The effective fine structure constant and other SM running couplings

Fred Jegerlehner
DESY Zeuthen and Humboldt Univ. Berlin
fjeger@physik.hu-berlin.de
www-com.physik.hu-berlin.de/~fjeger/alphaQEDc19.tar.gz
*

December 2019 update
see also Flavorseparation.pdf

*for href's to work use evince or Adobe Reader for display

Abstract

Description of FORTRAN package `alphaQEDc19` [1] For latest changes read CHANGES and installation details INSTALLandRUN.
For recent updates go-to »2019 extensions and updates« below.

A modified package for calculating the effective electromagnetic fine structure constant is available in real (for space-like regime) and complex (for time-like regime) version. It provides statistical and systematic errors separately now. A number of functions have been renamed as `name.f -> namex.f` `name.f -> name19.f` in order not to conflict with the alphaQED16 from 20/12/2016, which provided total error only for the hadronic shifts `der` for $\Delta\alpha^{(5)}_{\rm had}(s)$ and `deg` for $\Delta\alpha^{(5)}_{2\,\rm had}(s)$ provided by `hadr5n.f`. The World Average (WA) compilation for $R(s)$ provided by `Rdat_fun.f` always included statistical and systematic errors separately (also in earlier versions). Newly I also include an integration routine intRdatx.f, which allows you to calculate weighted integrals of $R(s)$ with appropriate error estimates.

[1] For more background see and the more recent contribution to arXiv:1905.05078

Note that R^n data are sorted in typical energy ranges $n = 1, ...$, originally corresponding to different generations of experiments. Data are considered to be correlated within these ranges and independent otherwise.

The covariance matrix for each range n is given by

$$C_{ij}^n = \begin{cases} (\delta_{i\,\mathrm{sta}}^n)^2 + (\delta_{i\,\mathrm{sys}}^n)^2 & \text{for} \quad j = i \\ \delta_{i\,\mathrm{sys}}^n \cdot \delta_{j\,\mathrm{sys}}^n & \text{for} \quad j \neq i \end{cases} , \quad i, j = 1, \cdots, N_n ,$$

where $\delta_{i\,\mathrm{sta}}^n$ and $\delta_{i\,\mathrm{sys}}^n$ denote the statistical and systematic error, respectively, of R_i^n, where i labels the data points. This has been available in previous versions of `alphaQED` since 2009 and later.

List of main routines: cp Makefile19c ==> Makefile

Rdat_fun.f	$R(s)$	
test_Rdat.f	test of Rdat_fun.f	Makefile_Rdat
hadr5n19.f	$\Delta\alpha^{(5)}_{\rm had}(s)$ & $\Delta\alpha^{(5)}_{2\,\rm had}(s)$	
alphaQEDr19.f	$\alpha(s)$ real and related	Makefile19r
alphaQEDc19.f	$\alpha(s)$ complex and related	Makefile
alpha2SMc19.f	$\alpha_2(s)$ complex and related	Makefile prog=alpha2SMc19
ACWMsin2theta.f	$\sin^2\theta_{\rm eff}(s)$ and related	Makefile prog=ACWMsin2theta
intRdatx.f	integrals of $R(s)$: $a_\mu^{\rm had}$, $\Delta\alpha^{(5)}_{\rm had}(s)$, Euclidean corr.	Makefile_iRdx
amuviaEC.f	$a_\mu^{\rm HVP-LO}$ via Euclidean corr.	Makefile_amuviaEC

Drivers (providing run specific parameters if required):

test_Rdat.f	test_Rdat_gnu.sh	testRXYgnuplot.in
alphaQEDr19.f	./alphaQEDr19	
alphaQEDc19.f	alphaQEDc19_gnu.sh	alphaQEDgnuplot.in
alpha2SMc19.f	./alpha2SMc19	
ACWMsin2theta.f	./ACWMsin2theta	
intRdatx.f	intRdatx_clean.sh, intRdatx.sh, ECintR.sh	intRdat.inp, intRdat.inp0, ECintR.inp0
amuviaEC.f	./amuviaEC	

For a list of all fortran code files see: `routines.list`

Important note concerning VP subtraction: in obtaining the undressed $R(s)$ [imaginary part of $\text{Im}\,\Pi_\gamma(s)$]. For narrow resonances ω, ϕ, etc I use PDG parameters for masses and width (which are the physical ones) and one has to

undress the corresponding physical Breit-Wigner and the undressed BW has of course different parameters. However, this is not unambiguous. I have performed VP subtraction using space-like $\alpha(-s)$ because for narrow resonances iterating VP subtraction with the time-like $\alpha(s)$ does not converge for some J/ψ and Υ resonances and as long as one is interested in integrals like $a_\mu^{\rm had}$ away from any resonance position the effect from over and under-shooting at resonances should cancel out. This can be checked for the cases where the time-like subtraction iteration converges. I have always pointed out that local resonance shapes are not unambiguous and for any change of the resonance parameters the calculation has to be redone from scratch in principle. Another difference one obtains if one is using the resonance data directly and not the PDG. I decided to trust the PDG parametrization in my analysis. Some of the points I have discussed in my Capri talk, where I also say what precisely I have done. For details also see:
F. Jegerlehner, *The Anomalous Magnetic Moment of the Muon*, Springer Tracts Mod. Phys. **274**, pp.1 (2017); arXiv:1705.00263 [hep-ph] (Frascati Proc.); EPJ Web Conf. **118**, 01016 (2016) (Capri Proc.)

Note: in case you only need the hadronic shifts $\Delta\alpha^{(5)}_{\rm had}(s)$ and/or $\Delta\alpha^{(5)}_{2\,\rm had}(s)$ you only need to compile and link

hadr5x19.f as standalone version including all tables

to your computer code.

Download and Installation

First read INSTALLandRUN for latest implementation.

New: I include the rhad FORTRAN package by Harlander and Steinhauser in the alphaQEDc19 package on a sudirectory ./rhad. This should simplify including pQCD $R(s)$ results where needed. This supersedes instructions concerning rhad implementation given elsewhere.

Download link: ⁎⋙ [alphaQEDc19.tar.gz]

➠ alphaQEDc19 [FUNCTION funalpqedcx] providing the real and imaginary parts of the subtracted photon vacuum polarization including hadronic, leptonic and top quark contributions as well as the weak part (relevant at ILC energies)

I recommend to utilize the `alphaQEDc19` package which provides $\alpha(s)$ and $\alpha_2(s)$ as a complex function of the square c.m. energy scale s. The new feature is that statistical and systematic uncertainties are provided separately. In the previous versions only the World Average compilation of $R(s)$ included statistical and

systematic errors separately.

A much simpler real version `alphaQEDr19` is also provided. I recommend to use this version for space-like momentum transfer only, where $\alpha(s)$ and $\alpha_2(s)$ are real in any case.

From this R-compilation Tables for $\Delta\alpha^{\rm had}(s)$ and $\Delta\alpha_2^{\rm had}(s)$ have been calculated by means of a program intRdatx.f which now is included in the package.

The real parts to the hadronic contributions to vacuum polarization shift (renormalized VP function) are accessible by linking

hadr5x19.f as standalone version including all tables

In order to get the complex renormalized VP function or $\alpha(s)$ or $\alpha_2(s)$ the full alphaQEDc19 package must be installed.

alphaQEDc19 the complex version

This version includes an $R(s)$ compilation (representing the imaginary part) in addition to $\Delta\alpha^{\rm had}(s)$ (often taken to be real). This requires first to install the `rhad` package written by Harlander and Steinhauser (FORTRAN package version rhad-1.01 (March 2009 issue)). For actualization see INSTALLandRUN.

Download link: ⁎⋙ [rhad-1.01.tar]

- install rhad-1.01 → ../rhad-1.01/ directory
- untar file `alphaQEDc19.tar.gz`
- copy files `rqcdHSn.f` to rhad subdirectory ../rhad-1.01/ and compile them [gfortran -c rqcdHSn.f]. The updated version includes the subroutine rqcdHS3x(...) which allows for flavor recombination needed for the complex version of `alpha2SMc19` providing $\Delta\alpha_2^{\rm had}(s)$.

- edit `Makefile19c`; adjust directories, load path [compiler?] etc.
- copy `Makefile19c` to `Makefile`
- make
- run ./alphaQEDc19 → writes results to files fort.1 - fort.4 (various real parts) and fort.11 - fort.14 (various imaginary parts)

  which you may plot with your favorite plot program[2]
- in user program use one of the function calls

[2] My plot-file headers contain the following information

```
number of lines following before first data entry
number data sets (curves)
number of data points
comment lines
```

data set

change the `intRdatx.f` code for the needs of your favorite plot program

Delta_alpha = cggvapx(s,cerror,cerrorsta,cerrorsys)

alpha_complex=funalpqedcx(s,cerror,cerrorsta,cerrorsys,contributionflag)

require to include in the user program the header code as commented in function routine `funalpqedcx.f`

Available variants: Main program `alphaQEDc19.f` and subroutine `alphaQEDcx_sub.f` to be called [Input: s ; $s = \pm E^2$; E in GeV]

```
call alphaQEDcx(s,cvpt_new,cvpt_hig,cvpt_low,alphac,alphah
 & ,alphal,alphact,alphaht,alphalt,calept,cahadr,caltop)
```

`cvpt_new`	$\Delta\alpha^{(5)}_{\rm had}(s)$ central value
`cvpt_hig`	$\Delta\alpha^{(5)}_{\rm had}(s)$ + total error
`cvpt_low`	$\Delta\alpha^{(5)}_{\rm had}(s)$ - total error
`alphac`	$\alpha(s)$ central value no top quark
`alphah`	high value
`alphal`	low value
`alphact`	$\alpha(s)$ central value incl. top quark
`alphaht`	high value
`alphalt`	low value
`calept`	$\Delta\alpha$ leptonic
`cahadr`	$\Delta\alpha$ hadronic 5 quarks
`caltop`	$\Delta\alpha$ top quark

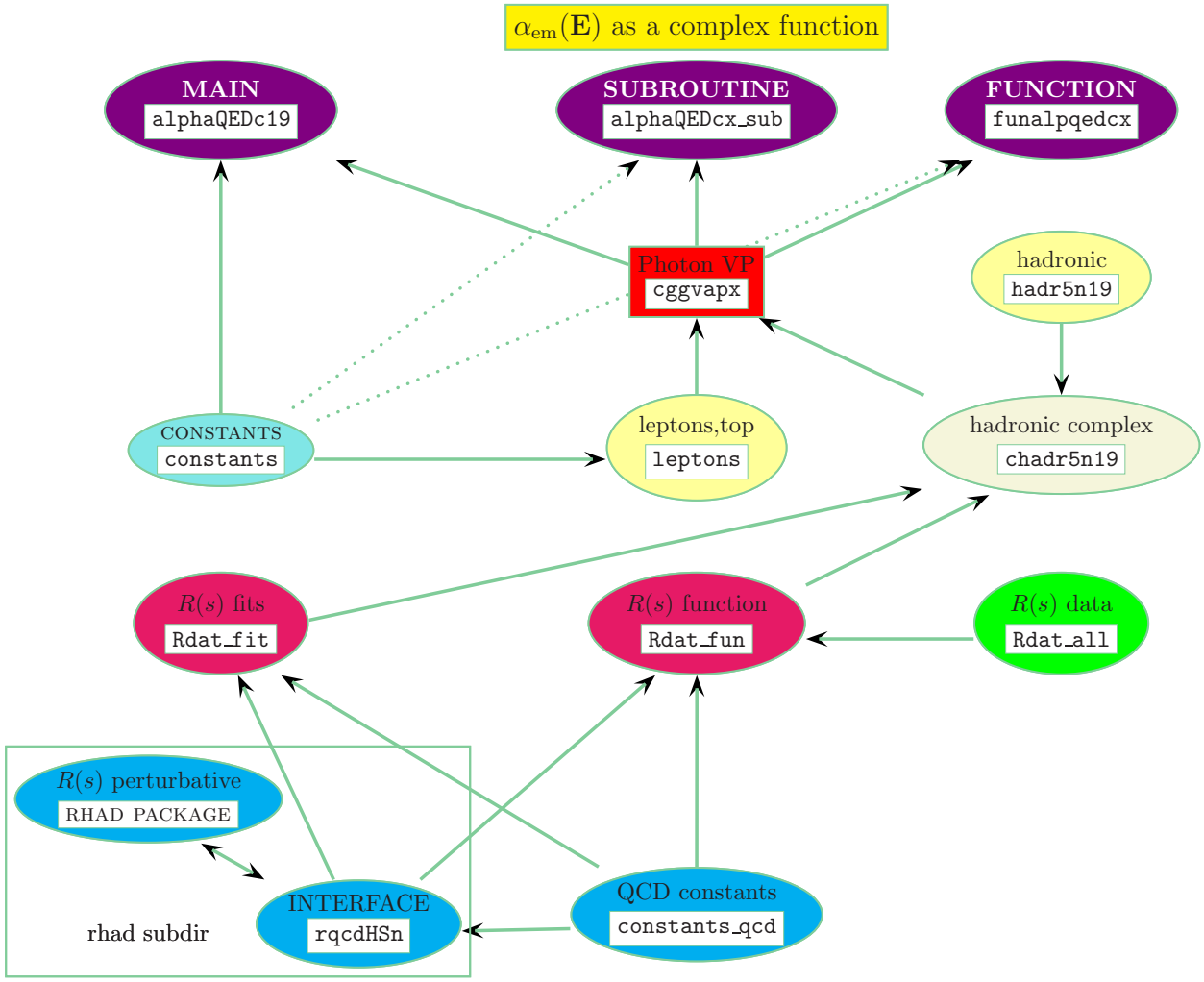
- copy and edit following header part from `funalpqedcx.f` (calling cggvapx) to your main program

```
********************* part to be included in main program (edited) **************
c check cuts CHPTCUT,EC,ECUT   ! specify where to use CHPT and pQCD in constants_qcd
      double precision pi1,ALINP,ERRAL,EINP,MTOP1 ! get QCD parameters as set in constants_qcd
c QCD parameters required to calculare imaginary part (R) and top contribution
      include 'common.h'
      common/var1/pi1,ALINP,ERRAL,EINP,MTOP1
      common /parm/st2,als,mtop
      COMMON/SMOO/Ismooth
c decomment next line if needed (provided also by cggvapx)
c      common /resu/dalept,dahadr,daltop,Dalphaweak1MSb   ! real results
      common /cres/calept,cahadr,caltop,cDalphaweak1MSb  ! complex results
      call constants()
      call constants_qcd()
c      sin2ell=0.23153 ! pm 0.00016 LEPEEWG Phys Rep 427 (2006) 257
c decommment next line and change default as appropriate in main
      st2=0.23153d0   ! Reference value for weak mixing parameter
      als=ALINP       ! alpha strong
      mtop=MTOP1      ! top quark mass
      Ismooth=0       ! flag for imaginary part: 0= R(s) data; 1= R(s) fits
c following two entried set as default in chad5n19.f
c      IRESON=1        ! include narrow resonances in fits
c      iresonances=1   ! include narrow resonances in data sets
      call Rdata()
c      call resonances_renoreal()
      call resonances_renocomplex()
```

```
C LEPTONflag=all,had,lep,ele,muo,tau -> iLEP=-3,-2,-1,1,2,3
C Default:
C      LEPTONflag='all'
C      iLEP  = -3  ! for sum of leptons + quarks
C decomment in main the following 2 lines for changing the default
c      LEPTONflag='all'   ! choose had,lep,top,ele,muo,tau
c      iLEP=LFLAG(LEPTONflag)
************************************************************************
```

For an extended version including resonances data see corresponding header of `alphaQEDc19.f`

The program structure is displayed in the following diagram:



More on rhad package: now already included in alphaQEDc19 see INSTALLandRUN!

Original settings:

Download the Harlander-Steinhauser program rhad from http://www.rhad.de/

expanding the tar file creates directory ./rhad-1.01/ with all files.

Copy the interfaces rqcdHSn.f and rqcdHS3x.f into that directory and compile it like the “example” provided with rhad package

.. by Robert V. Harlander and Matthias Steinhauser,

.. Comp. Phys. Comm. 153 (2003) 244,

.. CERN-TH/2002-379, DESY 02-223, hep-ph/0212294.

.. Changes: [March 30, 2009] v1.01

.. - exact results at α^4 included

choose option lverbose = .false. in parameters.f

you may also want to switch off warnings on non-perturbative regions

just before format statement 2003 in rhad.f, I use iwarn=0 flag to switch it off

```
if (iwarn.eq.1) then
   write(6,2003) dsqrt(scms)
endif
```

also change in parameters.f iunit = 6 to something not used by the `alphaQEDc19` package

alphaQEDr19 the real version

This version works without the $R(s)$ compilation and does not require the `rhad` package.

- untar file `alphaQEDc19.tar.gz`
- edit `Makefile19r`; adjust directories, load path [compiler?] etc.
- copy `Makefile19r` to `Makefile`
- make
- run ./alphaQEDr19 → writes results to files fort.1 - fort.4 and fort.11 - fort.14 (various real parts and detail results)
  which you may plot with your favorite plot program
- in user program use one of the function calls

```
Delta_alpha = dggvapx(s,error,errorsta,errorsys)
alpha_real=funalpqedrx(s,error,errorsta,errorsys,contributionflag)
```

which require to include in the user program the header code as commented in function routine `funalpqedrx.f`

Available variants: Main program `alphaQEDr19.f` and subroutine `alphaQEDrx_sub.f` to be called
[Input: s ; $s = \pm E^2$; E in GeV]

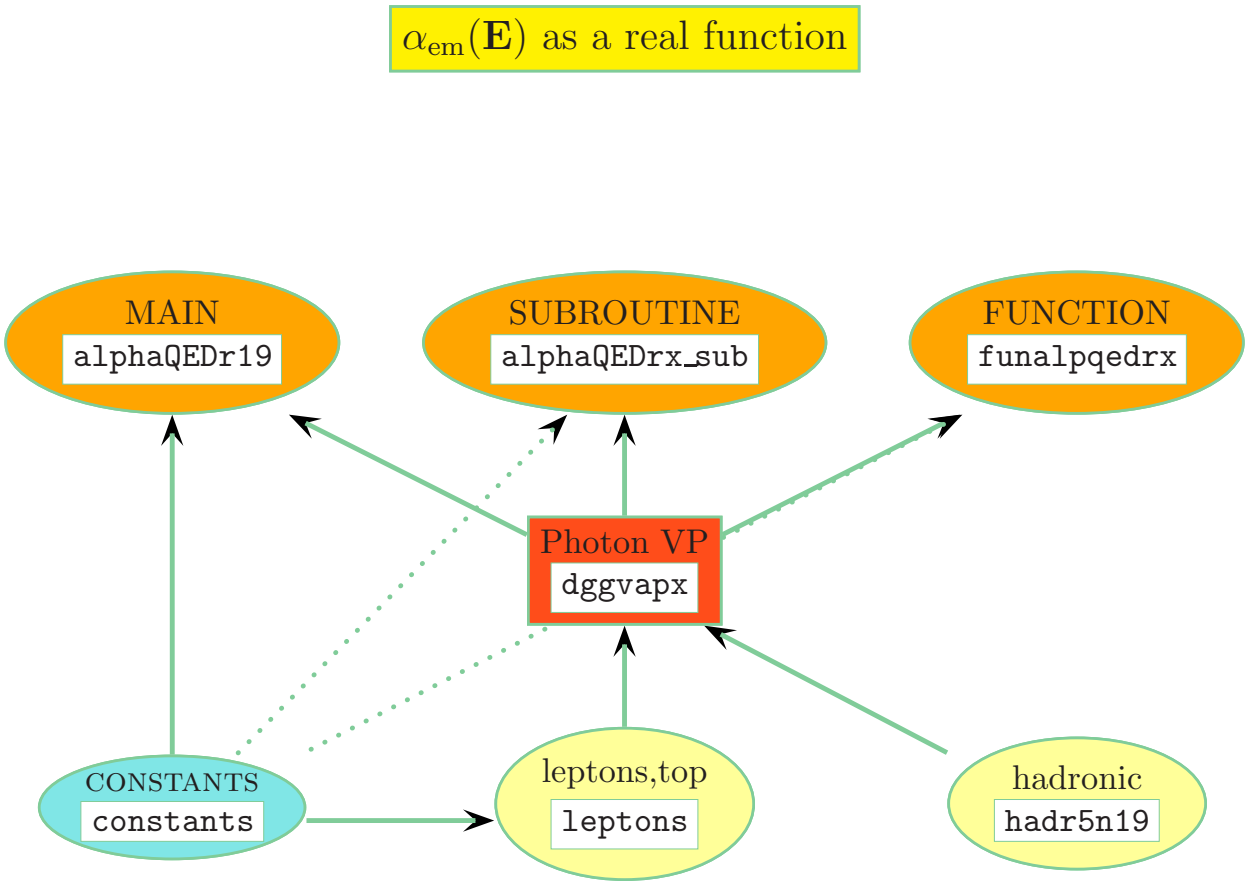
```
call alphaQEDrx(s,dvpt_new,dvpt_hig,dvpt_low,alphac,alphah
& ,alphal,alphact,alphaht,alphalt,dalept,dahadr,daltop)
```

`dvpt_new`	$\Delta\alpha^{(5)}_{\rm had}(s)$ central value
`dvpt_hig`	$\Delta\alpha^{(5)}_{\rm had}(s)$ + total error
`dvpt_low`	$\Delta\alpha^{(5)}_{\rm had}(s)$ - total error
`alphac`	$\alpha(s)$ central value no top quark
`alphah`	high value
`alphal`	low value
`alphact`	$\alpha(s)$ central value incl. top quark
`alphaht`	high value
`alphalt`	low value
`dalept`	$\Delta\alpha$ leptonic
`dahadr`	$\Delta\alpha$ hadronic 5 quarks
`daltop`	$\Delta\alpha$ top quark

- copy and edit following header part from `funalpqedrx.f` (calling dggvapx) to the main program

```
********************* part to be included in main program **************
      double precision pi1,ALINP,ERRAL,EINP,MTOP1    ! get QCD parameters as set in constants_qcd
c some QCD parameters required to calculare top contribution
      include 'common.h'
      common/var1/pi1,ALINP,ERRAL,EINP,MTOP1
      common /parm/st2,als,mtop
      common /resu/dalept,dahadr,daltop,Dalphaweak1MSb
      COMMON/IOSW/iOSW
      call constants()
      call constants_qcd()
c      sin2ell=0.23153 ! pm 0.00016 LEPEEWG Phys Rep 427 (2006) 257
      st2=0.23153d0   ! Reference value for weak mixing parameter
      als=ALINP       ! alpha strong
      mtop=MTOP1      ! top quark mass
C LEPTONflag=all,had,lep,ele,muo,tau -> iLEP=-3,-2,-1,1,2,3
C Default:
C      LEPTONflag='all'
C      iLEP  = -3  ! for sum of leptons + quarks
C Decomment in main the following 2 lines for changing the default
c      LEPTONflag='all'   ! choose had,lep,top,ele,muo,tau
c      iLEP=LFLAG(LEPTONflag)
*****************************************************************************
```

The program structure is displayed in the following diagram:



My alphaQED versus alpha2SM packages

Corresponding $SU(2)_L$ coupling $\alpha_2 = g^2/4\pi$ ➠ alpha2SMr19 and alpha2SMc19

Corresponding functions:

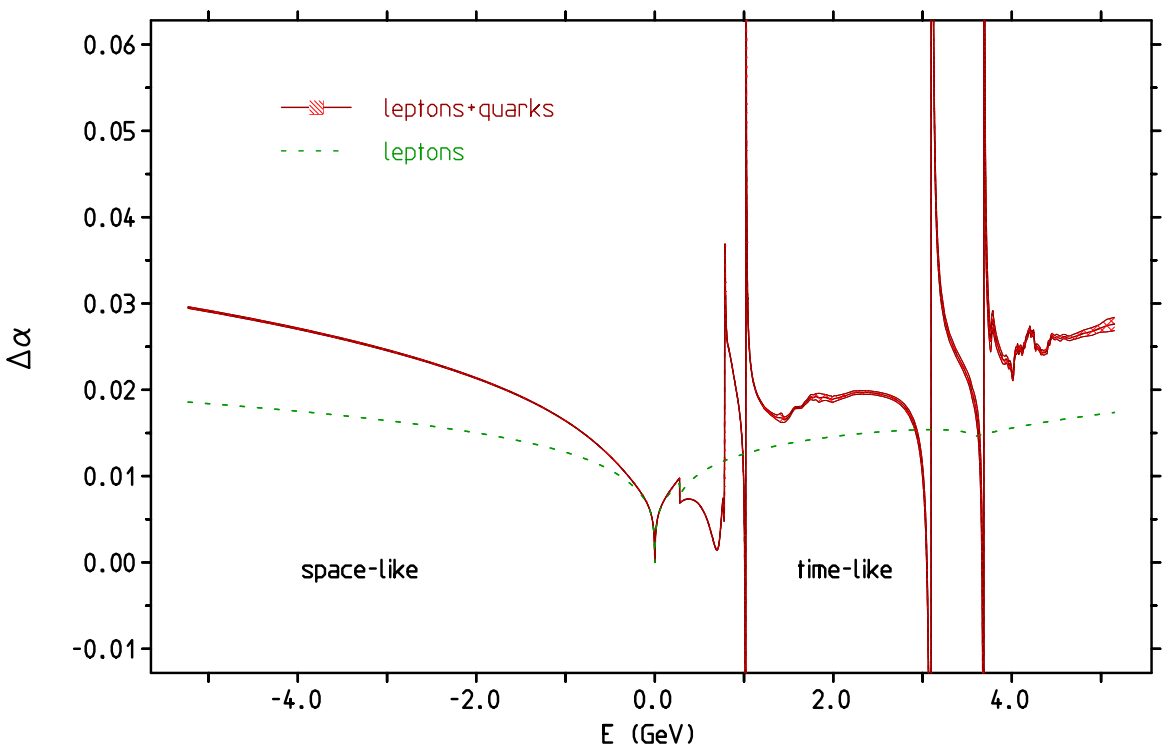
α	α_2	comment
dggvapx	degvapx	use in space-like region
cggvapx	cegvapx	use in time-like region
funalpqedcx	funalp2smcx	
alphaQEDc19	alpha2SMc19	

Delta_alpha2 = cegvapx(s,cerror,cerrorsta,cerrorsys)

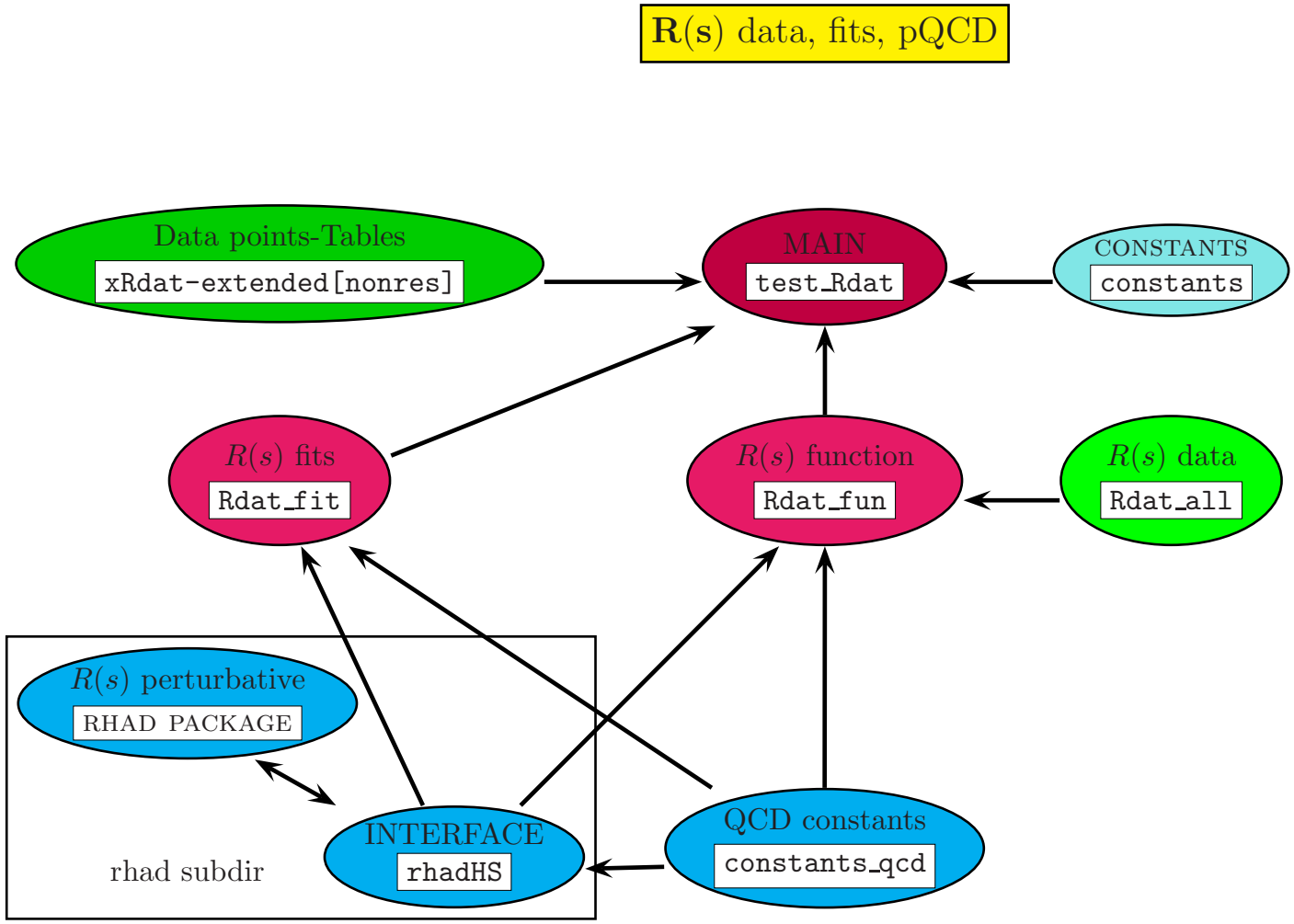
alpha2_complex=funalp2smcx(s,cerror,cerrorsta,cerrorsys,contributionflag)

provides in addition the corresponding imaginary parts.

Sample Plots:



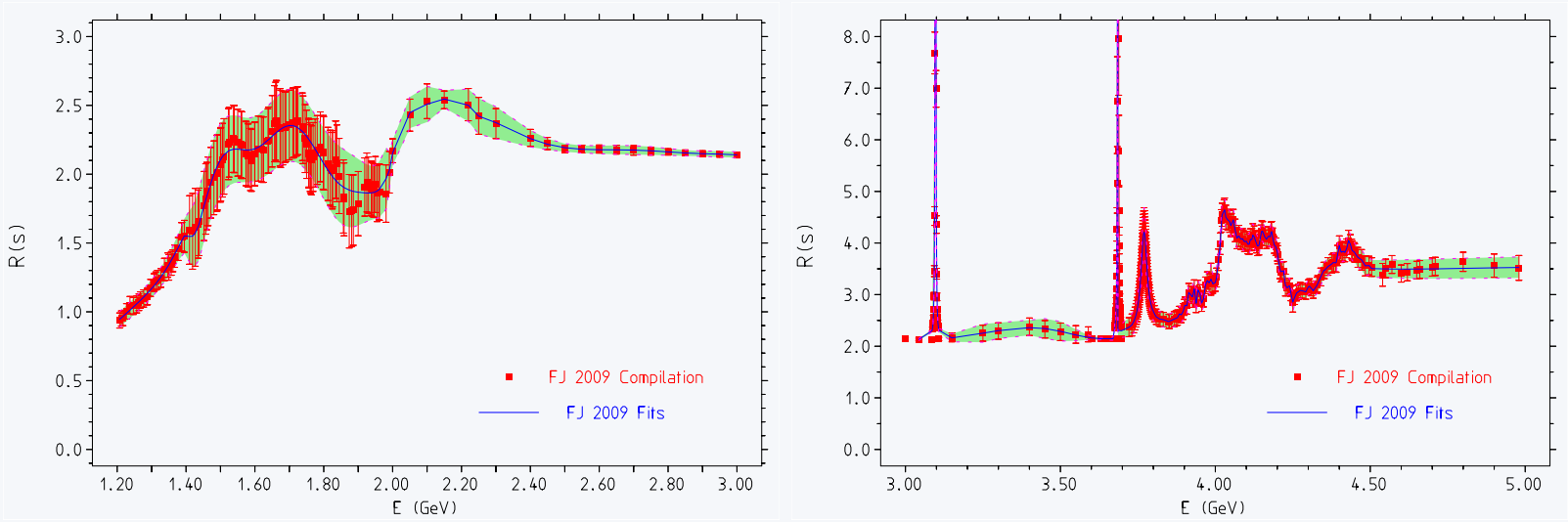
Shift of the effective fine structure constant $\Delta\alpha$ as a function of the energy scale in the time–like region $s > 0$ ($E = \sqrt{s}$) vs the space–like region $-s > 0$ ($E = -\sqrt{-s}$). The band indicates the uncertainties



Sample program `test_Rdat.f` for extracting $R(s)$ data, fits and pQCD calculation (using gnuplot preset from testRXYgnuplot.in):

```
make prog=test_Rdat
./test_Rdat_gnu.sh  emin emax iresonances IRESON  [default 0.689 1.06 1 1]
```

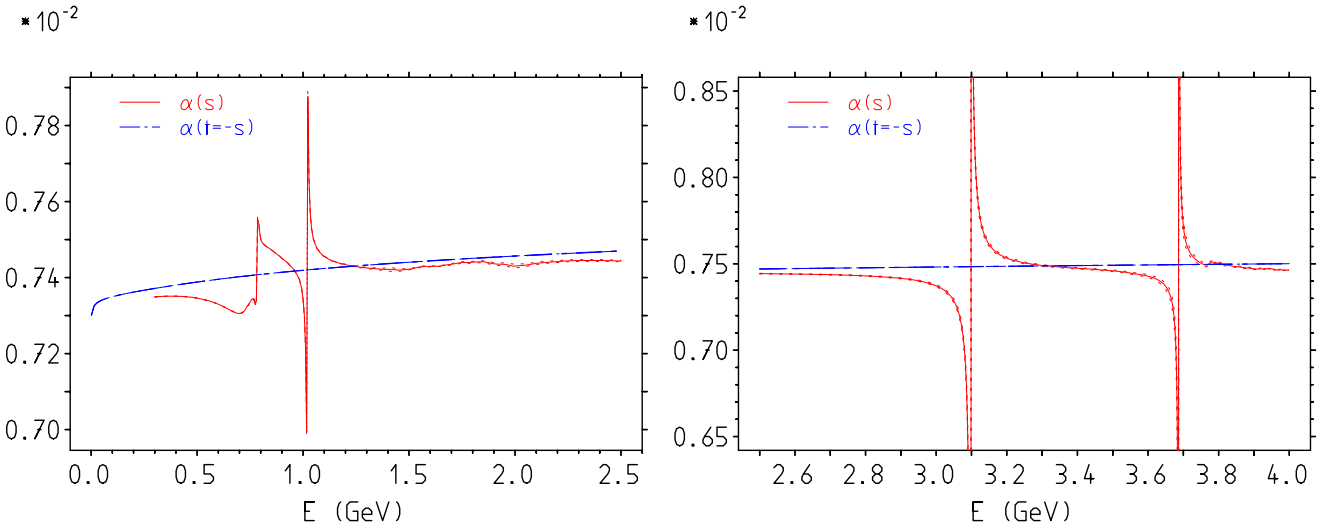
Sample results:



$R(s)$ $e^+e^- \to$ hadrons data vs. Chebyshev polynomial fits

[no fit for $\psi_3 \ldots \psi_6$ region yet]

The time-like vs space-like effective charge

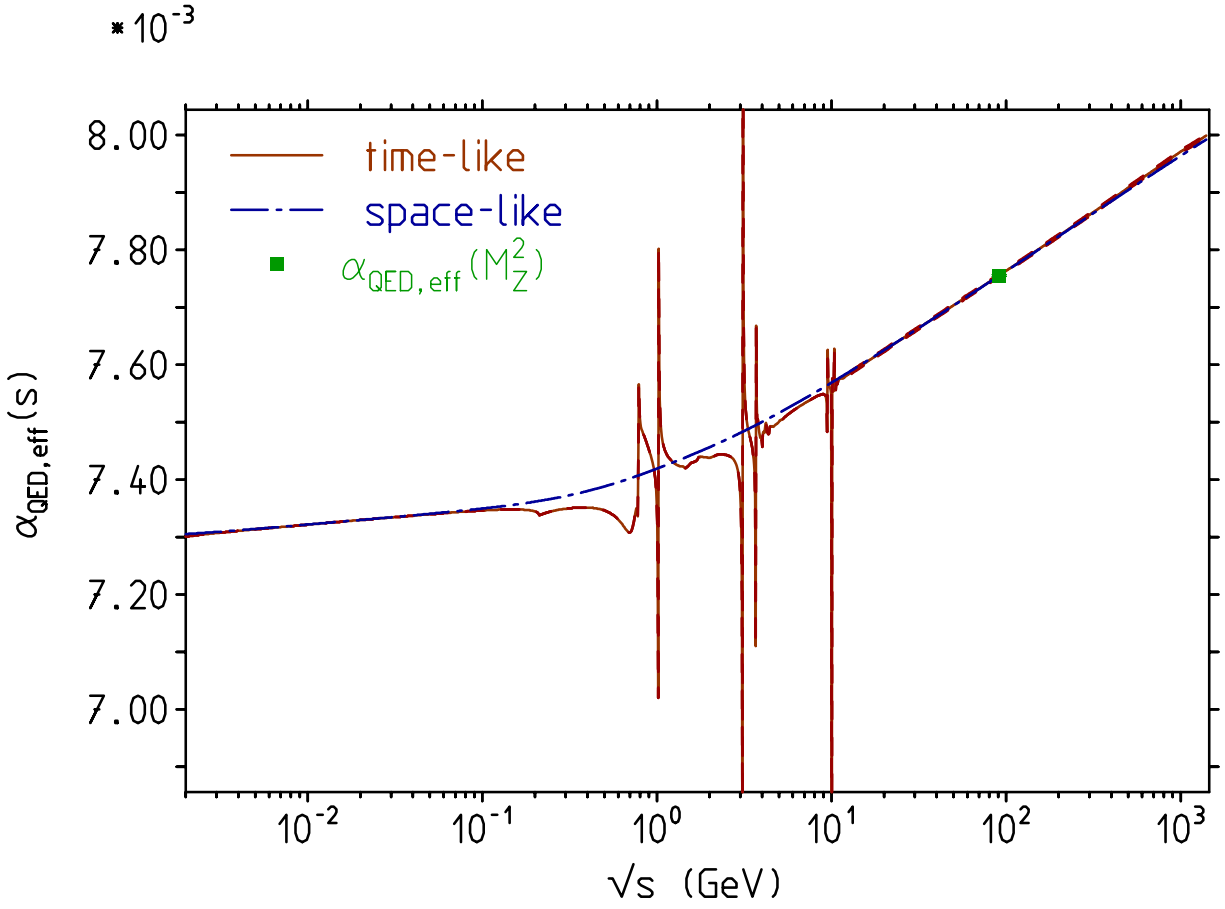


Note that the smooth space-like effective charge agrees rather well with the non-resonant "background" above the Φ (kind of duality)

No proof that this cannot produce non-negligible shifts!

Time-like VP-subtraction cannot be implemented locally near OZI suppressed resonances: $J/\psi, \psi'$ and $\Upsilon_1, \Upsilon_2, \Upsilon_3$

$\alpha_{\rm QED,eff}$: time-like vs. space-like



$\alpha_{\rm QED,eff}$ duality: $\alpha_{\rm QED,eff}(s)$ is varying dramatically near resonances, but agrees quite well in average with space-like version

The coupling α_2, M_W and $\sin^2\Theta_f$

How to measure α_2:

- charged current channel M_W ($g \equiv g_2$):

$$M_W^2 = \frac{g^2 v^2}{4} = \frac{\pi \alpha_2}{\sqrt{2} G_\mu}$$

- neutral current channel $\sin^2\Theta_f$

In fact here running $\sin^2\Theta_f(E)$: LEP scale $\Longleftrightarrow$ low energy $\nu_e e$ scattering

$$\sin^2\Theta_e = \left\{ \frac{1-\Delta\alpha_2}{1-\Delta\alpha} + \Delta_{\nu_\mu e,\text{vertex+box}} + \Delta\kappa_{e,\text{vertex}} \right\} \sin^2\Theta_{\nu_\mu e}$$

The first correction from the running coupling ratio is largely compensated by the ν_μ charge radius which dominates the second term. The ratio $\sin^2\Theta_{\nu_\mu e}/\sin^2\Theta_e$ is close to 1.002, independent of top and Higgs mass. Note that errors in the ratio $\frac{1-\Delta\alpha_2}{1-\Delta\alpha}$ can be taken to be 100% correlated and thus largely cancel.

Above result allow us to calculate non-perturbative hadronic correction in $\gamma\gamma$, γZ, ZZ and WW self energies, as

$$\begin{aligned}
\Pi^{\gamma\gamma} &= e^2 \quad \hat{\Pi}^{\gamma\gamma} \\
\Pi^{Z\gamma} &= \frac{eg}{c_\Theta} \quad \hat{\Pi}_V^{3\gamma} \quad - \quad \frac{e^2 s_\Theta}{c_\Theta} \quad \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{ZZ} &= \frac{g^2}{c_\Theta^2} \quad \hat{\Pi}_{V-A}^{33} \quad - \quad 2\,\frac{e^2}{c_\Theta^2} \quad \hat{\Pi}_V^{3\gamma} \quad + \quad \frac{e^2 s_\Theta^2}{c_\Theta^2} \quad \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{WW} &= g^2 \quad \hat{\Pi}_{V-A}^{+-}
\end{aligned}$$

with $\hat{\Pi}(s) = \hat{\Pi}(0) + s\,\hat{\pi}(s)$. Leading hadronic contributions:

$$\begin{aligned}
\Delta\alpha_{\rm had}^{(5)}(s) &= -e^2\,[\mathrm{Re}\,\hat{\pi}^{\gamma\gamma}(s) - \hat{\pi}^{\gamma\gamma}(0)] \\
\Delta\alpha_{2\,\rm had}^{(5)}(s) &= -\frac{e^2}{s_\Theta^2}\left[\mathrm{Re}\,\hat{\pi}^{3\gamma}(s) - \hat{\pi}^{3\gamma}(0)\right]
\end{aligned}$$

which exhibit the leading hadronic non-perturbative parts, i.e. the ones involving the photon field via mixing. $\Delta\alpha_{\rm had}^{(5)}(s)$ and $\Delta\alpha_{2\,\rm had}^{(5)}(s)$ via e^+e^--data and isospin arguments [(u,d), s flavor separation]:

$$\Pi_{ud}^{3\gamma} = \frac{1}{2}\,\Pi_{ud}^{\gamma\gamma} \quad ; \quad \Pi_s^{3\gamma} = \frac{3}{4}\,\Pi_s^{\gamma\gamma}$$

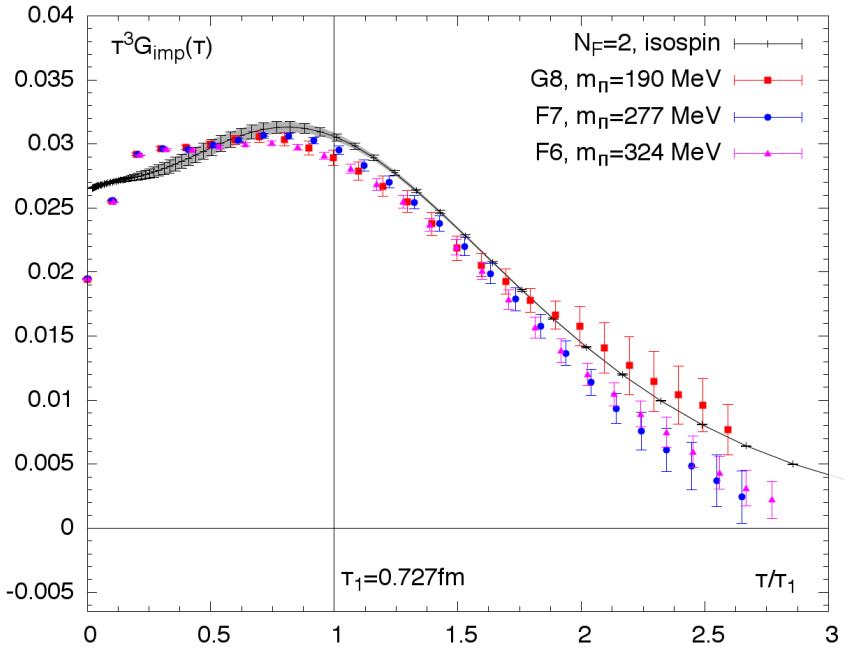
$$\Pi^{\gamma\gamma} = \Pi^{(\rho)} + \Pi^{(\omega)} + \Pi^{(\phi)} + \cdots \quad \Rightarrow \quad \Pi^{3\gamma} = \frac{1}{2}\,\Pi^{(\rho)} + \frac{3}{4}\,\Pi^{(\phi)} + \cdots$$

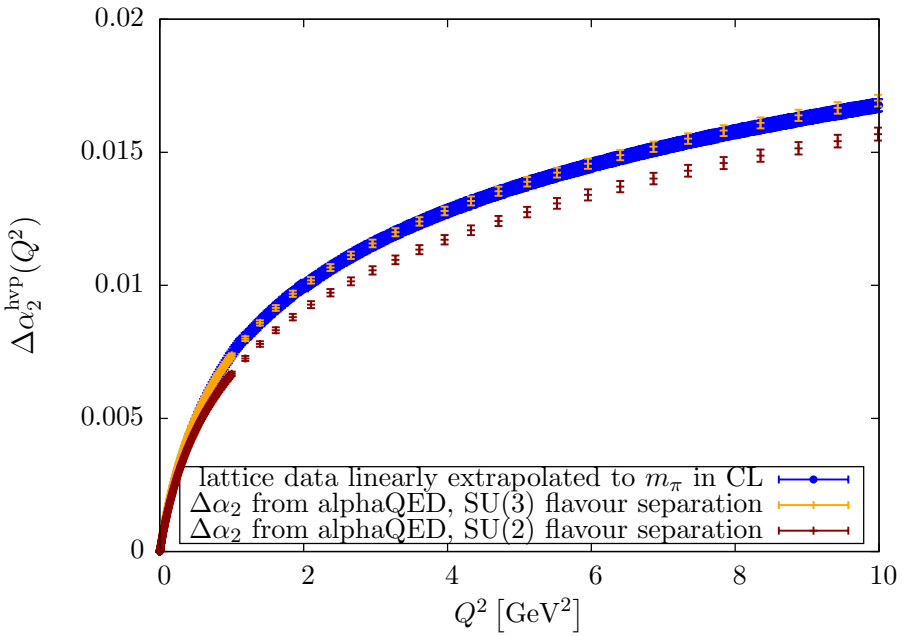
Flavor separation assuming OZI violating terms to be small $\Rightarrow$ perturbative rewighting $\Rightarrow$ disagrees with lattice QCD results!!!

Note that the "wrong" perturbative weighting

$$\Pi_{ud}^{3\gamma} = \frac{9}{20}\,\Pi_{ud}^{\gamma\gamma}\,; \quad \Pi_{s}^{3\gamma} = \frac{3}{4}\,\Pi_{s}^{\gamma\gamma}$$

has been proven to clearly mismatch lattice results, while the correction $\frac{9}{20} \Rightarrow \frac{10}{20}$ is in good agreement. This also means the OZI suppressed contributions should be at the 10% level and not negligibly small. For more details see.

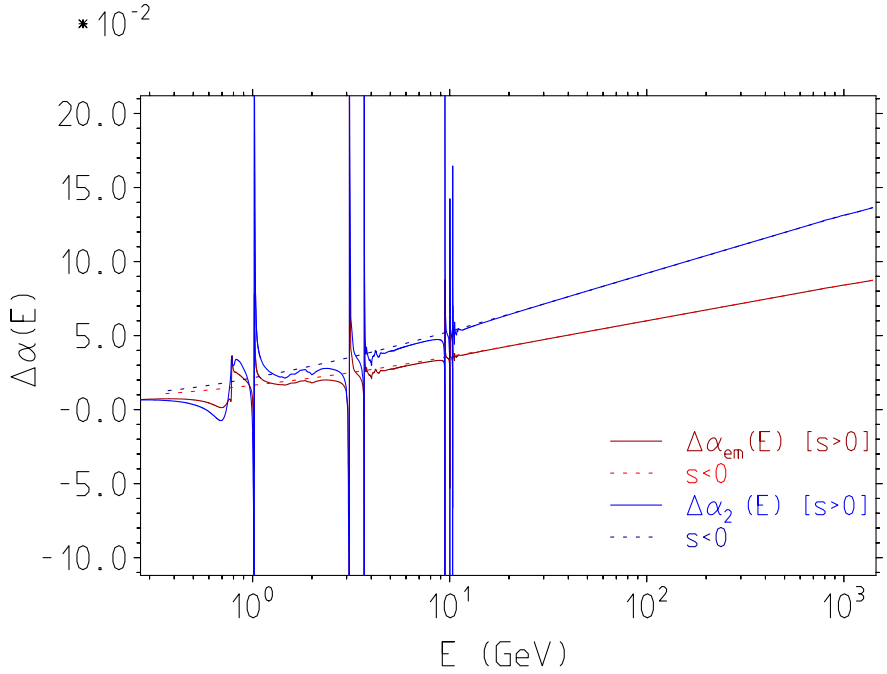




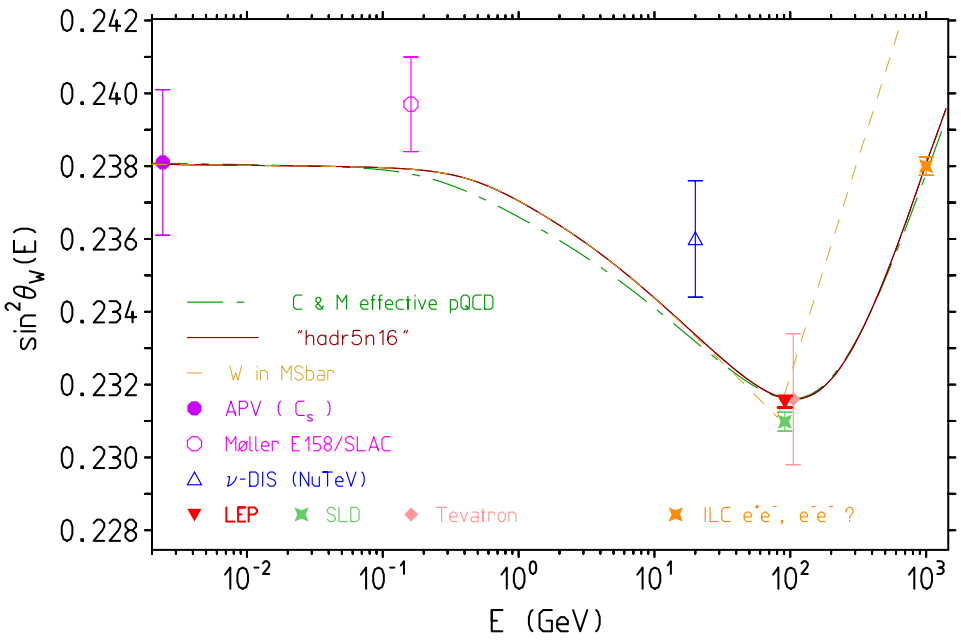
Testing flavor separation H. Meyer et al. [l], arXiv:1312.0035, K. Jansen et al. arXiv:1505.03283[r]

Note: gauge boson SE potentially very sensitive to New Physics (oblique corrections)

➠ new physics may be obscured by non-perturbative hadronic effects; need to fix this!

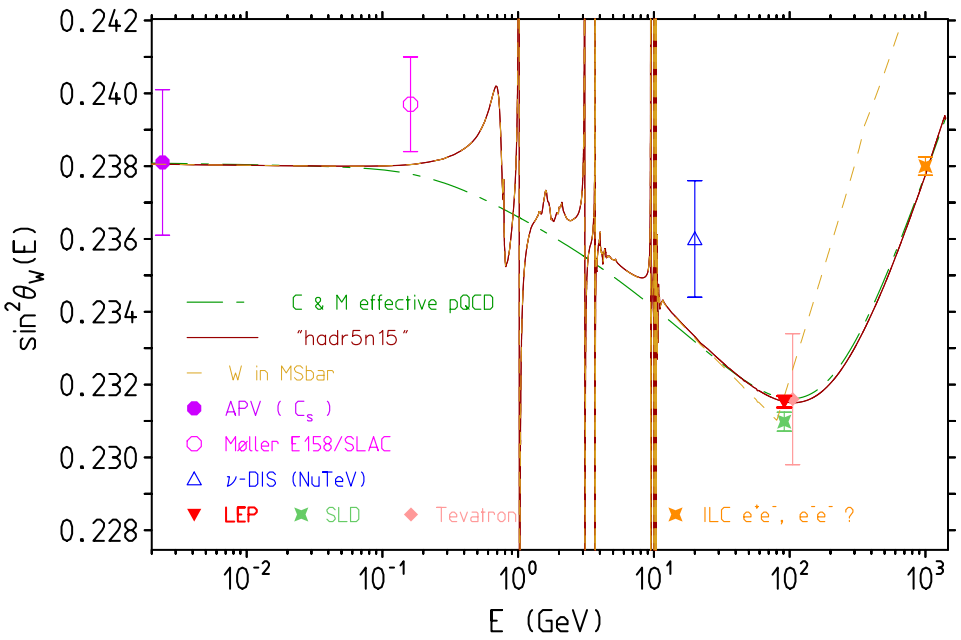


$\Delta\alpha_{\rm em}(E)$ and $\Delta\alpha_2(E)$ as functions of energy E in the time-like and space-like domain. The smooth space-like correction (dashed line) agrees rather well with the non-resonant "background" above the ϕ-resonance (kind of duality). In resonance regions as expected "agreement" is observed in the mean, with huge local deviations.



$\sin^2\Theta_W(Q)$ as a function of Q in the space-like region. Hadronic uncertainties are included but barely visible. Uncertainties from the input parameter $\sin^2\theta_W(0) = 0.23822(100)$ or $\sin^2\theta_W(M_Z^2) = 0.23153(16)$ are not shown. Future ILC/FCC measurements at 1 TeV would be sensitive to Z', H^{--} etc.

Except from the LEP and SLD points (which deviate by 1.8 σ), all existing measurements are of rather limited accuracy unfortunately!



$\sin^2\Theta_W(E)$ as a function of E in the time-like region. Note that $\sin^2\theta_W(0)/\sin^2\theta_W(M_Z^2) = 1.02876$ a 3% correction established at $6.5\,\sigma$.

$\sin^2\Theta_{\rm eff}$

exhibiting a specific dependence on the gauge boson SEs

is an excellent monitor for New Physics

Integrating $R(s)$

The `intRdatx.f` program may be used to calculate integrals of the hadronic cross section data:

Relevant files: intRdatx.f, intRdatx.inp, intRdatx.sh, Makefile_iRdx, intRdatx_clean.sh

Requires `NAG` library!

- edit `Makefile_iRdx`; adjust directories, load path [compiler?] etc.
- similarly customize `intRdatx.sh`
- run ./intRdatx.sh 0, which executes
  - make -f Makefile_iRdx

- ./intRdatx < intRdatx.inp

- Main results in files: dalhads.dat, intRdatx.tab

```
c
c*********************************************************************************
c*                                                                               *
c*    --- intRdatx ---                                Sun Jul 16 01:58:16 2017  *
c*                                                                               *
c*****************************# FJ@HU Berlin #*********************************
c
c  "Muon anomaly"                Update version (29/03/2017)
c
c ISCAN:  0, Ismooth: 0, IGS: 0, DSET: WAVE, iplot: 0
c IKE,Emin,Emax,NP,N,EPSREL:  0, 0.0000E+00, 0.0000E+00,   1, 10, 1.00E-02,
c ALS(sta)(sys):  0.1184( 0.0007)( 0.0000), MZINP:   91.1876, MTOP:    173.50
c NF,IOR,ICHK: 5,4,2,ISU2: 0,NFL: 5,INAG: 1,inotail:  0, sin2W:   1.00000
c CHPTCUT,EC,ECUT: 0.3180,     5.20,    11.50
c
 Result :   686.42     0.89     4.03     4.13
```

```
Timestamp:Sun Jul 16 01:58:16 2017

E:   0.0000000000000000


der contributions tab


  Chpt         & ( 0.28, 0.32) &      3.49 ( 0.13) ( 0.06)[ 0.14]&  4.2\% &
  $\rho$       & ( 0.32, 0.63) &    129.08 ( 0.54) ( 0.87)[ 1.02]&  0.8\% &
  $\omega$     & ( 0.42, 0.81) &     35.35 ( 0.29) ( 0.69)[ 0.75]&  2.1\% &
  $\phi$       & ( 1.00, 1.04) &     34.86 ( 0.28) ( 0.38)[ 0.47]&  1.4\% &
  $J/\psi$     & ( 3.10, 4.50) &      7.74 ( 0.41) ( 0.37)[ 0.55]&  7.1\% &
  $\Upsilon$   & ( 9.40,11.10) &      0.10 ( 0.00) ( 0.01)[ 0.01]&  7.4\% &
   had         & ( 0.63, 1.40) &    397.98 ( 0.33) ( 2.94)[ 2.95]&  0.7\% &
   had         & ( 1.40, 2.00) &     34.58 ( 0.25) ( 2.55)[ 2.56]&  7.4\% &
   had         & ( 2.00, 3.20) &     22.21 ( 0.09) ( 0.24)[ 0.25]&  1.1\% &
   had         & ( 3.20, 3.60) &      2.96 ( 0.01) ( 0.02)[ 0.03]&  1.0\% &
   had         & ( 3.60, 5.20) &      8.61 ( 0.04) ( 0.12)[ 0.13]&  1.5\% &
  pQCD         & ( 5.20, 9.50) &      6.29 ( 0.00) ( 0.01)[ 0.01]&  0.1\% &
```

```
 had        & ( 9.50,11.50) &      0.85 ( 0.00) ( 0.01)[ 0.01]&  1.5\% &
pQCD        &(11.5,$\infty$)&      1.96 ( 0.00) ( 0.00)[ 0.00]&  0.0\% &
data        & ( 0.28,11.50) &    677.80 ( 0.90) ( 4.09)[ 4.19]&  0.6\% &
total       &( 0.3,$\infty$)&    686.04 ( 0.90) ( 4.09)[ 4.19]&  0.6\% &


tab summary der contributions


der (err):  686.04386411195424    0.9047595475556616     4.0875945393348960
test der:   686.04386642475095      (   4.1865282033840847      )

Result :   686.04386411195424      (   4.1865282701653390      )
```

Before running `intRdatx` run `intRdatx_clean.sh` to remove old data files (some get saved)!!!

Running intRdatx using prefixed options:

```
./intRdatx.sh x
```

x=0,1,...,15; x=0 individual setting via `intRdatx.inp`

x=1,2,3 and 4,5,6 creates include files for `hadr5n19.f`

For Euclidean/space-like quantities, one also may specify the number of flavors to differ from $n_f = 5$ (either in intRdat.inp0 or) as a second parameter:

```
./intRdatx.sh x nf
```

nf=3,4,5 (note: nf=2 results are calculated in any case and written to intRdatx_n2.dap as plot-data.

flag		quantity	files created
0	=	input set from intRdatx.inp	`dalhads.dat, intRdatx.tab`
1	=	hadr5n time-like low der	`dalhadslow19_5.f`
2	=	hadr5n space-like der	`dalhadt19_5.f`
3	=	hadr5n time-like high der	`dalhadshigh19_5.f`
4	=	hadr5n time-like low deg	`deghadslow19_5.f`
5	=	hadr5n space-like deg	`deghadt19_5.f`
6	=	hadr5n time-like high deg	`deghadshigh19_5.f`
7	=	hadr5n time-like low deg3	`de3hadslow19_5.f`
8	=	hadr5n space-like deg3	`de3hadt19_5.f`
9	=	hadr5n time-like high deg3	`de3hadshigh19_5.f`
10	=	amuhad(K3c) 1st integral scan	`k3chadslow19.f input for 14,15`
11	=	hadr5n space-like high der	`dalhadthigh19_5.f`
12	=	hadr5n space-like high deg	`deghadthigh19_5.f`
13	=	hadr5n space-like high de3	`de3hadthigh19_5.f`
14	=	amuhad(K3c) 2nd integral	`k3chadres19.f`
15	=	amuhad(K3c) 2nd integral error	`k3chaderr19.f`

Options file: `intRdatx.inp`

CHPTCUT,EC,ECUT,ETAILCUT
ISCAN,Ismooth,IGS,DSET,iplot, ISU2 ,irennew, NFL ,INAG,inotail
IKE ,Emin,Emax,NP,N,EPSREL,XAXIS
ALS,EST,ESY,MZINP,MTOP,NF,IOR,ICHK
NNLO,IERK3C

Example:

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',0, 0 ,-1, 5 ,1,0
 0 ,1.0d0,1.d0,1,20,1d-2,'LIN'
0.1184,0.0007,0.000,91.1876,171.3,5,4,2
0, 0
# Muon g-2 LO
```

./intRdatx.sh 0 ==> dalhads.dat

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',1, 2 ,-1, 5 ,1,0
 1 ,-120.0d0,-0.d0,10,20,1d-2,'LIN'
0.1184,0.0007,0.000,91.1876,171.3,5,4,2
0, 0
# deg space-like -120.0 to 0.0
```

./intRdatx.sh 0 ==> intRdatx_3g.dap

more standard examples in `intRdatx.inp`

Channel 10 provides 1st integral for the double $R(s)$-integral for K3c NNLO amu kernel; channels 14 and 15 perform the second untegrals. 14 for the central value, 15 for the error (see chapter 5.1.13 of my book "The Anomalous Magnetic Moment of the Muon").

channel 10 read input data from intRdat.inp10 make sure it exist and exhibits the

proper settings

channels 14,15 read input data from intRdat.inp12 make sure it exist and exhibits the proper settings

```
' Kernel type IKE='
  0 (g-2) muon,
  1 (delta r),
  2 Adler function,
  3 (g-2) tau,
  4 (g-2) ele,
NNLO=0:                        NLLO=1:
  5 (g-2) muon HOa K2a,          5 (g-2) muon K3a,
  6 (g-2) muon HOb K2b,          6 (g-2) muon K3b,
  7 (g-2) ele  HOa K2a,          7 (g-2) muon K3LbL,
  8 Euclidean Correlators t^3*G_2(t)
  9 Euclidean Correlators t^4*G_3(t)
  10 Adler slope at s=0
```

```
 11 (g-2) muon K3c 1st integral
 12 (g-2) muon K3c 2nd integral
 13 (g-2) ele NNLO K3a

ISCAN=1,2,-1: use given X arrays (energy scan);
      1=time-like low,
      2=time-like high,
     -1=space-like
      0= choose
ISU2 =0[alpha],
      1[apha_2,old] obsolete,
      2[alpha_2,new],
      3[Pi33,old] obsolete,
      4[Pi33,new],
      old= SU(3) flavor splitting (obsolete),
      new= separate u,d [SU(2)]+ s from data
```

Flags:

variable	options	default/standard
1st line	CHPT and pQCD ranges	
CHPTCUT	CHPT tail from threshold to CHPTCUT	0.318d0
EC	pQCD window from EC to M_{Υ_1}	5.20d0
ECUT	pQCD above ECUT	11.500d0
ETAILCUT	upper limit dispersion integral $>$ 1.0d3	1.0d3
	cuts if set 0 = use default	
2nd line	options how to calculate?	
ISCAN	see list above	0
Ismooth	0=R data, 1=R fit	0
IGS	$\pi\pi$ channel: 0 data, 1 Gounaris-Sakurai parametrization	0
DSET	DSET=WAVE,CMD2 GSfit WA or CMD2 based	'WAVE'
iplot	table/plot output: iplot=0/1	0
ISU2	see list above	0
irennew	VP subtraction (resonances)	-1
NFL	number of flavors NFL=2,3,4,5?	5
INAG	SIMPSON OR NAG? INAG=0,1	0
inotail	pQCD tail for NF=2 inotail=0,1,-1 (notail=)	0

3rd line	Kernel, energy range, ...	
IKE	see list	
Emin,Emax,NP		
momentum transfer: E>0 time-like , E<0 space-like		
if NP =1 : table for E=Emin (Emax redundant)		
if E=Emin < Emax and NP > 1 data file for graph		
N	N number of subintervals for SIMPSON-Rule integration	20
EPSREL	relative error demanded for NAG-Routine integrals	1.0d-2
XAXIS	linear vs logarithmic energy axis	'LIN'
4th line	pQCD parameters	
ALS	$\alpha_s(\mu)$	$\alpha_s(M_Z)$
EST	stat error	
ESY	syst error	
MZINP	μ	91.1876d0
MTOP	top mass	173.21d0
NF	NF=5(no top), 6(with top)	5
IOR	IOR=1,2,3,4 order of pQCD	4
ICHK	ICHK=0,1,2 pQCD: 0(m=0), 1 (95) [CK], 2 (00) [HS]	2
	[CK] Chetyrkin, Kühn, [HS] Harlander, Steinhauser (rhad)	

5th line	NNLO flags	
NNLO	switch to NNLO for muon (g-2)	0
IERK3C	IERK3C=1 integrate VP, IERK3C=3 integrate error	1
6th line	if run via intRdatx.sh overwrite ISCAN and ISU2 as selected by script	
ISCAN		0
ISU2		0

Euclidean Correlators

make -f Makefile_iRdx

./intRdatx_clean.sh

./intRdatx < intRdatEC.inp1

intRdatEC.inp1=intRdatx.inp preset as:

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',1,  0  ,0,  3  ,1,0
 8 ,0.072d0,40.d0,200,20,1d-3,  'LIN'
0.1184,0.0007,0.000,91.1876,171.3,5,4,2
0,0
# Euclidean Correlators
```

Output data: intRdatx_3g.dap intRdatx_33.dap intRdatx_gg.dap intRdatx_n2.dap

Variants:	IKE	=	8,9	
	ISU2	=	0,2,4	i.e. $<\gamma\gamma>$,$<\gamma 3>$,$<33>$
	NFL	=	3,4,5,6	i.e. number of quark flavors
	NF	=	5,6	i.e. pQCD number of quark flavors
	XAXIS	=	'LIN','LOG'	

I also include a script file for EC's

./ECintR.sh ggxy where x=3,4,5 and y=lin,log

providing a set of standard parameters and producing

ECgg5lin.dap, ECgg4lin.dap, ECgg3lin.dap ECgg5log.dap, ECgg4log.dap, ECgg3log.dap

in the subdirectory ./dat/

Note: in order to get $\langle\gamma\gamma\rangle$ correlators for $N_f < 5$ one has to use the channel $\langle 3\gamma\rangle$, (based on flavorseparation and reweighting with results written to `intRdatx_3g.dap`,) in conjunction with the flag `itestflavorsplitting`=1 = flavorseparation without reweighting. This option is automatically selected when ISU2=0 and NFL<5

data files may be plotted with gnuplot

```
gnuplot

gnulpot> plot 'ECgg5lin.dap' skip 16 using 1:2

gnulpot> plot "ECgg5log.dap" skip 16 using 1:2:($2-$3):($2+$3) with yerrorbars ls 1
```

or

cd dat

gnuplot ECintR_gnu.in

Integrating the Euclidean Correlators

$$a_\mu^{\rm HVP-LO} = 4\,\alpha^2\,m_\mu \int\limits_0^\infty {\rm d}t\, t^3\, G(t)\,\overline{K}(t)$$

with kernel

$$\overline{K}(t) = \frac{2}{m_\mu\, t^3} \int\limits_0^\infty \frac{{\rm d}Q}{Q}\, f(Q^2) \left[(Q/E_0)^2 - 4\,\sin^2\left(\frac{1}{2}\,Q/E_0\right)\right]_{E_0=1/t}$$

and

$$f(s) = \frac{1}{m_\mu^2}\, rZ(r)^3\, \frac{1 - rZ(r)}{1 + rZ(r)^2}\;;\quad Z(r) = \frac{\left(\sqrt{r^2+4r} - r\right)}{2r}\;;\quad ;\quad r = s/m_\mu^2\;.$$

Using EC data include file `ECgg5oft.f`, created via "`./ECintR.sh gg5log`", the program `amuviaEC.f` calculates $a_\mu^{\rm HVP_LO}$:

make -f Makefile_amuviaEC

./amuviaEC

Output:
==> $685.58(1.2990)(4.8504) \times 10^{-10}$
from Euclidean time correlator for the HVP LO contribution (obtained as a 2-step integration) is in very good agreement with the result of the direct integration of $R(s)$
==> $686.04(0.9048)(4.0876) \times 10^{-10}$.
In order to create EC data file `ECgg5oft.f` use extended range option

```
-8,0.0072d0,100.d0,400,20,1d-2,'LOG'
```

in order not to miss contributions from tails.

Note: `ECgg5oft.f` is a copy of output file `dalhads.dat` from running `ECintR.f` (dalhads.dat requires some editing of the array boundary e.g. removing last entry)

A shell script `compare.sh` allows one to compare all fortran files on the current directory with the corresponding files in the directory supplied by the user.

2019 extensions and updates

********** 2019 *** NEW, NEW, NEW *** 2019 **********

see also CHANGES and INSTALLandRUN

In short:

basic datasets updated concern `Rdat_upd.f RGG0321.f, R3G0321.f, R330321.f, RN20321.f`

as well as hadr5n19.f including `dalhadshigh19_5.f dalhadslow19_5.f dalhadt19_5.f`,
which replace `dalhadshigh17.f dalhadslow17.f dalhadt17.f` of alphaQEDc17.

asked for by Kohtaroh Miura from Mainz University:
intRdatx.f extended to allow calculation of hadronic α and α_2 shifts for different flavors NFL=3,4,5 in Euclidean region for comparison with lattice VP calculations; intRdatx.sh adapted correspondingly

Set number of flavors NF in intRdat.inp0 then running
./intRdatx.sh 2 creates output for the different flavors `intRdatx_n3,4,5.dap`

with $\Delta\alpha_{\rm had}$ plot data

./intRdatx.sh 5 creates output for the different flavors `intRdatg3_n3,4,5.dap`

with $\Delta\alpha_{2\,\rm had}$ plot data

Fortran arrays representing Euclidean VP data can be created for different flavors as well by specifying NFL in `intRdat.inp0`

output dalhads.dat is copied to dalhadt19_3.f for NFL=3 etc. now

For the space-like low $\Delta\alpha_{\rm had}$ and $\Delta\alpha_{2\,\rm had}$ on may also run

./intRdatx.sh 2[5] 2[3,4,5]

where $2 provides the number of flavors NFL. This option [$2 set] reads input from `intRdat.inp1`

In details:

Updated R compilation

Region 1.2 to 2 GeV data updated; test-ground exclusive vs inclusive R measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty. Newly, all old data skipped, essentially only BaBar remains, see Fig.

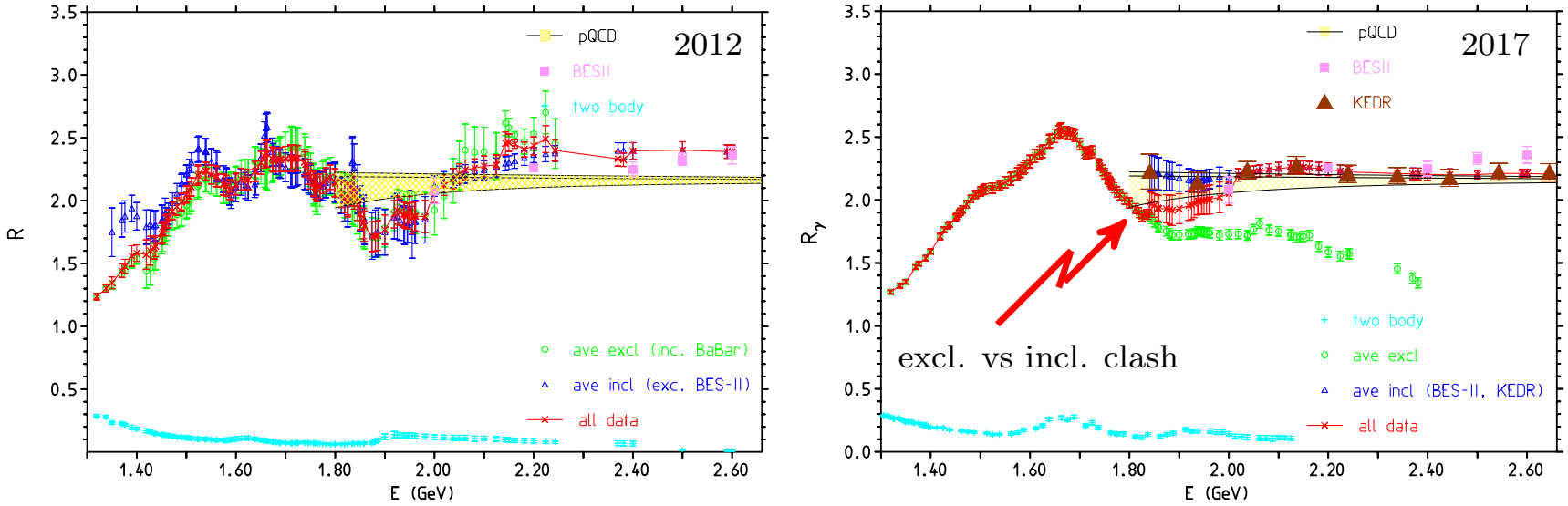


Figure 1: Illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

Accordingly:
all R-data files Rdat_upd.f included in Rdat_all.f and , R3G0321.f, RN20321.f, R330321.f RGG0321.f and all fit-parameter files had to be updated. This concerns all includefiles *.h entering the Rxxx_fit.f functions

Comment: As expected because of the resonant 4π channels there is a resonance enhancement (the $\rho(1700)$) overshooting pQCD predictions near 1.7 GeV. According to quark hadron duality, a compensating undershooting must follow at increasing energy before the pQCD level is expected to be reached. The exclusive data beautifully show a valley after the resonance enhancement as it should be. Therefore the lowest KEDR inclusive point seems to lie way to high, while the summed exclusive result misses reaching the pQCD level when approaching the 2 GeV region, where reliable inclusive measurements from BES II and KEDR exist. Taking a weighted average between exclusive and inclusive data seems to give a reasonable shape for $R(s)$ in the 2 GeV region.

Calculating Euclidean time correlators for different numbers of flavors

using the updated version of intRdatx.f

adapting intRdatx.inp

input is

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',0,0,-1,{\color{red}NFL},0,0
8,0.0072d0,100.d0,400,10,1d-2,'LOG'
0.1184,0.0007,0.000,91.1876,173.2,5,4,2
0 0
# Euclidean Correlators t^3*G_2(t)
```

where NFL=2,3,4,5

running via intRdatx.sh: `./intRdatx.sh 0`

each run produces dalhads.dat which is copied to fortran data tables ECgg2oft19.f, etc., where 2→3,4,5 as includefiles for amuviaEC.f

In order to get $a_{\mu\,\mathrm{had}}^{(5)}$ run amuviaEC.f

compile with Makefile_amuviaEC

amuviaEC.f now asks for number of flavors nf=2,3,4,5?

Calculating $\Delta\alpha^{\mathrm{had}}(-Q^2)$ for different numbers of flavors

using the updated version of intRdatx.f

$\Delta\alpha^{\mathrm{had}}(-Q^2)$ now available for flavor numbers NF=2,3,4,5

running via intRdatx.sh: `./intRdatx.sh 2`

writes results of space-like hadronic shifts to plot data files:

intRdatx_n2.dap, intRdatx_n3.dap, intRdatx_n4.dap, intRdatx_n5.dap

For crosscheck: der from hard5 may be tested by using testhadr5x [make prog=testhadr5x and ./testhadr5x writes result for der on fort.1 and for deg on fort.2].

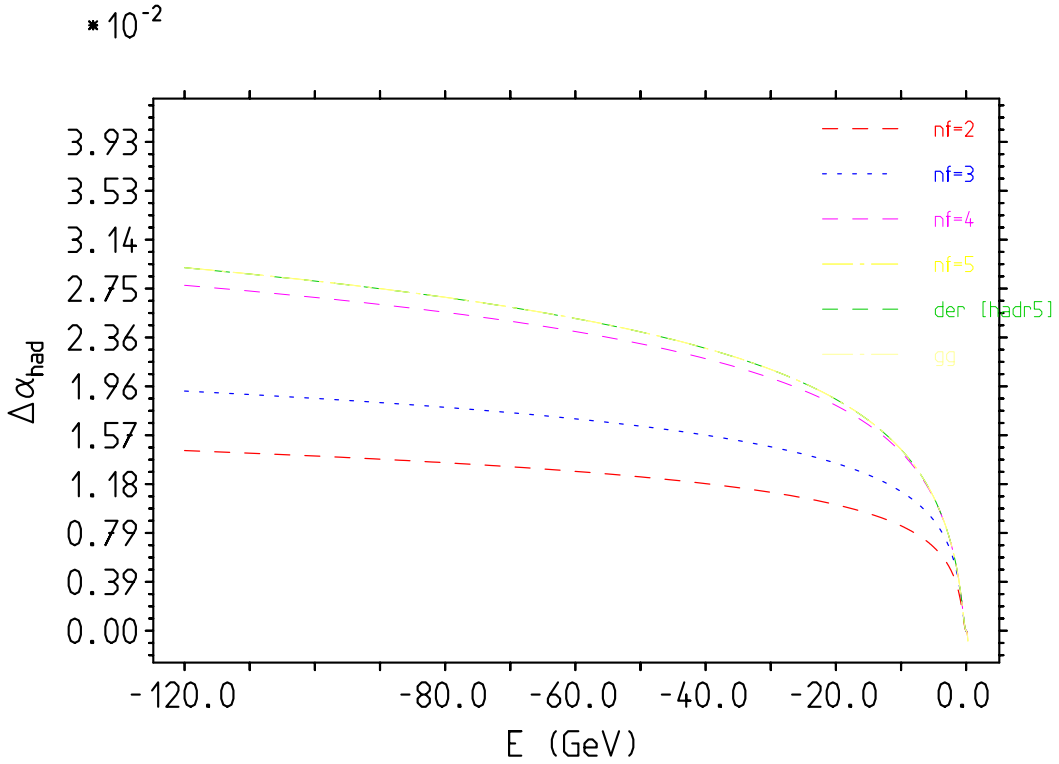


Figure 2: The renormalized Euclidean correlator $\langle\gamma\gamma\rangle(Q^2)$ denoted by $\Delta\alpha_{\rm had}(-E)$ for $n_f = 2, 3, 4, 5$ as a function of the space-like energy transfer denoted by $-E$.

Note that similarly testhadr5a tests the table entries of hadr5x19.f.

The full QCD $n_f = 6$ result, which besides $\Delta\alpha^{(5)}_{\rm had}$ includes the top-quark contributions, is available form alphaQEDc19.

Calculating $\Delta\alpha_2^{\rm had}(-Q^2)$ for different numbers of flavors

using the updated version of intRdatx.f

$\Delta\alpha_2^{\rm had}(-Q^2)$ now available for flavor numbers NF=2,3,4,5

running via intRdatx.sh: `./intRdatx.sh 5`

writes results of space-like hadronic shifts to plot data files:

intRdatg3_n3.dap, intRdatg3_n4.dap, intRdatg3_n5.dap

For crosscheck: get deg from hard5 using to testhadr5x

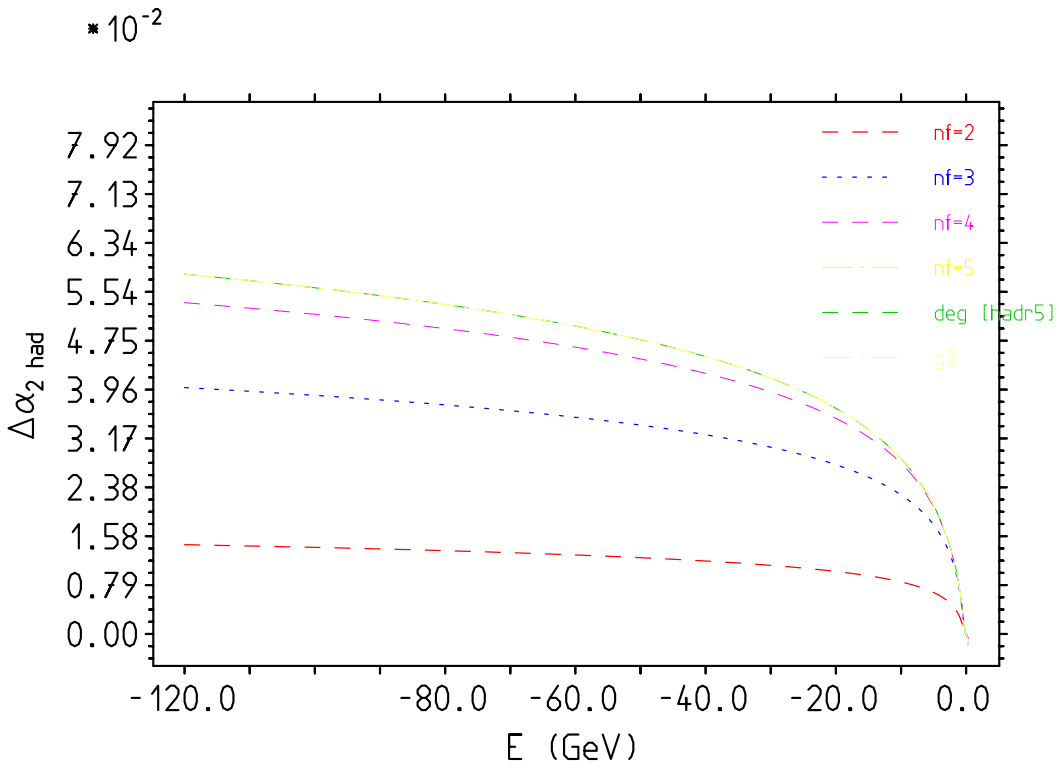


Figure 3: The renormalized Euclidean correlator $\langle\gamma\gamma\rangle(Q^2)$ denoted by $\Delta\alpha_{\rm had}(-E)$ for $n_f = 2, 3, 4, 5$ as a function of the space-like energy transfer denoted by $-E$.

The full QCD $n_f = 6$ result, which besides $\Delta\alpha^{(5)}_{2\,\rm had}$ includes the top-quark contributions, is available form alphaQEDc19.

The standalone version of hadr5n19.f is hadr5x19.f

which includes the include-files dalhad...19_5.f and deghad...19_5.f already.

Note: hadr5n19.f includes

- dalhadslow19_5.f which includes deghadslow19_5.f
- dalhadshigh19_5.f which includes deghadshigh19_5.f
- dalhadt19_5.f which includes deghadt19_5.f and also dalhadthigh19_5.f, deghadthigh19_5.f

These includefiles have been generated with the help of

./intRdatx.sh 1,2,3,4,5,6,11,12

This will change hadr5n19.f only if new data compilations for Rdat_upd.f included in Rdat_all.f and/or , R3G0321.f etc. become available.