arXiv:hep-ph/0310361 v2 18 Nov 2003

DESY 03-180
SFB/CPP-03-50
hep-ph/0310361
October 2003

$O(\alpha\alpha_s)$ relation between pole- and $\overline{\rm MS}$-mass of the t-quark*

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The $O(\alpha\alpha_s)$ contribution to the relationship between the $\overline{\rm MS}$- and the pole-mass of the t-quark propagator within the Standard Model is reviewed. At the same order also the corrections to the top-Yukawa coupling is discussed. We furthermore present the exact analytic expression for the gaugeless limit.

PACS numbers: 12.15.Lk 14.65.Ha 12.38.Bx 11.10.Gh

1. Introduction

The Standard Model (SM) belongs to the class of *renormalizable* quantum field theories [1] which means, in particular, that a restricted number of input parameters suffice for theoretical predictions of any process. The concrete choice of input parameters defines a specific *renormalization scheme*. The given set of independent parameters has to be extracted from an appropriate set of experimentally measured quantities. If we would be able to perform perturbative calculations to all orders all renormalization schemes would be equivalent. However, in practice, only the first few coefficients are known, so that predictions depend on the choice of the scheme. Such dependence on the truncation of the perturbative series is known as *scheme dependence*. In general, the difference between two schemes is of next higher order in the perturbation expansion. For higher order calculations those schemes are preferable for which the uncalculated higher order corrections are small. Of course, to find such a preferred scheme requires to perform calculations in different schemes[2]. Another possibility is to find

* Based on the talk given by M.Yu.K. at the XXVII International Conference of Theoretical Physics, Ustroń, Poland, September 15-21, 2003. Work supported in part by DFG under Contract SFB/TR 9-03 and by the European Community's Human Potential Programme under Contract HPRN-CT-2002-00311, EURIDICE .

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the scheme transition relations by calculating the input parameters in one scheme in terms of the input parameters of another scheme order by order in perturbation theory. For electroweak calculations a natural and generally accepted scheme is the so called *on-shell scheme* [3]–[8], where, in addition to the fine structure constant (and/or the Fermi constant), the pole masses of particles serve as input parameters. However, for quarks the pole mass suffers from renormalon contributions[9] which affect seriously the convergence of the perturbation expansion. This is one of the main reasons why for quarks the $\overline{\rm MS}$-mass appears to be a better input parameter. A good illustration of this point is the behavior of the QCD corrections to the ρ-parameter, which are large when $\Delta\rho$ is expressed in terms of the pole-mass. In contrast, in terms of the $\overline{\rm MS}$-mass the expansion coefficients are much smaller[10, 11].

The relation between pole- and $\overline{\rm MS}$-mass of quarks has been calculated including one-, two- and leading three-loop corrections. The one–loop results at $O(\alpha_s)$ and $O(\alpha)$ have been presented in [12] and e.g. in [7][1], respectively. The two–loop $O(\alpha_s^2)$ correction is given in [13], and the same result was obtained via regularization by dimensional reduction in [14]. The renormalized off-shell fermion propagator of order $O(\alpha_s^2)$ has been worked out in [15]. Only recently, in [16], the three–loop $O(\alpha_s^3)$ correction has been published. Finally, the two–loop $O(\alpha\alpha_s)$ and $O(\alpha^2)$ corrections have been calculated in the approximation of vanishing electroweak gauge couplings [17]. Our recent calculation [18], extends previous two–loop $O(\alpha\alpha_s)$ calculations of the gauge boson self–energies [19] and the SM $O(\alpha^2)$ corrections to the relation between the pole- and the $\overline{\rm MS}$-mass of the gauge bosons Z and W, presented in [20, 21].

2. Definitions

The definition of the top-quark pole mass has been discussed in [22]. In general, the position of the pole of a fermion propagator, which defines the pole-mass, is given by the formal solution for the momentum $\hat{p} = i\tilde{M}$, at which the inverse of the connected full propagator equal zero

$$i\hat{p} + m - \tilde{\Sigma}(p, m, \ldots) = 0 \; . \tag{1}$$

The "mass" $\tilde{M}$ is a complex number, i.e., $\tilde{M} \equiv M' - \frac{i}{2}\,\Gamma'$. The latter parameters are related to the pole mass M and the on-shell width Γ, which are parametrizing the pole of the squared transition matrix element $|T|^2$ analogous to the boson case, by (see [18])

$$\tilde{M}^2 = M^2 - iM\Gamma = M'^2 - \Gamma'^2/4 - iM'\Gamma' \; , \tag{2}$$

[1] (see also Eq. (B.5) in Appendix B of [21])

such that

$$M = \sqrt{M'^2 - \Gamma'^2/4}\,, \quad \Gamma = \frac{M'}{M}\Gamma'\,. \tag{3}$$

Since $M = M' + O(\alpha^2)$ and $\Gamma = \Gamma' + O(\alpha^2)$ for the $O(\alpha\alpha_s)$ terms considered in this paper we can identify $M = M'$ and $\Gamma = \Gamma'$ in the following.

For the remainder of the paper we will adopt the following notation: capital $M \simeq \mathrm{Re}\,\tilde{M}$ always denotes the pole mass; lower case m stands for the renormalized mass in the $\overline{\mathrm{MS}}$ scheme, while m_0 denotes the bare mass. The on–shell width is given by $\Gamma \simeq -2\,\mathrm{Im}\,\tilde{M}$. In addition we use e, g and g_s to denote the $U(1)_{\mathrm{em}}$, $SU(2)_{\mathrm{L}}$ and $SU(3)_{\mathrm{c}}$ couplings of the SM in the $\overline{\mathrm{MS}}$ scheme.

In perturbation theory (1) is to be solved order by order. For this aim we expand the self–energy function about the lowest order solution $\hat{p} = im_0$:

$$\tilde{\Sigma}(p, m, \ldots) = \tilde{\Sigma}\,|_{\hat{p}=im_0} + (i\hat{p}+m_0)\,\left[\tilde{\Sigma}'\right]\Big|_{\hat{p}=im_0} + \cdots = \Sigma + (i\hat{p}+m_0)\,\Sigma' + \cdots$$

To two loops we then have the solution

$$\frac{\tilde{M}}{m} = 1 + \Sigma_1 + \Sigma_2 + \Sigma_1\Sigma_1'\,, \tag{4}$$

where Σ_L is the bare ($m = m_0$) or $\overline{\mathrm{MS}}$–renormalized (m the $\overline{\mathrm{MS}}$–mass) L-loop contribution to the amplitude. According to Eq. (4) we need to calculate propagator-type diagrams up to two loops on–shell. In order to get manifestly gauge invariant results the Higgs tadpole diagrams must be included [4]. As we have elaborated in [21] the inclusion of the tadpoles is mandatory also for the self–consistency of the renormalization group (RG).

For our calculation all diagrams have been generated with the help of **QGRAF** [24]. The C-program **DIANA** [25] then was used together with the set of Feynman rules extracted from the package **TLAMM** [26] to produce the FORM input which is suitable for the package **ONSHELL2** [27] and/or for another package based on Tarasov's recurrence relations [28]. The relevant master-integrals have been calculated analytically[2] with the help of techniques developed recently in [30].

3. The gaugeless limit

The complete $O(\alpha\alpha_s)$ result of the calculation within the SM is given in our recent publication [18]. Here we present some details concerning the renormalization and present the complete $\mathcal{O}(\alpha\alpha_s)$ answer for the so called

[2] Of course, another possibility is to apply directly numerical programs, some of which are discussed in [29].

"gaugeless limit" approximation of the SM. This limit corresponds to the case $M_H, m_t \gg M_W$ and can be deduced from the Lagrangian of the SM in the approximation $g, g' \to 0, v^2 \neq 0$. It allows us a simplified calculation of the leading top-quark mass corrections to physical observables, like the ρ-parameter or corrections to $Z \to b\overline{b}$ decay [10, 17].

The mass renormalization constant Z_t in the $\overline{\rm MS}$ scheme at two loops may be written in the form

$$m_{t,0} \;=\; m_t(\mu^2)\, Z_t = m_t(\mu^2)\, \left(1 + \frac{g^2(\mu^2)}{16\pi^2}\frac{m_t^2}{m_W^2}\,\frac{1}{\varepsilon} Z_\alpha^{(1,1)} + \frac{\alpha_s(\mu^2)}{4\pi}\,\frac{1}{\varepsilon} Z_{\alpha_s}^{(1)} + \frac{\alpha_s(\mu^2)}{4\pi}\frac{g^2(\mu^2)}{16\pi^2}\frac{m_t^2}{m_W^2}\left(\frac{1}{\varepsilon} Z_{\alpha\alpha_s}^{(2,1)} + \frac{1}{\varepsilon^2} Z_{\alpha\alpha_s}^{(2,2)}\right) + \mathcal{O}(g^4, \alpha_s^2)\right), \quad (5)$$

where $\alpha_s = g_s^2/4\pi$ and

$$Z_\alpha^{(1,1)} \;=\; -\frac{3}{8}\frac{m_H^2}{m_t^2} + \frac{3}{8} + N_c\frac{m_t^2}{m_H^2}\,, \quad Z_{\alpha_s}^{(1)} = -3C_f. \quad (6)$$

In our calculation we obtained the two-loop renormalization constants $Z_{\alpha\alpha_s}^{(2,1)}$ and $Z_{\alpha\alpha_s}^{(2,2)}$

$$Z_{\alpha\alpha_s}^{(2,2)} = C_f\left[-9N_c\frac{m_t^2}{m_H^2} + \frac{9}{8}\frac{m_H^2}{m_t^2} - \frac{9}{4}\right], \quad Z_{\alpha\alpha_s}^{(2,1)} = C_f\left[2N_c\frac{m_t^2}{m_H^2} + \frac{3}{2}\right] \quad (7)$$

where, in the SM, $C_f = 4/3$, $N_c = 3$. We may use the SM renormalization group equations to cross-check the $1/\varepsilon^2$– and $1/\varepsilon$–terms [21, 23]. The coefficient $Z_{\alpha\alpha_s}^{(2,2)}$ may be calculated from the RG relations which allow one to predict the leading higher order poles in terms of the RG coefficients (see Eq.(4.39) in [18]). The terms proportional to $1/\varepsilon$ may be deduced from the RG equations calculated in the unbroken phase. It has been shown [21, 23] that in the $\overline{\rm MS}$ scheme we may write

$$m_t^2(\mu^2) = \frac{1}{2}\frac{Y_t^2(\mu^2)}{\lambda(\mu^2)}\; m^2(\mu^2)\;, \quad (8)$$

where m^2 and λ are the parameters of the symmetric scalar potential V

$$V = \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4$$

and Y_t is the top-quark Yukawa coupling. As a consequence we get the following relation for the anomalous dimension γ_t of the mass of the top-quark

$$\gamma_t \;=\; \gamma_Y + \frac{1}{2}\gamma_{m^2} - \frac{1}{2}\frac{\beta_\lambda}{\lambda}\;, \quad (9)$$

where the relevant RG results in the gaugeless limit up to $\mathcal{O}(g^4)$ are [31]

$$\gamma_{m^2} \equiv \frac{1}{m^2}\mu^2\frac{d}{d\mu^2}m^2 = \frac{1}{16\pi^2}\left[\lambda + 3Y_t^2\right] + 20\frac{g_s^2Y_t^2}{(16\pi^2)^2} ,$$
$$\beta_\lambda \equiv \mu^2\frac{d}{d\mu^2}\lambda = \frac{1}{(16\pi^2)}\left[2\lambda^2 + 6\lambda Y_t^2 - 18Y_t^4\right] + \frac{g_s^2Y_t^2}{(16\pi^2)^2}\left[40\lambda - 96Y_t^2\right] ,$$
$$\gamma_Y \equiv \frac{1}{Y_t}\mu^2\frac{d}{d\mu^2}Y_t = \frac{1}{16\pi^2}\left[\frac{9}{4}Y_t^2 - 4g_s^2\right] + 18\frac{g_s^2Y_t^2}{(16\pi^2)^2} . \qquad (10)$$

At the same time, the anomalous dimension γ_t can be related with the renormalization constant (see [20] for details). In our case we get

$$\gamma_t = \frac{m_t^2}{m_W^2}\left[\frac{g^2}{16\pi^2}Z_\alpha^{(1,1)} + \frac{g^2g_s^2}{(16\pi^2)^2}2Z_{\alpha\alpha_s}^{(2,1)}\right] + \frac{g_s^2}{16\pi^2}Z_{\alpha_s}^{(1)} .$$

Finally, the parameter relations $Y_t^2 = \frac{2m_t^2}{v^2}$, $\lambda = \frac{3m_H^2}{v^2}$, $g^2 = \frac{4m_W^2}{v^2}$, provide the bridge between Eqs. (9) and (10) and our Eqs. (6) and (7).

After performing the UV renormalization the MS renormalized amplitudes are finite. The relation between the top–propagator pole $\tilde{M}$ and the $\overline{\rm MS}$ mass m_t can be written as

$$\frac{\tilde{M}}{m_t} = 1 + \Sigma_{1,\overline{\rm MS}} + \left\{\Sigma_2 + \Sigma_1\Sigma_1^{'}\right\}_{\overline{\rm MS}} + \mathcal{O}(g^4, \alpha_s^2) , \qquad (11)$$

where

$$\Sigma_{1,\overline{\rm MS}} = \frac{\alpha_s}{4\pi}C_f\left[4 - 3\ln\frac{m_t^2}{\mu^2}\right] + \frac{g^2}{16\pi^2}\frac{m_t^2}{m_W^2}\left[Z_\alpha^{(1,1)}\ln\frac{m_t^2}{\mu^2} + \frac{(1+y^2)}{2y}\right.$$
$$\left. - N_c\frac{m_t^2}{m_H^2} - \frac{m_t^4}{8m_H^4}\ln(1+y) + \frac{m_t^2}{8m_H^2}\frac{(3+y^2)}{(1+y)}\ln y - i\pi\frac{1}{8}\right], \qquad (12)$$
$$\left\{\Sigma_2 + \Sigma_1\Sigma_1^{'}\right\}_{\overline{\rm MS}} = C_f\frac{\alpha_s}{4\pi}\frac{g^2}{16\pi^2}\frac{m_t^2}{m_W^2}\left(\ln^2\frac{m_t^2}{\mu^2}\left[\frac{9}{8}\frac{m_H^2}{m_t^2} - 9N_c\frac{m_t^2}{m_H^2} - \frac{9}{4}\right]\right.$$
$$+ \ln\frac{m_t^2}{\mu^2}\left[9 + 11N_c\frac{m_t^2}{m_H^2} - \frac{3m_t^4}{8m_H^4}\ln(1+y) + \frac{3m_t^2}{8m_H^2}\frac{(y^2+6y-3)}{(1+y)}\ln y + i\pi\frac{9}{8}\right]$$
$$+ \zeta_2\left\{\frac{3}{2y} + \frac{9}{2}y + \frac{3}{4}y^2\right\} - \frac{y}{(1+y)^2}\left\{\frac{11(1+y^2)(1+y)^2}{8y^2} + 8N_c\right\}$$
$$\left. + \frac{(1-y)^2}{y^2}\ln y\left[\ln(1-y) + \frac{1}{2}\ln(1+y)\right]\left[(1-y^2) - \frac{1}{2}(1+y^2)\ln y\right]\right.$$

$$
-\frac{1}{8}\frac{2+8y-10y^2-3y^3}{y}\ln^2 y+\frac{1}{8}\frac{(1+y)(6-63y+5y^2)}{y}\ln y
$$
$$
-\frac{1}{8}\frac{(1+y)^2(5-62y+5y^2)}{y^2}\ln(1+y)-\frac{3}{2}\zeta_2\ln(1+y)\frac{(1-y)^2(1+y^2)}{y^2}
$$
$$
+\frac{(1-y)(1+y)}{y^2}\left\{\frac{(5-28y+5y^2)}{4}\mathrm{Li}_2(-y)+(1-y)^2\mathrm{Li}_2(y)\right\}
$$
$$
+\frac{(1-y)^2(1+y^2)}{y^2}\left\{\frac{3}{2}\left[2\mathrm{Li}_3(y)+\mathrm{Li}_3(-y)\right]-\ln y\left[2\mathrm{Li}_2(y)+\mathrm{Li}_2(-y)\right]\right\}
$$
$$
-\frac{259}{16}-\frac{15}{4}\zeta_2-\frac{3}{2}\zeta_3-i\pi\left[\frac{17}{8}-\zeta_2\right]\Bigg), \qquad (13)
$$

where

$$
y=\frac{1-\sqrt{1-\frac{4m_t^2}{m_H^2}}}{1+\sqrt{1-\frac{4m_t^2}{m_H^2}}}\equiv\frac{1-\sqrt{1-\frac{6Y_t^2}{\lambda}}}{1+\sqrt{1-\frac{6Y_t^2}{\lambda}}}\,.
$$

As the top is an unstable particle the pole of the propagator has an imaginary part which is related up to a sign to the width Γ_t divided by two. In the gaugeless limit approximation it is equal to

$$
\frac{\Gamma_t}{M_t}=\frac{\alpha}{2\sin^2\theta_W^{OS}}\frac{1}{8}\frac{M_t^2}{M_W^2}\left[1+\frac{\alpha_s}{4\pi}C_f\,(5-8\zeta_2)\right]. \qquad (14)
$$

Very often the inverse of the relation (11) is required. To that end we have to solve the real part of (4) iteratively for m_t and to express all $\overline{\rm MS}$ parameters in terms of on-shell ones. The $\mathcal{O}(\alpha\alpha_s)$ solution to two loops reads

$$
\frac{m_t}{M_t}=1-\mathrm{Re}\Sigma_{1,\overline{\rm MS}}-\mathrm{Re}\left\{\Sigma_2+\Sigma_1\Sigma_1'\right\}_{\overline{\rm MS}}+\frac{2\alpha_s\alpha}{(4\pi)^2\sin^2\theta_W^{OS}}\left\{Z_{\alpha_s}^{(1)}\ \mathrm{Re}\ \Sigma_{1,\overline{\rm MS}}^{\alpha}\right.
$$
$$
\left.+\Sigma_{1,\overline{\rm MS}}^{\alpha_s}\left(\frac{M_t^2}{M_W^2}\left[\Delta X_\alpha^{(1)}+Z_\alpha^{(1,1)}+\ln\frac{M_t^2}{\mu^2}\left(\frac{3}{8}+2N_c\frac{M_t^2}{M_H^2}\right)\right]+\mathrm{Re}\Sigma_{1,\overline{\rm MS}}^{\alpha}\right)\right\}, \qquad (15)
$$

where $\Sigma_{1,\overline{\rm MS}}^{j}$ $(j=\alpha,\alpha_s)$ means that only the "j" part of the one-loop MS renormalized amplitude is to be taken into account and $\Delta X_\alpha^{(1)}$ denotes the real part of the derivative of the one-loop amplitude with respect to the top-mass:

$$
\Delta X_\alpha^{(1)}=\frac{M_H^4}{8M_t^4}\ln(1+Y)-2N_c\frac{M_t^2}{M_H^2}-\frac{(1+Y)(3+Y)}{8}\ln Y+\frac{(1-Y)^2}{4Y}\,,
$$

with $Y = \frac{1-\sqrt{1-\frac{4M_t^2}{M_H^2}}}{1+\sqrt{1-\frac{4M_t^2}{M_H^2}}}$. It is interesting to compare the result (15) calculated at $\mu = M_t$ with a similar relation calculated in the full SM (see Eq.(5.57) in [18]). The difference can be written in the following form

$$\frac{m_t^{SM}(M_t) - m_t^{GL}(M_t)}{M_t} = \frac{\alpha}{4\pi \sin^2\theta_W^{OS}} \frac{M_t^2}{M_W^2} \left[a \frac{M_t^2}{M_H^2} \left(1 - 4\frac{\alpha_s}{4\pi} C_f\right) + b + c\frac{\alpha_s}{4\pi} C_f \right] \tag{16}$$

where the constants a, b, c depend only on the values of the masses of the gauge bosons W, Z and the top-quark. Numerically, taking the input parameter values $M_W = 80.419$ GeV, $M_Z = 91.188$ GeV and $M_t = 174.3$ GeV, we obtain

$$\begin{aligned} a &= -\frac{1}{2}\frac{M_W^4}{M_t^4}\left(1 - 3\ln\frac{M_W^2}{M_t^2}\right) - \frac{1}{4}\frac{M_Z^4}{M_t^4}\left(1 - 3\ln\frac{M_Z^2}{M_t^2}\right) \sim -0.21934\,, \\ b &= -0.07978\,, \quad c = -0.429164\,. \end{aligned} \tag{17}$$

4. $O(\alpha\alpha_s)$ correction to the top-Yukawa coupling

In general, the concept of a quark mass is convention dependent. In electroweak theory we have the possibility to consider instead of the mass of the top-quark, for example, the top-Yukawa coupling. The one-loop electroweak corrections $\mathcal{O}(\alpha)$ to the relation between the Yukawa coupling and the pole parameters has been calculated first in [32]. Our result [18] allows us to extract the $\mathcal{O}(\alpha\alpha_s)$ correction to the top-Yukawa coupling. The starting point is the relation between the Fermi constant G_F and the pole parameters of the SM, which may be written in the following form [3]:

$$\sin^2\theta_W M_W^2 (1 - \overline{\Delta}r) = \frac{\pi\alpha(M_Z)}{\sqrt{2}G_F}\,, \quad \overline{\Delta}r \equiv \Delta r - \Delta\alpha\,,$$

where for Δr we use parametrization proposed in [33]. Using the transition from the on-shell to the $\overline{\rm MS}$ parameters of the SM (see [23] for details) we get the following expression for $v_{MS}^2(\mu^2) \equiv 1/\sqrt{2}G_F(\mu^2)$:

$$v_{MS}^2(\mu^2) = \frac{1}{\sqrt{2}G_F}\frac{1}{1-\overline{\Delta}r}\left[\frac{m_W^2(\mu^2)}{M_W^2}\right]\left[\frac{\alpha(M_Z)}{\alpha_{MS}(\mu^2)}\right]\left[\frac{\sin^2\theta_W^{MS}(\mu^2)}{\sin^2\theta_W^{OS}}\right], \tag{18}$$

such that we have

$$\frac{y_t(\mu^2)}{M_t 2^{3/4} G_F^{1/2}} = \frac{m_t(\mu^2)}{M_t}\sqrt{(1-\overline{\Delta}r)\frac{\alpha_{MS}(\mu^2)}{\alpha(M_Z)}\frac{M_W^2}{m_W^2(\mu^2)}\frac{\sin^2\theta_W^{OS}}{\sin^2\theta_W^{MS}(\mu^2)}}\,. \tag{19}$$

This is our basic expression. Expanding each relation in powers of the coupling constants α and α_s the correction to the top-Yukawa coupling can be extracted at the given order. Let us introduce the following decomposition of the renormalization constants

$$\frac{m_t(\mu^2)}{M_t} - 1 = \delta^{\alpha} + \delta^{\alpha_s} + \delta^{\alpha\alpha_s} + \cdots, \frac{\alpha_{MS}(\mu^2)}{\alpha(M_Z)} - 1 = Z_e^{\alpha} + Z_e^{\alpha\alpha_s} + \cdots$$
$$\frac{\sin^2\theta_W^{MS}(\mu^2)}{\sin^2\theta_W^{OS}} - 1 = Z_\theta^{\alpha} + Z_\theta^{\alpha\alpha_s} + \cdots, \frac{m_W^2(\mu^2)}{M_W^2} - 1 = Z_W^{\alpha} + Z_W^{\alpha\alpha_s} + \cdots$$
$$\overline{\Delta} r = \overline{\Delta} r^{\alpha} + \overline{\Delta} r^{\alpha\alpha_s} + \cdots \qquad (20)$$

and shortly describe each term. The first relation corresponds to (5.57) in [18]. The relation between $\overline{\rm MS}$ and on-shell values of the electric charge, Z_e, includes besides the perturbative corrections also the nonperturbative contribution from the hadrons [34]. So, in our notation, the factor $Z_e^{\alpha\alpha_s}$ includes only the perturbative contribution from the massive top-quark, at the same time the factor Z_e^{α} includes the contribution from five massless quarks, the massive leptons and the nonperturbative contribution (a recent numerical value is given in [35]). The renormalization constant of the Weinberg angle θ_W is related with the renormalization of the masses of the Z- and W-bosons via

$$Z_\theta^{\alpha} + Z_\theta^{\alpha\alpha_s} \equiv -\frac{\cos^2\theta_W^{OS}}{\sin^2\theta_W^{OS}}\left[Z_W^{\alpha} - Z_Z^{\alpha} + Z_W^{\alpha\alpha_s} - Z_Z^{\alpha\alpha_s}\right],$$

where the perturbative contributions to $Z_W^{\alpha\alpha_s}, Z_Z^{\alpha\alpha_s}$ can be extracted from [19] (see also [21]) and nonperturbative effect is given in [36]. The $O(\alpha)$ and $O(\alpha\alpha_s)$ corrections to $\overline{\Delta} r$ are given in [3] and [19], respectively, and $\overline{\Delta} r^{\alpha\alpha_s}$ includes only the perturbative contribution from the quarks.

Using the decomposition (20) we deduce

$$\frac{y_t(\mu^2)}{M_t 2^{3/4} G_F^{1/2}} - 1 = \delta^{\alpha} + \delta^{\alpha_s} + \frac{1}{2}\left[Z_e^{\alpha} - Z_W^{\alpha} - Z_\theta^{\alpha} - \overline{\Delta} r^{\alpha}\right]$$
$$+\delta^{\alpha\alpha_s} + \frac{1}{2}\delta^{\alpha_s}\left[Z_e^{\alpha} - Z_W^{\alpha} - Z_\theta^{\alpha} - \overline{\Delta} r^{\alpha}\right] + \frac{1}{2}\left[Z_e^{\alpha\alpha_s} - Z_W^{\alpha\alpha_s} - Z_\theta^{\alpha\alpha_s} - \overline{\Delta} r^{\alpha\alpha_s}\right]. \qquad (21)$$

The first line of the relation (21) corresponds to Eq. (2.13) of [32]. In the second line, the first term is our recent result (see Eq. (5.57) in [18]), the second term is simply the product of the one-loop QCD correction $\delta_{\alpha_s} = 16/3 - 4\ln\frac{m_t^2}{\mu^2}$ and the one-loop electroweak corrections and the last terms can be extracted from [19]. The result is relatively lengly and will be presented in a forthcoming publication.

Acknowledgments. We would like to thank the organizers of the Conference for their very warm hospitality. M.Yu.K. is specially grateful to Janusz Gluza for his help concerning the participation at this conference. Furthermore, we thank I. Ginzburg and G. Passarino for interesting discussions. M.K.'s research was supported in part by RFBR grant # 04-02-17192.

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