Implications of low and high energy measurements on SUSY models

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The SM: too good and too bad!

- SM works too well?, where is the Higgs?, What is missing in muon $g - 2$?
- Cold Dark Matter (21%) missing
- Too much Dark Energy (74%): Higgs VEV? Why is the Cosmological Constant so tiny?
- Baryonic Matter (4.4%) “stuff we are made of” 99% hadronic binding energy (frozen energy)
  Asymmetry: $n_B/n_{\bar{B}} \sim 0$; $n_B/n_\gamma \sim 10^{-9}$
- Baryogenesis: missing $B$ violation, amount of $CP$ violation?
Supersymmetric extension of the SM

bosons (forces) ↔ fermions (matter)

The SUSY algebra [graded Lie algebra]

\[ \{ Q_\alpha, \overline{Q}_\beta \} = -2 (\gamma^\mu)_{\alpha\beta} P_\mu ; \ P_\mu = (H, \vec{P}) \]

\( P_\mu \) the generators of space–time translations, \( Q_\alpha \) four component Majorana (neutral) spinors and \( \overline{Q}_\alpha = \left( Q^+ \gamma^0 \right)_\alpha \) the Pauli adjoint, is the only possible non–trivial unification of internal and space–time symmetry in a quantum field theory. The Dirac matrices in the Majorana representation play the role of the structure constants.

Unbroken SUSY: zero vacuum energy cosmological constant!), \( a_{\mu}^{\text{SUSY}} = 0 \), etc.

In real life: SUSY broken:
The SUSY breaking lifts the degeneracy between particles and sparticles and
essentially makes all sparticles to be heavier than all particles.

- SM hierarchy problem: SUSY protects scalars to be light
  actually minimal SUSY extensions require
  \[ m_H < M_Z + \text{radiative corrections} \leq 140 \text{ GeV} \]
  chiral symmetry \rightarrow\ fermions light ;
  gauge symmetry \rightarrow\ gauge bosons light ;
  supersymmetry \rightarrow\ scalars light

- SUSY allows for grand unification broken at low scale (\sim 1 \text{ TeV})
a perfect candidate for observed $a^\text{exp}_\mu - a^\text{the}_\mu$

SUSY+R-parity: stable lightest SUSY particle (LSP) good candidate for astrophysical established dark matter
LSP: lightest neutralino $\tilde{\chi}_1^0$

general MSSM more than 100 new parameters
Super Symmetric extension of SM: in particular MSSM

- Need two Higgs doublets! and $R$–party ($Z_2$ invariance), dark matter candidate.
- Doubling spectrum of SM plus second Higgs doublet ($H^0, A^0, H^\pm$)
- Free parameters $\tilde{\chi}$ masses and mixings, $\mu$ and $\tan \beta$
- MSSM has particular type of 2HDM Higgs sector:
  - Type II 2HDM share relevant properties for $(g_\mu - 2)$

Constraints on SUSY breaking:

- CMSSM: SUSY-GUT soft breaking masses universal at GUT scale
- NUHM1: as CMSSM with non-universal Higgs masses
- VCMSSM: very constrained CMSSM $A_0 = B_0 + m_0$ imposed on tri- and bi-linear soft breaking terms
- mSUGRA: super gravity induced SUSY breaking $m_{3/2} = m_0$ at bare level
There are a lot of “SUSY’s”

- General MSSM has $>100$ free parameters

- CMSSM – “constrained” and, related but even more constrained mSUGRA, and others
  - These models assume many degeneracies of masses and couplings in order to restrict the number of parameters
  - Typically, $m_0$, $m_{1/2}$, $\text{sign}(\mu)$, $\tan\beta$, $A$ (or even more)

- Then there is R–parity – is sparticle number conserved (dark matter candidate!)?

- And, many ways to describe symmetry breaking
Low energy monitor: the muon anomaly

**Dirac:** $g_\mu = 2\ ,\ a_\mu$ muon anomaly

\[
\gamma(q) \mu(p') = (-ie) \bar{u}(p') \left[ \gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu} F_2(q^2) \right] u(p)
\]

\[
F_1(0) = 1\ ;\ F_2(0) = a_\mu
\]

Precision measurement of $a_\mu$ provides most sensitive test of magnetic helicity flip transition

\[
\bar{\psi}_L \sigma_{\mu\nu} F^{\mu\nu} \psi_R \quad (\text{dim 5 operator})
\]

- test of quantum structure
- monitor for new physics
Present:

\[ a^\text{Exp.}_\mu = 1.16592080(63) \times 10^{-3} \quad a^\text{The.}_\mu = 1.16591790(65) \times 10^{-3} \]

\[ \delta a^\text{NP?}_\mu = a^\text{Exp.}_\mu - a^\text{The.}_\mu = (290 \pm 90) \times 10^{-11}, \]

Uncertainties: experiment 0.54ppm = 6.3 \times 10^{-10}
theory 0.57ppm = 6.5 \times 10^{-10}

What is it?

- statistical fluctuation of experimental result
- underestimated systematic error
- missing higher-order SM contributions
- underestimated theory error (incl. possible computational)
- physics beyond SM
Remark on hadronic VP (see D. Nomura’s talk)

recent: $\tau$ vs. $e^+e^-$ puzzle resolved

F.J. & R. Szafron, $\rho - \gamma$ interference:

\[ -i \Pi^{\mu\nu}_{\gamma\gamma}(q) = \ldots + \ldots \]

\[ v_0(s) = r_{\rho\gamma}(s) R_{\text{IB}}(s) v_-(s) \]

$\tau$ require to be corrected for missing $\rho - \gamma$ mixing!

results obtained from $e^+e^-$ data is what goes into $a_\mu$
<table>
<thead>
<tr>
<th>$\tau$ decays</th>
<th>ALEPH 1997</th>
<th>$390.75 \pm 2.65 \pm 1.94$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ALEPH 2005</td>
<td>$388.74 \pm 4.00 \pm 2.07$</td>
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<tr>
<td></td>
<td>OPAL 1999</td>
<td>$380.25 \pm 7.27 \pm 5.06$</td>
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<td>CLEO 2000</td>
<td>$391.59 \pm 4.11 \pm 6.27$</td>
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<td></td>
<td>Belle 2008</td>
<td>$394.67 \pm 0.53 \pm 3.66$</td>
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<td></td>
<td>$\tau$ combined</td>
<td>$391.06 \pm 1.42 \pm 2.06$</td>
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<tr>
<th>$e^+e^-+CVC$</th>
<th>CMD-2 2006</th>
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<tr>
<td></td>
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<td>KLOE 2008</td>
<td>$380.21 \pm 0.34 \pm 3.27$</td>
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<td></td>
<td>KLOE 2010</td>
<td>$377.35 \pm 0.71 \pm 3.50$</td>
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<td></td>
<td>BABAR 2009</td>
<td>$389.35 \pm 0.37 \pm 2.00$</td>
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<td></td>
<td>$e^+e^-$ combined</td>
<td>$385.12 \pm 0.87 \pm 2.18$</td>
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$I=1$ part of $a_{\mu}^{\text{had}}[\pi\pi]$
\[
\tau \text{ decays}
\]

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I=1 part of \(a_\mu^{\text{had}}[\pi\pi]\)
M. Benayoun et al. HLS effective resonance Lagrangian model global fits

- **incl. ISR**
  - **DHMZ10** \((e^+e^-)\)
    - \(180.2 \pm 4.9\)  
    - [3.6 \(\sigma\)]
  - **DHMZ10** \((e^+e^-+\tau)\)
    - \(189.4 \pm 5.4\)  
    - [2.4 \(\sigma\)]
  - **JS11** \((e^+e^-+\tau)\)
    - \(179.7 \pm 6.0\)  
    - [3.4 \(\sigma\)]
  - **HLMNT11** \((e^+e^-)\)
    - \(182.8 \pm 4.9\)  
    - [3.3 \(\sigma\)]

- **excl. ISR**
  - **DHea09** \((e^+e^-)\)
    - \(178.8 \pm 5.8\)  
    - [3.5 \(\sigma\)]
  - **A** \((e^+e^-+\tau)\)
    - \(173.4 \pm 5.3\)  
    - [4.3 \(\sigma\)]
  - **B** \((e^+e^-+\tau)\)
    - \(175.4 \pm 5.3\)  
    - [4.1 \(\sigma\)]

- **experiment**
  - **BNL-E821** (world average)
    - \(208.9 \pm 6.3\)

\(a_\mu \times 10^{10} = -11659000\)
New Physics contributing to $a_\mu$

New physics contributions: (examples)

a) $\gamma$

Possible New Physics contributions: neutral boson exchange: a) scalar or pseudoscalar and c) vector or axialvector, flavor changing or not, new charged bosons: b) scalars or pseudoscalars, d) vector or axialvector

(a) Case: $m_\mu = M \ll M_0$

(b) Case: $m_\mu \ll M_0 = M$
New physics typically:

\[ a_{\mu}^{\text{NP}} = C \frac{m_{\mu}^2}{M_{\text{NP}}^2} \]

where naturally \( C = \mathcal{O}(\alpha/\pi) (\sim \text{lowest order } a_{\mu}^{\text{SM}}) \);

Typical New Physics scales required to satisfy \( \Delta a_{\mu}^{\text{NP}} = \delta a_{\mu} \):

<table>
<thead>
<tr>
<th>(C)</th>
<th>1</th>
<th>(\alpha/\pi)</th>
<th>((\alpha/\pi)^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_{\text{NP}})</td>
<td>2.0^{+0.4}_{-0.3} \text{ TeV}</td>
<td>100^{+21}_{-13} \text{ GeV}</td>
<td>5^{+1}_{-1} \text{ GeV}</td>
</tr>
</tbody>
</table>

- Different extensions of the SM yield very different effects in \( a_{\mu} \)
- \( a_{\mu} \) very good monitor to hint and/or constrain possible SM extensions
General: $\delta a_\mu^{NP} = C \left( \frac{m_\mu^2}{M^2} \right)$ \quad $C \approx 0.0001 \ldots 1$ $\Rightarrow a_\mu$ very useful monitor

Classification:

$C = O(\frac{\alpha}{4\pi})$

very small contributions! $\quad \delta a_\mu \sim 29 \times 10^{-10}$ for $M < 100$ GeV

$C = O(1)$

new strong interaction

technicolor, . . . Czarnecki, Marciano 01

large contributions! $\quad \delta a_\mu \sim 29 \times 10^{-10}$ for $M > 1$ TeV

$C = O(\tan \beta \frac{\alpha}{4\pi})$

Supersymmetry, 2HDMs

fits precisely! $\quad \delta a_\mu \sim 29 \times 10^{-10}$ for $M \sim 300$ GeV, $\tan \beta \sim 10$

Most promising New Physics scenario: SUSY
Leading SUSY contributions to $g - 2$ in supersymmetric extension of the SM.

- $\tilde{m}$ lightest SUSY particle; SUSY requires two Higgs doublets
- $\tan \beta = \frac{v_1}{v_2}$, $v_i = \langle H_i \rangle$; $i = 1, 2$

\[
a_\mu^{\text{SUSY}} \approx \frac{\text{sign}(\mu M_2)\alpha(M_Z)}{8\pi \sin^2 \Theta_W} \frac{5 + \tan^2 \Theta_W}{6} \frac{m_\mu^2}{M_{\text{SUSY}}^2} \tan \beta \left(1 - \frac{4\alpha}{\pi} \ln \frac{M_{\text{SUSY}}}{m_\mu}\right)
\]
with $M_{\text{SUSY}}$ a typical SUSY loop mass and the sign is determined by the Higgsino mass term $\mu$, RG improved.

- $\tan \beta \sim m_t/m_b \sim 40$ [4 – 40]

**Role for LHC searches:**

3 $\sigma$ deviation in muon g-2 (if real) requires $\text{sign}(\mu)$ positive and $\tan \beta$ preferable large

**Note:**

- $\text{sign}(\mu)$ cannot be obtained from LHC (hadron collider)
- $\tan \beta$ can’t be pinned down by LHC (hadron collider)

so muon g-2 important hint for constraining SUSY parameter space (if SUSY)

**More scenarios:**
two Higgs doublet models
\( \tan \beta \) enhanced contributions:

\[
a^{(2)2\text{HDM}}_{\mu}(h) \simeq \frac{\sqrt{2}G_\mu m_\mu^2}{4\pi^2} \tan^2 \beta \frac{m_\mu^2}{m_h^2} \left( \ln \frac{m_h^2}{m_\mu^2} - \frac{7}{6} \right) > 0 , \quad \checkmark
\]

\[
a^{(2)2\text{HDM}}_{\mu}(A) \simeq \frac{\sqrt{2}G_\mu m_\mu^2}{4\pi^2} \tan^2 \beta \frac{m_\mu^2}{m_A^2} \left( -\ln \frac{m_A^2}{m_\mu^2} + \frac{11}{6} \right) < 0 . \quad \times
\]

Potentially large 2–loop contributions: Barr-Zee type diagrams:

\[
a^{\text{bos,2L}}_{\mu}(\text{MSSM/2HDM} - \text{SM}) < 3 \times 10^{-11} , \text{ parameter range } m_A \gtrsim 50 \text{ GeV}, \tan \beta \lesssim 50.
\]

sequential fermions (4th family?)

Present bounds \( m_L > 100 \text{ GeV heavy lepton}, m_{b'} \gtrsim 200 \text{ GeV heavy quark.} \)

Note \( a_\mu(\tau) \simeq 42 \times 10^{-11} \) only!
• Grand Unified Theories: $Z'$, $W'$, leptoquarks etc
  Present bounds $M_{Z', W'} > 600 – 800$ GeV depending on the GUT scenario.
  Essentially rescaling weak SM contribution with

  $$(M_W/M_{W'})^2 \sim 0.01, \text{ i.e., 1\% of } 19.5 \times 10^{-10} \text{ only, too small to be of relevance.}$$

• extra dimensions, graviton excitations etc (with KK-parity DM candidate)
  Integrating out the extra coordinates under the hypothesis of factorization one obtains
  $\overline{M}_{Pl} = M_{Pl}/\sqrt{8\pi} = (\sqrt{8\pi G_N})^{-1} = 2.4 \times 10^{18}$ GeV ($G_N$ Newton’s gravitational constant) from the relation

  $$\overline{M}_{Pl}^2 = \overline{M}_D^{2+\Delta} V_\Delta = \overline{M}_D^{2+\Delta} (2\pi R)^\Delta = M_D^{2+\Delta} R^\Delta.$$ 

  Phenomenology: $1/R$ is about $\sim 300$ GeV or larger, contributions to $g - 2$ small, model dependent, towers of Kaluza-Klein excitations:
Barbieri, Hall and Nomura

\[ \Delta a_{\mu}^{(2) \text{KK}} = - \frac{g^2}{192} \frac{11 - 18s^2}{12c^2} (m_\mu R)^2 \]

numerically, for \( 1/R = 370 \pm 70 \text{ GeV} \), it is given by

\[ \Delta a_{\mu}^{(2) \text{KK}} = -0.07^{+0.04}_{-0.02} \cdot a_{\mu}^{(2) \text{EW}} = -(13.6^{+7.1}_{-4.0}) \times 10^{-11} \]

Appelquist, Chang and Dobrescu model (universal extra dimension),

\[ \Delta a_{\mu}^{(2) \text{KK}} = -0.276 \cdot a_{\mu}^{(2) \text{EW}} = -53.7[-24.8] \times 10^{-11} \cdot \]

which again for any sensible value of \( R \) yields a result well inside the uncertainties of the SM prediction.

- Little Higgs Models of EW symmetry breaking
Contributions at most $a^\text{LH}_\mu \sim 10 \times 10^{-11}$.

Littlest Higgs model with T-parity (also DM candidate) $a^\text{LHT}_\mu < 12 \times 10^{-11}$.
The new muon $g - 2$: Fermilab E989

- $\delta a_\mu = 16 \times 10^{-11}$ by 2015
- Magnetic field: $\frac{\delta \langle B \rangle_\mu}{\langle B \rangle_\mu} \leq 2 \times 10^{-8}$
- Requires 10% error on HLbL
- HLbL white paper in progress

Present:

- $a^{\text{exp}}_\mu = 116 592 089(63) \times 10^{-11}$; $a^{\text{SM}}_\mu = 116 591 793 \pm 51 \times 10^{-11}$

E989: statistics $21 \times$; total error factor 4 more precise

$$\left\{ \begin{array}{l}
\sigma_{\text{stat}} = 0.1 \text{ ppm} \\
\sigma_{\text{syst}} = 0.1 \text{ ppm}
\end{array} \right\} \sigma_{\text{tot}} = 0.14 \text{ ppm}$$

- $a^{\text{exp}}_\mu = 116 59x xxx(16) \times 10^{-11}$
Challenge requires improvements of hadronic effects:

- hadronic VP ➔ more experiments!, lattice QCD
- hadronic light-by-light scattering ➔ improving effective resonance Lagrangian approaches, lattice QCD, more experimental $\gamma\gamma \rightarrow \text{hadrons}$ constraints

Time horizon for next step in improvement: 5 years

Will provide important information on Physics Beyond the SM scenarios!

Provided deviation is real $3\sigma \rightarrow 9\sigma$ possible?

If SUSY:

$$\delta a_\mu \leftrightarrow \text{sign}(\mu) \text{ and } \tan \beta$$

If not SUSY or 2HDM may be even more interesting!
In any case establishing a new theory replacing SM likely is a long way to go and requires efforts on very different levels.

Low energy precision (muon g-2, MEG), B-decays, LHC and ILC all needed.
Scanning SUSY parameter space

selection of Supersymmetric Parameter Spaces

[Hertzog, Stöckinger 08]
constraints from LEP, B-physics, g-2, cosmic relict density K. Olive et al.

$m_0$ scalar mass \hspace{1cm} m_{1/2} \text{ gaugino mass}
\( \tan \beta \) measurement

[Hertzog, Miller, de Rafael, Roberts, Stöckinger 07; Rauch 08 (Sfitter)]

important input for pinning down SUSY parameter space for Collider searches!

In conjunction with other SM precision tests:
SUSY looks favored by data!
global fit in the constrained MSSM including data from $g - 2$, $B$ physics, and cosmic relic density

[O. Buchmueller, ... , Weber, Weiglein, arXiv:0707.3447]

- SUSY does not yield better global fit!
New Bounds form Direct Searches at the LHC
K. Olive et al. as above.
$M_h = 110^{+8}_{-10}$ GeV
Some critical comments

- minimal super symmetrization of SM alone fails (FCNC, proton decay, etc), no dark matter, no coupling unification etc.

- what saves SUSY is R-parity, sparticle conservation: SM particles $R_p = +1$ sparticles $R_p = -1$ → stable LSP dark matter candidate (weakly interacting only), density of right size

- normal matter in universe dominated by nucleons, mass 99% frozen energy normal matter density of universe not what we obtain from electroweak SB (Higgs mechanism)

- what if dark matter is not due to stable heavy elementary particle, but again a form of frozen energy e.g. SU(4) confined sates (no fermions → no DM stars, bosonic DM e.g $U(4) = SU(4) \otimes U(1)_V$) stability natural like B conservation.

SUSY improves agreement with experiment for $a_\mu$ and $M_W$ observables!

However: global fit $\chi^2$ barely improved. Dark matter is of course different, but not a
consequence of supersymmetrizing the SM, rather a consequence of assuming $R$–parity in addition.

<table>
<thead>
<tr>
<th>Dark Matter</th>
<th>Value</th>
<th>Reference</th>
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</thead>
<tbody>
<tr>
<td>SM</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>SPS1a'</td>
<td>0.10</td>
<td>[SPA Report 05]</td>
</tr>
<tr>
<td>Exp.</td>
<td>0.11(1)</td>
<td>[WMAP 08]</td>
</tr>
</tbody>
</table>
Outlook

SUSY: sales two for one of SM

Is it science or fiction?
LHC may rule out some of the very constrained SUSY models

if Higgs is found and Higgs mass heavier than expected the most severe constraint.

many scenarios: can hardly be disproven, may be more difficult to be established than expected.

new Fermilab \((g - 2)_\mu\) experiment can much more severely constrain new physics scenarios or parameter spaces of beyond the SM models.

ILC as we know indispensable for detailed studies as e.g. if SUSY which breaking scenario is realized.

Note SUSY and GUT are uncorrelated symmetry concepts, GUT assumptions almost always made in SUSY extensions of the SM maybe not be realistic.
Now we are waiting for the big surprises!