements of the covariance matrix are visualized.

We may fit now $|F_\pi|^{2\,I=1}_{(\tau)}(s)$ with the Gounaris–Sakurai formula (8) with no ω term, i.e., with $\delta = 0$, in order to obtain $m_{\rho^\pm}$ and $\Gamma_{\rho^\pm}$. We would like to emphasize that it is important to "zoom in the ρ" appropriately in determining $m_{\rho^\pm} - m_{\rho^0}$ and $\Gamma_{\rho^\pm} - \Gamma_{\rho^0}$, i.e., we have to perform the fits at fixed background in the ρ dominated region between 610.5 MeV and 961.5 GeV (cmd-2 range). This simple leading effect analysis yields the results in Tab. 1[3]. As can be seen the τ data give consistently larger values for both mass and width of the charged ρ. The evidence is far from impressive, between ALEP and CMD-2 we have $\Delta m_\rho = 2.7 \pm 0.8$ and $\Delta \Gamma_\rho = 1.3 \pm 1.0$. The two parameter fits are not of good quality and a more elaborate analysis would be needed to come to more precise conclusions. An analysis based on more recent preliminary ALEP data, yields the slightly larger values $\Delta m_\rho = 3.1 \pm 0.9$ and $\Delta \Gamma_\rho = 2.3 \pm 1.6$ [24], which are consistent however with our findings.

Now we suppose that the systematic deviations seen in the $\rho^\pm$ parameters include

[2]The velocity factor correction of course only applies when, as frequently has been done, the wrong velocity was used in (4) in the extraction of the charged channel form factor.

[3]For the e^+e^- channel we fit the data including the ω. Fits after subtraction of the $I=0$ part yield practically the same result.

electromagnetic iso–spin breaking which we have to correct for. We now may ask two questions. The first is: how does the test–ratio look like is we replace $m_{\rho^\pm}$ and $\Gamma_{\rho^\pm}$ in the τ–data fit by the more appropriate m_{ρ^0} and Γ_{ρ^0} ? The second is: what mass and width do we get if we fit them in the τ–data parameterization such that the test–ratio is comes out close to unity? Not too surprisingly we find them close to the ones fitted by CMD-2: It makes the ratio unity within 0.1 % ! Uncertainties may be obtained from the ones in the parameterizations. Of course keeping the background fixed the result looks pretty trivial. In fact fitting all parameters of the GS formula simultaneously in the much wider range of τ data, as has been performed in [21], yields results which look very similar to the ones discussed above. The parameters obtained are very strongly correlated and all of them may be affected by some iso–spin braking effects. A much more elaborate analysis would be necessary to actually establish tight experimental values for possible iso–spin breakings in these parameters.

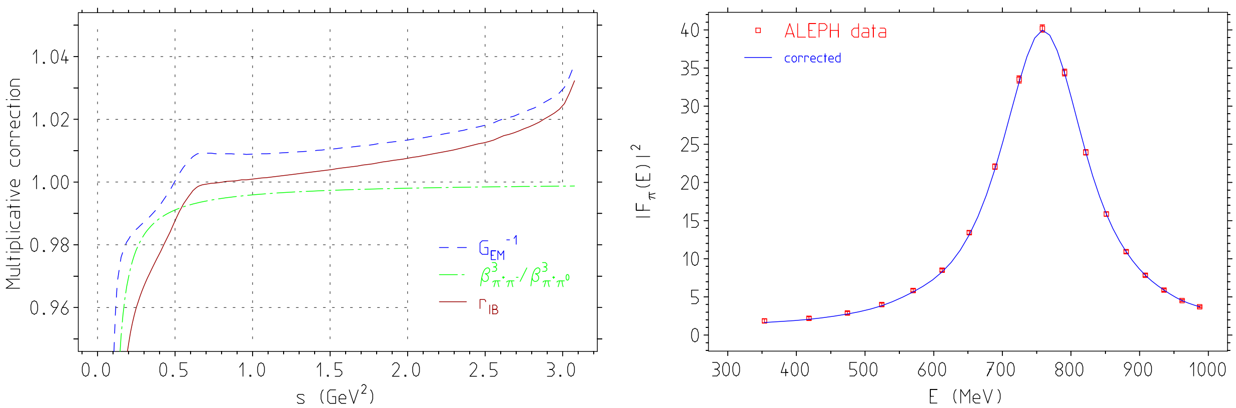

Figure 2: Iso-spin corrections applied to the τ data: left the corrections [9], with r_{IB} defined in (9), and right the effect on the ALEPH data.

We conclude that the τ vs. e^+e^- discrepancy very likely is an iso–spin breaking effect which has not been accounted for correctly in previous analyzes. This also would establish a significant differences for $m_{\rho^\pm} - m_{\rho^0}$ and $\Gamma_{\rho^\pm} - \Gamma_{\rho^0}$. Of course, what it means is the τ data cannot be utilized to calculate a_μ^{had} without reference to the e^+e^- data. Also, since now substantially correlated, the inclusion of the τ–data is much less straight forward. The question is how much they still can contribute to reduce the uncertainties in the evaluation of a_μ^{had}. This also makes it very likely that the e^+e^-–data based evaluations are the more trustworthy ones. After the correction in the normalization of the CMD-2 data we get the leading hadronic contribution to the anomalous magnetic moment of the muon. We now obtain

$$a_\mu^{\rm had(1)} = (694.8 \pm 8.6) \ \times \ 10^{-10} \qquad [e^+e^- - \text{data based}]. \tag{10}$$

With this estimate we get

$$a_\mu^{\rm the} = (11\,659\,179.4 \pm 8.6_{\rm had} \pm 3.5_{\rm LBL} \pm 0.4_{\rm QED+EW}) \times 10^{-10} \tag{11}$$

which compares to the most recent experimental result [25]

$$a_\mu^{\rm exp} = 11\,659\,203 \pm 8) \times 10^{-10} \quad . \tag{12}$$

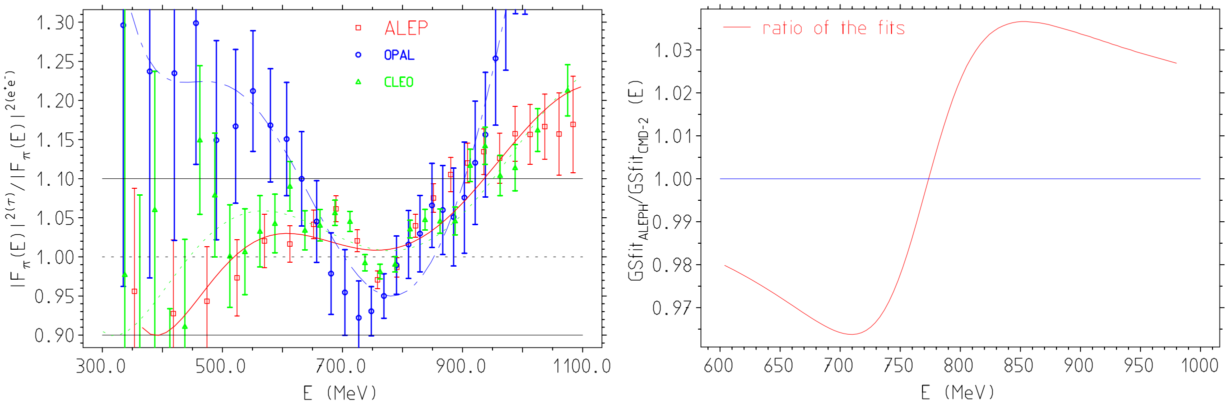

Figure 3: Left: The ratio between τ–data sets from ALEPH, OPAL and CLEO and the $I = 1$ part of the CMD-2 fit of the e^+e^-–data. The curves which should guide the eye are fits of the ratios using 8th order Tschebycheff polynomials. Right: Ratio of the ALEPH vs. CMD-2 fits differing by mass and width only (see Tab. 1). By the iso-spin violation correction $(m_{\rho^-}, \Gamma_{\rho^-}) \to (m_{\rho^0}, \Gamma_{\rho^0})$ of the τ–data this ratio becomes trivially equal to unity.

The "discrepancy" $|a_\mu^{\rm the} - a_\mu^{\rm exp}| = (23.6 \pm 12.3) \times 10^{-10}$ corresponds to a deviation of about 1.9 σ. For other recent estimates we refer to [10, 26] .

3 Summary and Conclusion

Since recently we have in each case two reasonably consistent sets of data: the ALEPH and CLEO τ–data sets on the one hand and the CMD-2 and KLOE (still preliminary) e^+e^-–data sets on the other hand. The τ–data samples are about 10% higher than the e^+e^- ones in the tail above the ρ. This can be clearly seen in Fig. 3. Assuming that the experiments are essentially correct we think that a 10% increase in a resonance tail can easily be attributed to a 1% increase in the energy scale. Since it is very unlikely a problem of energy calibration, the only explanation remains that the resonance parameters must be different in the charged and the neutral channel.

Our analysis shows that to a large extent we may understand the e^+e^- vs. τ discrepancy as an iso-spin breaking effect coming from the fact that mass and width of charged and neutral ρ–mesons, as naively expected, are different and thus that the τ–data must be mapped to the neutral channel parameters before they can be utilized in addition to the e^+e^-–data. We thus assume that a main part of the problem is due to additional iso-spin breaking effects and not primarily an experimental one. Of course there are also experimental problems which hopefully will be resolved by forthcoming experiments.

The fact that the ρ–mass difference is found to be of size comparable to the well–known pion mass difference $m_{\rho^\pm} - m_{\rho^0} \sim m_{\pi^\pm} - m_{\pi^0} \sim 4.6$ MeV seems to be rather plausible as the corresponding bound states have the same quark content and in the neutral case the electromagnetic interaction contributes more to the binding energy and thus lowers its mass. Since we do not have an independent evaluation of the charged and neutral ρ–meson parameters, the iso–spin correction needed in order the τ–data to be useful for the evaluation of the hadronic contribution to the muon anomalous magnetic moment cannot be performed at sufficient precision at the moment. Nevertheless, the τ–data still provide important cross checks and last but not least Ref. [8] triggered a discussion which forced all parties to check more carefully what they have done. One impact was that also the results in the e^+e^-–channel had to be corrected.

The main point is that there must be such effects which were not correctly treated so far. Maybe the available data do not allow us to pin down a solid value for the mass and width differences (all GS parameters in fact must be subject to iso-spin breaking and the data may not suffice to come to a definite conclusion). I think the main point is that the derivative of a_μ with respect to m_ρ and Γ_ρ is rather large and hence no stable result can be given if the iso-spin breaking in these parameters are not known with sufficient precision.

Note that in spite of the fact that the dominating ρ–peak is shifted downwards, due to the correction which we have to apply to the τ–data, the s^{-2} weighted a_μ–integral does not increase. It rather decreases, because the width also substantially decreases by the correction and actually over-compensates the effect of the shift in the mass.

Our conclusion: very likely we are back to <u>one</u> prediction for a_μ which is the e^+e^-–based value at about 2σ below the experimental result! Unfortunately, at present, we do not have a precise enough understanding of the iso-spin violations to be able to utilize the τ–data for the evaluation of the hadronic contribution to $g-2$ of the muon.

Acknowledgments

It is a pleasure to thank H. Leutwyler, A. Stahl, K. Mönig and S. Eidelman for numerous fruitful discussions.

References

[1] R. R. Akhmetshin *et al.* [the CMD-2 Collaboration], "Reanalysis of hadronic cross section measurements at CMD-2," hep-ex/0308008.

[2] R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **527** (2002) 161.

[3] R. Barate *et al.* [ALEPH Collaboration], Z. Phys. C **76** (1997) 15; Eur. Phys. J. C **4** (1998) 409.

[4] K. Ackerstaff *et al.* [OPAL Collaboration], Eur. Phys. J. C **7** (1999) 571.

[5] S. Anderson *et al.* [CLEO Collaboration], Phys. Rev. D **61** (2000) 112002.

[6] Y. S. Tsai, Phys. Rev. D **4** (1971) 2821 [Erratum-ibid. D **13** (1976) 771].

[7] S. Eidelman and F. Jegerlehner, Z. Phys. C **67** (1995) 585; F. Jegerlehner, Nucl. Phys. Proc. Suppl. **51C** (1996) 131; J. Phys. G **29** (2003) 101.

[8] R. Alemany, M. Davier, A. Höcker, Eur. Phys. J. C **2** (1998) 123.

[9] V. Cirigliano, G. Ecker and H. Neufeld, Phys. Lett. B **513** (2001) 361; JHEP **0208** (2002) 002.

[10] M. Davier, S. Eidelman, A. Höcker and Z. Zhang, Eur. Phys. J. C **27** (2003) 497; hep-ph/0308213.

[11] K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66** (2002) 010001.

[12] J. Bijnens and P. Gosdzinsky, Phys. Lett. B **388** (1996) 203.

[13] G.J. Gounaris and J.J. Sakurai, Phys. Rev. Lett. **21** (1968) 244.

[14] A. Denig [the KLOE Collaboration], hep-ex/0211024; M. Incagli [the KLOE Collaboration], *The Hadronic Cross Section Measurement at KLOE*, presented at the EPS 2003 Conference, Aachen; B. Valeriani, presented at "SIGHAD03 Worskhop on Hadronic Cross Section at Low Energy ", Pisa, Italy, October 8th-10th, 2003.

[15] E. P. Solodov [BABAR collaboration], in *Proc. of the* e^+e^- *Physics at Intermediate Energies Conference* ed. Diego Bettoni, eConf **C010430** (2001) T03 [hep-ex/0107027]; M. Davier, presented at "SIGHAD03 Worskhop on Hadronic Cross Section at Low Energy ", Pisa, Italy, October 8th-10th, 2003.

[16] G. Rodrigo, H. Czyż, J. H. Kühn and M. Szopa, Eur. Phys. J. C **24** (2002) 71.

[17] W. J. Marciano and A. Sirlin, Phys. Rev. Lett. **61** (1988) 1815.

[18] E. Braaten and C. S. Li, Phys. Rev. D **42** (1990) 3888.

[19] R. Decker and M. Finkemeier, Nucl. Phys. B **438** (1995) 17.

[20] H. Czyż and J. H. Kühn, Eur. Phys. J. C **18** (2001) 497.

[21] M. Davier, eConf **C0209101** (2002) WE03 [hep-ex/0301035].

[22] A. Hoefer, J. Gluza and F. Jegerlehner, Eur. Phys. J. C **24** (2002) 51.

[23] F. Jegerlehner, hep-ph/0308117.

[24] M. Davier, presented at "SIGHAD03 Worskhop on Hadronic Cross Section at Low Energy ", Pisa, Italy, October 8th-10th, 2003.

[25] G. W. Bennett *et al.* [Muon g-2 Collaboration], Phys. Rev. Lett. **89** (2002) 101804 [Erratum-ibid. **89** (2002) 129903].

[26] K. Hagiwara, A. D. Martin, D. Nomura and T. Teubner, Phys. Lett. B **557** (2003) 69; presented at the EPS 2003 Conference, Aachen.