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Full two-loop electroweak corrections to the pole masses of gauge bosons[‡]

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Full two-loop electroweak corrections to the pole masses of gauge bosons

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We discuss progress in SM two-loop calculations of the pole position of the massive gauge-boson propagators.

1. INTRODUCTION

An important type of "universal" (process independent) corrections relevant for complete electroweak 2-loop calculations of physical processes are the contributions from on-shell gauge boson self-energies which incorporate the relation between bare, $\overline{\rm MS}$ and on-shell (pole) masses (2-loop renormalization constant in on-shell scheme). Such calculations are important to scrutinize theoretical uncertainties which might obscure e.g. the indirect Higgs mass bounds obtained by the LEP experiments. Most interesting are $2 \to 2$ fermion processes, which in future eventually may be investigated with much higher precision at TESLA in case the Giga$Z$ [1] option would be realized. Here, we outline the complete 2-loop SM calculation of the pole position of the massive gauge-boson propagators presented in [2,3].

The position of the pole s_P and the wave-function renormalization constant Z_2 of the propagator of a massive gauge-boson are defined by the relation

$$\frac{1}{s - m_0^2 - \Pi(p^2, m_0^2, \cdots)} \simeq \frac{Z_2}{s - M^2 + iM\Gamma} \qquad (1)$$

for $s \simeq s_P$ where $\Pi(p^2, \cdots)$ is the transversal part of the one-particle irreducible self-energy and m_0^2 is the bare mass and we have adopted the standard parameterization of the pole $s_{P,V} = M_V^2 - iM_V\Gamma_V$, where Γ_V is the width of the unstable gauge-boson.

This equation can be solved perturbatively, order by order. Up to two loops the solution reads

$$s_P = m^2 + \Pi^{(1)} + \Pi^{(2)} + \Pi^{(1)}\Pi^{(1)\prime}\,, \qquad (2)$$

$$Z_2^{-1} = 1 - \Pi^{(1)\prime} - \Pi^{(2)\prime} - \Pi^{(1)}\Pi^{(1)\prime\prime}\,, \qquad (3)$$

where $\Pi^{(L)}$ is the bare or $\overline{\rm MS}$ -renormalized L-loop contribution to Π, the prime (double prime) denotes the derivative (second derivative) with respect to p^2 at $p^2 = m^2$. One of the remarkable properties of (2) is that the pole position is well-defined in terms of self-energy diagrams and its derivatives at momentum (square) equal to the bare or the $\overline{\rm MS}$ mass (square) which, by construction, are real parameters.

For the Z-boson the equation for the position of the pole is modified due to $\gamma - Z$ mixing

$$s_P - m_Z^2 - \Pi_{ZZ}(s_P) - \frac{\Pi_{\gamma Z}^2(s_P)}{s_P - \Pi_{\gamma\gamma}(s_P)} = 0\,.$$

The 2-loop wave-function renormalization constant in this case is equal to

$$Z_{ZZ}^{-1} = 1 - \Pi_{ZZ}^{(1)\prime} - \Pi_{ZZ}^{(2)\prime} - \Pi_{ZZ}^{(1)}\Pi_{ZZ}^{(1)\prime\prime} - \frac{2}{m_Z^2}\Pi_{\gamma Z}^{(1)}\Pi_{\gamma Z}^{(1)\prime}\,.$$

We would like to stress, that in order to obtain gauge invariant results the tadpole contributions have been included [4].

2. CALCULATIONS

At the 2-loop level we have about 1000 1PI diagrams each for the Z- and the W-propagator. Our calculation is largely automatized: we use **QGRAF** [5] to generate the diagrams and then the C-program **DIANA** [6] to produce for each diagram the input suitable for our **FORM** packages.

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Two-loop propagator type diagrams with several masses can be reduced to a restricted set of so-called master-integrals by using a complete set of recurrence relations given in [7]. We have used this approach only for the calculation of the 2-loop massless fermion corrections, where analytical results for the master-integrals are available [8]. As compared to the existing calculation of the massless fermion contribution, performed in [9], we apply Tarasov's recurrence relations [7] which allows us to reduce the number of master-integrals to a minimal set. The latter includes integrals which may be evaluated by using the package **ON-SHELL2** [10] plus the following new prototypes: (using standard notation [2,3])

- ZZ**:**

$F_{W0W00}, F_{0000W}, V_{H00Z}, V_{W00W}, J_{00\{H,W\}}$.

- WW**:**

$F_{Z0W00}, F_{0000Z}, V_{W00Z}, V_{\{H,Z\}00W}, J_{00\{H,Z\}}$.

We have worked out an independent analytical calculation of these master integrals by using a method developed in [3]. One of the crucial points of our approach is the observation [11] that the analytical results for diagrams with two-massive cuts have a very simple form if written in terms of new variables. For diagrams with two equal internal masses m^2 and arbitrary external momentum p^2 the new variable is $y = \frac{1-\sqrt{\frac{z}{z-4}}}{1+\sqrt{\frac{z}{z-4}}}$, where $z = p^2/m^2$. When the external momentum belongs to one of the internal masses $p^2 = m^2$, the new variable is $\chi = \frac{1-\sqrt{1-4x}}{1+\sqrt{1-4x}}$, where $x = m^2/M^2$. To speed up the numerical evaluation of on-shell gauge-boson self-energies, an expansion in $\sin^2\theta_W$ may be used, which converges well and simplifies the results considerably. However, it is not a naive Taylor expansion due to the presence of threshold singularities of the form $\ln^j \sin^2\theta_W$. We therefore briefly outline the corresponding expansions for the relevant master-integrals by giving two examples. The integral F_{W0W00} $[z = 1/\cos^2\theta_W]$ depends on the small parameter $s \equiv 1 - 1/z$ via the variable (see [3])

$$y = \frac{1}{1-s}\left(\frac{1}{2} - s + \mathrm{i}\frac{\sqrt{3}}{2}\sqrt{1-\frac{4}{3}s}\right) = \left(\sum_{j=0} s^j\right)$$

$$\left[\exp\left(i\frac{\pi}{3}\right) - s - s\frac{\mathrm{i}}{\sqrt{3}}\sum_{n=0}\binom{2n}{n}\left(\frac{s}{3}\right)^n \frac{1}{n+1}\right]$$

which we expand first and then substitute it into the analytical result in order to get the series expansion for F_{W0W00}. The integral $F_{Z0W00}[x = \cos^2\theta_W]$ depends on s via the variable

$$\chi = \frac{1}{1-s}\left(-\frac{1}{2} + s + \mathrm{i}\frac{\sqrt{3}}{2}\sqrt{1-\frac{4}{3}s}\right) = \left(\sum_{j=0} s^j\right)$$

$$\left[\exp\left(i\frac{2\pi}{3}\right) + s - s\frac{\mathrm{i}}{\sqrt{3}}\sum_{n=0}\binom{2n}{n}\left(\frac{s}{3}\right)^n \frac{1}{n+1}\right]$$

and we proceed as in the previous case. However, in all other cases, the master-integrals which show up are not know analytically so far. They can be calculated numerically, by using one of the approaches and/or programs developed for the 2-loop self-energies [12]. The full list of master-integrals occurring in the 2-loop calculation of the pole-mass of the gauge-bosons (for the photon propagator see [13]) reads:

- ZZ**:**

$F_{WWWWH}, F_{ttttH}, F_{ZWHWZ}, F_{ZHHZZ}, F_{ZZHHH}$, $F_{WWWWZ}, F_{ZtHtt}, F_{W0W0t}, F_{WtWt0}, F_{t0t0W}$, $V_{WZWW}, V_{WHWW}, V_{HWWZ}, V_{HttZ}, V_{ZWWH}$, $V_{ZttH}, V_{tHtt}, V_{W0tW}, V_{t0Wt}, V_{tZtt}, V_{HHZZ}, V_{ZZZH}$, $J_{ZWW}, J_{HWW}, J_{0WW}, J_{0tt}, J_{Ztt}, J_{Htt}, J_{0WZ}$, J_{ZHH}, J_{0Wt} .

- WW**:**

$F_{00ttZ}, F_{W0Ztt}, F_{W0Htt}, F_{WWZZH}, F_{WWHZZ}$, $F_{WWHHH}, F_{WZZWW}, F_{WZHWW}, F_{WHHWW}$, $F_{WtZ00}, V_{00Wt}, V_{0Ztt}, V_{0Htt}, V_{WHHH}, V_{WWWZ}$, $V_{WWWH}, V_{WttZ}, V_{WttH}, V_{WZZH}, V_{WZHZ}$, $V_{Z0tW}, V_{H0tW}, V_{ZWHW}, V_{HWZW}, V_{ZWZW}$, $V_{HWHW}, J_{0WZ}, J_{0Wt}, J_{0WH}, J_{0Zt}, J_{0Ht}, J_{WHH}$, $J_{WZZ}, J_{Wtt}, J_{WZH}$.

To keep control of gauge invariance we adopt the R_ξ gauge with three different gauge parameters ξ_W, ξ_Z and ξ_γ. In most cases exact analytic results in terms of known functions are not available. Thus, instead of working with the exact formulae (which only can be evaluated numerically, at present) we resort to some approximations, namely, we perform appropriate series ex-

pansions in (small) mass ratios [15]. For diagrams with several different masses it is possible that several small parameters are available. In this case we apply different asymptotic expansions (see [14]) one after the other. Specifically, we expand in the gauge parameters about $\xi_i = 1$, in $\sin^2\theta_W$ and, for diagrams with Higgs or/and top-quark lines, in m_V^2/m_H^2 or/and m_V^2/m_t^2. Numerical results are obtained using the packages **ON-SHELL2** [10] and **TLAMM** [16] (see [2,3]).

Renormalizing the pole-mass at the 2-loop level requires to calculate the 1-loop renormalization constants for all physical parameters (charge and masses), as well as the 2-loop mass-renormalization constant itself. Not needed are the wave-function and ghost sector renormalizations.

The full 2-loop relation between pole and $\overline{\rm MS}$ parameters can be written in the form

$$s_{P,V} = m^2_{V,0} + \Pi^{(1)}_{V,0} + \Pi^{(2)}_{V,0} + \Pi^{(1)}_{V,0}\Pi^{(1)\prime}_{V,0} + \left[\sum_j (\delta m^2_{j,0})^{(1)} \frac{\partial}{\partial m^2_{j,0}} + (\delta e_0)^{(1)} \frac{\partial}{\partial e_0}\right] \Pi^{(1)}_{V,0}, \quad (4)$$

where the sum runs over all species of particles, $(\delta e_0)^{(1)}$ and $(\delta m^2_{j,0})^{(1)}$ are the 1-loop counter-terms for the charge and physical masses in the $\overline{\rm MS}$ -scheme. The derivatives in (4) correspond to the subtraction of sub-divergencies. The genuine 2-loop mass counter-term is obtained by expanding $m^2_{V,0}$ in terms of the renormalized mass.

Summing all 2-loop contributions we restore gauge invariance of the position of the complex pole before UV renormalization. After UV-renormalization the propagator pole is represented in terms of finite amplitudes. For explicite results we refer to [2,3]. For the numerical evaluation we were using the first six coefficients in the weak angle $\sin^2\theta_W$ and in the mass ratios m_V^2/m^2_{heavy}.

3. RENORMALIZATION GROUP EQUATIONS

We now consider the SM renormalization group (RG) equations. We will denote on-shell masses by capital M and $\overline{\rm MS}$ masses by lowercase m. We adopt the following definitions for the RG functions: for all dimensionless coupling constants, like g, g', g_s, e, λ, the β-function is given by $\mu^2 \frac{\partial}{\partial \mu^2} g = \beta_g$ and for all mass parameters (a mass or the Higgs v.e.v. v) the anomalous dimension γ_{m^2} is given by $\mu^2 \frac{\partial}{\partial \mu^2} \ln m^2 = \gamma_{m^2}$. Using the fact that s_P is RG-invariant: $\mu^2 \frac{d}{d\mu^2} s_P \equiv 0$, we are able to calculate the anomalous dimension of the gauge bosons masses from our finite results. At the same time, a typical relation between bare- and $\overline{\rm MS}$ -masses has the form

$$m^2_{V,0} = m^2_V(\mu)\,(1 + \sum_{k=1} Z_V^{(k)} \varepsilon^{-k})$$

such that the RG functions may be calculated directly from the UV counter-terms $\gamma_V = \sum_j \frac{1}{2} g_j \frac{\partial}{\partial g_j} Z_V^{(1)}$, $(j = g, g_s)$. In addition, the UV counter-terms satisfy relations connecting the higher order poles with the lower order ones:

$$\left(\gamma_V + \sum_j \beta_{g_j} \frac{\partial}{\partial g_j} + \sum_i \gamma_i m_i^2 \frac{\partial}{\partial m_i^2}\right) Z_V^{(n)} = \frac{1}{2} \sum_j g_j \frac{\partial}{\partial g_j} Z_V^{(n+1)}. \quad (5)$$

In the SM it is interesting to compare the RG equations calculated in broken phase with the ones obtained in the unbroken phase. Let us remind that at the tree-level the vacuum expectation value v is given by $v^2 \equiv \frac{m^2}{\lambda}$, where m^2 and λ are the parameters of the symmetric scalar potential. The masses of the gauge-bosons are equal to $m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2$ and $m_W^2 = \frac{1}{4} g^2 v^2$, respectively. The fact that these relations are RG invariant on the level of the bare quantities implies the relations

$$\gamma_W - 2\frac{\beta_g}{g} = \gamma_{m^2} - \frac{\beta_\lambda}{\lambda}\,, \quad (6)$$

$$\gamma_Z - \gamma_{m^2} + \frac{\beta_\lambda}{\lambda} = 2\left(\cos^2\theta_W \frac{\beta_g}{g} + \sin^2\theta_W \frac{\beta_{g'}}{g'}\right), \quad (7)$$

where the 2-loop RG functions $\beta_g, \beta_{g'}, \beta_\lambda, \gamma_{m^2}$ have been calculated in the unbroken phase in [17]. We have verified in the $\overline{\rm MS}$ scheme, that these relations are valid up to 2-loop order in the broken phase with the same RG functions. Thus the RG equations for the $\overline{\rm MS}$ masses in the broken

theory can be written as

$$m_W^2(\mu^2) = \frac{1}{4}\frac{g^2(\mu^2)}{\lambda(\mu^2)} m^2(\mu^2) \,,$$
$$m_Z^2(\mu^2) = \frac{1}{4}\frac{g^2(\mu^2)+g'(\mu^2)}{\lambda(\mu^2)} m^2(\mu^2) \,,$$
$$m_H^2(\mu^2) = 2m^2(\mu^2) \,,$$
$$m_t^2(\mu^2) = \frac{1}{2}\frac{y_t^2(\mu^2)}{\lambda(\mu^2)} m^2(\mu^2) \,, \qquad (8)$$

where y_t is the top-quark Yukawa coupling (the other Yukawa couplings are kept zero). The $\overline{\rm MS}$ Fermi constant

$$\hat{G}_F(\mu^2) \equiv \frac{\sqrt{2}\, e^2(\mu^2)}{8 m_W^2(\mu^2) \sin^2\theta_W(\mu^2)} \qquad (9)$$

satisfies the following RG equation

$$\mu^2 \frac{\partial}{\partial \mu^2} \ln \hat{G}_F(\mu^2) = \frac{\beta_\lambda}{\lambda} - \gamma_{m^2} \,. \qquad (10)$$

The knowledge of the anomalous dimensions γ_V allow us to write expression for the pole positions $s_{P,V}$ with explicit factorization of the RG logarithms. What we get is

$$s_{P,V} = m_V^2 \left(1 - g^2 \gamma_V^{(1)} L_a\right) + g^2 X_{V,1}$$
$$+ g^4 m_V^2 \left(C^{(2,2)} L_a^2 - C^{(2,1)} L_a\right) + g^4 X_{V,2} \,, \qquad (11)$$

where $\mu^2 \frac{\partial}{\partial \mu^2} \ln m_a^2 = \gamma_a^{(1)} g^2 + \gamma_a^{(2)} g^4$, $L_a = \ln \frac{\mu^2}{m_a^2}$,

$$2C_V^{(2,2)} = \left[\theta + \sum_j \gamma_{m_j}^{(1)} m_j^2 \frac{\partial}{\partial m_j^2}\right] \gamma_V^{(1)} \,, \qquad (12)$$

$$C_V^{(2,1)} = \gamma_V^{(2)} + \gamma_V^{(1)} \gamma_a^{(1)}$$
$$+ \frac{1}{m_V^2}\left[2\beta_g^{(1)} + \sum_j \gamma_{m_j}^{(1)} m_j^2 \frac{\partial}{\partial m_j^2}\right] X_{V,1} \,, \qquad (13)$$

and $\theta = \gamma_V^{(1)} + 2\beta_g^{(1)}$, $\mu^2 \frac{\partial}{\partial \mu^2} g = \beta_g^{(1)} g^3$. In contrast to QCD, $\gamma_V^{(1)}$ and $C_V^{(i,j)}$ have non-polynomial structure in the massless coupling constants which originated from the tadpole contributions. $X_{V,1}$ has been calculated long time ago [4] and $X_{V,2}$ are our results which, for the numerical evaluation, we approximated by finite series. The functions $X_{V,j}$ we have represented in terms of $\overline{\rm MS}$ parameters. We note that the amplitudes $X_{V,j}$ entering (11) have no explicit μ dependence.

4. SCHEME DEPENDENCE

Our results reveal terms of unexpectedly high powers of the Higgs and the top-quark masses in (11), arising from 2-loop corrections. In fact the purely bosonic diagrams yield m_H^4/m_V^4 terms and the (t,b) quark-doublet ($m_t \gg m_b$) contributes $m_t^6/(m_H^2 m_V^4)$ power corrections. At a first glance, such terms contradict Veltman's screening theorem [19] which states, that the L-loop Higgs dependence of a physical observable is bounded by $(m_H^2)^{L-1} \ln^L m_H^2$ for large Higgs masses. However, this theorem only applies to physical observables like cross sections and asymmetries. If we consider quantities like Δr (which is an observable in the on-shell scheme) in the $\overline{\rm MS}$ scheme the screening theorem does not hold in general. To illustrate this, let us compare the 1-loop EW corrections to the Fermi constant [18] in the on-shell and the $\overline{\rm MS}$ scheme[2]:

$$\begin{aligned} G_F &= \frac{\pi\alpha}{\sqrt{2}} \frac{1}{M_W^2 \sin^2\Theta_W} \left(1 + \Delta r_{OS}^{(1)}\right) \\ &= \hat{G}_F(\mu^2) \left(1 + \Delta r_{MS}^{(1)}\right) \end{aligned} \qquad (14)$$

so that

$$\Delta r_{MS}^{(1)} = \Delta r_{OS}^{(1)} + \left[\frac{m_W^2}{M_W^2} - 1\right] + \left[\frac{\sin^2\theta_W}{\sin^2\Theta_W} - 1\right] + \Delta\alpha \,.$$

The 1-loop correction $\Delta r_{OS}^{(1)}$ in the limit of a heavy Higgs boson has a logarithmic Higgs mass dependence only . In contrast, in the $\overline{\rm MS}$ scheme, higher powers of the Higgs mass are showing up, because $\frac{m_W^2}{M_W^2} - 1$ contributes the term

$$\Delta r_{MS}^{(1)} \sim -\frac{\sqrt{2}\hat{G}_F(\mu^2) m_W^2}{16\pi^2} \frac{7}{2} \frac{m_H^2}{m_W^2} + O(m_H^2) \,,$$

while all other corrections exhibit logarithmic behavior in the Higgs mass, only[3]. Our relation (11) between parameters of two different schemes does *not* relate physical observable but it is very important for analysis of the uncertainties coming

[2] We should mention that $\Delta r_{MS}^{(1)}$ introduced in (14) is different from $\Delta\hat{r}$ defined via a hybrid scheme (couplings $\overline{\rm MS}$, masses on-shell) in [20].

[3] The hybrid quantity $\Delta\hat{r}^{(1)}$ does not show extra powers of the Higgs mass since masses are kept on-shell.

from higher order effects [21]. In particular, our estimations of the 2-loop boson contribution to Δr [2] is very close to the results of real calculations [22,23].

5. CONCLUSION

By an explicite 2-loop calculation we have shown that (to this order): **1**. The position of the complex pole s_p of a the gauge-boson (Z, W) propagator is a gauge invariant quantity after inclusion of the Higgs tadpole contributions (see also [24]). **2**. The renormalized on-shell self-energies are infrared finite. This derives from the fact that within dimensional regularization, which allows to regularize UV and IR singularities by the same ε ($\varepsilon = (4-d)/2 \to 0$) parameter, the singular $1/\varepsilon$ terms are absent after UV renormalization. **3**. The inclusion of the tadpoles is important for the renormalization group invariance and for the gauge invariance of the parameter renormalizations. **4**. By our calculation we have proven that the $\overline{\rm MS}$ renormalization scheme is self consistent and works properly in case of unstable particles. **5**. Our results for the 2-loop mass renormalization constants in the on-shell and the $\overline{\rm MS}$ scheme can be applied in calculations of physical quantities in both of these schemes at the 2-loop level. Examples of such calculations, where results of our paper [2] have been used, are the computations of the bosonic 2-loop contributions to the Muon life-time presented in [23].

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