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HADRONIC EFFECTS IN $(g-2)_\mu$ and $\alpha_{\rm QED}(M_Z)$: STATUS AND PERSPECTIVES[a]

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I review recent evaluations of the hadronic contributions to the muon anomalous magnetic moment $(g-2)_\mu$ and to the effective fine structure constant $\alpha(M_Z)$. A new estimate for the hadronic shift $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027782 \pm 0.000254$ is presented which implies $\alpha^{-1}(M_Z) = 128.913 \pm 0.035$. It is based on a recent perturbative calculation of the Adler function which includes mass effects up to three-loops in a MOM scheme and requires little ad hoc assumptions. I then discuss perspectives for possible improvements in estimations of $a_\mu^{\rm had}$ which we expect from the Φ–factory Daphne at Frascati.

1 Status

I briefly review the status of the estimations of hadronic *vacuum polarization effects*. I have presented a similar report some time ago in Ref. [1] and the present contribution should be considered as an Addendum to my previous summary. Vacuum polarization effects play a role in many places in physics (charge screening). Their precise knowledge is most important for the interpretation of high energy precision experiments where theoretical predictions depend on the effective fine structure constant $\alpha(s) = \alpha/(1-\Delta\alpha(s))$ as well as for low energy precision experiments like for the anomalous magnetic moment a_μ, one of the most precisely measured quantities in physics. The photon vacuum polarization amplitude $\Pi'_\gamma(q^2)$ is defined by

$$\Pi^\gamma_{\mu\nu}(q) = i\int d^4x e^{iqx} < 0|TJ^\gamma_\mu\ (x)\ J^\gamma_\nu\ (0)\ |0>$$

[a] *Fourth International Symposium on RADIATIVE CORRECTIONS: Application of Quantum Field Theory to Phenomenology*
(RADCOR 98: Barcelona, Catalonia, Spain 8-12 Sep 1998),
Ed. J. Solà, World Scientific, Singapore, 1999.

$$= -\left(q^2\, g_{\mu\nu} - q_\mu q_\nu\right)\ \Pi'_\gamma\ (q^2) \tag{1}$$

and determines the full photon propagator

$$-g_{\mu\nu}\frac{i}{q^2}\frac{1}{1+e^2\Pi'_\gamma(q^2)} \tag{2}$$

as well as the shift in the fine structure constant

$$\Delta\alpha(q^2) = e^2\left(\Pi'_\gamma(0) - \Pi'_\gamma(q^2)\right) \quad . \tag{3}$$

The latter is subtracted at zero momentum, i.e., in the classical limit. At the one–loop order in perturbation theory one obtains

$$e^2\Pi'_\gamma(q^2) = \frac{\alpha}{3\pi}\sum_f Q_f^2 N_{cf}\left(\ln\frac{\mu^2}{m_f^2} + \frac{5}{3} + y + \left(1+\frac{y}{2}\right)\beta\ln\frac{\beta-1}{\beta+1}\right) \tag{4}$$

with μ the $\overline{\rm MS}$ scale, N_{cf} the color factor, $y = 4m_f^2/s$ and $\beta = \sqrt{1-y}$.

The electromagnetic current J_μ^γ is the sum of a leptonic and a hadronic part. The leptonic part can be calculated in perturbation theory and is known in QED up to three–loops [2]: $\Delta\alpha_{\rm lep} = 0.031497687$ (2–loop fraction $\sim 2.5\times10^{-3}$, 3–loop fraction $\sim 3.4\times10^{-5}$). The quark contribution cannot be calculated reliably in perturbation theory because of low energy strong interaction effects. Fortunately existing e^+e^- data allow us to evaluate the hadronic contributions by means of the dispersion integral

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\mathrm{P}\!\int_{4m_\pi^2}^{E_{\rm cut}^2} ds'\frac{R_\gamma^{\rm data}(s')}{s'(s'-s)} + \mathrm{P}\!\int_{E_{\rm cut}^2}^{\infty} ds'\frac{R_\gamma^{\rm pQCD}(s')}{s'(s'-s)}\right) \tag{5}$$

where

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^-\to\gamma^*\to{\rm hadrons})}{\sigma(e^+e^-\to\gamma^*\to\mu^+\mu^-)} = 12\pi{\rm Im}\Pi'_\gamma(s) \tag{6}$$

has been measured in e^+e^-–annihilation experiments up to about $E_{\rm cut}\sim 40$ GeV. By virtue of the asymptotic freedom of QCD the high energy tail can be safely calculated using pQCD [3].

Taking a conservative attitude we have compiled the existing e^+e^-–data (non–perturbative) and evaluated Eq. (5) for $\sqrt{s}\lesssim E_{\rm cut}\sim 40$GeV. After including the perturbative tail we found [4]

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02804 \pm 0.00064 \qquad \text{(Eidelman, Jegerlehner 95)}$$

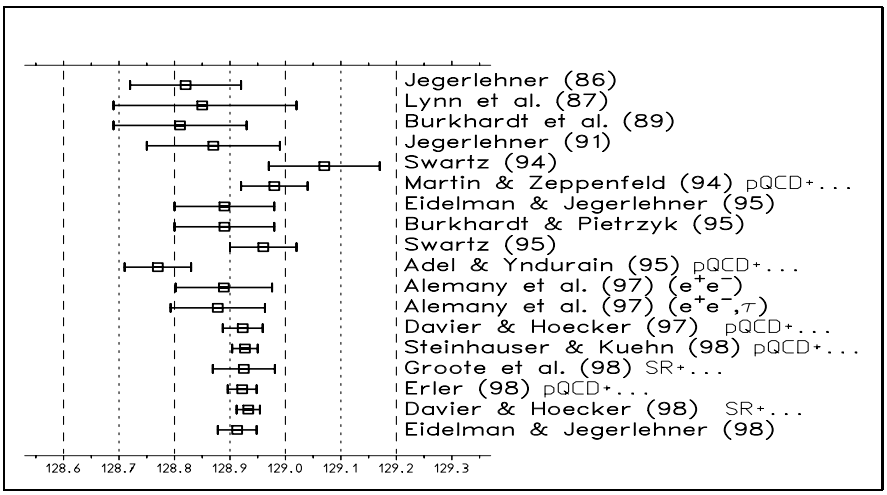

Figure 1: Evaluations of $1/\alpha(M_Z)$.

which leads to $\alpha^{-1}(M_Z) = 128.89 \pm 0.09$. It is obviously safe to use pQCD in the continuum above the ψ resonances and below the Υ resonances (from 5.0 GeV to M_Υ) and in the continuum above the Υ resonances above about 12 GeV [5]. We then obtain [4] $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02801 \pm 0.00058$. Utilizing the fact that the vector–current hadronic τ–decay spectral functions are related to the iso–vector part of the e^+e^-–annihilation cross–section via an iso-spin rotation [6,7], one can obtain information from τ–decay spectra to reduce the uncertainty of e^+e^- hadronic cross–sections at energies below the τ–mass [8,9]. The inclusion of the τ data, first performed in Ref. [10], unfortunately lead to a marginal improvement only

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02810 \pm 0.00062 \qquad \text{(Alemany, Davier, Höcker 98)}$$

The large uncertainty of about 2.3% is due to large systematic errors of the total hadronic cross–section measurements. To overcome this disturbing fact more recent estimations [11,12,13,14,15] rely more on pQCD than on data. Since hadronic τ–decays seem to be described very well by pQCD one assumes that perturbation theory describes the quantity $R_\gamma(s)$ down to $M_\tau \sim 1.8$ GeV with good accuracy as well, i.e., $E_{\rm cut} \sim 1.8$ GeV is now a popular choice. The low energy tail below 1.8 GeV, the narrow resonances (ω, ϕ, J/ψ's and Υ's) and the charm resonance region from 3.7 to 5.0 are included separately, still using experimental data. While the authors of Ref. [12] use the published charm data

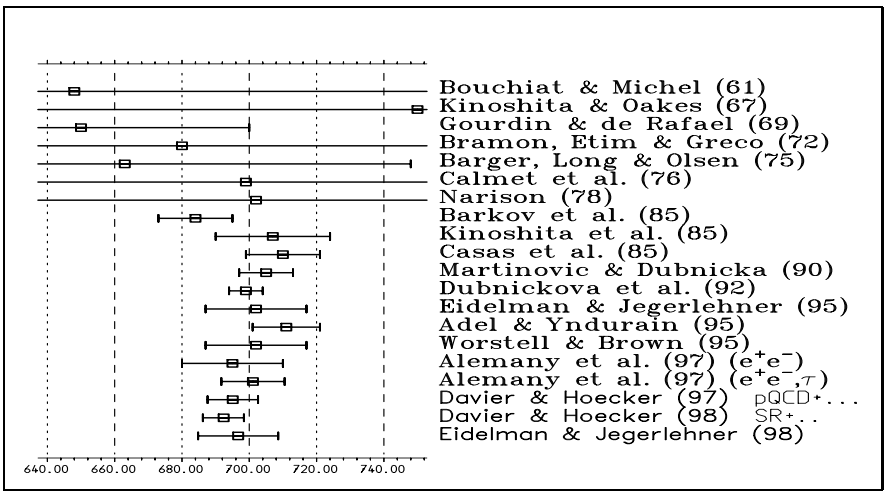

Figure 2: Different estimates of $a_\mu^{\rm had} \times 10^{10}$.

directly, obtaining [12]

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02778 \pm 0.00026 \qquad \text{(Davier, Höcker 98a)}$$

the authors of Refs. [11,13] utilize the charm data only after re–normalizing individual data sets by factors accounting for the mismatch in normalization: $< R_\gamma^{\rm data}(s)/R_\gamma^{\rm pQCD}(s) >$ averaged over small intervals just below and just above the charmonium resonance domain. The result here is [13]

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02777 \pm 0.00017 \qquad \text{(Kühn, Steinhauser 98)}$$

Another approach is based on techniques which admit suppressing contributions from data in problematic regions by a contour trick [14,15]. This yields [15]

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.02763 \pm 0.00016 \qquad \text{(Davier, Höcker 98b)}$$

and $\alpha^{-1}(M_Z) = 128.933 \pm 0.021$.

In most cases more or less obvious theoretical uncertainties typical in hadron physics are not included because they are hard to be specified. The recent reevaluations are collected together with older results in Fig. 1.

The second quantity of interest is the hadronic contribution to $a_\mu \equiv \frac{g_\mu - 2}{2}$

which is determined from the same data by the dispersion integral

$$a_\mu = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{4m_\pi^2}^{E_{\rm cut}^2} ds\, \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int\limits_{E_{\rm cut}^2}^{\infty} ds\, \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right) \qquad (7)$$

The kernel $\hat{K}(s)$ is a known smooth bounded function growing from 0.63... at threshold to 1 at ∞. Note the extra $1/s$–enhancement of contributions from low energies in Eq. (7) as compared to Eq. (5). Our updated evaluation reads

$$a_\mu^{\rm had} = (696.7 \pm 11.9) \times 10^{-10} \qquad \text{(Eidelman, Jegerlehner 98)}$$

For this observable the use of τ spectral–functions in addition to e^+e^- cross–sections leads to a dramatic reduction of the uncertainty [10]. Extended use of pQCD [12] and application of QCD sum rule techniques [15] leads to further improvements

$$a_\mu^{\rm had} = (701.1 \pm 9.4) \times 10^{-10} \qquad \text{(Alemany, Davier, Höcker 98)}$$
$$a_\mu^{\rm had} = (695.1 \pm 7.5) \times 10^{-10} \qquad \text{(Davier, Höcker 98a)}$$
$$a_\mu^{\rm had} = (692.4 \pm 6.2) \times 10^{-10} \qquad \text{(Davier, Höcker 98b)}$$

Again recent reevaluations are shown together with older results in Fig. 2. For a theory update of properties of a_μ we refer to Ref. [18].

2 Comments on the improvements

1) A substantial reduction of uncertainties of $a_\mu^{\rm had}$ is possible by using τ–decay spectra to reduce the uncertainty of e^+e^- hadronic cross–sections at energies below the τ–mass [8,9]. Conservation of the iso–vector current (CVC) allows us to relate non–strange τ–decay data to e^+e^-–annihilation data by an iso-spin rotation:

$$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+e \to X^0$$

where X^- and X^0 are related hadronic states. The e^+e^- cross–section is then given by

$$\sigma_{e^+e^- \to X^0}^{I=1} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \le M_\tau \qquad (8)$$

The τ spectral function v_1 is obtained from the normalized invariant mass-squared distribution $\rho\,(X^-) \equiv (1/N_{X^-}\, dN_{X^-}/ds)$ of the τ vector channel

$X^-\nu_\tau$ by

$$v_{1,X^-} = A\,\frac{B(\tau^- \to X^-\nu_\tau)}{B(\tau^- \to e^-\nu_\tau\bar{\nu}_e)}\,\rho\,(X^-)\left[\left(1-\frac{s}{M_\tau^2}\right)^2\left(1+\frac{2s}{M_\tau^2}\right)\right]^{-1} \qquad (9)$$

where

$$A = \frac{M_\tau^2}{6|V_{ud}|^2\,(1+\delta_{\rm EW})}$$

with $|V_{ud}| = 0.9752 \pm 0.0007$ the CKM mixing matrix element and $\delta = 0.0194$ the electroweak radiative corrections [19]. With the precision of the validity of CVC, this allows to improve the $I = 1$ part of the e^+e^- cross–section which by itself is not a directly measurable quantity. It mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ–resonance contribution) which is dominating in $a_\mu^{\rm had}$ (72%). Problems of combining data are not too serious for this particular observable/mode which is pure $I = 1$ and hence does not require to separate data into iso-spin components. The precision is then only a matter of how well the corresponding exclusive channels can be measured. With the error estimates discussed in [10] it was possible to reduced the error of a_μ considerably.

In any case the following points should be kept in mind:
(i) the usual problems of matching an averaging data has to be dealt with. For details we refer to [4,10].
(ii) In general the use of τ data requires a splitting of the e^+e^- data into the $I = 0$ and $I = 1$ parts. There is no precise method known how to do this, except for counting even and odd numbers of pions. In any case this introduces an additional systematic error which is hard to estimate in a clean manner.
(iii) The previous point as well as the CVC assumption rely on estimates of the size of iso-spin breaking effects. This problem is absent when using e^+e^- data solely. Iso-spin breaking effects originate from the difference $m_u - m_d$ in the masses of the u and d quarks and from electromagnetic interaction, i.e., an imperfect treatment of QED corrections of the various exclusive channels, electromagnetic contributions to the hadron masses and widths etc.
In a recent paper [7] Eidelman and Ivanchenko estimate the validity of CVC by a detailed comparison of τ channels,

$$\tau^- \to X^-\nu_\tau\ ,\quad X^- = \pi^-\pi^0, \pi^-3\pi^0, (3\pi)^-\pi^0, \cdots$$

with corresponding e^+e^- channels. The results are given in the Table 1.

Table 1: Branching Ratios of $\tau^- \to X^- \nu_\tau$ in percent.

Hadronic State X	World Average 1996	CVC Predictions
$\pi^-\pi^0$	25.24 ± 0.16	24.74 ± 0.79
$\pi^- 3\pi^0$	1.14 ± 0.14	1.07 ± 0.10
$(3\pi)^-\pi^0$	4.25 ± 0.09	4.36 ± 0.55
Total	30.63 ± 0.23	30.17 ± 1.00

The difference between the CVC prediction and the measured branching ratio of τ to 2-π and 4-π states is consistent with zero within the errors (-0.46 $\pm$ 1.00)% or -1.5% relatively. There is no reason **not** to take this 1.5% into account while calculating $R^{I=1}(s)$ from the τ spectra.

2) Substantial progress in pQCD calculations, which now includes quark mass effects up to three–loops [20], allows us to apply pQCD in regions where mass effects are important. Many authors now assume pQCD to be valid down to M_τ, and hence that

$$\sigma_{\rm tot}(e^+e^- \to \text{hadrons}) \sim \sigma_{\rm tot}(e^+e^- \to \text{quarks and gluons})$$

The assumption seems to be supported by

- the apparent applicability of pQCD to τ physics. In fact the running of $\alpha_s(M_\tau) \to \alpha_s(M_Z)$ from the τ mass up to LEP energies agrees well with the LEP value. The estimated uncertainty may be debated, however.
- non–perturbative (NP) effects if parametrized as prescribed by the operator product expansion (OPE) of the electromagnetic current correlator [21] are seemingly small [22,12].

3) Let us consider possible NP effects separately. A way to parametrize NP effects at sufficiently large energies and away from resonances is provided by the OPE applied to Eq. (1). Due to non–vanishing gluon and light quark condensates [21] one finds the leading power corrections

$$\Pi_\gamma^{\prime\,\rm NP}(Q^2) = \frac{4\pi\alpha}{3} \sum_{q=u,d,s} Q_q^2 N_{cq} \cdot \left[\frac{1}{12} \left(1 - \frac{11}{18} a \right) \frac{< \frac{\alpha_s}{\pi} GG >}{Q^4} \right.$$

$$+ 2 \left(1 + \frac{a}{3} + \left(\frac{11}{2} - \frac{3}{4} l_{q\mu}\right) a^2\right) \frac{< m_q \bar{q} q >}{Q^4} \qquad (10)$$
$$+ \left(\frac{4}{27} a + \left(\frac{4}{3}\zeta_3 - \frac{257}{486} - \frac{1}{3} l_{q\mu}\right) a^2\right) \sum_{q'=u,d,s} \frac{< m_{q'} \bar{q'} q' >}{Q^4} \Bigg]$$
$$+ \cdots$$

where $a \equiv \alpha_s(\mu^2)/\pi$ and $l_{q\mu} \equiv \ln(Q^2/\mu^2)$. $< \frac{\alpha_s}{\pi} GG >$ and $< m_q \bar{q} q >$ are the scale-invariantly defined condensates. The terms beyond leading order in α_s have been calculated from the results which were obtained in Refs. [23], [24] and [25]. Sum rule estimates of the condensates yield typically $< \frac{\alpha_s}{\pi} GG > \sim (0.389\ \mathrm{GeV})^4$, $< m_q \bar{q} q > \sim -(0.098\ \mathrm{GeV})^4$ for $q = u, d$ and $[< m_q \bar{q} q > \sim -(0.218\ \mathrm{GeV})^4$ for $q = s\ ]$ with rather large uncertainties (see Fig. 4 below).

Note that the expansion Eq. (10) is just a parametrization of the high energy tail of NP effects associated with the existence of non–vanishing condensates. There are other kind of NP phenomena like bound states, resonances, instantons and what else. The dilemma with Eq. (10) is that it works only for Q^2 large enough and it has been successfully applied in heavy quark physics. It fails do describe NP physics at lower Q^2, once it starts to be numerically relevant pQCD starts to fail because of the growth of the strong coupling constant (see Fig. 4 below).

4) Some improvements are based on assuming global duality and using sum rules (SR). Starting point is the OPE

$$< h|J_\mu(x)\, J_\nu(0)|h' > \simeq \sum_i C_{\mu\nu\, i}(x)\, H_i$$

where $H_i =< h|\mathcal{O}_i(0)|h' >$ is a matrix element of an operator $\mathcal{O}_i$ between hadronic states h, h', and $C_{\mu\nu\, i}(x)$ are the Wilson coefficients. The global duality assumption asserts that all non–perturbative physics resides in the H_i's and $C_{\mu\nu\, i}(x)$ is given by pQCD. Phenomenological tests infer that this works at the 10% level. Much better tests are needed to allow for precise confirmation as required for application in precision calculations.
I would like to point out that the following schemes have **no** justification:

- $\sigma_{\mathrm{had}} = \sigma_{\mathrm{pQCD}} + \sigma_{\mathrm{resonances}}$
- local duality: i.e., duality applied to energy intervals (resonance regions)

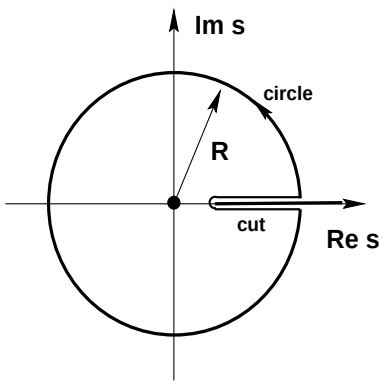

Figure 3: Integral contour.

Another warning I would like to make concerns pitfalls in the use of dispersion relations. Often one encounters arguments of the following type: consider a function, $\Delta\Pi'(s) = \Pi'(s) - \Pi'(0)$ say, which is an analytic function order by order in perturbation theory. Analyticity then infers that the contour integral along a path shown in Fig. 3 vanishes. Therefore

$$\int_{\rm cut} \frac{ds}{s} \Delta\Pi'(s) = -\int_{\rm circle} \frac{ds}{s} \Delta\Pi'(s)$$

For a large enough circle we can apply pQCD on the right hand side and thus obtain the integral of our interest, which exhibits the non-perturbative physics. What is usually forgotten is that the uncertainty is of the order of $\delta = 2\pi R\varepsilon$ with ε being the small error expected from the truncation of the perturbative series. δ easily can turn out to be large (due to R large) such that we are not able to make a safe estimate for the wanted integral. Since analyticity is true order by order in perturbation theory, we precisely reproduce the perturbative answer for the left hand side if we use perturbation theory on the right hand side. The additional use of the OPE to include the condensates, in my opinion, does not make the estimate much more convincing. While analyticity is a very powerful theoretical concept it is difficult to be applied in numerical problems, because, small perturbations in one place typically cause large variations at remoter locations.

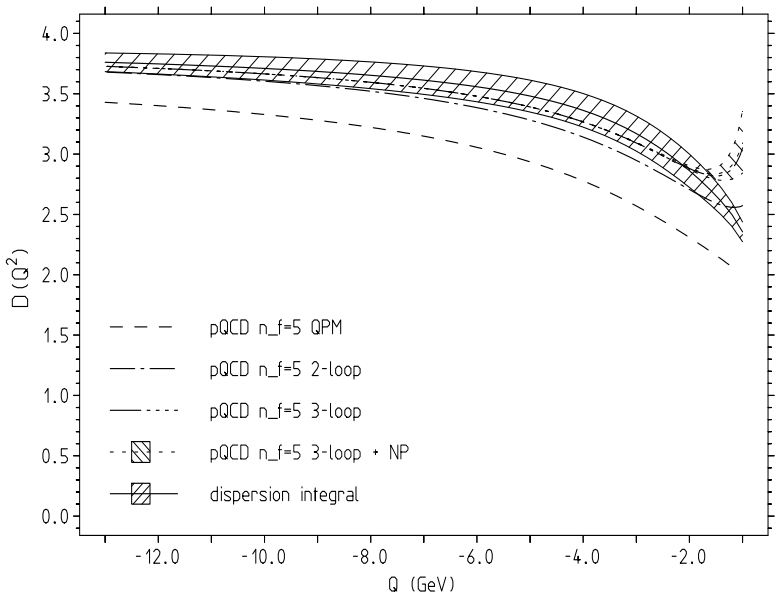

Figure 4: Adler function: data versus theory (see Ref.[28]).

3 My estimate of $\Delta\alpha^{(5)}_{\rm had}(-M_Z^2)$

Here I propose a method for estimating $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ which is free of assumptions which are not under control. In a recent paper [28] we investigated the validity of pQCD by means of the Adler function in the Euclidean region where pQCD is supposed to work best (far from resonances and thresholds). Results obtained from the evaluation of the dispersion integral are very well described by pQCD down to about 2.5 GeV. In this analysis it turned out to be crucial to take into account the exact mass dependence up to three loops. For the three–loop case this was obtained by using the series expansions presented in [20] together with Padé improvement, which from a practical point of view yields the exact behavior in the Euclidean region. Because the precise mass–dependence is important we utilize the gauge invariant background field MOM scheme calculation presented recently in Ref. [27]. The result is shown in Fig. 4. With this result at hand, we then calculate in the Euclidean region

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} \quad (11)$$

and obtain, for $s_0 = (2.5\,\mathrm{GeV})^2$ where $\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} = 0.007541 \pm 0.000254$,

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.027737 \pm 0.000254 \;\text{ and }\; \alpha^{-1}(-M_Z) = 128.917 \pm 0.035$$

for the Euclidean effective fine structure constant a value I would use in phenomenological applications without hesitation. The virtues of this analysis are obvious:

- no problems with the physical threshold and resonances
- pQCD is used only in the Euclidean region and not below 2.5 GeV. For lower scales pQCD ceases to describe properly the functional dependence of the Adler function[28] (although the pQCD answer remains within error bands down to about 1.6 GeV).
- no manipulation of data must be applied and we need not use global or local duality. That contributions of the type Eq. (10) are negligible has been shown long time ago in Ref. [22]. More recently this was confirmed in [12]. This, however, does not proof the absence of other kind of non-perturbative effects.

Remaining problems are the following:
a) contributions to the Adler function up to three–loops all have the same sign and are substantial. Four– and higher–orders could still add up to non-negligible contribution. An error for missing higher order terms is not included. The scheme dependence $\overline{\rm MS}$ versus background field MOM has been discussed in Ref. [27].
b) the link between space–like and time-like region is the difference

$$\Delta = \Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.000045 \pm 0.000002\,,$$

which can be calculated in pQCD. It accounts for the $i\pi$–terms from the logs $\ln(-q^2/\mu^2) = \ln(|q^2/\mu^2|) + i\pi$. One may ask the question whether these terms should be resummed at all, i.e., included in the running coupling. Usually such terms tend to cancel against constant rational terms which are not included in the renormalization group (RG) evolution. It is worthwhile to stress here that the running coupling is **not** a true function of q^2 (or even an analytic function of q^2) but a function of the RG scale μ^2. The coupling as it appears in the Lagrangian in any case must be a constant, albeit a μ^2–dependent one, if we do not want to end up in conflict with basic principles of quantum field theory. The effective identification of μ^2 with a particular value of q^2 must be

understood as a subtraction (reference) point.
The above result was obtained using the gauge invariant background–field MOM renormalization scheme, presented recently in Ref. [27]. In the transition from the $\overline{\rm MS}$ to the MOM scheme we adapt the rescaling procedure described in [27], such that for large μ

$$\overline{\alpha}_s((x_0\mu)^2) = \alpha_s(\mu^2) + 0 + O(\alpha_s^3)\ . \tag{12}$$

This means that x_0 is chosen such that the couplings coincide to leading and next–to–leading order at asymptotically large scales. Numerically we find $x_0 \simeq 2.0144$. Due to this normalization by rescaling the coefficients of the Adler–function remain the same in both schemes up to three–loops. In the MOM scheme we automatically have the correct mass dependence of full QCD, i.e., we have automatic decoupling and do not need decoupling by hand and matching conditions like in the $\overline{\rm MS}$ scheme. For the numerical evaluation we use the pole quark masses [29] $m_c = 1.55$GeV, $m_b = 4.70$GeV, $m_t = 173.80$GeV and the strong interaction coupling $\alpha_{s\,\overline{\rm MS}}^{(5)}(M_Z) = 0.120 \pm 0.003$. For further details we refer to [28].

4 Improvements expected from Daphne

For the rather dramatic progress on the theory–driven reduction of uncertainties of $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ and a_μ, discussed in Sec. 1, little compelling theoretical or experimental justification is available to remove doubts concerning the existence of further unaccounted theoretical uncertainties. Therefore it is important to pursue the experimental program to reduce uncertainties of $R_\gamma(s)$ measurements. New low energy data are expected from BES, CMD II and DAΦNE in the near future. On a time scale of a few years the Φ–factory Daphne is the most promising facility. Therefore one of the challenges for Daphne is presented by the measurement of the total e^+e^-–hadronic cross–section at $\sqrt{s} \lesssim 2$ GeV, which provides the information on the hadronic contributions to $(g-2)_\mu$ and to a lesser extent to $\alpha(M_Z)$. Considerations from precision physics suggest that the goal should be to measure the cross-section at the level of a few permille. This could allow a reduction of the error on a_μ such as to be closer to the contribution from virtual light-by-light scattering (see e.g. [18]). To extract the total cross-section a certain amount of modeling is necessary, since, in contrast to LEP, at low energies the theoretical shape of the total cross-section cannot be calculated from first principles. Important cross-checks will be obtained by studying connections between form-factor measurements at Daphne and τ-lepton decays (CLEO, ALEPH, OPAL, DELPHI, BABAR) through iso-spin

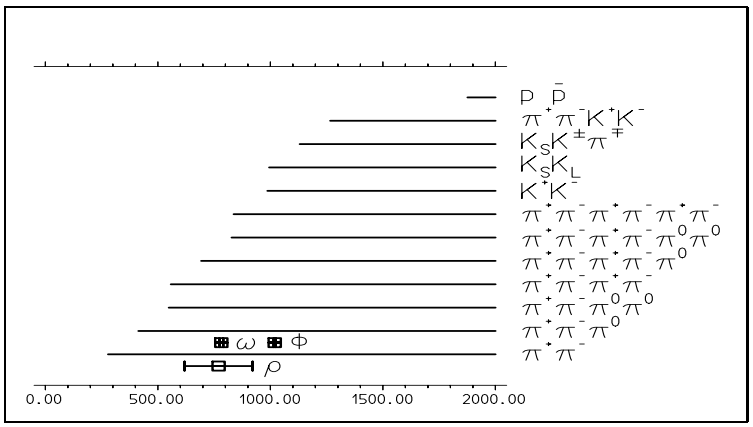

Figure 5: Multi–particle channels in e^+e^-–annihilation at low energy.

relations between multi-pion final states, which we mentioned before.

There are two stages: at the Φ–resonance one can measure the pion form factor $|F_\pi(s)|$ for $\sqrt{s} \lesssim 1$ GeV via hard photon tagging

$$\Phi \to \gamma^* + \gamma(\text{real, hard}) \to \pi^+\pi^- + \gamma(\text{real, hard})$$

At a second stage, after a non–trivial Daphne upgrade, one expects to be able to perform an energy scan from threshold up to about 2 GeV, which allows for a "direct" measurement of $R_\gamma(s)$. Unfortunately the measurement of this inclusive quantity at low energies is not as easy as at LEP/SLD. Because of competing 2 body channels e^+e^-, $\mu^+\mu^-$, $\pi^+\pi^-$, K^+K^-, $\cdots$, which must be unambiguously separated, a good particle identification is required. This means that the inclusive cross–section can be obtained only by summing up the exclusive channels. This also requires a detailed theoretical understanding of the individual channels at low energies ($\lesssim 2$ GeV). In Fig. 5 we show the spectrum of channels which contribute the total hadronic cross–section. The contributions from the exclusive channels to $a_\mu^{\rm had}$ are listed in Table 2. In order to reduce the uncertainty from the region below 2 GeV to the 0.3% level the accuracy required for the individual channels is given in the last column.

Table 2: Contributions to $\tilde{a}_\mu^{\rm had} = a_\mu^{\rm had} \times 10^{10}$ from exclusive channels.

channel	$\tilde{a}_\mu^{\rm had}$	acc.	channel	$\tilde{a}_\mu^{\rm had}$	acc.
$\rho, \omega \to \pi^+\pi^-$	506	0.3%	3π	4	10%
$\omega \to 3\pi$	47	$\sim 1\%$	K^+K^-	4	$\downarrow$
ϕ	40	$\downarrow$	$K_S K_L$	1	·
$\pi^+\pi^-\pi^0\pi^0$	24	·	$\pi^+\pi^-\pi^+\pi^-\pi^0$	1.8	·
$\pi^+\pi^-\pi^+\pi^-$	14	·	$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	0.5	·
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	5	10%	$p\bar{p}$	0.2	·
2 GeV $\leq E \leq M_{J/\psi}$	22				
$M_{J/\psi} \leq E \leq M_\Upsilon$	20				
$M_\Upsilon < E$	$\lesssim 5$				

Theoretical work with the aim to calculate radiative corrections at the level of precision as indicated in Table 2 is in progress.

5 Summary and conclusions

The future of precision physics depends to a large extent on the uncertainties of hadronic vacuum polarization effects. Present and future are different for $a_\mu^{\rm had}$ and $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$:

$\underline{a_\mu^{\rm had}}$: uncertainty from $\pi^+\pi^-, \rho^0$ dominates

Table 3: Uncertainties of $a_\mu^{\rm had}$ in units 10^{-11}.

δa_μ	input	δa_μ	input
~ 156	e^+e^- data [4,10]	~ 102	e^+e^-, τ data [10]
$\sim$ 60	Daphne + additional τ data	$\sim$ 40	BNL a_μ–experiment [18]
$\sim$ 62	theory-driven guess [15]		

$\underline{\Delta\alpha^{(5)}_{\rm had}(M_Z^2)}$: uncertainty distributed everywhere below Υ

Uncertainties of e^+e^- data range from 5% to 20% at present which yields $\delta\Delta\alpha \sim 0.00065$ (data) [4,10]. Suppose we could improve this to a 1% measure-

ment in systematics and with high enough statistics, this would lead us to $\delta\Delta\alpha \sim 0.00028$ (data), which has to be confronted with the theory driven estimate $\delta\Delta\alpha \sim 0.00016$ (theo) [13,15]. There is little hope that this precision can ever be reached by an experiment.
Alternative strategies:

- try to determine $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ by a direct measurement
- simplest: find the Higgs and determine m_H precisely; this would allow us to determine $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ from the precise value of $\sin^2\Theta^\ell_{\rm eff}$ measured at LEP/SLD.
- For what concerns theory, which is essentially pQCD, we need better estimates of uncertainties (scheme dependence, higher orders etc. which are a problem at M_τ), extended studies in MOM schemes [27], analytic extensions of the $\overline{\rm MS}$ scheme [30] etc. are needed.

In the meantime I propose to use in phenomenological applications an estimate of the kind I presented in Sec. 3.

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