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Facts of life with γ_5

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Abstract

The increasing precision of many experiments in elementary particle physics leads to continuing interest in perturbative higher order calculations in the electroweak Standard Model or extensions of it. Such calculations are of increasing complexity because more loops and/or more legs are considered. Correspondingly efficient computational methods are mandatory for many calculations. One problem which affects the feasibility of higher order calculations is the problem with γ_5 in dimensional regularization. Since the subject thirty years after its invention is still controversial I advocate here some ideas which seem not to be common knowledge but might shed some new light on the problem. I present arguments in favor of utilizing an anticommuting γ_5 and a simple 4–dimensional treatment of the hard anomalies.

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1. Introduction

The electroweak Standard Model (SM) [1] has been extremely successful in the interpretation of LEP/SLC data and higher order effects typically amount to 10 σ deviations if not taken into account [2]. These precise predictions are only possible due to the renormalizability [3] of the SM and the by now very precise knowledge of the relevant input parameters. Last but not least the relevant coupling constants are small enough such that perturbation theory mostly works very well.

The formal proofs of renormalizability of the SM [4] often relied on the assumption that a gauge invariant regularization exists. The question whether such a regularization exists is non–trivial because of the chiral structure of the fermions involved. At present the only regularization, which makes elaborate computations of radiative corrections feasible, is the dimensional regularization (DR) scheme [5, 6] which is well-defined for field theories with vectorial gauge symmetries only. However, in theories exhibiting chiral fermions, like the electroweak SM, problems with the continuation of the Dirac matrix γ_5 to dimensions $D \neq 4$ remain open within this context and several modifications of the 't Hooft–Veltman DR have been proposed [7, 8, 9, 10, 11, 12, 13, 14, 15]. It turns out that starting from the standard SM-Lagrangian and using a γ_5, which does not anticommute with the other Dirac matrices γ^μ, leads to "spurious anomalies" which violate chiral symmetry and hence gauge invariance. These anomalies would spoil renormalizability if we would not get rid of them by imposing "by hand" the relevant Ward-Takahashi (WT) [16] and Slavnov-Taylor (ST) [17, 18] identities order by order in perturbation theory [13, 19, 20, 21, 22]. At first sight this might not look to be a serious problem, however, violating the symmetries of the SM makes practical calculations much more difficult and tedious than they are anyway.

The problems of course are related to the existence of the Adler–Bell–Jackiw (ABJ) anomaly [23], which must cancel in the SM in order not to spoil its renormalizability [24].

Surprisingly, the prescriptions proposed and/or used by many authors continue to be controversial [9, 11, 13, 15, 20, 25, 26, 27, 28, 29, 30, 31, 32], and hence it seems to be necessary to reconsider the problem once again. We shall emphasize, in particular, the advantage of working with chiral fields. The consequences of working as closely as possible with chiral fields, it seems to me, has not been stressed sufficiently in the literature so far.

As a matter of principle it is important to mention two other approaches which both work in $D = 4$ dimensions. i) In quantum field theories on the lattice a recent breakthrough was the discovery of exact chiral invariance on the lattice [33] which circumvents the Nielsen–Ninomiya no–go theorem [34]. A well defined regularization which preserves simultaneously chiral–and gauge–symmetries is thus known and could be applied to the SM. ii) The algebraic renormalization of the electroweak SM to all orders [35] within the Bogoliubov-Parasiuk-Hepp-Zimmermann (BPHZ) framework is a mathematically well defined scheme, which is much more involved because it breaks the symmetries at intermediate stages and hence leads to much longer expressions which are extremely tedious to handle in practice. In cases of doubt this is the only known scheme which is free of ambiguities and works directly in 4–dimensional continuum field theory. For a detailed discussion of practical aspects of algebraic renormalization we refer to [36] and references therein.

For perturbative calculations in the continuum we have to stick as much as possible to the

more practical route of dimensional regularization. In the following tensor quantities in $D = 4$ dimensions are supposed to be defined by interpolation of $D = 2n$ ($n \geq 2$, integer) dimensions to dimensions below $D = 4$. It is well known that the γ–algebra, the so called "naive dimensional regularization" (NDR) *

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \cdot \mathbf{1} \ , \quad g^\mu{}_\mu = D \ , \quad \mathrm{AC}(\mu) \equiv \{\gamma^\mu, \gamma_5\} = 0 \tag{1}$$

for dimensions of space–time $D = 4 - 2\epsilon$, $\epsilon \neq 0$ is inconsistent with

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) \neq 0 \ . \tag{2}$$

The latter condition is often considered to be necessary, however, for an acceptable regularization since at $D = 4$ we must find

$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5) = 4\mathrm{i}\varepsilon^{\mu\nu\rho\sigma} \ . \tag{3}$$

Generally, for γ_5 odd traces one obtains trace conditions from the cyclic property of traces. They are not fulfilled automatically, as we shall see, and hence the algebra is ill-defined in general. Considering $\mathrm{Tr}(\prod_{j=0}^{4} \gamma^{\mu_j}\gamma_5\gamma^\alpha)$ cyclicity requires

$$\mathrm{Tr}(\prod_{j=0}^{4} \gamma^{\mu_j}\mathrm{AC}(\alpha)) - 2\sum_{i=0}^{4}(-1)^i g^{\alpha\mu_i}\mathrm{Tr}(\prod_{j=0,\, j\neq i}^{4} \gamma^{\mu_j}\gamma_5) = 0 \ . \tag{4}$$

Contraction with the metric tensor $g_{\alpha\mu_0}$ yields

$$2\,(g^\alpha{}_\alpha - 4)\ \mathrm{Tr}(\prod_{j=1}^{4} \gamma^{\mu_j}\gamma_5) + \mathrm{Tr}(\prod_{j=1}^{4} \gamma^{\mu_j}\mathrm{AC}(\gamma)) = 0 \tag{5}$$

with $\mathrm{AC}(\gamma) \equiv \gamma_\alpha\mathrm{AC}(\alpha)$. Thus $g^\alpha{}_\alpha = D \neq 4$ together with (2) implies $\mathrm{AC}(\mu) \neq 0$. However, non-anti-commutativity of γ_5 is in conflict with the chiral structure and hence with gauge invariance of the SM, in general. It is the purpose of this note to study the possibility of restoring gauge invariance by employing chiral fields systematically.

Before doing so, let me point out that the above purely algebraic argumentation is to naive if fact. Kinematic tensors, like the γ_5–odd traces, occur as factors of singular amplitudes and the isolated consideration of the individual factor is more than problematic. In other words, we must ask whether

$$\epsilon\ \mathrm{Tr}(\prod_{j=1}^{4} \gamma^{\mu_j}\gamma_5)\ \times\ \frac{1}{\epsilon} = \mathrm{Tr}(\prod_{j=1}^{4} \gamma^{\mu_j}\gamma_5)\ \text{ or }\ = 0\ \times\ \frac{1}{\epsilon} = 0\ ?$$

Of course, what is the correct answer to the question we learn by imposing the appropriate WT identities.

*I read it as "normal dimensional regularization"

2. Formally gauge invariant Feynman rules

Obviously only terms involving γ^μ in the standard SM Lagrangian can be affected by a non–anticommuting γ_5. As an example we consider the leptonic part, given by

$$\begin{aligned}\mathcal{L}_\ell &= \bar{\ell}_{\rm R} i\gamma^\mu \left(\partial_\mu + ig' B_\mu\right)\ell_{\rm R} + \bar{\nu}_{\ell{\rm R}} i\gamma^\mu \partial_\mu \nu_{\ell{\rm R}} \\ &+ \bar{L}_\ell i\gamma^\mu \left(\partial_\mu + i\frac{g'}{2}B_\mu - ig\frac{\tau_a}{2}W_{\mu a}\right) L_\ell \end{aligned} \tag{6}$$

using standard notation. As usual the chiral fields

$$\ell_{\rm R} = \Pi_+ \ell \ , \ \ \nu_{\ell{\rm R}} = \Pi_+ \nu_\ell \ , \ \ L_\ell = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_{\rm L} = \Pi_- \begin{pmatrix} \nu \\ \ell \end{pmatrix} \tag{7}$$

may be represented in terms of the lepton fields $\ell(x)$ and the neutrino field $\nu_\ell(x)$ with the help of the chiral projectors

$$\Pi_\pm \equiv \frac{1}{2}\left(\mathbf{1} \pm \gamma_5\right) . \tag{8}$$

In order that $\Pi_\pm$ are Hermitean projection operators γ_5 must have the properties

$$\gamma_5^2 = \mathbf{1} \ , \ \ \gamma_5^+ = \gamma_5 \ . \tag{9}$$

Furthermore, we demand $\Pi_\pm$ to be chiral projectors also for the adjoint $\bar{\psi} = \psi^+\gamma^0$ of a Dirac field ψ. This implies

$$\{\gamma^0, \gamma_5\} = 0 \ . \tag{10}$$

By Lorentz covariance in the 4–dimensional physical subspace the latter condition extends to

$$\{\gamma^\mu, \gamma_5\} = 0 \ \ \text{for} \ \ \ \mu = 0,1,2,3 \ . \tag{11}$$

It is easy to verify that $\mathcal{L}_\ell$ is invariant under local $SU(2)_{\rm L} \otimes U(1)_{\rm Y}$ gauge transformations, irrespective of $\mathrm{AC}(\mu) \neq 0$. Since the chiral fields have the simple transformation properties

$$\begin{aligned} L_\ell &\rightarrow \exp\{-i/2\left(g'\beta - g\tau_a\omega_a\right)\Pi_-\} L_\ell = \exp\{-i/2\left(g'\beta - g\tau_a\omega_a\right)\} L_\ell \\ \ell_{\rm R} &\rightarrow \exp\{-ig'\beta\Pi_+\}\ell_{\rm R} = \exp\{-ig'\beta\}\ell_{\rm R} \\ \nu_{\rm R} &\rightarrow \nu_{\rm R} \ , \end{aligned} \tag{12}$$

the invariance of $\mathcal{L}_\ell$ follows immediately from the properties of $\Pi_\pm$ alone.

We notice that in utilizing chiral fields there seems to be no conflict with the non–anti-commutativity of γ_5 and the formal validity of the ST–identities.

Usually, one prefers to write Feynman rules in terms of the Dirac fields ℓ and ν_ℓ. The standard Feynman rules are obtained using the relations

$$\bar{\psi}\Pi_\mp \gamma^\mu \Pi_\pm \psi = \bar{\psi}\gamma^\mu \Pi_\pm \psi \ , \tag{13}$$

which are valid only, provided $\mathrm{AC}(\mu) = 0$.

If $\mathrm{AC}(\mu) \neq 0$ in $D \neq 4$ dimensional space–time, the above relations no longer hold and hence the standard Feynman rules manifestly violate gauge invariance. The correct relations, replacing (13), read

$$\bar{\psi}\Pi_{\mp}\gamma^{\mu}\Pi_{\pm}\psi = \frac{1}{2}\bar{\psi}\ (\Gamma^{\mu} \pm \Gamma_5^{\mu})\ \psi\ , \tag{14}$$

with

$$\Gamma^{\mu} \equiv \gamma^{\mu} - \frac{1}{2}\mathrm{AC}(\mu)\gamma_5 = \frac{1}{2}\left(\gamma^{\mu} - \gamma_5\gamma^{\mu}\gamma_5\right) \tag{15}$$

and

$$\Gamma_5^{\mu} = \frac{1}{2}[\gamma^{\mu}, \gamma_5] = \Gamma^{\mu}\gamma_5\ . \tag{16}$$

We notice that by definition all Γ's are anticommuting with γ_5

$$\{\Gamma^{\mu}, \gamma_5\} \equiv 0\ . \tag{17}$$

According to (14) the proper expressions for the vector current and for the axial–vector current read

$$V^{\mu}(x) = \bar{\psi}\Gamma^{\mu}\psi = \bar{\psi}\gamma^{\mu}\psi - \frac{1}{2}\bar{\psi}\mathrm{AC}(\mu)\gamma_5\psi \tag{18}$$

and

$$A^{\mu}(x) = \bar{\psi}\Gamma^{\mu}\gamma_5\psi = \bar{\psi}\gamma^{\mu}\gamma_5\psi - \frac{1}{2}\bar{\psi}\mathrm{AC}(\mu)\psi\ , \tag{19}$$

respectively. It might be worthwhile to point out that the standard form of the axial current $\bar{\psi}\gamma^{\mu}\gamma_5\psi$ is not Hermitean when $\mathrm{AC}(\mu) \neq 0$. The above consideration also shows how anomalies may come about in the vector current when $\gamma_5\gamma^{\mu}\gamma_5 \neq -\gamma^{\mu}$.

The fermion kinetic term changes to

$$\bar{\psi}i\Gamma^{\mu}\partial_{\mu}\psi = \bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi - \frac{1}{2}\bar{\psi}i\mathrm{AC}(\mu)\gamma_5\partial_{\mu}\psi\ . \tag{20}$$

Correspondingly, the free massless fermion fields must satisfy the field equation

$$\left(\gamma^{\mu} - \frac{1}{2}\mathrm{AC}(\mu)\gamma_5\right)\partial_{\mu}\psi = 0\ . \tag{21}$$

This formally implies that the conserved canonical Noether currents are precisely the ones given above.

By the field equation the fermion spinors satisfy

$$\begin{aligned}\left(\not{k} - \frac{1}{2}\mathrm{AC}(k)\gamma_5 - m\right)\ u(k,s) &= 0\\ \left(\not{k} - \frac{1}{2}\mathrm{AC}(k)\gamma_5 + m\right)\ v(k,s) &= 0\end{aligned} \tag{22}$$

and the free fermion propagator reads ($\mathrm{AC}(k) \equiv k_{\mu}\mathrm{AC}(\mu)$)

$$S_F(k) = \frac{1}{\not{k} - \frac{1}{2}\mathrm{AC}(k)\gamma_5 - m + i0} = \frac{\not{k} - \frac{1}{2}\mathrm{AC}(k)\gamma_5 + m}{K^2 - m^2 + i0} \tag{23}$$

with

$$K^2 \equiv k^2 - \frac{1}{4}\mathrm{AC}(k)\mathrm{AC}(k)\ . \tag{24}$$

Formally, we have obtained chiral and gauge invariant Feynman rules for non-anticommuting γ_5. Eqs. (18), (19) and (23) replace the standard expressions valid for $\mathrm{AC}(\mu) = 0$.

3. Non-existence of a chirally invariant DR

The gauge invariant Feynman rules presented in the preceding section do not permit a regularization by continuation in the dimension D when $\mathrm{AC}(\mu)$ is chosen compatible with the trace condition (2). This can be proven as follows. First we consider the Dirac algebra extended to $D = 2n$ ($n \geq 2$, integer). In this case 2^n–dimensional representations of the γ–algebra are well known [8]. A basis for the algebra is given by the set of matrices $\mathbf{1}$, γ_5 and the antisymmetrized products $\gamma^{[\mu_1 \cdots \mu_p]}$ associated with p–dimensional subspaces of M_D. We will split the $SO(1, D-1)$ vectors (tensors) into 4–dimensional vectors $p^\mu_{\|} = \hat{p}^\mu$ ($\mu = 0, 1, 2, 3$), in the physical subspace M_4, and their orthogonal complements $p^\mu_\perp = \bar{p}^\mu$ ($\mu = 4, \ldots, D-1$). If we impose the trace condition (2) in the physical subspace (see Eq. (11) above) we obtain the 't Hooft–Veltman algebra [5]:

$$\mathrm{AC}(\mu) = \begin{cases} 0 & ;\ \mu = 0, 1, 2, 3 \\ 2\bar{\gamma}^\mu \gamma_5 & ;\ \mu = 4, \ldots, D-1 \end{cases} \tag{25}$$

with $\gamma_5 = \frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\hat{\gamma}^\mu\hat{\gamma}^\nu\hat{\gamma}^\rho\hat{\gamma}^\sigma$. Here, it is important to notice that $\mathrm{AC}(\mu)$ is a matrix of rank $\bar{\epsilon} \equiv D - 4$. The matrix–elements themselves are of order $O(1)$. As a consequence higher products of AC-terms are <u>not</u> of higher order in $\bar{\epsilon}$ for $D \to 4$. This is the reason why the extra terms needed to restore the Ward-Takahashi identities cannot be considered as perturbations. They affect the free part of the Lagrangian! and hence the form of the fermion propagators, as shown above. The symmetry at the end can only be there if the free and the interacting parts of the Lagrangian match appropriately.

We are now ready to reconsider the fermion propagator (23). Using (25), we get for the scalar product (24)

$$K^2 = k^2 - \bar{k}^2 = \hat{k}^2 \tag{26}$$

and thus

$$S_F(k) = \frac{\hat{\not{k}} + m}{\hat{k}^2 - m^2 + i0} \tag{27}$$

takes its 4–dimensional form, independent of D! It is then impossible to regularize fermion-loop integrals by continuation in D. The crucial point is that the consistency with the trace condition requires that in (24) the extra term proportional to $\mathrm{AC}(k)^2$ like AC is a matrix of rank $\bar{\epsilon} \equiv D - 4$ and not a correction of order $O(\epsilon^2)$ in the ϵ–expansion!

The problem may be reconsidered in terms of the Γ–algebra defined by (15), which may be associated to <u>any</u> γ–algebra:

$$\{\Gamma^\mu, \Gamma^\nu\} = 2g^{\mu\nu} \cdot \mathbf{1} - \frac{1}{4}\{\mathrm{AC}(\mu), \mathrm{AC}(\nu)\}\ ,\quad \{\Gamma^\mu, \gamma_5\} = 0\ . \tag{28}$$

For any Γ–algebra in order to be closed, we must require

$$\{\Gamma^\mu, \Gamma^\nu\} \ = \ 2G^{\mu\nu} \cdot \mathbf{1} \tag{29}$$

for some symmetric $D \times D$–matrix G, which satisfies

$$g^\mu{}_\nu G^{\nu\rho} = G^{\mu\rho}\ . \tag{30}$$

The trace condition (4) must hold with the replacements

$$(\gamma^\mu,\ g^{\mu\nu},\ \mathrm{AC}(\mu) \neq 0) \to (\Gamma^\mu,\ G^{\mu\nu},\ \mathrm{AC}(\mu) = 0) \tag{31}$$

which implies

$$g_{\mu\nu} G^{\mu\nu} = G^\mu{}_\mu = 4\ . \tag{32}$$

Assuming G to have block–diagonal form

$$G = \left(\begin{array}{c|c} \hat{g} & 0 \\ \hline 0 & \bar{g} \end{array}\right) \tag{33}$$

the condition (2) can be satisfied with a singular metric G only:

$$\bar{g} = 0\ ,\ \ G = \hat{g} \tag{34}$$

where $\hat{g}$ must be the Minkowski metric. Thus, starting from the 't Hooft–Veltman scheme, we are lead to a dimensional reduction (DRED) scheme [10] by adding just some terms in the Feynman rules which vanish in $D = 4$.

As a result, the Γ–form of the 't Hooft–Veltman algebra is identical to the 4–dimensional Dirac algebra. In other words, using the 't Hooft–Veltman algebra (in its D–dimensional form) together with the chiral fields, which are adapted to the gauge symmetry, "non-regularization" of fermion–loops is implied. Again, a regularization can only be obtained by giving up either the trace condition (2) or gauge invariance.

This last statement, of course, is not terribly new. What we have shown is that the Dirac algebra assuming anticommuting γ_5 on the one hand and the 't Hooft–Veltman algebra on the other hand are not really different, since the latter can always be rewritten in the anticommuting Γ–form by means of the relations (15) and (16). In any case, for theories involving γ_5, "dimensional regularization" compatible with (4), does not provide well–defined integrals for loops involving fermion lines. This has been noticed by 't Hooft and Veltman in their original paper [5] where they state: "the usual ambiguity of choice of integration variables is replaced in our formalism by the ambiguity of location of γ_5 in the trace". Statements to the contrary, frequently found in the literature, are misleading. Usually, extra "prescriptions" about where to put the γ_5 in a particular calculation are proposed. These prescriptions, however, do not resolve the problem of mathematical inconsistencies, i.e., they still require an explicit check and the restoration of the Ward-Takahashi identities.

The use of chiral fields provides an unambiguous rule for the proper location of the γ_5–matrices before generalization to $D \neq 4$. Unfortunately, this has lead to the "non-regularization" by dimensional continuation when the $D \neq 4$ trace condition (2) is imposed, which in turn essentially implies the 't Hooft–Veltman scheme.

If we violate gauge invariance by the naive application of the 't Hooft–Veltman prescription, we have to restore the symmetry by imposing the relevant Ward–Takahashi identities and fixing appropriate counter terms. But this precisely amounts to including the extra $\mathrm{AC}(\mu)$ terms given in Eqs. (18) and (19). Which in turn is nothing but another way of utilizing the naive anticommuting γ_5.

4. Conclusion for the practitioner

According to our considerations above we are left with two possible strategies:

i) $\mathrm{AC}(\mu) \neq 0$: the chirally improved 't Hooft–Veltman scheme

If we insist on the trace condition (2) the gauge invariance must be manifestly broken in order to obtain the "pseudo regularization" by dimensional continuation. Again we start at the level of the chiral fields but must avoid the non–regularization by treating the AC–terms in the free part of the Lagrangian as interaction terms, i.e., we use the standard D–dimensional Fermi propagator

$$S_F(k) = \frac{\not{k} + m}{k^2 - m^2 + i0} \tag{35}$$

together with the chiral currents (18, 19) as our "chiral Feynman rules". Since $\mathrm{AC}(\mu) \neq 0$, the choice of the Fermi propagator (35) amounts to adding the symmetry breaking term

$$\Delta\mathcal{L}_{\mathrm{SB}} = \frac{1}{2}\bar{\psi} i \mathrm{AC}(\mu)\gamma_5 \partial_\mu \psi = \bar{\psi} i \bar{\gamma}^\mu \partial_\mu \psi \tag{36}$$

to the Lagrangian. Besides the fact that this operator has no 4–dimensional representation, it is not a higher order term for $D \neq 4$ as it would be necessary for treating it as a counter-term (perturbation). Expanding $\Delta\mathcal{L}_{\mathrm{SB}}$ perturbatively amounts to the assumption that $\mathrm{AC}(\mu) = O(\epsilon)$ in the sense of matrix elements, which conflicts with (2). As we have mentioned earlier, (2) requires $\mathrm{AC}(\mu)$ to be a matrix of rank $\bar{\epsilon} = D - 4$ with matrix elements of order $O(1)$. A mathematically satisfactory way out of the dilemma within the framework of DR is not possible as a result of the existence of the ABJ–anomaly.

Our considerations show that "quasi gauge invariant" Feynman rules may be obtained for non-anticommuting γ_5 provided $\mathrm{AC}(\mu)$ is treated as a perturbation i.e. $\mathrm{AC}(\mu) = O(\epsilon)$. Examples are briefly considered in the Appendix. Results turn out to be AC–independent in this case. AC–invariance may be used as a helpful tool for checking the gauge invariance of fermionic loop contributions to amplitudes. Usually such checks are possible only by explicit consideration of WT- and/or ST-identities. We stress, once again, that any approach which treats the AC–term as a perturbation conflicts with the trace condition (2) at some point. Ignoring this point leads to "standard" confusions, frequently appearing in the literature. While working with the 't Hooft–Veltman prescription in the standard form requires the subsequent check of the Ward-Takahashi identities,after utilizing the chiral version of the Feynman rules we may restrict ourselves to check the hard anomaly diagrams.

Since amplitudes exhibiting spurious anomalies only may be chiralized either by our chirally improved Feynman rules or by imposing the Ward-Takahashi identities which makes them AC–invariant we obviously may directly choose the scheme $\mathrm{AC}(\mu) = 0$, which is our second and preferred option:

ii) AC(μ) = 0: the quasi self-chiral scheme

From a practical point of view an acceptable computational scheme should avoid spurious anomalies in the first place. This is possible only if the trace condition (2) is given up. Gauge invariance can be preserved then by using an anticommuting γ_5. This has been noticed in [7] (see also [25, 26, 28, 39]).

We observe that taking chiral fields seriously on a formal level, the only consistent way to avoid the above non-regularization is the simple one: use anticommuting γ_5 from the very beginning, i.e., choose the NDR algebra (1). Since $\Gamma^\mu \equiv \gamma^\mu$ in this case we do not get the non-regularization of the fermion propagators. The ABJ–anomaly must be considered separately as we are going to discuss now [†].

In the gauge invariant approach, closed fermion loops exhibiting γ_5 odd traces and hard anomalies, cannot be obtained by dimensional continuation, merely, γ_5 odd traces are to be considered as intrinsically 4-dimensional quantities. Since charge conjugation properties and the related Bose symmetry are not automatically satisfied one has to account left- and right-circulation of the fermions in closed loops separately. In any case Adler's approach [40] can be utilized to resolve the remaining ambiguities. For this purpose, let us briefly consider the ABJ–anomaly [23] exhibited by the current correlator $< 0|T\{V^\mu(x_1)V^\nu(x_2)A^\lambda(y)\}|0 >$ of two vector currents and an axial–vector current. The one–loop diagrams are shown in Fig. 1.

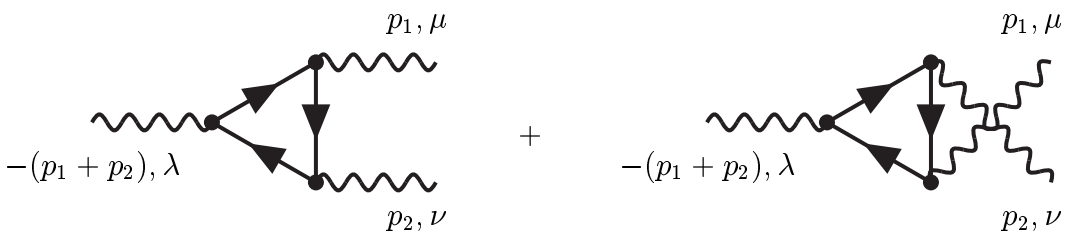

Figure 1: The VVA triangle diagrams.

In $D = 4$, working as usual in momentum space, we may perform a covariant decomposition of the third rank pseudotensor which depends on the two independent momenta p_1 and p_2:

$$\begin{aligned} A^{\mu\nu\lambda}(p_1,p_2) &= \varepsilon^{\mu\nu\lambda\alpha}\ (p_{1\alpha}\,A_1 + p_{2\alpha}\,A_2) \\ &+ \varepsilon^{\mu\lambda\alpha\beta}\,p_{1\alpha}p_{2\beta}\ (p_1^\nu\,A_3 + p_2^\nu\,A_4) \end{aligned}$$

[†]The terminology introduced in [9, 13] which calls a scheme "consistent" if it respects the trace condition (2) and "inconsistent" otherwise is definitely misleading by the considerations presented in this paper. Since we cannot satisfy the Ward-Takahashi identities and the trace condition simultaneously we have the choice which one we want to consider more fundamental. Something has to be restored at the end by hand in any case. To put into place the model independent ABJ–anomalies, is by far simpler, than restoring the chiral symmetry which is broken by non–NDR schemes.

$$
\begin{aligned}
&+ \quad \varepsilon^{\nu\lambda\alpha\beta}\, p_{1\alpha} p_{2\beta}\, \left(p_1^\mu\, A_5 + p_2^\mu\, A_6\right) \\
&+ \quad \varepsilon^{\mu\nu\alpha\beta}\, p_{1\alpha} p_{2\beta}\, \left(p_1^\lambda\, A_7 + p_2^\lambda\, A_8\right)
\end{aligned} \tag{37}
$$

where the amplitudes A_i are Lorentz scalars. We now impose

- Bose symmetry (i.e. consider the sum of the two diagrams of Fig. 1):

$$
A^{\mu\nu\lambda}(p_1, p_2) = A^{\nu\mu\lambda}(p_2, p_1)
$$

which implies

$$
\begin{aligned}
A_1(p_1,p_2) &= -A_2(p_2,p_1), \quad A_3(p_1,p_2) = -A_6(p_2,p_1), \\
A_4(p_1,p_2) &= -A_5(p_2,p_1), \quad A_7(p_1,p_2) = +A_8(p_2,p_1).
\end{aligned} \tag{38}
$$

- Vector current conservation:

$$
p_{1\mu} A^{\mu\nu\lambda} = p_{2\nu} A^{\mu\nu\lambda} = 0
$$

which implies

$$
\begin{aligned}
A_1 &= -\left(p_2^2\, A_4 + p_1 p_2\, A_3\right) \\
A_2 &= -\left(p_1^2\, A_5 + p_1 p_2\, A_6\right) \ .
\end{aligned} \tag{39}
$$

We thus find that the amplitudes A_1 and A_2 are determined uniquely in terms of the A_i, $i = 3, \ldots, 6$. The crucial observation, made by Adler long time ago [40], is that the amplitudes A_i, $i = 3, \ldots, 8$, have dimension $d_{\text{eff}} = 1-3 = -2$ and hence are represented by convergent integrals. In contrast, A_i, $i = 1, 2$, have dimension $d_{\text{eff}} = 1 - 1 = 0$ (logarithmically divergent) and thus require regularization and renormalization. However, imposing Bose symmetry and vector current conservation uniquely determines the two regularization/renormalization dependent amplitudes in terms of the other convergent and hence unambiguous ones, i.e., the result is unique without need to refer to a specific renormalization scheme. The divergence of the axial–vector current takes the form

$$
-(p_1 + p_2)_\lambda A^{\mu\nu\lambda} = 2mR^{\mu\nu} + 8\pi^2 p_{1\alpha} p_{2\beta} \varepsilon^{\alpha\beta\mu\nu} \neq 0
$$

where the first term on the r.h.s. is the normal term which vanishes for vanishing fermion mass m while the second term is the mass independent anomaly. Formal axial–vector current conservation in the limit of vanishing fermion mass would require

$$
A_1 - A_2 - (p_1^2 + p_1 p_2)\, A_7 - (p_2^2 + p_1 p_2)\, A_8 = 0 \ ?
$$

with A_1 and A_2 fixed already by vector current conservation, this expression as we know does not vanish but yields the famous axial–vector current anomaly. All true anomalies, i.e.,quantum effects like the triangle anomaly which cannot be removed by adding a corresponding counter term to the Lagrangian, are well known to be related to the triangle diagram. Besides the triangle diagram itself they appear by tensor reduction from one–loop box and pentagon diagrams and diagrams which contain the one–loop anomalous graphs as subgraphs.

The Adler–Bardeen non-renormalization theorem [37] of the one–loop anomalies implies that matters are under control provided Bose symmetry and vector current conservation are imposed, if necessary by hand. In DR it has been reconsidered in [38, 39]. Last but not least we must have the anomaly cancelation, possible by virtue of the quark lepton duality, in order to have the SM renormalizable [24].

Summary: we have shown that different γ_5–schemes may be related by adding suitable terms in the D–dimensional Lagrangian which vanish at $D = 4$. In any scheme we can mimic chiral fields by the appropriate choice of the Feynman rules. We consider this to be crucial since the physical SM derives via a Higgs mechanism from a symmetric phase which exhibits chiral fermions only. The corresponding "chiral completion" (see (18,19)) of the Feynman rules cannot make a consistent scheme inconsistent or vice versa. Avoidable (often called "spurious") anomalies are then absent. Our arguments strongly support the application of the NDR scheme (1), i.e., the D–dimensional γ–algebra together with a strictly anticommuting γ_5, together with the simple 4–dimensional treatment of the hard anomalies discussed above. The NDR is easily implemented into computer codes and is by far the most convenient and efficient approach in calculations of radiative corrections. Removable anomalies are avoided and hence a tedious procedure of restoration of WT- and ST-identities is not needed.

The rules advocated here have been utilized successfully in the last twenty years by many authors at the one– and the two–loop level and beyond. Most SM calculations of higher order effects adopted the NDR scheme without encountering any inconsistencies. Of course, the NDR scheme has been advocated by several authors [7, 11, 25, 26, 28, 39] (see also [30]) in the past. I hope the present paper contributes to clarify part of the ongoing controversy.

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Appendix: Calculations with $\mathrm{AC}(\mu) \neq 0$ in the SM: Two examples.

We [41] have verified explicitly that all spurious anomalies disappear from fermion propagators and fermion form factors at one-loop order for the case where we use Feynman rules as proposed in Sec. 4. in case $\mathrm{AC}(\mu) \neq 0$. As explained earlier, in order to avoid the "non-regularization" of fermion lines, we must treat AC as a perturbation $\mathrm{AC}(\mu) = O(\epsilon)$ and work to linear order in AC. All calculations have been performed in the 't Hooft gauge with an arbitrary gauge parameter ξ, which makes possible direct analytical checks of gauge invariance. We only summarize the structure of the results.

The irreducible self-energy $\Sigma(k)$ we obtained has the following form

$$\Sigma(k) = \left(\not{k} - m - \frac{1}{2}\mathrm{AC}(k)\gamma_5 \right) A + \frac{1}{2}[\not{k}, \gamma_5]\, B + mC \; . \tag{40}$$

This implies that the mass- and wave-function renormalization are completely AC–independent:

$$\delta m = -mc_0 \ , \ \ \sqrt{Z_2} = 1 - \frac{1}{2}a_0 - \frac{1}{2}b_0\gamma_5 \ . \tag{41}$$

Here the wave-function renormalization constant is given by the matrix

$$\sqrt{Z_2} = \sqrt{Z_R}\,\Pi_+ \ + \ \sqrt{Z_L}\,\Pi_- \tag{42}$$

where $\sqrt{Z_R}$ and $\sqrt{Z_L}$ are the independent wave–function renormalizations of the right–handed and left–handed fields, respectively. Thus the renormalized self–energy reads

$$\Sigma_r(k) = \left(\not{k} - m - \frac{1}{2}\mathrm{AC}(k)\gamma_5\right)\ (A - a_0) + \frac{1}{2}[\not{k}, \gamma_5]\ (B - b_0) + m\ (C - c_0) \tag{43}$$

with $A - a_0$, $B - b_0$ and $C - c_0$ finite, and hence

$$\Sigma_r(k) = (\not{k} - m)\ (A - a_0) + \not{k}\gamma_5\ (B - b_0) + m\ (C - c_0) + O(\epsilon) \tag{44}$$

By contrast, using standard Feynman rules, we obtain

$$\begin{aligned} \Sigma(k) \ &= \ (\not{k} - m)\ A + \frac{1}{2}[\not{k}, \gamma_5]\ B + mC \\ &+ \ \mathrm{AC}(k)\gamma_5\ D + [\not{k}, \mathrm{AC}(\gamma)]\ E + [\not{k}, \mathrm{AC}(\gamma)]\ \gamma_5\ F \end{aligned} \tag{45}$$

for the bare self-energy. In this case it is not possible to perform the renormalization in the standard way without imposing the Ward-Takahashi identities first, which must lead to the form (40).

Similar results can be found for form-factors. The following applies to the $\bar{\ell}\ell\gamma$ and $\bar{\ell}\ell Z$ vertices. The general form of the irreducible vertices reads

$$\Pi^\mu(p_1, p_2) = \left(\gamma^\mu - m - \frac{1}{2}\mathrm{AC}(\mu)\gamma_5\right)\ F_1 + \frac{1}{2}[\gamma^\mu, \gamma_5]\ F_2 + p_1^\mu F_3 + p_2^\mu F_4 \ . \tag{46}$$

We notice that the only surviving AC–term is $\mathrm{AC}(\mu)\gamma_5$ which appears in the canonical from (15) as in the Born term. Thus the vertex renormalization can be performed in an AC–independent way, i.e., the renormalized vertex is given by

$$\Pi^\mu_r(p_1, p_2) = \left(\gamma^\mu - m - \frac{1}{2}\mathrm{AC}(\mu)\gamma_5\right)\ (F_1 - c_1) + \frac{1}{2}[\gamma^\mu, \gamma_5]\ (F_2 - c_2) + p_1^\mu F_3 + p_2^\mu F_4 \tag{47}$$

with $F_1 - c_1$, $F_2 - c_2$, F_3 and F_4 finite. Hence, we have

$$\Pi^\mu_r(p_1, p_2) = \gamma^\mu\ (F_1 - c_1) + \gamma^\mu\gamma_5\ (F_2 - c_2) + p_1^\mu F_3 + p_2^\mu F_4 + O(\epsilon) \tag{48}$$

independent of any AC–term. In contrast, by applying standard Feynman rules, we find additional terms of the form $\mathrm{AC}(\mu)\gamma_5$, $[\gamma^\mu, \mathrm{AC}(\gamma)]$ and $\{\gamma^\mu, \mathrm{AC}(\gamma)\}\gamma_5$ which cannot be removed by renormalization, unless we impose the Ward-Takahashi identities first. In the chiral scheme we obtain gauge invariant form factors directly without imposing Ward-Takahashi identities by hand. Calculations in this "chiral" scheme in fact look very similar to the ones performed with anticommuting γ_5.

As a result of these findings we decided to work with an anti-commuting γ_5 henceforth, first at the one–loop level [42, 43], later at the two–loop level [44, 45, 46]. In most of these calculations we worked in the 't Hooft gauge with a free gauge parameter which allowed us to check explicitly the gauge invariance of on-shell matrix elements.

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