Introduction to Cosmology

(essentials of the lectures)

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General Setup

Cosmology = theory of universe at large ($\gtrsim 1$ Gigaparsec)

Content = pressure free “dilute gas of galaxies”, cosmic microwave background radiation, etc.: electromagnetically neutral stuff $\rightarrow$ gravity only

Dynamics = Gravity $\rightarrow$ Einstein’s General Relativity Theory

Cosmology’s metric space: assuming cosmological principle and Weyl’s postulate

Comoving coordinates = distinguished cosmological frame of reference in which matter is at rest
Cosmological principle: the cosmos looks homogeneous and isotropic \( \Rightarrow \) spatial part must be 3–dimensional space of constant curvature. Case of positive curvature can be viewed as a sphere in a 4–dimensional space:

\[
x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1/k = \text{constant}
\]

\[
x_4^2 = k - \vec{x}^2 = 1/k - r^2 \Rightarrow 2x_4 dx_4 = -2r dr
\]

with \( r = |\vec{x}| \). In spherical coordinates:

\((x_1, x_2, x_3) \rightarrow (r, \theta, \varphi)\) yields

\[
d\ell^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2
\]

\[
= dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) + \frac{r^2 dr^2}{1/k - r^2}
\]

\[
= \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)
\]

We can rescale this spatial line elements with the factor \( R^2 \) and restrict \( k \) to values 0, \( \pm 1 \):

\[
d\ell^2 = R^2 \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right)
\]

then \( Rr = \) physical radius, \( r = \) coordinate radius and the space has curvature \( k/R^2 \).
The parameter $k$ characterizes the type of the geometry of the spaces of constant curvature.

<table>
<thead>
<tr>
<th>Type</th>
<th>Geometry</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = +1$</td>
<td>spherical</td>
<td>positive</td>
</tr>
<tr>
<td>$k = 0$</td>
<td>euclidean</td>
<td>no (flat)</td>
</tr>
<tr>
<td>$k = -1$</td>
<td>hyperbolic</td>
<td>negative</td>
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</tbody>
</table>

- Cosmological principle + Weyl’s postulate (stuff in universe follows hydrodynamical flow)

The complete Robertson-Walker line element then may be written as

$$ds^2 = c^2 dt^2 - R^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right)$$

Determines all measurements of space and time in the universe!

- to every point there exists a local rest frame: co-moving system, co-moving coordinates
- co-moving observer measures $t$ as Eigentime
- $t$ is called the cosmic time
- $\rho(P)$: total energy density
  $p(P)$: isotropic pressure
  in the co-moving system only depend on $t$
  (cosmological principle, Weyl postulate)
Propagation of light and spectral red shift: $d^2 s = 0$

in radial direction: $d\theta = 0$; $d\varphi = 0$ \rightarrow \frac{cdt}{R(t)} = \frac{dr}{\sqrt{1-kr^2}}$

Spectral ratio:

\[
\zeta = 1 + z = \frac{\nu_1}{\nu_0} = 1 + \frac{\Delta \lambda}{\lambda_1}; \quad \Delta \lambda = c(\Delta t_0 - \Delta t_1)
\]

$z$ is the spectral shift:

\[
z = \frac{\Delta \lambda}{\lambda_1} \begin{cases} z > 0 & \text{red shift} \\ z < 0 & \text{blue shift} \end{cases}
\]

\[
\frac{1}{c} \int_{r_0}^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_0}^{t_1} \frac{dt}{S(t)}
\]

independent of $t$.

The $r$ integral yields

\[
\int_{0}^{r} \frac{dr'}{\sqrt{1-k r'^2}} = \begin{cases} \arcsin r & ; \quad k = 1 \\ r & ; \quad k = 0 \\ \arcsinh r & ; \quad k = -1 \end{cases}
\]

Red shift:

\[
\frac{\Delta t_0}{S(t_0)} = \frac{\Delta t_1}{S(t_1)} \quad \text{hence} \quad \zeta = 1 + z = \frac{\Delta t_0}{\Delta t_1} = \frac{S(t_0)}{S(t_1)}
\]

Result: the time dependent scale factor $S(t)$ implies a red shift

\[
z = \frac{S(t_0)}{S(t_1)} - 1
\]
Distances and Hubble's law:

The coordinate distance between an event \((t_1, r_1)\) and the observer \((t_0, r_0 = 0)\) is \(r_1\).

The proper distance \(D_1\) can be defined by setting \(dt = 0\): \((\tau = t_0 - t_1)\)

\[
D_1 = S(t_0) \int_0^{r_1} \frac{dr}{\sqrt{1 - kr^2}}
\]

\[
= S(t_0) \left\{ r_1 + k \frac{r_1^3}{6} + \cdots \right\}
\]

\[
= c \int_{t_1}^{t_0} dt \frac{S(t_0)}{S(t)}
\]

\[
= c \tau \left\{ 1 + \frac{\tau}{2} H_0 + \frac{\tau^2}{6} H_0^2 (2 + q_0) + \cdots \right\}
\]

Inverting this relation, we obtain

\[
\tau = \frac{D_1}{c} - \frac{H_0}{2} \left( \frac{D_1}{c} \right)^2 + \frac{H_0^2}{6} (1 - q_0) \left( \frac{D_1}{c} \right)^3 + \cdots
\]

and the red shift is given by

\[
z = H_0 \frac{D_1}{c} + \frac{H_0^2}{2} (1 + q_0) \left( \frac{D_1}{c} \right)^2 + \cdots
\]

Here, \(D_1\) is the present distance (observation time) of the galaxy.

The form of the relation exhibits the two key parameters \(H_0\) and \(q_0\) and

\[
z = H_0 \frac{D_1}{c}
\]

is the Hubble law.

The latter would be valid as an exact relation if the expansion would be
non-accelerated: \( q_0 = 0 \) such that

\[
S(t) = S(t_0) \left( 1 + (t - t_0) H_0 \right) ; \quad q_0 = 0
\]

Depending on the value of \( q_0 \) we have:

- \( q_0 > 0 \) the expansion is decelerated
- \( q_0 < 0 \) the expansion is accelerated

Observed: first by Hubble 1929:

Present status: \( H_0 = 74.2 \pm 3.6 \text{ (km/s)/Mpc} \) [HST 2009]

In 1968, the first good estimate of \( H_0 \), 75 km/s/Mpc, was published by Allan Sandage, but it would be decades before a consensus was achieved.

Proved universal expansion!!!

Measuring \( \dot{S}(t_0) \) as well as \( \ddot{S}(t_0) \):
\[ H_0 = H(t_0) = \frac{\dot{S}(t_0)}{S(t_0)} \] \text{ Hubble constant}

\[ q_0 = q(t_0) = -\frac{\ddot{S}(t_0) S(t_0)}{(\dot{S}(t_0))^2} \] \text{ deceleration parameter}

A closer look: deviation from linear relation, from observations one presently extracts:

\[
H_0 = 74.2 \pm 3.6 \text{ km/s Mpc (HST)} \quad q_0 = -0.60 \pm 0.02
\]

\[
71 \pm 4 \text{ km/s Mpc (WMAP)}
\]

Type Ia super nova reveal an accelerated expansion of our universe

Requires positive cosmological constant \((\Lambda > 0)\)!

Proves accelerated expansion!!!

Supported by power spectrum of CMB fluctuations!

Note: Cosmological constant not so important for evolution of universe in the
past, but determines its future! (see Freidmann’s equations)

For \( q_0 = 0 \) the age of the universe is

\[
H_0^{-1} = (13.7 \pm 0.2) \cdot 10^9 \text{ years [Hubble age]}
\]

the time back a Big Bang must have been taking place.

The corresponding horizon of the universe, where the escape velocity reaches the speed of light \( c \) is

\[
D_{\text{max}} = \frac{c}{H_0} \simeq 3.34 \text{ Gpc}
\]

about \( 10.9 \cdot 10^9 \) light years.

Note: observable Cosmology commences at \( \gtrsim 1 \text{ Gpc} \) and ends at the horizon 3.34 Gpc.
Friedmann equation

Homogeneous, isotropic matter distribution: Einstein’s field equations $\Rightarrow$ Friedmann’s solution

\[ H^2 = \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G \rho_{\text{tot}}}{3} - k \frac{c^2}{R^2} \]

- $G$: Gravitational constant
- $\rho_{\text{tot}}$: Total energy-matter density
- $R(t)$: Radius of the universe
- $\dot{R}(t)$: Expansion rate ($\dot{R} \equiv dR/dt$)
- $k$: 0, $\pm 1$ Type of geometry

Cosmological constant $\Lambda$:

\[ \rho_{\text{tot}}|_{\Lambda=0} + \rho_{\text{vacuum}} \rightarrow \rho_{\text{tot}} ; \quad \rho_{\text{vacuum}} = \frac{\Lambda}{8\pi G} \]

For simplicity we take: $\Lambda = 0$ (was not relevant in past, just starts to be relevant)

Observations require $\Lambda > 0$ (dark energy)!
Friedmann’s equation: derivation

Energy balance: per unit mass (mass in spherical shell = $4\pi R^2 dR \rho$)

Kinetic energy + potential energy = constant

$$\frac{1}{2} v^2 - \frac{G M(R)}{R} = \text{constant}$$

$$M(R) = \frac{4\pi}{3} R^3 \rho, \quad v = \frac{dR}{dt} \equiv \dot{R}$$

yields, after division by $R^2$ and multiplication with 2,

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8\pi G \rho}{3} = -k \frac{c^2}{R^2}$$
Basic dynamical equations of Cosmology

\[ \dot{R}^2 = \frac{\Lambda}{3} R^2 + \frac{8\pi G}{3} \rho R^2 - k c^2 \] : Friedmann equation

\[ \frac{d}{dR} (\rho R^3) = -3p R^2 \] : Energy balance

\[ p = p(\rho) \] : Equation of state

The equation of state of matter and radiation is determined by the thermodynamic properties of the energy distribution:

- gas, liquid, solid, dust, radiation.

For the state of matter electromagnetic, weak and strong interaction forces play the key role.

Of interest for cosmology:

Radiation:

\[ p = \frac{1}{3} \rho c^2 \] dominates at very high temperatures

Matter pressure-free:

\[ p = 0 \] after cooling down and dilution by expansion

Vacuum energy:

\[ p = -\rho c^2 \] after cooling in future becomes dominating term
Observable quantities:

Curvature \[ K(t) = k/R^2(t) \]

Hubble constant \[ H(t) = \dot{R}(t)/R(t) \]

Deceleration parameter \[ q(t) = -(\ddot{R}(t) R(t))/\dot{R}^2(t) \]

Friedmann equation:

\[
K(t) = \frac{\kappa}{2} (\rho(t) + p(t)) - \frac{H^2(t)}{c^2} (q(t) + 1) \\
\lambda = \frac{\kappa}{2} (\rho(t) + 3p(t)) - 3 \frac{H^2(t)}{c^2} q(t)
\]

\[ (\lambda = \Lambda/c^2, \kappa = \frac{8\pi G}{3c^2}) \]
Matter dominated universe

Universe as a gas cloud of galaxies

Non-relativistic gas, kinetic energy negligible relative to rest mass: dust

\[ p \sim 0 \]

Density of non-relativistic particles is proportional to \( 1/R^3 \)

\[ \rho(t) R^3(t) = \rho_0 R_0^3 = \text{constant} \]

\[ \frac{\dot{R}^2}{c^2} = \frac{\kappa}{3} \rho R^2 - k = \frac{C_{\text{mat}}}{R} - k \]

\[ C_{\text{mat}} \equiv \frac{\kappa}{3} \rho R^3 = \text{constant} \]

elementary integrable

\[ k = 1 : \quad R(t) = \frac{C_{\text{mat}}}{2} (1 - \cos \eta), \quad ct = \frac{C_{\text{mat}}}{2} (\eta - \sin \eta) \]

\[ k = 0 : \quad R(t) = \left( \frac{9}{4} C_{\text{mat}} \right)^{1/3} (ct)^{2/3} \]

\[ k = -1 : \quad R(t) = \frac{C_{\text{mat}}}{2} (\cosh \eta - 1), \quad ct = \frac{C_{\text{mat}}}{2} (\sinh \eta - \eta) \]

Behavior for small \( t \):

\[ R(t) \simeq \left( \frac{9}{4} C_{\text{mat}} \right)^{1/3} (ct)^{2/3}, \quad k = 0, \pm 1 \]

independent of \( k \) ! all three cases: Big-Bang solutions

Cold “Big-Bang” !!!
Behavior for large $t$:

$k = 1$ : cyclic, cycle $ct_c = \pi C_{\text{mat}}$

$k = 0$ : $R(t) = \left(\frac{9}{4} C_{\text{mat}}\right)^{1/3} (ct)^{2/3}$

$k = -1$ : $R(t) \simeq ct$

The product $M = \rho R^3$ is constant and corresponds to the total mass.

Galaxies, clusters of galaxies, dark matter ($\nu \ll c$)
Einstein - de Sitter Universe

\( k = 0 \) is the limiting case in between closed and open universe

**No curvature**

\[ K(t) \equiv 0 \]

**Deceleration parameter**

\[ q_{\text{EdS}}(t) \equiv 1/2 \]

**Matter density**

\[ \rho_{\text{EdS}}(t) = \frac{3}{\kappa} \frac{H^2(t)}{c^2} \]

\[ C_{\text{mat}} \equiv \frac{\kappa}{3} \rho R^3 \]

\[ = \frac{\kappa}{3} \frac{3}{\kappa} \frac{H^2(t)}{c^2} R^3 \]

\[ = \frac{\kappa}{3} \frac{3}{\kappa} \frac{H^2(t)}{c^2} \frac{9}{4} C_{\text{mat}}(ct)^2 \]

Hence cosmic time is:

\[ t = \frac{2}{3} H^{-1}(t) \]

**Today:**

\[
\begin{align*}
H_0 & \sim 0.52 \times 10^{-28} \text{ cm}^{-1} \times c \\
\rho_{0\text{EdS}} & \sim 4.6 \times 10^{-30} \text{ gr/cm}^3 \times c^2 \\
t_{0\text{EdS}} & = \frac{2}{3} H_0^{-1} \sim 13 \times 10^9 \text{ years}
\end{align*}
\]

⇒ **Age of the universe if it would be flat!**

(CMB says: universe is flat!)

**Generally:**

\[ \rho(t) = \rho_{\text{EdS}}(t) \cdot 2q(t) \Rightarrow \begin{cases} 
\rho > \rho_{\text{EdS}} \text{ if } k = +1 \\
\rho < \rho_{\text{EdS}} \text{ if } k = -1 
\end{cases} \]
Critical density: $\rho_c \equiv \rho_{EdS}$

Density parameter in units of the critical density:

$$\Omega_0 = \frac{\rho_0}{\rho_c}$$

$\Omega = 1$

Critical density $\Rightarrow$ Space just remains open, infinite, flat.

$\Omega > 1$

Lots of matter $\Rightarrow$ Space closes under gravitational attraction!
Gravity wins and stops expansion. The universe collapses into itself, universe dies from heat.

$\Omega < 1$

Little matter $\Rightarrow$ Gravity not sufficient to stop expansion, universe expands forever, space open, universe dies from cold.
Radiation dominated universe

the universe as a fireball

\[ p = \frac{1}{3} \rho c^2 \]

Since \( E_\gamma \sim R^{-1} \) we have

energy density of relativistic particles is proportional to \( 1/R^4 \)

\[ \rho(t) R^4(t) = \rho_0 R^4_0 = \text{constant} \]

\[ \frac{\dot{R}^2}{c^2} = \frac{\kappa}{3} \rho R^2 - k = \frac{C^2_{\text{rad}}}{R^2} - k \]

\[ C^2_{\text{rad}} = \frac{\kappa}{3} \rho R^4 = \text{constant} \]

elementary integrable

\[ \begin{align*}
  k = 1 : & \quad R(t) = \sqrt{C^2_{\text{rad}} - (C_{\text{rad}} - ct)^2} \\
  k = 0 : & \quad R(t) = \sqrt{2 C_{\text{rad}} ct} \\
  k = -1 : & \quad R(t) = \sqrt{(C_{\text{rad}} + ct)^2 - C^2_{\text{rad}}} 
\end{align*} \]

Behavior at small \( t \): \( R(t) \approx \sqrt{2 C_{\text{rad}} ct} \), \( k = 0, \pm 1 \)

independent of \( k \) ! all three cases: Big-Bang solutions

Hot Big-Bang !!!

HOT BIG BANG
Behavior at large \( t \):

\[
\begin{align*}
    k = 1 & : \text{ cyclic, cycle } \frac{ct}{c} = 2C_{\text{rad}} \\
    k = 0 & : \quad R(t) = \sqrt{2} C_{\text{rad}} \frac{ct}{c} \\
    k = -1 & : \quad R(t) \simeq ct
\end{align*}
\]

The product \( S = \rho R^4 \) is constant and corresponds to the total entropy.

Radiation, photons, neutrinos, highly relativistic particles \((v \sim c)\)
For radiation in thermal equilibrium (black body radiation) the Stefan-Boltzmann law:

\[ \rho_{\text{radiation}} = a T^4 c^2, \quad a = 8.4 \times 10^{-36} \text{ gr/cm}^3 (\circ K)^4 \]

with \( \rho R^4 = \text{constant} \) follows

\[ \rho \propto T^4 \propto \frac{1}{R^4} \]

or

\[ T = T_0 \frac{R_0}{R} \]

Isotropic microwave radiation = cosmic black body radiation

\[ T_0 \simeq 2.725 \circ K \]

Big-Bang radiation!

Planck's spectrum: energy density (\( \omega = 2\pi \nu \))

\[ u_\omega = \frac{1}{\pi^2} \frac{\omega^2}{c^3} \epsilon(T), \quad \epsilon(T) = \frac{1}{2} \hbar \omega + \frac{\hbar \omega}{\exp \frac{\hbar \omega}{kT} - 1} \]
Four-velocity field (liquid, gas):
\[ u^\mu = \frac{dx^\mu}{ds} \]

in the co-moving system
\[ u^\mu = (1, \vec{0}) \]

particle current:
\[ j^\mu(x) = n u^\mu \]

Energy-Momentum-Tensor:
\[ T^{\mu\nu}(x) = (\rho + p) u^\mu u^\nu - p g^{\mu\nu} \]

Einstein equation: \((\lambda = \Lambda/c^2, \kappa = \frac{8\pi G}{3c^2})\)
\[ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} R - \lambda g_{\mu\nu} = \kappa T_{\mu\nu} \]

For RW-metric ➔ Friedmann’s equation!
Thermodynamics

- $T$: Temperature
- $k$: Boltzmann constant
- $kT$: Thermal energy
- $\epsilon = \rho c^2$: Energy density
- $n$: Particle density
- $p$: Pressure
- $m$: Particle mass
- $g$: Spin factor (number of degrees of freedom)
- $\mu$: Chemical potential

$$\mu_a + \mu_b = \mu_c + \mu_d$$

Particle number in thermal equilibrium:

- Fermions:
  $$n = \frac{g}{2\pi^2 \hbar^3} \int_{mc^2}^{\infty} E \sqrt{E^2/c^2 - m^2 c^2} \frac{E dE}{\exp\left(\frac{E - \mu}{kT}\right) + 1}$$

- Bosons:
  $$n = \frac{g}{2\pi^2 \hbar^3} \int_{mc^2}^{\infty} E \sqrt{E^2/c^2 - m^2 c^2} \frac{E dE}{\exp\left(\frac{E - \mu}{kT}\right) + 1}$$

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- Fermions:
  $$p = \frac{g}{2\pi^2 \hbar^3} \int_{mc^2}^{\infty} \sqrt{E^2/c^2 - m^2 c^2} \frac{E dE}{\exp\left(\frac{E - \mu}{kT}\right) + 1}$$

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Density of states in momentum space (Quantum Mechanics):

\[
\frac{d^3p}{\hbar^3} = \frac{d^3p}{(2\pi\hbar)^3} = \frac{p^2dpd\Omega}{(2\pi\hbar)^3}
\]

Integrated over angles in case of isotropic distribution:

\[
\int d\Omega \frac{p^2dpd}{(2\pi\hbar)^3} = \frac{p^2dp}{2\pi^2\hbar^3}
\]

Since \( p = \sqrt{E^2/c^2 - m^2c^2} \) we have \( pdp = EdE/c^2 \) and therefore

\[
\frac{1}{2\pi^2\hbar^3} \sqrt{E^2/c^2 - m^2c^2} \frac{E}{c^2} dE
\]

is the density of states in the energy distribution, while

\[
\frac{1}{\exp \left(\frac{E-\mu}{kT}\right) + 1}
\]

is the statistical occupation probability (Quantum statistics).

Fermions can occupy a state 0- or 1-times (Pauli-Principle), leads to weight factor \( \left( \exp \left(\frac{E-\mu}{kT}\right) + 1 \right)^{-1} \)

Bosons can occupy any state arbitrarily often, leads to the weight factor \( \left( \exp \left(\frac{E-\mu}{kT}\right) - 1 \right)^{-1} \)
Limiting cases

In the relativistic limit \((kT \gg mc^2)\) one has

\[
\begin{align*}
n_{\text{Bosons}} &= \frac{4}{3} n_{\text{Fermions}} = \frac{\zeta(3)}{\pi^2} \left(\frac{kT}{\hbar c}\right)^3 \\
\epsilon_{\text{Bosons}} &= \frac{8}{7} \epsilon_{\text{Fermions}} = \frac{\pi^2 g}{30} \left(\frac{kT}{\hbar c}\right)^3 kT \\
\sigma_{\text{Bosons}} &= \frac{8}{7} \sigma_{\text{Fermions}} = \frac{2\pi^2 g}{45} \left(\frac{kT}{\hbar c}\right)^3 k
\end{align*}
\]

In the non-relativistic limit \((kT \ll mc^2)\) the difference between Fermions and Bosons disappears:

\[
n = g \left(\frac{kT}{\hbar c}\right)^3 \sqrt{\left(\frac{mc^2}{2\pi kT}\right)^3} \exp\left(-\frac{mc^2}{kT}\right)
\]

\[
s = \frac{mc^2 n}{T}, \quad \epsilon = nmc^2, \quad p = nkT
\]

Energy density of the relativistic components:

Stefan-Boltzmann law

\[
\rho c^2 = \frac{\pi}{30} g^* \left(\frac{\hbar c}{kT}\right)^3 kT
\]

with \(g^*\) the effective number of relativistic degrees of freedom = sum of all contributions of the relativistic particle species.
Degrees of freedom, particle species

for $kT \gg 300\text{GeV}$

all particles of the Standard Model are in thermodynamic equilibrium

free quarks and gluons

all masses negligible

all particles at the same temperature

Fermions:
Leptons : $e^\pm(2), \mu^\pm(2), \tau^\pm(2), 3\nu(1), 3\bar{\nu}(1)$
Quarks : $3(u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}, b\bar{b}, t\bar{t})(2)$

Bosons:
$W^\pm(3), Z^0(3), \gamma(2), \text{Higgs}(1), 8\text{ Gluons}(2)$

Fermions : $90 \cdot \frac{7}{8} = \frac{315}{4}$
Bosons : 28
Total : $g = \frac{427}{4} = 106.75$

With decreasing temperature also $g$ decreases,
Particles freeze out as a consequence of their mass.
Gravitation, relativity (electromagnetism) and quantum theory each are characterized by a typical fundamental constant ⇒

Basic constants of physics: $G$, $c$, $\hbar$ (Planck 1913)

They may be combined into

“smallest” distance: Planck length:

$$\ell_{P1} = \sqrt{\frac{\hbar G}{c^3}} = 1.616252(81) \cdot 10^{-33} \text{ cm}$$

“biggest” mass: Planck mass:

$$m_{P1} = \sqrt{\frac{\hbar}{cG}} = 2.2 \cdot 10^{-5} \text{ gr}$$

shortest time: Planck time:

$$t_{P1} = \ell_{P1}/c = 5.4 \cdot 10^{-44} \text{ sec}$$
Using $E = mc^2$ we obtain:

highest energy $m_{Pl} = \sqrt{\frac{\hbar c}{G}} = 1.22 \cdot 10^{19} \text{ GeV}$

**Planck scale**

Using $E_{\text{thermal}} = k_B T$ we obtain:

highest temperature

$$\frac{m_{Pl} c^2}{k_B} = \sqrt{\frac{\hbar c^5}{Gk_B^2}} = 1.416786(71) \cdot 10^{32} \text{ °K}$$

(Temperature of the Big Bang)

Classical cosmology based on GRT stops to be valid for $t \lesssim t_{Pl}$!

Big Bang singularity $R(0) = 0$ artifact of classical Big Bang Cosmology model.

At the Planck scale: unification of gravity with the other forces quantum gravity expected. No compelling answers, no working theory.
History of the Universe

The further we look back to the past, the universe appears to be compressed more and more. We therefore expect the young universe was very dense and hot:

At Start a Light-Flash: ➳ **Big-Bang (fireball)**
Light quanta very energetic, all matter totally ionized, all nuclei disintegrated. **Elementary particles only!**: \( \gamma, e^+, e^-, p, \bar{p}, \cdots \)

**Processes:**
\[
2\gamma \leftrightarrow e^+ + e^- \\
2\gamma \leftrightarrow \bar{p} + p \\
\vdots
\]

- Particle–Antiparticle Symmetry!

**Digression into high energy physics: example LEP**
$e^+ e^- \text{ Annihilation at LEP}$

45.5 GeV

Electron
(Matter)

Positron
(Antimatter)

$E = 2m_{\text{eff}} c^2$

Mini Big Bang!
What happens in Electron Positron Collisions?

Matter and Antimatter annihilate to pure light or heavy light (Z’s), which re-materializes in new forms of matter (mostly showers of unstable particle).

Matter:  
- Electron $e^-$  J. J. Thomson 1897  
- Proton $p$  E. Rutherford 1911  
- Neutron $n$  J. Chadwick 1932  

Relativity Theory  
A. Einstein 1905

Quantum Theory  
W. Heisenberg, E. Schrödinger 1925  
P. Dirac, M. Born

Antimatter:  
- Positron $e^+$  P. Dirac 1928, C.D. Anderson 1932  
- Antiproton $\bar{p}$  E. Segré, O. Chamberlain 1955  
- Antineutron $\bar{n}$  B. Cork et al. 1956/58

Science Fiction as Reality!

Theory:  
Matter ↔ Antimatter Symmetry

To every particle there exists an antiparticle of opposite charge

Nature: in our universe: antimatter is missing!
Energy versus temperature correspondence:

\[ 1^\circ K \equiv 8.6 \times 10^{-5} \text{eV} \]

(Boltzmann constant \( k_B \))

\[ \downarrow \]

temperature of an event

\[ T \sim 1.8 \times 10^{11} \times T_\odot \]

In nature such temperatures only existed in the very early universe:

\[
t = \frac{2.4}{\sqrt{g^*(T)}} \left( \frac{1 \text{MeV}}{k_B T} \right)^2 \text{sec.}
\]

\[ \downarrow \]

\[ t \sim 0.3 \times 10^{-10} \text{ sec. after B.B.} \]

**early universe**

\[ 0^\circ K \equiv -273.15^\circ \text{C} \quad \text{absolute zero temperature} \]

\[ T_\odot \sim 5700^\circ \text{K} \quad \text{surface temperature of the Sun} \]

\( g^*(T) \) number of highly relativistic degrees of freedom at given \( T \)
History of the Universe (continuation)

\[10^{-43} \text{ sec.} \quad \text{quantum-chaos}\]

\[2 \times 10^{-6} \text{ sec.} \quad \text{nucleon–antinucleon annihilation}\]

\[2 \text{ sec.} \quad \text{electron–positron annihilation}\]

\[200 \text{ sec.} \quad \text{Helium synthesis}\]

\[500'000 \text{ years} \quad \text{from now on matter dominates}\]

\[700'000 \text{ years} \quad \text{Hydrogen recombination}\]

\[13 \text{ Billion years} \quad \text{today}\]

\[n_B/n_\gamma \simeq 10^{-9}\]

\(\Rightarrow\) recombination

\(\Rightarrow\) annihilation

\(\Rightarrow\) hydrogen

\(\Rightarrow\) helium

\(\Rightarrow\) electron

\(\Rightarrow\) positron

\(\Rightarrow\) neutrino

\(\Rightarrow\) mass

\(\Rightarrow\) density

\(\Rightarrow\) atoms

\(\Rightarrow\) galaxies

\(\Rightarrow\) clusters

\(\Rightarrow\) universe

\(\Rightarrow\) photons

\(\Rightarrow\) charges

\(\Rightarrow\) neutral

\(\Rightarrow\) matter

\(\Rightarrow\) galaxies
"Annihilation Drama of Matter"

$10^{-35}$ sec.

$X, \bar{X}$-Decay: $\Rightarrow$

\[
\begin{align*}
X & \quad \cdots \quad q \quad e^- \quad \nu \\
\bar{X} & \quad \cdots \quad \bar{q} \quad e^+ \quad \bar{\nu}
\end{align*}
\]

\[
\begin{align*}
\{ & q : \bar{q} = 1,000,000,001:1 \\
e^- : e^+ & = 1,000,000,001:1
\}
\]

$10^{-30}$ sec.

LEP events

$q\bar{q} \rightarrow \gamma\gamma$:

$10^{-4}$ sec.

$e^+ e^- \rightarrow \gamma\gamma$:

1 sec.
Nucleo-Synthesis

quark-gluon-plasma

$\rightarrow$

$N$ nucleons (proton $p$, neutron $n$)

Some protons $p$ and neutrons $n$ form Helium nuclei

After 700 000 years: light atoms (electrically neutral)

$\begin{align*}
\text{Hydrogen} & : \text{Helium} \\
H : He & \sim 3 : 1
\end{align*}$

together 98% of baryonic matter!

Where are the $\gamma$’s?

$\Rightarrow$ Cosmic Microwave Background ($T \simeq 2.725^\circ K$)
CMB Discovery

Penzias & Wilson 1965, Nobel Prize 1978

to a full proof that the isotropic Cosmic Microwave Background indeed follows a Planck distribution COBE satellite

![Graph showing intensity vs wavelength with a peak at approximately 2.7 K.]

(Mather 1990)

\[ T_0 = (2.725 \pm 0.02) \, \text{K} \]
A compilation of CMB black-body spectral data

☆ CMB the Proof and the Noise

(Smoot 1990)

G.F. Smoot, J.C. Mather, Nobel Prize 2006
$\Omega_B$ from Big Bang nucleosynthesis provides strong limits for the existence of new light particles (not neutrinos only) and also yields e.g. a bound for the $\tau$ neutrino mass:

$$m_{\nu_{\tau}} \lesssim 0.5\,\text{MeV}$$

$\Rightarrow$ excludes range 0.5 MeV to 25 MeV

(25 MeV is the present limit from laboratory experiments)

Nucleosynthesis of the light elements is very sensitive to details. Only works if standard cosmology describes correctly the history of the universe back to

$$t \gtrsim 0.01\ \text{sec. after B.B.}$$

i.e. up to temperatures equivalent to

$$T \lesssim 10\ \text{MeV}.$$
Dark Matter

Galaxy rotation curves! missing mass!
CMB temperature fluctuations

\[ \frac{\delta T}{T} \sim 10^{-5} \]

Spherical harmonics analysis of temperature field \( \Delta T(\hat{n}) \Rightarrow \text{power spectrum} \) 
\( \ell (\ell + 1) C_\ell \)
Acoustic peaks: baryons oscillating in ripples of dark matter;
angular size $\phi \sim \frac{180^\circ}{\ell} \Rightarrow \ell \sim \frac{180^\circ}{\phi}$

- oscillations damped by Silk damping at large $\ell$
- low $\ell \leq 5$ obscured by cosmic variance
- low $\ell > 5$ Sachs-Wolfe effect: gravitational fluctuations red shift partly eaten up by simultaneous change in Hubble rate

- 1st main peak horizon at last scattering $\Leftrightarrow$ total energy/mass content
- 2nd peak total baryon density (in agreement with result from Big Bang nucleosynthesis of light elements)
- 3rd peak gives total dark matter density

specific information in height of the peaks!
- dark energy 70% in conjunction with constraint from deceleration parameter in SN Ia redshifts
- dark matter (as required independently by many other observations): 25%
- baryonic matter: $\sim 4.5\%$
- neutrinos 0.3%

Remark: ☐ Astronomers estimate $\Omega_0 = 0.2 \pm 0.1$

Resulting model: $\Lambda$CDM–model
Inflationary world model: $\Omega_0 = 1$

$\Omega_B = \Omega(\text{Nucleons}) \lesssim 0.05$

??? $\Omega_X \sim 0.95$ ???

??? What are to other 95% of the universe made of ???

New in recent years: Einstein’s GRT must be modified to include cosmological constant $\Lambda \Rightarrow$ interpreted as dark energy

A model for dark energy is the SM Higgs mechanism!

$$\Delta T_{\mu\nu} = g_{\mu\nu} V_{\text{Higgs}}(v) = g_{\mu\nu} \frac{m_H^2}{8\sqrt{2} G_F}$$
the Higgs potential (in unitary gauge) and

\[ v = \langle H \rangle \]

Higgs vacuum expectation value.

Equation of state of Higgs condensate

\[ p_H = -\rho_H c^2. \]

Precisely the right pattern but 60 orders of magnitude too big!

The cosmological constant question is not why is there a cosmological constant, rather why is it so small?

Other sources, quantum vacuum fluctuations, quark condensates, other spontaneous symmetry breaking at higher scales etc. Also \( V_{\text{Higgs}}(0) = 0 \) assumed above, no justification!

What about the missing dark matter?
WIMPS? 
(Weakly Interacting Massive Particles)

WIMPs determine the type of structures one observes in the universe: distribution of galaxies, clusters of galaxies, voids, ...

WIMPs ⇒

weak=weak force ⇒ decouple earlier

⇒ can build structures earlier (clumping matter)

**HDM: Hot Dark Matter**

\( m \approx 50 \text{ eV} \)

**Candidates:** Light Massive Neutrinos

(neutrinos have tiny masses, provide established form of DM!)

Contribution too small, only can account for tiny fraction of DM

**CDM: Cold Dark Matter**

\( m \sim 10^5 \sim 10^{12} \text{ eV} \)

**Candidates:** NEUTRALINO

in SUSY-theories with conserved R-parity
Comparison

COBE/WMAP data
(large scales)

Data on distribution of galaxies
(small scales)

\[ \Omega_{\text{tot}} = 1.006(6) \]
\[ \Omega_A = 0.74(3) \]
\[ \Omega_M = 0.26(2) \]
\[ \Omega_{\text{CDM}} = 0.21(2) \]
\[ \Omega_B = 0.044(4) \]
\[ \Omega_\gamma = 4.48(4) \cdot 10^{-5} \]
\[ \Omega_\nu = 0.0009 - 0.048 \]

\[ \Omega_{\text{CDM}} = \Omega_M - \Omega_B - \Omega_\nu \]

\[ \Omega_{\text{HDM}} = \Omega_\nu \left( \sum_\nu m_\nu \lesssim 1 \text{ eV} \right) \]

PDG 2010
Problem of fine-tuning

Time evolution of the density function:

\[ \Omega(t) = \frac{1}{1 - x(t)} \]

\[ x(t) = \frac{3k/R^2}{8\pi G\rho} \]

In order that the density today has a value 10 times higher or 10 times lower than the critical value one has to assume that at Planck times the density was coinciding to about 60 digits with the critical value 1.

\[ |\Omega(10^{-43}\text{sec.}) - 1| \lesssim 10^{-60} \]

Solution by Inflation

Cure: add an inflation term, which blows up the early universe, such that it looks flat today:

\[ \frac{8\pi}{3m_{Pl}^2} \left( V(\phi) + \frac{1}{2} \phi^2 \right) \]

to be added to the right hand side of Friedmann's equation (must be the dominating for small times)
Conditions for the possibility of the baryogenesis: (A. Sacharov 1967)

① Baryon number violating processes must exist (B–L violation!)

② CP violation! Cronin, Fitch 1964 (NP 1980)
   (Violation of time-reversal symmetry)
   first seen in neutral Kaon decays 0.3% effect, more
   recently established in B-meson physics $\sim 100\%$ effect
   CP violation precisely as predicted by SM of elementary
   $\Rightarrow$ B-factories: BaBar and Belle

③ Thermal far from equilibrium (non-stationary time
   evolution (as predicted by Friedmann’s equations
   [GRT]))
   is true as the universe is expanding

Standard Model of elementary particle physics cannot
explain these facts: $\Rightarrow$ **NEW PHYSICS REQUIRED likely at
energies not yet accessible by experiment (except LHC may
reveal new physics and hopefully does)!**

GUT’S, SUSY, STRING’S ???
in GUT’s: X–bosons and leptoquarks!
$\Rightarrow$ ATLAS, CMS and LHCb experiments at the
LHC
After baryogenesis and $e^+e^-$-annihilation:

$$\frac{n\bar{p}}{n_p} \simeq 0; \quad \frac{n_p}{n_\gamma} \simeq 10^{-9}$$

At few seconds after B.B. and temperature $T \sim 10^{10}$ °K dropping fast

$n_e = n_p$ and $n_B = n_P + n_n$ conserved neutrons, protons in thermal equilibrium

\[
\begin{align*}
n + \nu & \rightleftharpoons p + e^- , \\
n + e^+ & \rightleftharpoons p + \bar{\nu} , \\
n & \rightleftharpoons p + e^- + \bar{\nu}
\end{align*}
\]

① Ratio $n_n/n_p$ is given by the Boltzmann equation:

$$\frac{n_n}{n_p} = e^{-(m_p-m_n)c^2/k_BT}$$

which at $T \sim 10^{10}$ °K gives $n_n/n_p = 0.223$.

② Below $T \sim 10^{10}$ °K, no new neutrons are formed and the ratio is fixed. Yet it is too hot for deuterium to form, so protons and neutrons remain free. The free neutrons decay into protons via $\beta$–decay ($n \rightarrow p + e^- + \nu_e$, half life 617 seconds). About 4 minutes after B.B. temperature has dropped to $T \sim 10^9$ °K, and deuterium can form. At this point, neutron decay has rebalanced the neutron to proton ratio to $n_n/n_p = 0.164$.

③ Now time is ready for nucleosynthesis. At $T \sim 10^9$ °K, deuterium will survive. So nuclear reactions will form deuterium, tritium, and helium:
Now per proper volume: $n_{He}$ helium, $n_H$ hydrogen nuclei: close to all neutrons are in helium: $n_{He} = n_n/2$,

$n_H = n_p - 2 n_{He} = n_p - n_n \Rightarrow \text{fractional abundance by weight of helium } Y = 4n_{He}/(4n_{He} + n_H) = 2n_n/(2n_n + n_p - n_n) = 2n_n/(n_p + n_n) = 2x/(1 + x)$,

with $xn_n/n_p = 0.164$ at $T \sim 10^9 \, ^\circ \text{K}$ we have $Y = 0.282$. Observation close to 25%.

$D$ is a steep function of baryon number $n_B \Rightarrow$

Baryometer $\Omega_B = 0.02$

Cosmic concordance: one value of $n_B$ predicts light element abundances $^4\text{He}$, $^3\text{He}$, $^7\text{Li}$ rather well and in agreement with value form CMB!
Under conditions after the B.B. (low density of baryons after baryon antibaryon annihilation) only adding up nucleon by nucleon could be successful. Start of nucleosynthesis hindered by small binding energy of Deuteron [bottleneck]. Formation of $D$ requires low enough temperature (at about $T = 0.1 \, \text{MeV}$) but in the meantime neutrons density decreases by neutrons decaying. At the end most of the remaining neutrons survive bounded in the most sable of the light elements $^4\text{He}$.

The lack of stable elements of masses 5 and 8 makes it very difficult for BBN to progress beyond Lithium and even Helium. Need conditions found in stars for formation of the heavy elements.

The Bottlenecks in BBN

- Elements of order $^\odot$ and $^\boxplus$ missing (totally unstable)
- Barrier for production of heavier elements in the early universe
- heavy elements must have been produced in stars (extreme temperature and pressure)
The Standard Cosmological Model

- Standard $\Lambda$CDM cosmological model is an exceedingly successful phenomenological model

- Rests on three pillars
  - Inflation: sources all structure
  - Cold Dark Matter: causes growth from gravitational instability
  - Cosmological Constant: drives acceleration of expansion that are poorly understood from fundamental physics

- $\Lambda$CDM and its generalizations to dark energy and slow-roll inflationary models is highly predictive and hence highly falsifiable.

Wayne Hu’s summary
The hot Big Bang is supported very well by many facts. The picture of the universe is one of the outstanding intellectual achievements of the 20th century. It bridges physics at smallest scales with physics at largest scales - the cosmic bridge. It involves mysteries about its beginning and leads us into an uncertain future.

Last but not least: Standard Model of particle physics cannot explain

- Dark matter,
- Baryogenesis [matter vs antimatter asymmetry] (missing B violation, missing amount of CP violation),
- Does not explain why cosmological constant is so small
- Why universe is flat [Inflation is beyond the SM physics]

One of today’s motivations and a great challenge for high energy particle physics

Find what’s beyond the SM

- dark matter
- B-violating interactions
- and all that ....
Dark Ages after last scattering (epoch of recombination):
between 150 million to 1 billion years reionization of atoms takes place.
The first stars and quasars form from gravitational collapse. The intense
radiation they emit reionizes the intergalactic gas (hydrogen) of the
surrounding universe. From this point on, most of the universe is composed
of plasma. Stars cannot be seen until reionization becomes negligible due to
ongoing expansion and decreasing matter density (dilution).
As far as we can see.