HU-EP-06/24, DESY 06-134
SFB/CPP-06-38
hep-ph/0608329
Nucl. Phys . Proc. Suppl. 162 (2006) 22
August 2006

Precision measurements of $\sigma_{\rm hadronic}$ for $\alpha_{\rm eff}(E)$ at ILC energies and $(g-2)_\mu$

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*Work supported by DFG Sonderforschungsbereich Transregio 9-03 and in part by the European Community's Human Potential Program under contract HPRN-CT-2002-00311 EURIDICE and the TARI Program under contract RII3-CT-2004-506078.

Precision measurements of $\sigma_{\mathrm{hadronic}}$ for $\alpha_{\mathrm{eff}}(E)$ at ILC energies and $(g-2)_\mu$

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A more precise determination of the effective fine structure constant $\alpha_{\mathrm{eff}}(E)$ is mandatory for confronting data from future precision experiments with precise SM predictions. Higher precision would help a lot in monitoring new physics by increasing the significance of any deviation from theory. At a future e^+e^-–collider like the ILC, as at LEP before, $\alpha_{\mathrm{eff}}(E)$ plays the role the static zero momentum $\alpha = \alpha_{\mathrm{eff}}(0)$ plays in low energy physics. However, by going to the effective version of α one loses about a factor 2×10^2 at $E = m_\mu$ to 10^5 at $E = M_Z$ in precision, such that for physics at the gauge boson mass scale and beyond $\alpha_{\mathrm{eff}}(E)$ is the least known basic parameter, about a factor 20 less precise than the neutral gauge boson mass M_Z and by about a factor 60 less precise than the Fermi constant G_F. Examples of precision limitations are $\alpha_{\mathrm{eff}}(m_\mu)$ which limits the theoretical precision of the muon anomalous magnetic moment a_μ and $\alpha_{\mathrm{eff}}(M_Z)$ which limits the accuracy of the prediction of the weak mixing parameter $\sin^2\Theta_f$ and indirectly the upper bound on the Higgs mass m_H. An optimal exploitation of a future linear collider for precision physics requires an improvement of the precision of $\alpha_{\mathrm{eff}}(E)$ by something like a factor ten. We discuss a strategy which should be able to reach this goal by appropriate efforts in performing dedicated measurements of $\sigma_{\mathrm{hadronic}}$ in a wide energy range as well as efforts in theory and in particular improving the precision of the QCD parameters α_s, m_c and m_b by lattice QCD and/or more precise determinations of them by experiments and perturbative QCD efforts. Projects at VEPP-2000 (Novosibirsk) and DANAE/KLOE-2 (Frascati) are particularly important for improving on $\alpha_{\mathrm{eff}}(M_Z)$ as well as $\alpha_{\mathrm{eff}}(m_\mu)$. Using the Adler function as a monitor, one observes that we may obtain the hadronic shift $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ as a sum $\Delta\alpha^{(5)}_{\mathrm{had}}(-s_0)^{\mathrm{data}} + \Delta\alpha^{(5)}_{\mathrm{had}}(s_0, M_Z^2)^{\mathrm{pQCD}}$ where the first term includes the full non-perturbative part with the choice $s_0 = (2.5\ \mathrm{GeV})^2$ or larger. In such a determination low-energy machines play a particularly important role in the improvement program. We present an up-to-date analysis including the recent data from KLOE, SND, CMD-2 and BABAR. The analysis based on e^+e^-–data yields $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027593 \pm 0.000169$ [$\alpha^{-1}(M_Z^2) = 128.938 \pm 0.023$] (splitting with $s_0 = (10\ \mathrm{GeV})^2$ to reduce dependence on m_c), $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.027607 \pm 0.000225$ [$\alpha^{-1}(M_Z^2) = 128.947 \pm 0.035$] (standard approach), and $a_\mu^{\mathrm{had}} = (692.1 \pm 5.6) \times 10^{-10}$. The continuation of $\alpha_{\mathrm{eff}}(E)$ from the Z mass scale to ILC energies may be obtained by means of perturbative QCD. We emphasize the very high improvement potential of the VEPP-2000 and DANAE/KLOE-2 projects.

1. INTRODUCTION

Of all non-perturbative hadronic effects entering the predictions of a host of electroweak precision observables, the main effect enters via the effective fine-structure "constant" $\alpha(E)$ [1,2]. It is the non-perturbative hadronic contribution to charge screening by vacuum polarization (VP) which limits its precision. The value of $\alpha(E)$ is of interest for a wide range of energies, well known examples are $\alpha(M_Z)$ or $\alpha(m_\mu)$ where the latter accounts for the leading hadronic contribution in the muon anomaly $a_\mu \equiv (g-2)_\mu/2$ (see [3–10] for some recent e^+e^-–data based evaluations). While electroweak effects like lepton contributions are calculable in perturbation theory, for the strong interaction part (hadron- and/or quark-contributions) perturbation theory fails and a dispersion integral over e^+e^-–data encoded in

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \mathrm{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

provides a reliable approach to estimate the non-perturbative effects. Errors of data directly enter the theoretical uncertainties of any prediction

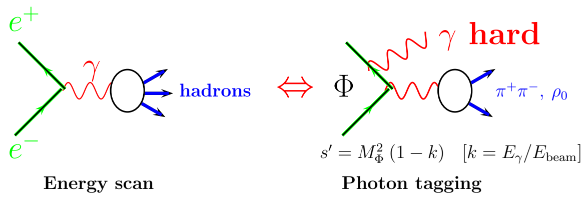

Figure 1. The two schemes of measuring $\sigma(e^+e^- \to \text{hadrons})$.

depending on $\alpha(E)$ at non-zero E. Evaluations of the above-mentioned dispersion relation have developed into an art of getting precise results still often from old cross–section measurements of poor precision. What is needed is a reduction of the present error by an order of magnitude within the next ten years. This is a new challenge for precision experiments on $\sigma(e^+e^- \to \text{hadrons})$ such as ongoing experiments KLOE, BABAR, and Belle which measure $\sigma_{\rm hadronic}$ via radiative return or photon tagging (see Fig.1) [11–13]. For future precision experiments, in many cases, we will need to know the running $\alpha_{\rm em}$ very precisely, desirably at the per mill level. Note that corrections are large and steeply increasing at low E, as may be learned from Fig.2.

An immediate question might be: why not measure $\alpha_{\rm eff}(E)$ directly, like the QCD running coupling $\alpha_s(s)$? The problem is that any measurement requires a normalizing process like Bhabha (Fig.3) which itself depends on $\alpha_{\rm eff}(t)$ and $\alpha_{\rm eff}(s)$. In fact one is always measuring something like

$$r(E) \propto \left(\alpha_{\rm eff}(s)/\alpha_{\rm eff}(t)\right)^2 \ , \tag{1}$$

with $t = -\frac{1}{2}\,(s-4m_e^2)\,(1-\cos\theta)$. Unless one is able to measure essentially at zero momentum transfer (~ 0 angle), because of the steep rise of $\alpha_{\rm eff}(E)$ at low energies a large fraction of the effect which we would like to determine drops out, especially the strongly rising low-energy piece, which includes substantial non-perturbative effects (see also [14]).

It should be noted that not only $\alpha_{\rm eff}(E)$ at high energies is of interest. As a logarithmically increasing function the problems show up at relatively low scales like for $\alpha_{\rm eff}(m_\mu)$, which determines the leading uncertainty of the muon anomaly a_μ (see Fig. 4). Another not so well known example is $\alpha_{\rm eff}(M_{\rm proton})$ at the proton mass scale which enters β–decay and affects the determination of the CKM element V_{ud} [15,16].

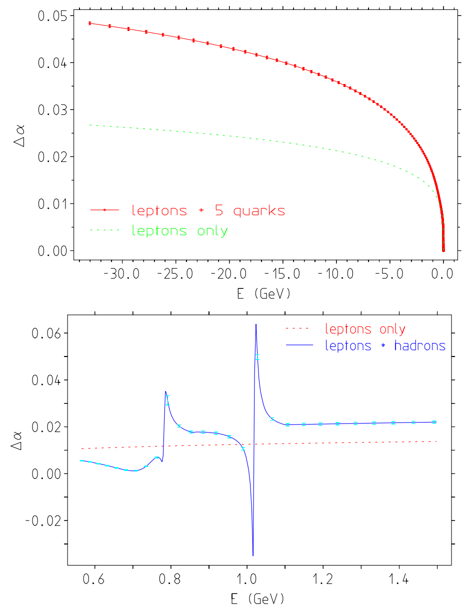

Figure 2. The running of $\alpha(E) = \alpha(0)/(1-\Delta\alpha)$ in terms of $\Delta\alpha$. The "negative" E axis is chosen to indicate space-like momentum transfer. The vertical bars at selected points indicate the uncertainty. In the time-like region the resonances lead to pronounced variations of the effective charge (shown in the $\rho-\omega$ and ϕ region).

In spite of the fact that the discussion presented here is not really new, the substantial amount of new data in the low energy region from KLOE, SND and CMD-2 and from BABAR leads to a noticeable change of the error profile of $\alpha_{\rm eff}(E)$ at different scales and it provides a strong motivation for the upcoming VEPP-2000 [17] and future DANAE/KLOE-2 [18] projects to continue precise measurements of $\sigma_{\rm hadronic}$ in the region from threshold up to 2.5 GeV.

2. $\alpha(M_Z)$ IN PRECISION PHYSICS

At higher energies for all processes which are not dominated by a single one photon exchange, $\alpha_{\rm eff}(E)$ enters in a complicated way in observables and cannot be measured in any direct way. In the SM many parameters are interrelated as im-

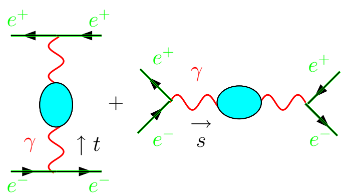

Figure 3. Running α in Bhabha scattering as a normalization process. The "blobs" are VP insertions into the tree level diagrams.

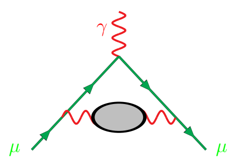

Figure 4. Leading hadronic contribution to $g_\mu - 2$

plied by the mass generating Higgs mechanism. A nice example are parameters showing up in typical four fermion and vector boson processes. Unlike in QED and QCD the SM is a spontaneously broken gauge theory which leads to a new kind of parameter interdependencies, which have been tested at LEP with high accuracy. Besides the fermion masses and mixing parameters the SM has only 3 independent parameters the gauge couplings: g and g' and the Higgs vacuum expectation value v, which may be fixed from the most precisely measured quantities, namely, α, G_μ and M_Z. All other dependent parameters are then predictions which can be tested and provide a monitor for new physics.

The impact of the hadronic uncertainties of $\alpha_{\rm eff}(M_Z)$ in physics of the heavy gauge bosons M_W and M_Z are well known and for a more detailed discussion I refer to my earlier articles [1]. As mentioned before, in place of α, which is known to a precision $\frac{\delta\alpha}{\alpha} \sim 3.6 \times 10^{-9}$, the effective $\alpha_{\rm eff}(M_Z)$ is needed and non-perturbative hadronic effects reduce its precision to $\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} \sim 1.6 \div 6.8 \times 10^{-4}$, while the other two basic parameters are known with much better accuracy $\frac{\delta G_\mu}{G_\mu} \sim 8.6 \times 10^{-6}$ and $\frac{\delta M_Z}{M_Z} \sim 2.4 \times 10^{-5}$. The uncertainty in $\alpha_{\rm eff}(M_Z)$ carries over to the W mass M_W and to the weak mixing parameter $\sin^2\Theta_f$ as

$$\begin{aligned}
\frac{\delta M_W}{M_W} &\sim \frac{1}{2}\frac{\sin^2\Theta_W}{\cos^2\Theta_W - \sin^2\Theta_W}\,\delta\Delta\alpha \sim 0.23\;\delta\Delta\alpha \\
\frac{\delta \sin^2\Theta_f}{\sin^2\Theta_f} &\sim \frac{\cos^2\Theta_f}{\cos^2\Theta_f - \sin^2\Theta_f}\,\delta\Delta\alpha \sim 1.54\;\delta\Delta\alpha
\end{aligned}$$

and to all kinds of observables which depend on these parameters. In particular the indirect bounds on the Higgs mass obtained from electroweak precision measurements are weakened thereby.

3. EVALUATION OF $\alpha(M_Z)$

The non-perturbative hadronic shift $\Delta\alpha^{(5)}_{\rm had}(s)$, due to the 5 light quark flavors, can be evaluated in terms of $\sigma(e^+e^- \to \text{hadrons})$ data via the well known dispersion integral (see [3] and references therein):

$$\begin{aligned}
\Delta\alpha^{(5)}_{\rm had}(s) &= -\frac{\alpha s}{3\pi}\left(\;\;\mathcal{P}\!\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R_\gamma^{\rm data}(s')}{s'(s'-s)} \right. \\
&\quad \left. + \;\mathcal{P}\!\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R_\gamma^{\rm pQCD}(s')}{s'(s'-s)} \right) \qquad (2)
\end{aligned}$$

where

$$R_\gamma(s) \equiv \frac{\sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})}{\frac{4\pi\alpha^2}{3s}}\;. \qquad (3)$$

A compilation of the data is shown in Fig. 5.
For the evaluation at $M_Z = 91.1876$ GeV we take

- $R(s)$ data up to $\sqrt{s} = E_{cut} = 5.2$ GeV and for Υ resonances region between 9.6 and 13 GeV,
- perturbative QCD [19,20] from 5.2 to 9.6 GeV and for the high energy tail above 13 GeV, as recommended in [21]. As a result we obtain

$$\begin{aligned}
\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) &= 0.027607 \pm 0.000225 \\
\alpha^{-1}(M_Z^2) &= 128.947 \pm 0.035\,. \qquad (4)
\end{aligned}$$

Contributions from various energy regions and the origin of the errors are shown in Fig. 6.

As we will explain below $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$, or more generally $\alpha_{\rm eff}(E)$ for $E > 2.5$ GeV, may be evaluated in a different way, by exploiting pQCD as much as possible in a well controlled fashion via

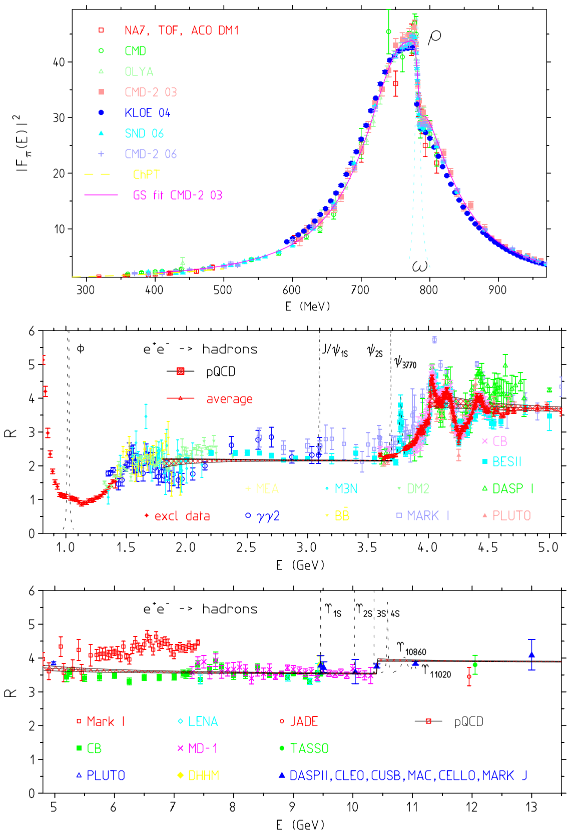

Figure 5. A compilation of the presently available experimental hadronic e^+e^-–annihilation data

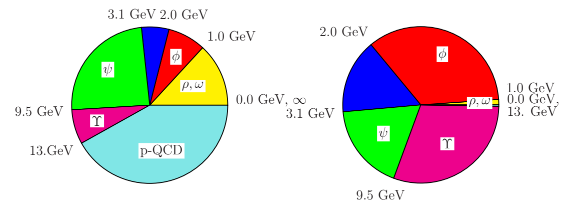

Figure 6. $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$: contributions (left) and errors^2 (right) from different regions

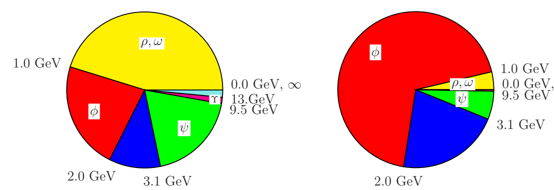

Figure 7. $\Delta\alpha^{(5)}_{\rm had}(-s_0)$: contributions (left) and errors^2 (right) from different regions

the Adler function. In this Adler function approach the complete non-perturbative part may be evaluated at $\sqrt{s_0} = 2.5$ GeV where

$$\Delta\alpha^{(5)}_{\rm had}(-s_0) \quad = \quad 0.007364 \pm 0.000101\,.$$

and $\Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)$ is reliably calculable using pQCD. The profile of $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is shown in Fig. 7; for more details we refer to Table 2 below.

4. A LOOK AT THE e^+e^-–DATA

In order to learn where results have to be improved we briefly have a closer look at the existing $e^+e^- \to$ hadrons cross-section data. Since our analysis [3] in 1995 data from MD1 [22], BES-II [23] and from CMD-2 [24] have lead to a substantial reduction in the hadronic uncertainties on $\Delta\alpha^{\rm had}$ and $a_\mu^{\rm had}$. More recently KLOE [25], SND [26] and CMD-2 [27] published new measurements in the region below 1.4 GeV. Unfortunately the agreement between the different experiments is not very satisfactory. Nevertheless, the progress is substantial, the low energy domain is no longer essentially dominated by one experiment (CMD-2 2003) and the errors of the combined data are reduced noticeably and existing problems can be settled by ongoing experiments. In the past few years also more data on the purely neutral channels $\pi^0\gamma, \pi^0\pi^0\gamma$, $\eta\gamma$, $\pi^0\eta\gamma$, $\eta'\gamma$, $\omega\pi^0$ mainly from SND [28] and CMD-2 [29] have been included. Substantial improvement on the ω and ϕ resonances, on 4π and 5π channels and a study of $\eta\pi^+\pi^-$, $\omega\pi^+\pi^-$ [30] contributed to the improvement. An important conclusion from the CMD-2 study [29] is that the ω- and ϕ-resonance decays completely saturate the cross sections of these channels. Consequently, there are no unaccounted contributions to the R value from such neutral channels (see [31] for a more detailed expert summary). Further data included 3π, 4π, KK and pp channels and come from SND, CMD-2, BES-II and CLEO [32–37]

As in other more recent evaluations, modes not measured directly have been included by estimating them using isospin relations and known branching fractions of decay modes. Included are (see also [5,6]) **(1)** $\pi^+\pi^-3\pi^0 \sim [2(\pi^+\pi^-)\,\pi^0 - \eta\pi^+\pi^-)]/2 + \eta\pi^+\pi^- \cdot \mathrm{BR}(\eta \to 2\pi^0)$, **(2)** $\eta\pi\pi$ not included already in the 5π modes, **(3)** $\omega(\omega \to \pi^0\gamma)\pi\pi$ including the $\omega\pi^0\pi^0$ mode via isospin $(= (\omega\pi^+\pi^- + \omega\pi^0\pi^0[= 1/2\omega\pi^+\pi^-]) \cdot \mathrm{BR}(\omega \to \pi^0\gamma)$, **(4)** unseen $KK\pi\pi$ modes using $e^+e^- \to K^0_SX$ data of DM1, which amounts to taking 2 times the total for the modes $K^0\bar{K}^0(\pi\pi)^0$ and $K^+K^-(\pi\pi)^0$ $(=2(K^0_SX - K^0_S\bar{K}^0_L - K^0_S\bar{K}^\pm\pi^\mp - K^+K^-\pi^0))$, $K^+K^-\pi^+\pi^-$ measured by DM1 and BABAR is evaluated from the data and hence is to be subtracted from the total $KK\pi\pi$ contribution, **(5)** missing isospin modes in 6π: $\pi^+\pi^-4\pi^0$ $(\simeq 0.093 \cdot 3(\pi^+\pi^-) + 0.031 \cdot 2(\pi^+\pi^-)\,2\pi^0)$, **(6)** $P\gamma$ $(P = \pi^0, \eta)$ (note $\pi^0\pi^0\gamma$ is a version of $\omega\pi^0$ already included), **(7)** $K^+K^-\pi^0$ (was missing in non-DM2 R compilations) also accounting for $K^0_LK^0_S\pi^0[= K^+K^-\pi^0]$. Channel measured for the first time include $K^+K^-\,2(\pi^+\pi^-)$ and $2(K^+K^-)$ (BABAR). For a useful review on the e^+e^- data we refer to [38] (up to 2003).

At higher energies data are particularly problematic in the region between 1.4 and 2.5 GeV. Fortunately, a new set of measurements is available now from radiative return experiments at BABAR [39] for the exclusive channels $e^+e^- \to \pi^+\pi^-\pi^0$, $\pi^+\pi^-\pi^+\pi^-$, $K^+K^-\pi^+\pi^-$, $2(K^+K^-)$, $3\ (\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$ and $K^+K^-2(\pi^+\pi^-)$. These data cover a much broader energy interval and extend to much higher energies than previous experiments. The compilation of all data in this range is shown in Fig. 9. Some of the early experiments measured exclusive processes channel by channel, up to 2.5 GeV including about 23 channels (see Fig.8) and others performed inclusive measurements for $R(n > 2)$ to which the two–body channels have to be added. The latter drops to a small contribution above 1.4 GeV. This is in favor of an inclusive strategy and helps to separate leptonic two prong events from the hadronic channels dominated by $n > 2$ events. In view of the many channels an inclusive measurement seems to be more feasible at a precision of about 1% which should be attempted in the improvement program. Table 1 gives a more detailed picture about the relevance of the various modes for the contribution to the range $2M_K < E < 2$ GeV.

In our analysis we are working throughout with renormalized bare cross sections (3) [3]. For older data the missing renormalizations are performed accordingly. Not performing these a posteriori radiative corrections would lead to a value for a_μ^{had} which is about 1 σ higher. Final state photon radiation for the low energy dominating $\pi^+\pi^-$ channel is included as given by scalar QED. In fact this procedure is less model dependent than it looks like in first place. The reason is that experiments have subtracted radiative events using the same model, while in fact at least the virtual hard photon effects are included in the measured cross sections, as virtual effects cannot be eliminated by cuts [41].

An intresting reconsideration of data in the J/ψ resonance region [42] demonstrates very good agreement between BES [23] and much older Crystal Ball [43] results and in fact taking into account only the two data sets leads to a reduction of the uncertainty in this region.

Table 1
Contributions to a_μ^{had} and $\Delta\alpha^{(5)}_{\mathrm{had}}(-s_0)$ from the energy region $2M_K < E < 2$ GeV. $X^* = X(\to \pi^0\gamma)$, $_{iso}$=evaluated using iospin relations.

channel X	a_μ^X	%	$\Delta\alpha^X$	%
$\pi^0\gamma$	0.04	0.04	0.00	0.03
$\pi^+\pi^-$	11.99	11.66	1.59	9.64
$\pi^+\pi^-\pi^0$	9.22	8.98	1.25	7.55
$\eta\gamma$	0.45	0.44	0.05	0.30
$\pi^+\pi^-2\pi^0$	19.27	18.75	3.79	22.93
$2\pi^+2\pi^-$	13.99	13.62	2.80	16.92
$\pi^+\pi^-3\pi^0$ $_{iso}$	1.17	1.14	0.26	1.56
$2\pi^+2\pi^-\pi^0$	1.94	1.88	0.43	2.60
$\pi^+\pi^-4\pi^0$ $_{iso}$	0.08	0.08	0.02	0.12
$\eta^*\pi^+\pi^-$	0.26	0.25	0.05	0.31
$2\pi^+2\pi^-2\pi^0$	1.70	1.65	0.42	2.54
$3\pi^+3\pi^-$	0.32	0.31	0.08	0.49
$\omega^*\pi^0$	0.77	0.75	0.13	0.78
K^+K^-	21.99	21.39	2.64	15.94
$K^0_SK^0_L$	13.17	12.82	1.49	8.99
$\omega^*\pi^+\pi^-$	0.09	0.08	0.02	0.12
$K^+K^-\pi^0$	0.35	0.34	0.08	0.49
$K^0_SK^0_L\pi^0$ $_{iso}$	0.35	0.34	0.08	0.49
$K^0_SK^\pm\pi^\mp$	1.08	1.05	0.25	1.49
$K^0_LK^\pm\pi^\mp$ $_{iso}$	1.08	1.05	0.25	1.49
$K^+K^-\pi^+\pi^-$	1.08	1.05	0.28	1.70
$K\bar{K}\pi\pi$ $_{iso}$	2.22	2.16	0.54	3.23
$p\bar{p}$	0.07	0.07	0.02	0.12
$n\bar{n}$	0.08	0.07	0.02	0.13
$\phi \to$ missing	0.03	0.03	0.00	0.02
sum	102.78	100.00	16.54	100.00
tot [sum in %]	692.00	[14.85]	73.65	[22.46]

Table 2
Contributions for $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\times 10^4$ (direct integration method) and $\Delta\alpha^{(5)}_{\rm had}(-s_0)\times 10^4$ (non-perturbative part in the Adler function method), with relative (rel) and absolute (abs) error in percent.

Energy range	$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)\times 10^4$	rel [%]	abs [%]	$\Delta\alpha^{(5)}_{\rm had}(-s_0)\times 10^4$	rel [%]	abs [%]
ρ,ω $(E<2M_K)$	36.23 [13.1](0.24)	0.7	1.1	33.29 [45.2](0.22)	0.7 %	4.6 %
$2M_K<E<2$ GeV	21.80 [7.9](1.33)	6.1	34.9	16.44 [22.3](0.83)	5.0 %	67.7 %
2 GeV $<E<M_{J/\psi}$	15.73 [5.7](0.88)	5.6	15.4	7.91 [10.7](0.44)	5.6 %	19.3 %
$M_{J/\psi}<E<M_\Upsilon$	66.95 [24.3](0.95)	1.4	18.0	13.94 [18.9](0.29)	2.0 %	8.1 %
$M_\Upsilon<E<E_{\rm cut}$	19.69 [7.1](1.24)	6.3	30.4	0.96 [1.3](0.06)	6.2 %	0.4 %
$E_{\rm cut}<E$ pQCD	115.66 [41.9](0.11)	0.1	0.3	1.09 [1.5](0.00)	0.1 %	0.0 %
$E<E_{\rm cut}$ data	160.41 [58.1](2.24)	1.4	99.7	72.55 [98.5](1.01)	1.4 %	100.0 %
total	276.07 [100.0](2.25)	0.8	100.0	73.64 [100.0](1.01)	1.4 %	100.0 %

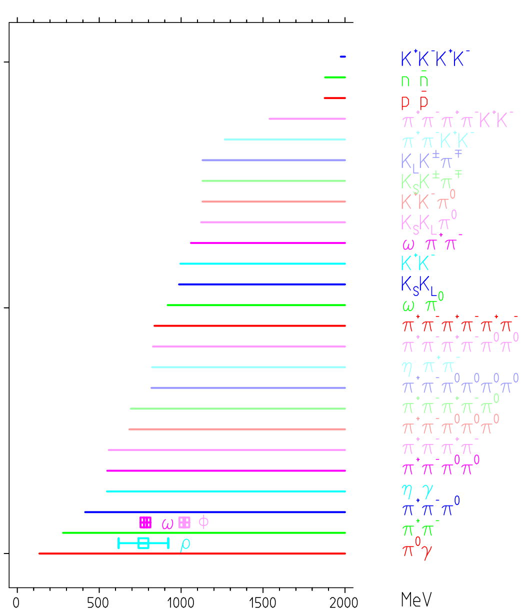

Figure 8. Thresholds for exclusive multi particle channels below 2 GeV

Substantial progress is expected in this region by the future VEPP-2000 [17] at Novosibirsk and a possible new facility at Frascati [18]. Not only the effective fine structure constant can be substantially improved, also the hadronic contribution to the muon $g-2$ may be improved (see Tab.3)

The present error profile of the various quantities of interest is compared in Fig.10.

5. $\Delta\alpha^{\rm had}$ AND THE ADLER FUNCTION

The Adler function is an ideal tool for disentangling perturbative from non–perturbative effects in the Euclidean region. It is defined by

$$D_\gamma(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

and thus for the hadronic part we may write

$$D_\gamma(Q^2) = Q^2 \int\limits_{4m_\pi^2}^{\infty} ds\, \frac{R_\gamma(s)}{\left(s+Q^2\right)^2}\,. \qquad (5)$$

While the time–like function $R_\gamma(s)$ is calculable in pQCD only by refering to quark–hadron duality, $D_\gamma(Q^2)$ is a smooth simple function both in terms of hadrons (dispersion integral over physical cross sections) and in terms of quarks (pQCD) such that the validity of pQCD can be examined directly. A comparison of the experimental vs. the pQCD Adler function Fig. 11 shows that pQCD in the Euclidean region works very well for $\sqrt{Q^2}\gtrsim 2.5$ GeV [44]. The point is that at energies above that "threshold" massive QCD works in any renormalization scheme provided mass effects are taken into account correctly[1]. The breakdown of pQCD is obviously due the fact that we are using the $\overline{\rm MS}$ running coupling $\alpha_s^{\overline{\rm MS}}(s)$ which grows

[1]Note that the curvature of $D_\gamma(Q^2)$ is almost solely due to the quark masses. In massless QCD in the $\overline{\rm MS}$ scheme $D_\gamma(Q^2)$ is essentially a constant depending on the number of flavors N_f.

Table 3
Contributions to $a_\mu^{\rm had} \times 10^{10}$ with relative (rel) and absolute (abs) error in percent.

Energy range	$a_\mu^{\rm had}$[%](error) $\times 10^{10}$	rel [%]	abs [%]
ρ, ω $(E < 2M_K)$	538.33 [77.8](3.65)	0.7 %	42.0 %
$2M_K < E < 2$ GeV	102.31 [14.8](4.07)	4.0 %	52.1 %
2 GeV $< E < M_{J/\psi}$	22.13 [3.2](1.23)	5.6 %	4.8 %
$M_{J/\psi} < E < M_\Upsilon$	26.40 [3.8](0.59)	2.2 %	1.1 %
$M_\Upsilon < E < E_{\rm cut}$	1.40 [0.2](0.09)	6.2 %	0.0 %
$E_{\rm cut} < E$ pQCD	1.53 [0.2](0.00)	0.1 %	0.0 %
$E < E_{\rm cut}$ data	690.57 [99.8](5.64)	0.8 %	100.0 %
total	692.10 [100.0](5.64)	0.8 %	100.0 %

to ∞ at $\sqrt{s} = \Lambda_{\rm QCD}^{\overline{\rm MS}}{}^2$. In any case it looks convincing to calculate [46]

$$\begin{aligned}\Delta\alpha_{\rm had}^{(5)}(M_Z^2) \;\; &= \;\; \Delta\alpha_{\rm had}^{(5)}(-s_0)^{\rm data} \qquad (6)\\ &+ \left[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-s_0)\right]^{\rm pQCD}\\ &+ \left[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)\right]^{\rm pQCD}\end{aligned}$$

where the first term only has to be calculated from the experimental data. The large perturbative second term now is rather sensitive to a precise knowledge of the QCD parameters α_s, m_c and m_b, however. For our pQCD evaluation we take the QCD parameters:

$\alpha_s(M_Z) = 0.1189 \pm 0.0010$ [47],

and quark masses (in GeV):

masses	non-lattice	lattice only
$\bar{m}_c(\bar{m}_c)$	1.24 ± 0.09 [48]	$1.30 \pm 0.03 \pm 0.20$ [49]
$\bar{m}_b(\bar{m}_b)$	4.20 ± 0.07 [48]	$4.20 \pm 0.10 \pm 0.10$ [51]

as reviewed in [50]. Lattice results now provide important cross checks of the phenomenological sum rule results which we will actually use.

In this approach (see [1] for a detailed discussion of the parameter dependences and error estimates) we obtain

$$\Delta\alpha_{\rm had}^{(5)}(-s_0)^{\rm data} \;\; = \;\; 0.007364 \pm 0.000101 \qquad (7)$$

and together with the second pQCD term we arrive at

$$\begin{aligned}\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) \;\; &= \;\; 0.027478 \pm 0.000144\\ \alpha^{-1}(-M_Z^2) \;\; &= \;\; 128.954 \pm 0.020\,.\end{aligned}$$

Finally, for the third term of (6) we obtain

$$\Delta = 0.000038 \pm 0.000002\,,$$

such that our final result reads

$$\begin{aligned}\Delta\alpha_{\rm had}^{(5)}(M_Z^2) \;\; &= \;\; 0.027547 \pm 0.000144\\ \alpha^{-1}(M_Z^2) \;\; &= \;\; 128.949 \pm 0.020\,. \qquad (8)\end{aligned}$$

We notice that this result already has a substantially lower error than the value obtained via the direct dispersion integral. Errors of the perturbative part have been taken to be 100% correlated (worst case). The sensitivity to the charm mass in particular is substantial:

parameter	range	pQCD uncertainty
α_s	0.1179 ... 0.1199	0.000017
m_c	1.15 ... 1.33	0.000078
m_b	4.13 ... 4.27	0.000008

but it can be reduced by choosing at higher scale like $\sqrt{s_0} = 10$ GeV to diminish the m_c dependence. The value obtained is given in the abstract as my best conservative estimate.

The conclusion of our analysis is that while in the time-like approach pQCD works well only

[2]One could attempt to go to an infrared finite renormalization scheme e.g. by redefining in pQCD

$$\alpha_s^{c-{\rm scheme}}(s) = \frac{\alpha_s^{\overline{\rm MS}}(s)}{1+\alpha_s^{\overline{\rm MS}}(s)/c}\,,$$

where c a constant $O(1)$. Now $\alpha_s^{c-{\rm scheme}}(s)$ goes to the finite value c (of your chioce) as $\sqrt{s} \to \Lambda_{\rm QCD}^{\overline{\rm MS}}$, which apparently would extend the validity of the pQCD result to lower energies. But this would be at the expense of a huge scheme dependence. The point is that α_s is not by itself an observable and unless one precisely specifies an observable which defines it, its meaning remains unclear. Also, this kind of regularization "by hand" introduces an infrared fixed point in the β-function which could be in conflict with confinement expectations. For a different point of view see [45] and references therein.

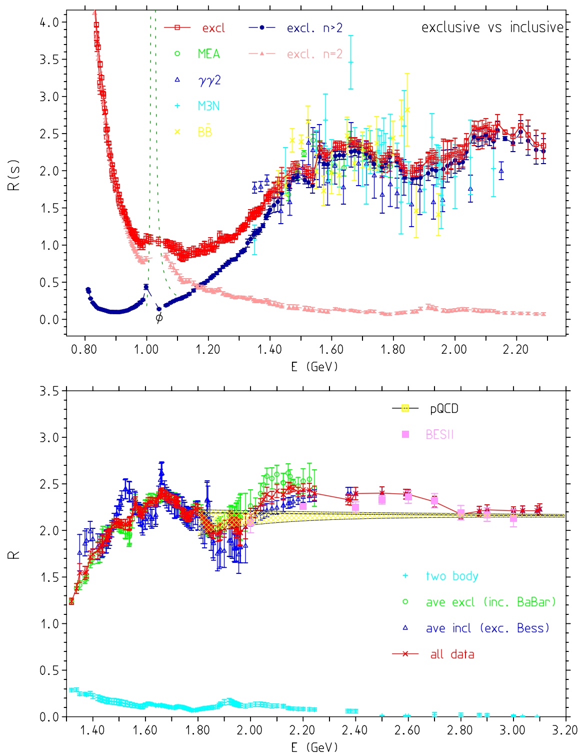

Figure 9. Status of exclusive and inclusive measurements in the most problematic region

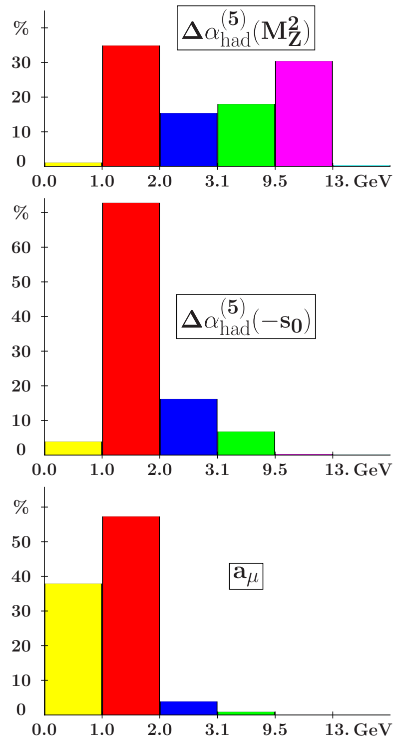

Figure 10. Comparison of error profiles between $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$, $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ and a_μ

in the "perturbative windows" $3.00-3.73$ GeV, $5.00-10.52$ GeV and $11.50-\infty$, in the space-like approach the plot of the Adler function shows that pQCD works well for all $Q^2=-q^2>2.5$ GeV.

For the future, in particular in view of ILC requirements, one should attemt to improve the determination of the effective $\alpha_{\rm em}$ by a factor 10 in accuracy. What may we expect to be realistic? We assume that dedicated cross section measurements are possible at the 1% level in the relevent energy regions. One then obtains the following estimates:

- with the direct integration of the data, and cuts applied as above, 58% of the hadronic contribution to $\alpha(M_Z)$ is obtained from data and 42% from pQCD. Given $\Delta\alpha^{(5)\,{\rm data}}_{\rm had}\times 10^4=160.41\pm 2.24$ (1.4% error) assuming a 1% overall accuracy the error would be ± 1.60. However, assuming that different independent experiments are performed at 1% accuracy for each region (divided up as in Tab. 2) and adding up errors in quadrature, one gets ± 0.83. The improvement factor from the data thus would be 2.7. The theory part $\Delta\alpha^{(5)\,{\rm pQCD}}_{\rm had}\times 10^4=115.66\pm 0.11$ already now has an 0.1% accuracy and would not be required to be improved.
- Using integration via the Adler function we have a 26% contribution from data and a 74% pQCD. The experimental part is $\Delta\alpha^{(5)\,{\rm data}}_{\rm had}\times 10^4=73.64\pm 1.01$ (1.5% accuracy) and a 1% overall accuracy would reduce the error to ± 0.74. Again, assuming that independent 1% accuracy measurements are possible for each region (divided up as in Tab. 2) and combining errors in quadrature would yield ± 0.40. Again the improvement factor is essentially as above 2.8 but 5.6 with respect to the present direct method which is usually adopted. The important point, however, is the very different error profile Fig. 10

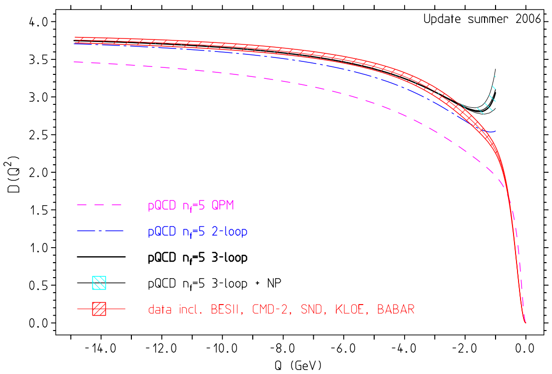

Figure 11. "Experimental" Adler-function versus theory (pQCD + NP) in the low energy region (as discussed in [44]). Note that the error includes both statistical and systematic ones, in contrast to Fig. 5 where only statistcal errors are shown.

in the two approaches. In the Adler function approach errors may be reduced to a large extent alone by low energy machines below 2.5 GeV.

- One important draw back is that the large pQCD part $\Delta\alpha_{\rm had}^{(5)\,{\rm pQCD}} \times 10^4 = 201.83 \pm 1.03$ uses pQCD down to much lower energies, and the parameter uncertainties become much more severe leading to a 0.5% accuracy "only". Here an improvement by a factor 5 would be desirable. There has been steady progress in the past and we have no doubt that much improvement will be possible in coming years.

6. CONCLUSION

The analysis presented above suggests that an improvement by a factor 5 in the error of $\alpha_{\rm eff}(M_Z)$ may be realistic. The strategy for reaching this goal:

- Adopt the Adler function approach to monitor and control non-perturbative strong interaction effects, i.e., write $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ in the form (6) and determine $\Delta\alpha_{\rm had}^{(5)}(-s_0)$ from data using the dispersion relation (2).
- The determination of $\Delta\alpha_{\rm had}^{(5)}(-s_0)$ requires a dedicated program of $\sigma(e^+e^- \to {\rm hadrons})$ cross section measurements especially at low energy machines attempting a precision of 1% in the total cross section. As much as possible the inclusive method should be pushed as it seems to be simpler and easier to reach the 1% accuracy. Detailed Monte Carlo simulation and detector studies are necessary to clarify the possibilities.
- Such experiments have to be accompanied by QED and SM calculations of processes like Bhabha scattering, μ–pair production and π–pair production at least at the level of complete two–loop QED plus resummations of leading higher order effects, and leading weak effects.
- The evaluation of the missing pieces needed to obtain e.g. $\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ may be performed using pQCD. This 74% piece must be evaluated at a precision at least at the 0.5% level. The four–loop pQCD calculation of the Adler function should be extended to include mass effects. Much more important is a substantial improvement in the precise determination of the QCD parameters α_s, m_c, and, to a lesser extent, m_b. Here lattice QCD must play a crucial role (see e.g. [52] and references therein).
- Obviously, to reach this goal requires a big effort especially on the experimental side. Of course the simple straightforward direct integration approach further will be applied and will provide an important cross check for the evaluation based on the splitting (6). However, to reach the same precision using this standard method would require much more experimental effort also at higher energies as may be concluded from Fig. 10.

In the near future progress will be possible by the radiative return experiments at KLOE, BABAR and Belle [40] up to $\sqrt{s} \sim 3$ GeV and by a new energy scan experiment at VEPP-2000 up to $\sqrt{s} = 2$ GeV. Further improvements of the accuracy of the R measurements in the range $3~{\rm GeV} < \sqrt{s} < 5$ GeV is also expected from CLEO-C[53] and from the τ-charm factory with BES-III [54] in Beijing. At Frascati a new facility DANAE/KLOE-2 planned could start data taking in 2010 and provide a further indispensable step in the improvement program on $\alpha_{\rm eff}(M_Z)$ and $a_\mu^{\rm had}$. Note that a 1% measurement in the range 1 to 2 GeV would reduce the error of (7) to $\Delta\alpha_{\rm had}^{(5)}(-s_0)^{\rm data} = 0.007364 \pm 0.000060$.

A challenging long term project for lattice QCD is the direct determination of the Adler function from first principles. First attempts were made in the past [55], however, the non-perturbative part is largely dominated by the proper inclusion of the $\pi\pi \to \rho$ resonance. The dominant low energy part is extremely sensitive to mass and width of the ρ [7], and to get them correct requires to perform simulations at the physical parameters of coupling and light quark masses.

Acknowledgments

Special thanks go to A. Denig, S. Eidelman, W. Kluge, K. Mönig, S. Müller, F. Nguyen, G. Pancheri and G. Venanzoni for numerous stimulating discussions. I am very grateful to H. Meyer, O. Tarasov and G. Venanzoni for carefully reading the manuscript. I also thank Frascati National Laboratory and the KLOE group for the kind hospitality extended to me. This work was supported in part by DFG Sonderforschungsbereich Transregio 9-03 and by the European Community's Human Potential Program under contract HPRN-CT-2002-00311 EURIDICE and the TARI Program under contract RII3-CT-2004-506078.

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Addendum: contributions for exclusive channels from threshold up to 2 GeV. Note Tab. 1 presents results from 1 to 2 GeV only.

Table 4
Contributions to $a_\mu^{\rm had}$ and $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ from the energy region $m_{\pi^0} < E < 2$ GeV. $X^* = X(\to \pi^0\gamma)$, $_{iso}$=evaluated using iospin relations.

channel X	a_μ^X	%	$\Delta\alpha^X$	%
$\pi^0\gamma$	3.32	0.48	0.24	0.33
$\pi^+\pi^-$	504.44	72.90	31.62	42.94
$\pi^+\pi^-\pi^0$	46.22	6.68	3.97	5.39
$\eta\gamma$	0.47	0.07	0.05	0.07
$\pi^+\pi^-2\pi^0$	19.59	2.83	3.83	5.20
$2\pi^+2\pi^-$	14.08	2.03	2.81	3.81
$\pi^+\pi^-3\pi^0$	0.86	0.12	0.19	0.26
$2\pi^+2\pi^-\pi^0$	1.32	0.19	0.30	0.41
$2\pi^+2\pi^-\eta$	0.11	0.02	0.03	0.04
$\pi^+\pi^-4\pi^0$	0.08	0.01	0.02	0.03
$\eta\pi^+\pi^-[*]$	0.42	0.06	0.08	0.11
$2\pi^+2\pi^-2\pi^0$	1.70	0.25	0.42	0.57
$3\pi^+3\pi^-$	0.32	0.05	0.08	0.11
$\omega\pi^0[*]$	0.81	0.12	0.13	0.18
K^+K^-	21.78	3.15	2.59	3.52
$K_S^0K_L^0$	13.17	1.90	1.49	2.02
$\omega\pi^+\pi^-[*]$	0.09	0.01	0.02	0.03
$K^+K^-\pi^0$	0.35	0.05	0.08	0.11
$[K_S^0K_L^0\pi^0]_{\rm iso}$	0.35	0.05	0.08	0.11
$K_S^0K^\pm\pi^\mp$	1.08	0.16	0.25	0.34
$[K_L^0K^\pm\pi^\mp]_{\rm iso}$	1.08	0.16	0.25	0.34
$K^+K^-\pi^+\pi^-$	1.01	0.15	0.26	0.36
$K^+K^-\pi^0\pi^0$	2.07	0.30	0.50	0.67
$[K\bar{K}\pi\pi]_{\rm iso}$	0.20	0.03	0.05	0.07
$K^+K^-\pi^+\pi^-\pi^0$	0.00	0.00	0.00	0.00
$K^+K^-\pi^+\pi^-\eta$	0.09	0.01	0.02	0.03
$K^+K^-2\pi^+2\pi^-$	0.00	0.00	0.00	0.00
$p\bar{p}$	0.07	0.01	0.02	0.03
$n\bar{n}$	0.08	0.01	0.02	0.03
$2K^+2K^-$	0.00	0.00	0.00	0.00
$\omega \to$ missing	0.09	0.01	0.01	0.01
$\phi \to$ missing	0.03	0.00	0.00	0.00
sum	635.27	91.80	49.43	67.11
tot	692.00	100.00	73.65	100.00