Hadronic Contributions

to the

Photon Vacuum Polarization

and their Role in

Precision Physics

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50 Years of Electroweak Physics

A symposium in honor of Professor Alberto

Sirlin's 70th Birthday

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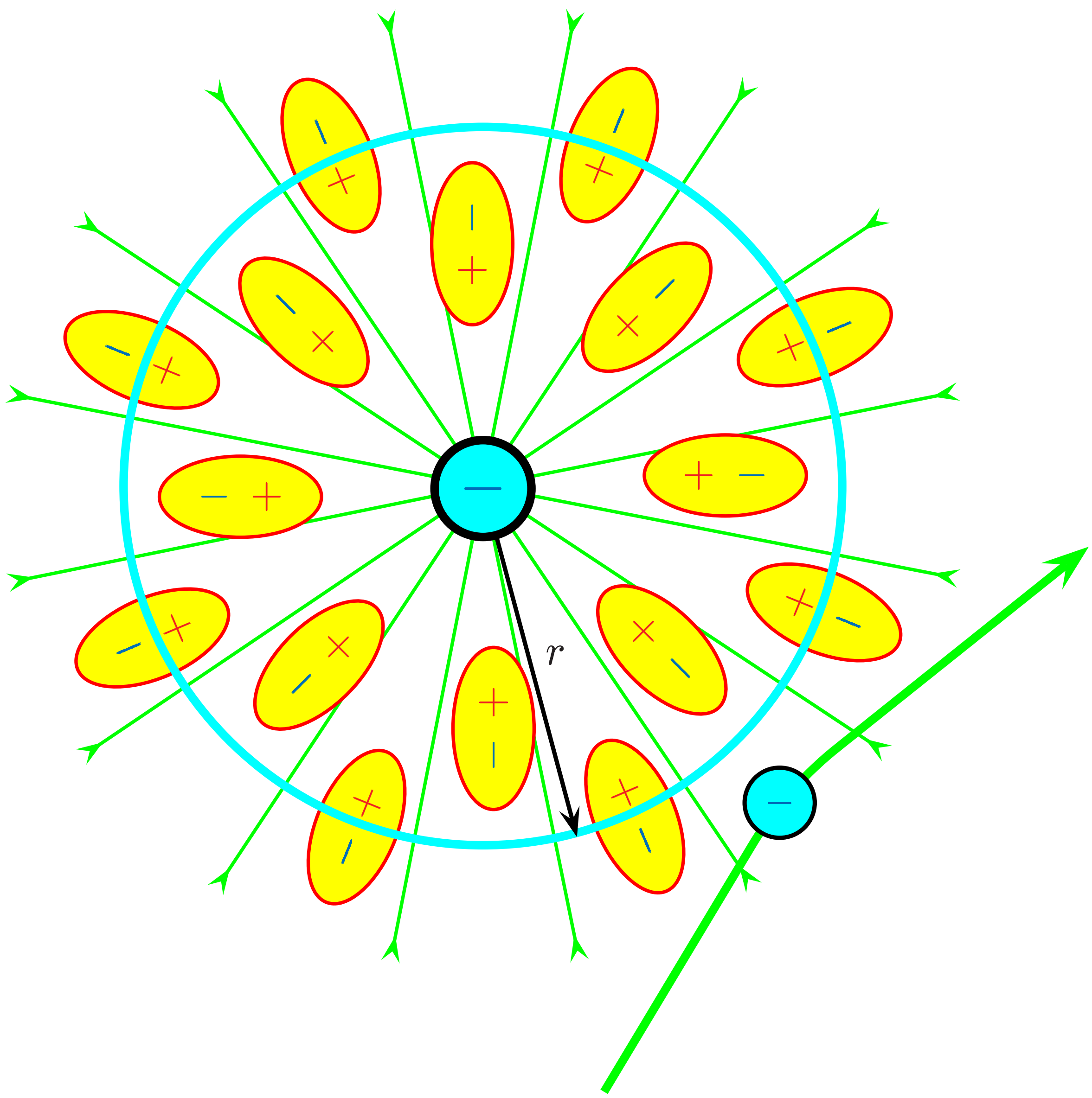
① Introduction

Vacuum polarization and charge screening



In a collision of impact energy E the effective charge is the charge contained in the sphere of radius $r \simeq 1/E$, which due to vacuum polarization is larger than the classical charge seen in a large shere ($r \to \infty$)

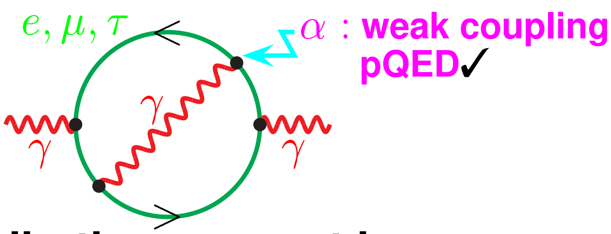
$\Downarrow$

charge screening **(charge renormalization).**

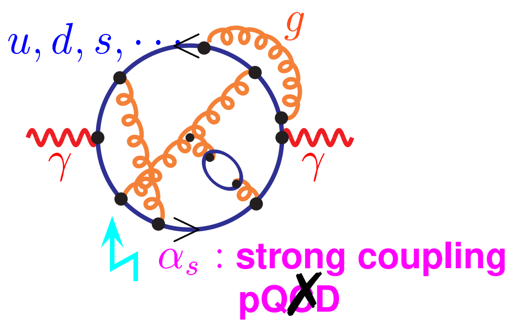
- The fine structure constant $\boxed{\alpha}$ of Quantum Electrodynamics and its extension to the electroweak Standard Model is one of the fundamental constant of nature.

- Due to charge screening effects (vacuum polarization) it is in fact a function $\boxed{\alpha(E)}$ of the energy scale E typical for a given process ($\alpha(0) = \alpha$).

- The contribution of the leptons to the vacuum polarization can be reliably calculated in perturbation theory, since $\alpha(0) = 1/137.036..$ is small.



- In contrast the quark/hadron contributions cannot be reliably calculated in perturbation theory because they exhibit strong interaction and the strong coupling $\alpha_s(E)$ increases at low energies (due to the asymptotic freedom=anti-screening) such that the perturbation expansion fails to “converge”.



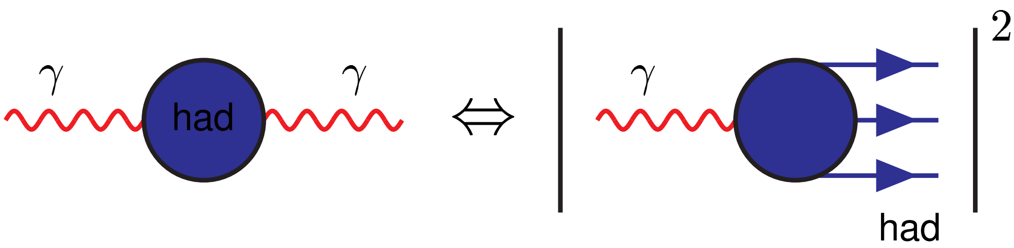
❑ The way out:

non–perturbative approach by dispersion relation

The total cross section

$$\sigma(e^+e^- \to hadrons)$$

for hadro-production in e^+e^-– annihilation directly determines the hadronic contribution to the photon vacuum polarization function $\Pi'_\gamma(q^2)$



$\Pi'_\gamma(q^2)$ $\quad\quad\quad\quad$ $\sim \sigma_{\rm tot}^{\rm had}(q^2)$

$$\Delta\alpha(s) \equiv -4\pi\alpha \mathrm{Re}\left[\Pi'_\gamma(s) - \Pi'_\gamma(0)\right]$$

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\quad \mathrm{P}\!\!\!\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R_\gamma^{\rm data}(s')}{s'(s'-s)} \right.$$
$$\left. + \quad \mathrm{P}\!\!\!\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R_\gamma^{\rm pQCD}(s')}{s'(s'-s)} \right)$$

where

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \mathrm{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi \mathrm{Im}\Pi'_\gamma(s)$$

(Davier, Höcker 97)

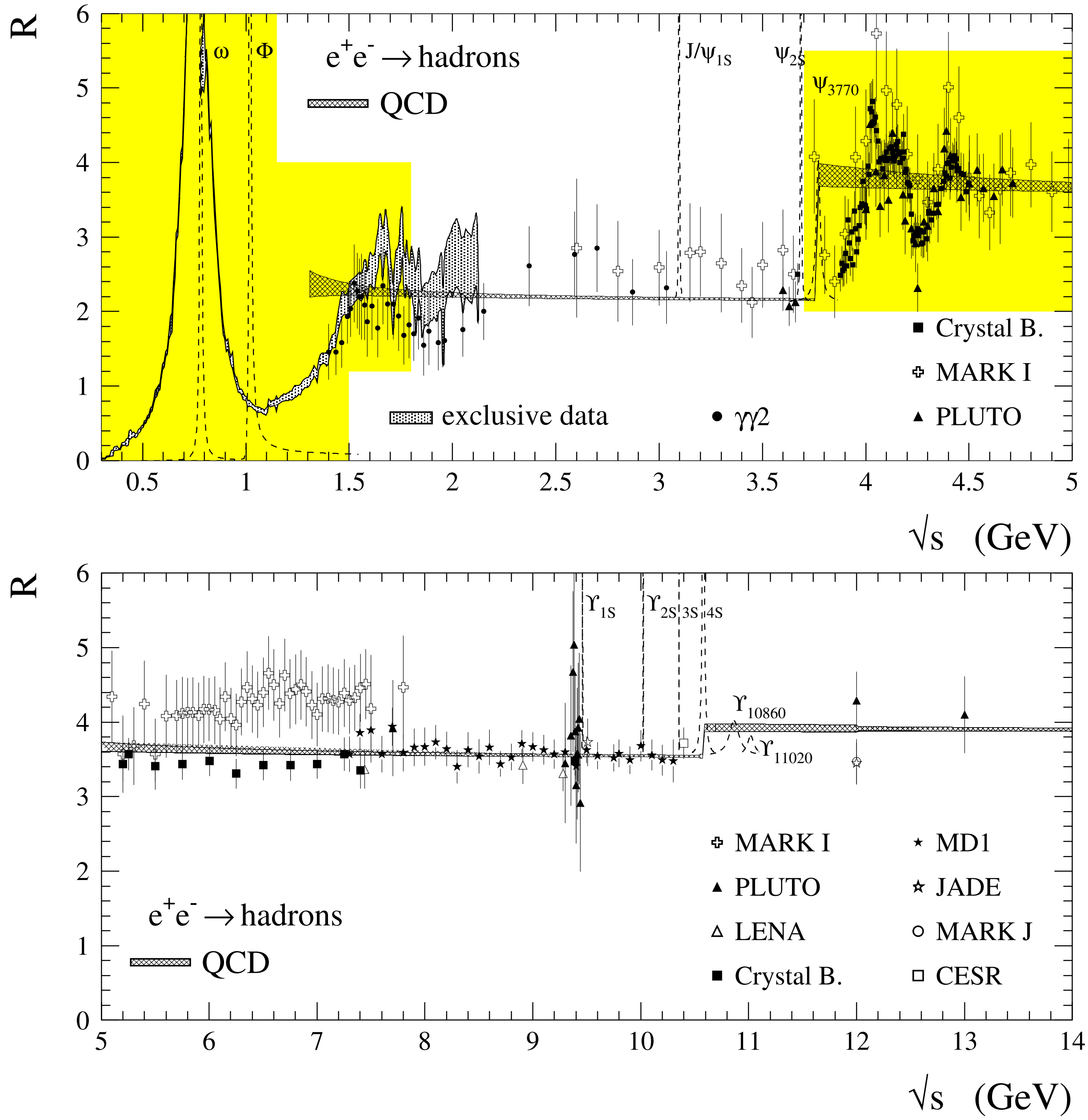


Figure 1: *Inclusive hadronic cross section ratio in e^+e^- annihilation versus the c.m. energy $\sqrt{s}$. Additionally shown is the QCD prediction of the continuum contribution as explained in the text. The shaded areas depict regions were experimental data are used for the evaluation of $\Delta\alpha_{\rm had}(M_Z^2)$ and $a_\mu^{\rm had}$ in addition to the measured narrow resonance parameters. The exclusive e^+e^- cross section measurements at low c.m. energies are taken from DM1,DM2,M2N,M3N,OLYA,CMD,ND and τ data from ALEPH (see Ref. [1] for detailed information).*

Precision physics basics:

α , G_μ, M_Z **most precise input parameters**

partially

$\Updownarrow$

non-perturbative

precision predictions

relationship $\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters**

for VB physics (Z,W) etc.

$$
\begin{array}{cccc}
\frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} \\
\frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} \\
\frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} \\
\frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 1.6 \div 6.8 & \times & 10^{-4}
\end{array}
$$

$\Rightarrow$ **now a limiting factor**

for precision physics

② Running fine structure constant

α running due to vacuum polarization (charge screening)

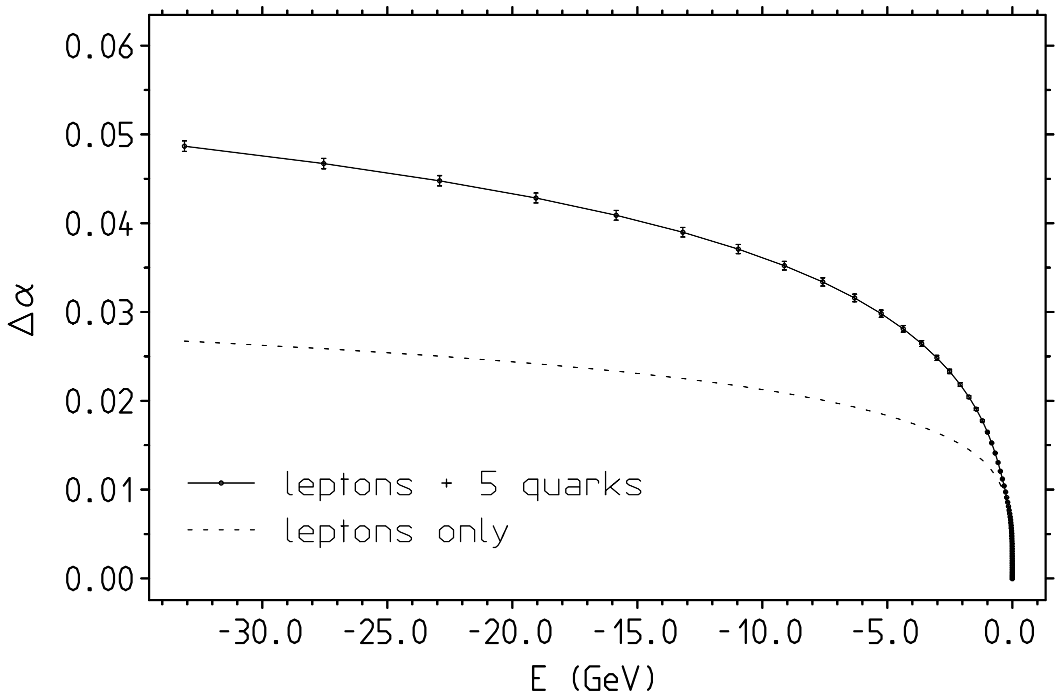
Effective fine structure "constant" $\alpha(E)$

appears in many places in physics! large EWRC !

Formal definition:

Effective QED coupling constant at scale $\sqrt{s}$:

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}$$



$$\Delta\alpha(M_Z) \sim 6\%$$

$$\Delta\alpha(5\,\mathbf{GeV}) \sim 3\%$$

$\sim 50\%$ **from leptons**

$\sim 50\%$ **from hadrons**

Formal definition:

Effective QED coupling constant at scale $\sqrt{s}$:

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)}$$

with

$$\Delta\alpha(s) = -4\pi\alpha \mathrm{Re}\left[\Pi'_\gamma(s) - \Pi'_\gamma(0)\right]$$

where $\Pi'_\gamma(s)$ is the photon vacuum polarization function

$$i \int d^4x\, e^{iq\cdot x} \langle 0|\mathrm{T} j^\mu_{em}(x) j^\nu_{em}(0)|0\rangle = -(q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi'_\gamma(q^2)$$

and $j^\mu_{em}(x)$ is the electromagnetic current.

Contributions:

- **Leptons**

Calculable in perturbation theory: Free lepton loop affected by small electromagnetic corrections.

$$\begin{aligned} \Delta\alpha_{\mathrm{leptons}}(s) = & \\ = & \sum_{\ell=e,\mu,\tau} \frac{\alpha}{3\pi}\left[-\frac{8}{3} + \beta_\ell^2 - \frac{1}{2}\beta_\ell(3-\beta_\ell^2)\ln\left(\frac{1-\beta_\ell}{1+\beta_\ell}\right)\right] \\ = & \sum_{\ell=e,\mu,\tau} \frac{\alpha}{3\pi}\left[\ln\left(s/m_\ell^2\right) - \frac{5}{3} + O\left(m_\ell^2/s\right)\right] \\ = & \; 0.03142 \text{ for } s = M_Z^2 \end{aligned}$$

where $\beta_\ell = \sqrt{1 - 4m_\ell^2/s}$.

❑ Quarks

The contribution of the five light quarks (u, d, s, c, b) is not calculable in perturbation theory: Free quark loop strongly modified by strong interactions at low energy. Way out: unitarity and the analyticity of $\Pi'_\gamma(s)$ allows us to write

(Cabibbo, Gatto 1961)

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha s}{3\pi}\left(\quad ⨍_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R_\gamma^{\rm data}(s')}{s'(s'-s)} + ⨍_{E^2_{\rm cut}}^{\infty} ds' \frac{R_\gamma^{\rm pQCD}(s')}{s'(s'-s)} \right)$$

where

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi \mathrm{Im}\Pi'_\gamma(s)$$

has been measured in e^+e^-–annihilation experiments up to about $E_{\rm cut} \sim 40$ GeV. By virtue of the asymptotic freedom of QCD the high energy tail can be safely calculated using pQCD (Gorishny, Kataev, Larin 91).

$E_{\rm cut} \gtrsim 12.GeV$

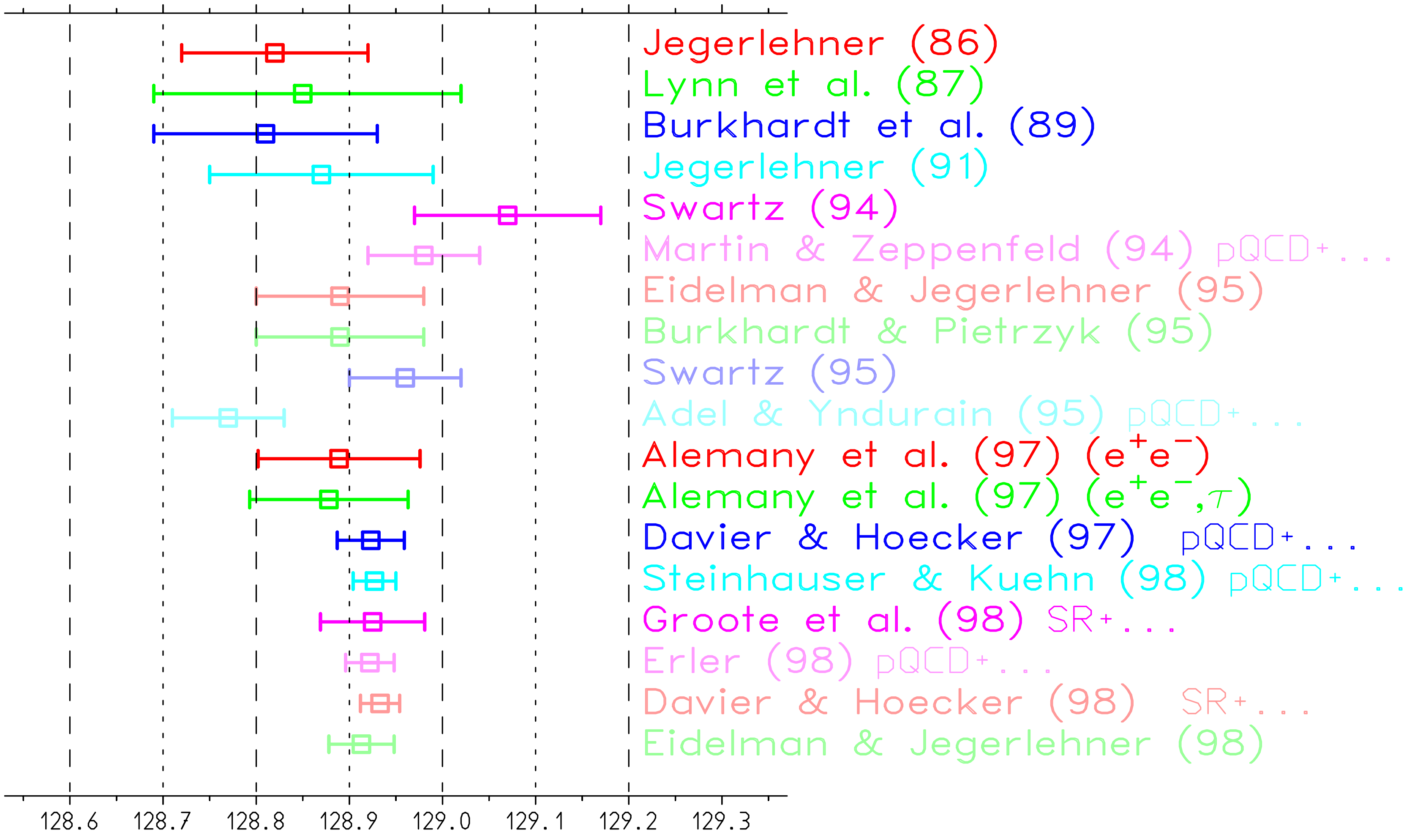
$$\begin{aligned} \Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.0280 \pm 0.0007 \quad \mathbf{E} \quad (EJ95) \\ &\to 0.02763 \pm 0.00016 \quad \mathbf{T} \quad (DH98) \end{aligned}$$

$E_{\rm cut} \sim 1.7GeV$

$$\begin{aligned} \alpha^{-1}(M_Z) &= 128.89 \pm 0.09 \quad \mathbf{E} \quad (EJ95) \\ &\to 128.933 \pm 0.021 \quad \mathbf{T} \quad (DH98) \end{aligned}$$

Evaluations of $\alpha(M_Z)$ by different authors (before 2000)

All values have been rescaled to $M_Z = 91.1887$ GeV. The small top quark and the W contributions are not included.



$\Delta\alpha$ in electroweak precision physics

Calculate $\sin^2\Theta_i$ **from** α, G_μ **and** M_Z **:**

$$\boxed{\sqrt{2}\, G_\mu\, M_Z^2\, \sin^2\Theta_i\, \cos^2\Theta_i = \pi\alpha\, \frac{1}{1-\Delta r_i}}$$

Alberto's Formula 1980

Various definitions of $\sin^2\Theta_i$**, which coincide at tree level: From weak gauge boson masses, from electroweak gauge couplings and from the neutral current coupling of the charged fermions:**

$$\begin{aligned}\sin^2\Theta_W &= 1-\frac{M_W^2}{M_Z^2}\\ \sin^2\Theta_g &= e^2/g^2\\ \sin^2\Theta_f &= \frac{1}{4|Q_f|}\left(1-\frac{v_f}{a_f}\right)\;,\quad f\neq\nu\end{aligned}$$

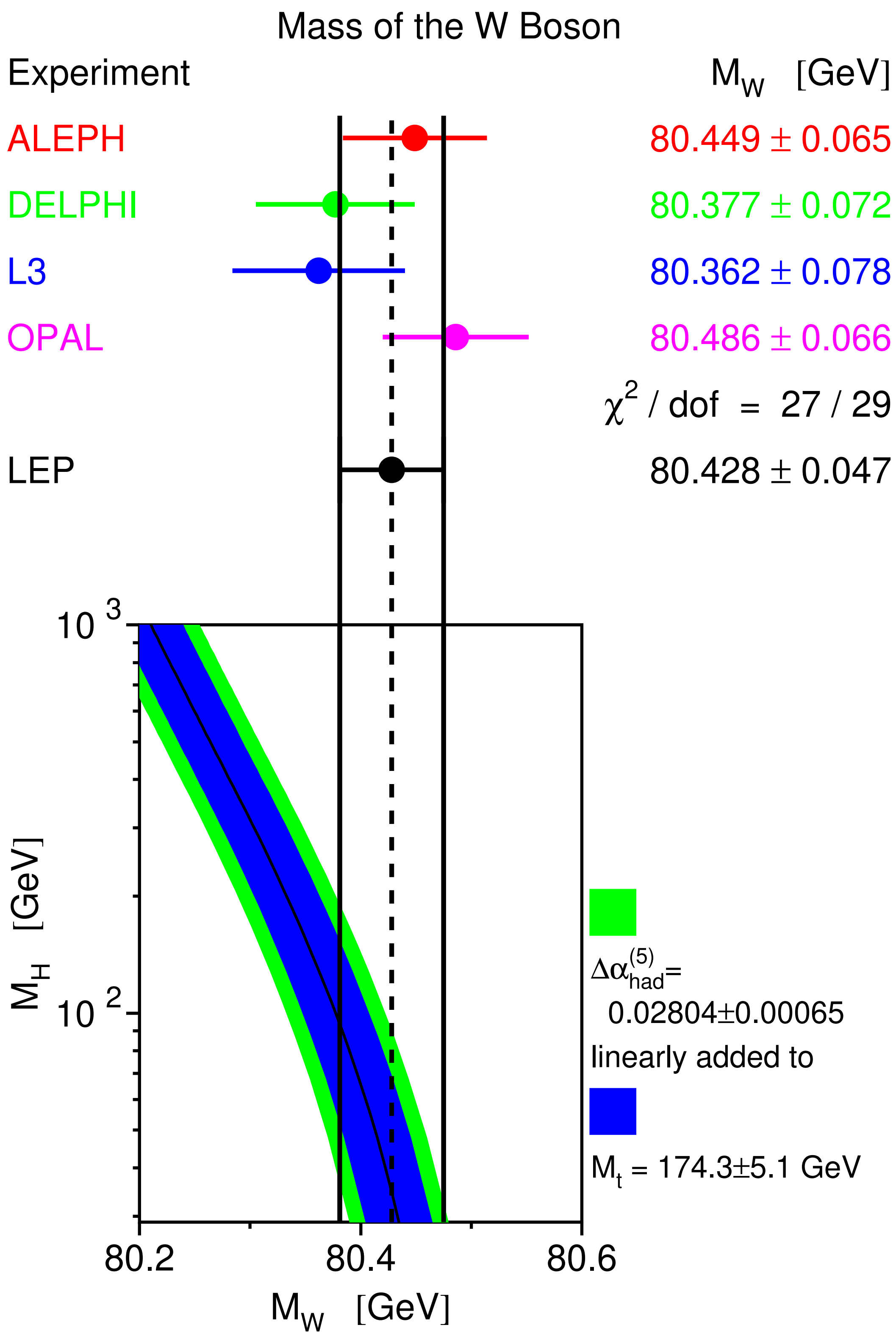
$$\begin{aligned}\Delta r_i &= \Delta r_i(\alpha,\, G_\mu,\, M_Z, m_H, m_{f\neq t}, m_t)\\ \Delta r_i &= \Delta\alpha - f_i(\sin^2\Theta_i)\,\Delta\rho + \Delta r_{i\,\mathrm{rem}}\end{aligned}$$

$\uparrow$

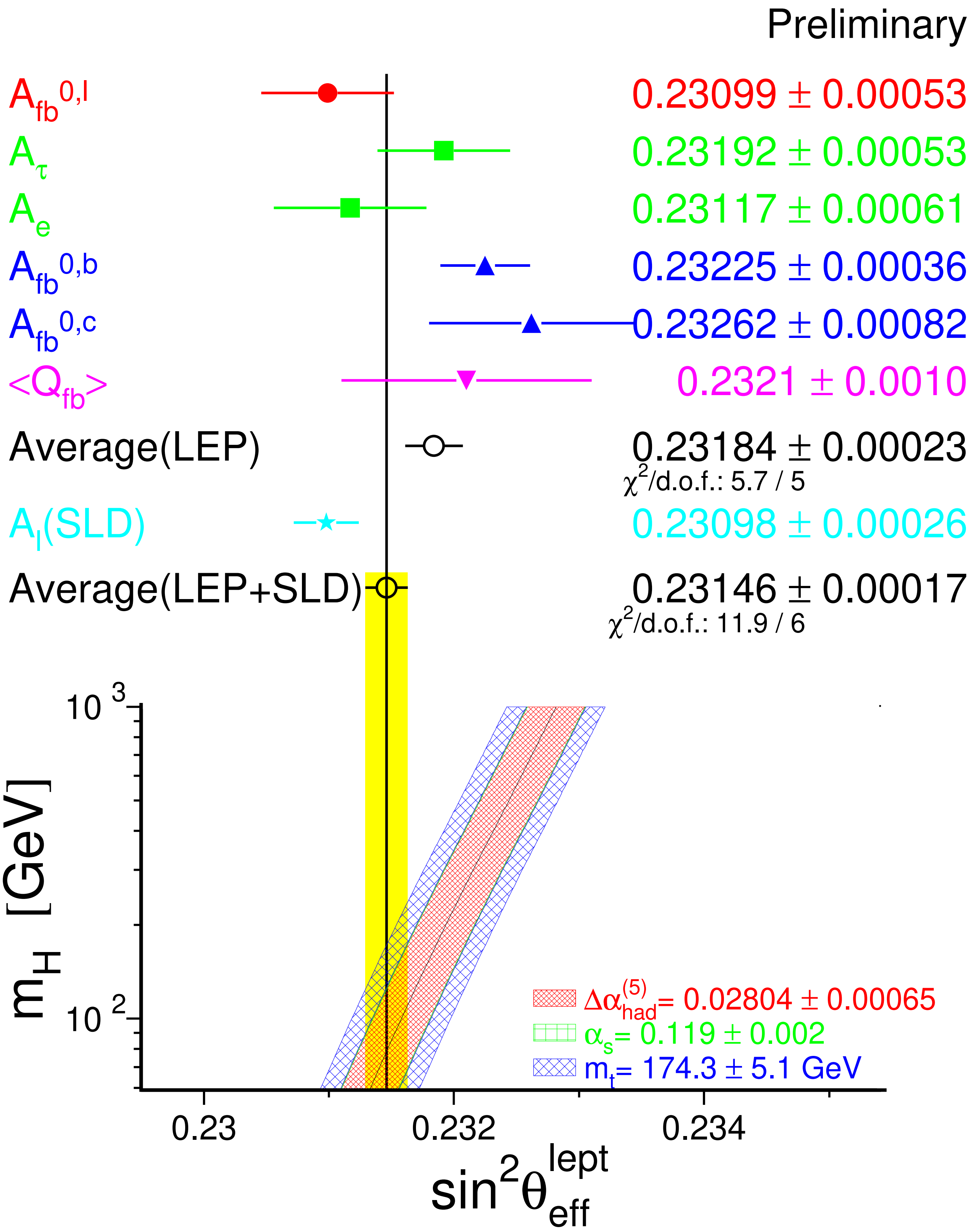
universal

$\rightarrow$ **predictions for** M_W, A_{LR}, A^f_{FB}, Γ_f, $\cdots$

$$\delta\Delta\alpha \to \delta M_W\;,\quad \delta\sin^2\Theta_i\;,\;\cdots$$



(LEP Electroweak Working Group: D. Abbaneo et al. 00)



(LEP Electroweak Working Group: D. Abbaneo et al. 00)

Sensitivity of indirect Higgs mass bounds for $m_H \sim 100$ GeV:

$$\frac{\delta m_H}{m_H} \sim 1300 \frac{\delta M_W}{M_W}$$
$$\frac{\delta m_H}{m_H} \sim 440 \frac{\delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{lep}}}{\sin^2\theta_{\mathrm{eff}}^{\mathrm{lep}}}$$

EW–fits all data 00:

$$\begin{aligned} M_W &= 80.385 \pm 0.022 \text{ GeV} \\ \sin^{2\,\mathrm{lep}}_{\mathrm{eff}} &= 0.23150 \pm 0.00016 \\ m_t &= 173.2 \pm 4.5 \text{ GeV} \end{aligned}$$

$\Rightarrow \quad \Rightarrow \quad \Rightarrow$

$$\begin{aligned} \delta m_H &\sim (35)^{+168}_{-\ 20}\ , \quad m_H < 203 \text{ GeV} \quad \text{from} \quad M_W \\ \delta m_H &\sim (75)^{+145}_{-\ 60}\ , \quad m_H < 220 \text{ GeV} \quad \text{from} \quad \sin^2\theta_{\mathrm{eff}}^{\mathrm{lep}} \end{aligned}$$

All data:

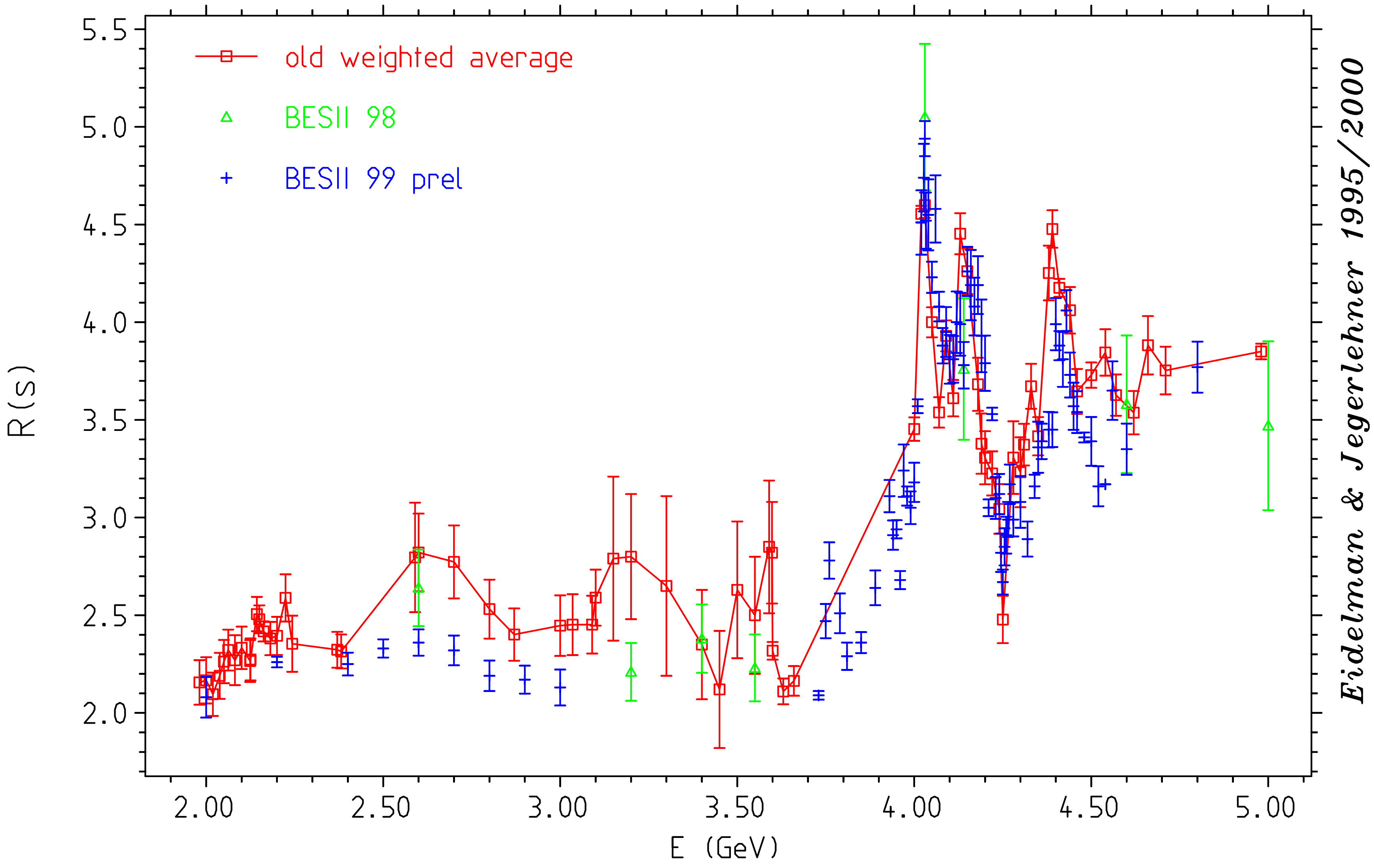
$$m_H = 77^{+\ 69}_{-\ 39}\ , \quad m_H < 215 \text{ GeV at } 95\% \text{ CL}$$

direct $\quad m_H > 113.2$ GeV at 95% CL

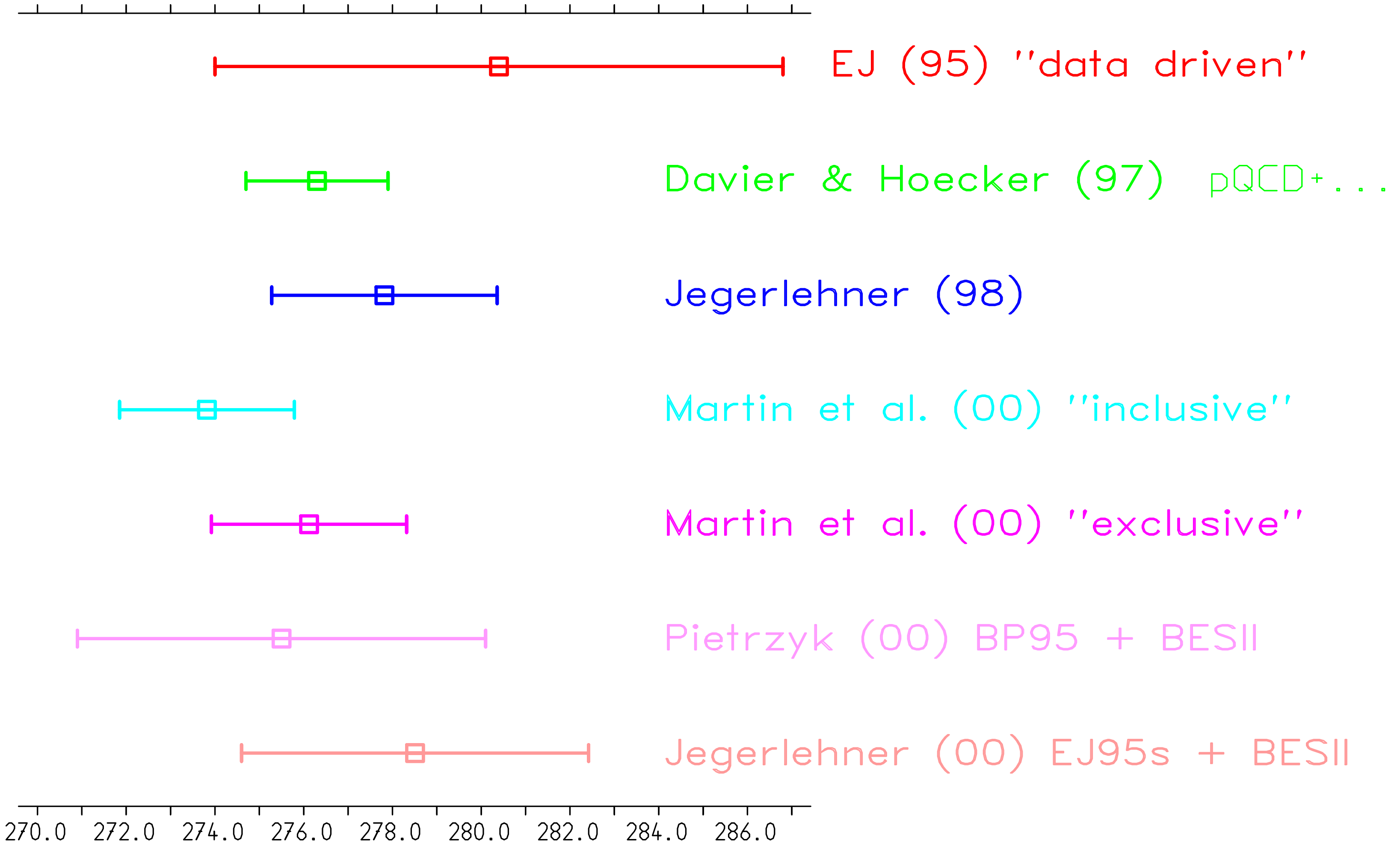
Future:

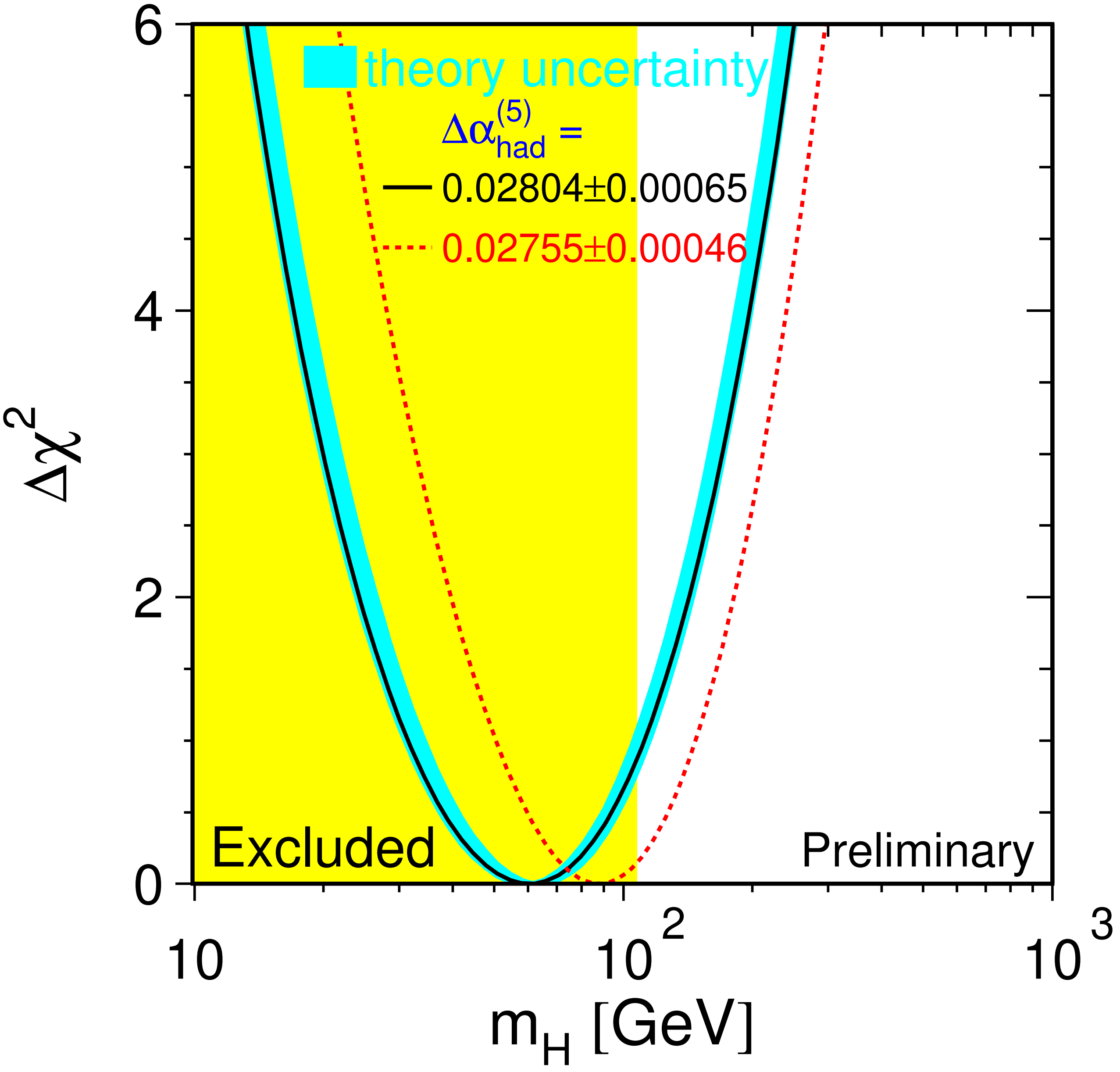
$$\delta m_t \sim 2 \text{ GeV}\ , \quad \delta\Delta\alpha \sim ???$$

New preliminary BESII data (summer 2000)



Recent evaluations of $\Delta\alpha(M_Z)$



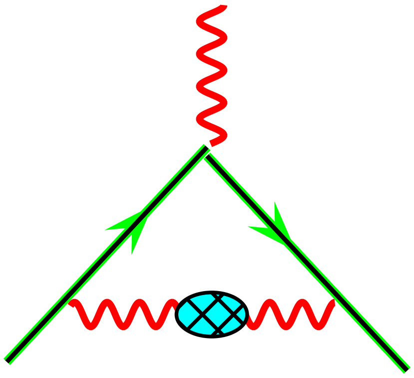


(LEP Electroweak Working Group: D. Abbaneo et al. 00)

❸ Hadronic contribution to $(g-2)$ of the muon

most precise test of quantum filed theory model (QED/SM)

❑ Leading order hadronic contributions



$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{4m_\pi^2}^{E^2_{\rm cut}} ds \, \frac{R_\gamma^{\rm data}(s)\; \hat{K}(s)}{s^2} + \int\limits_{E^2_{\rm cut}}^{\infty} ds \, \frac{R_\gamma^{\rm pQCD}(s)\; \hat{K}(s)}{s^2} \right)$$

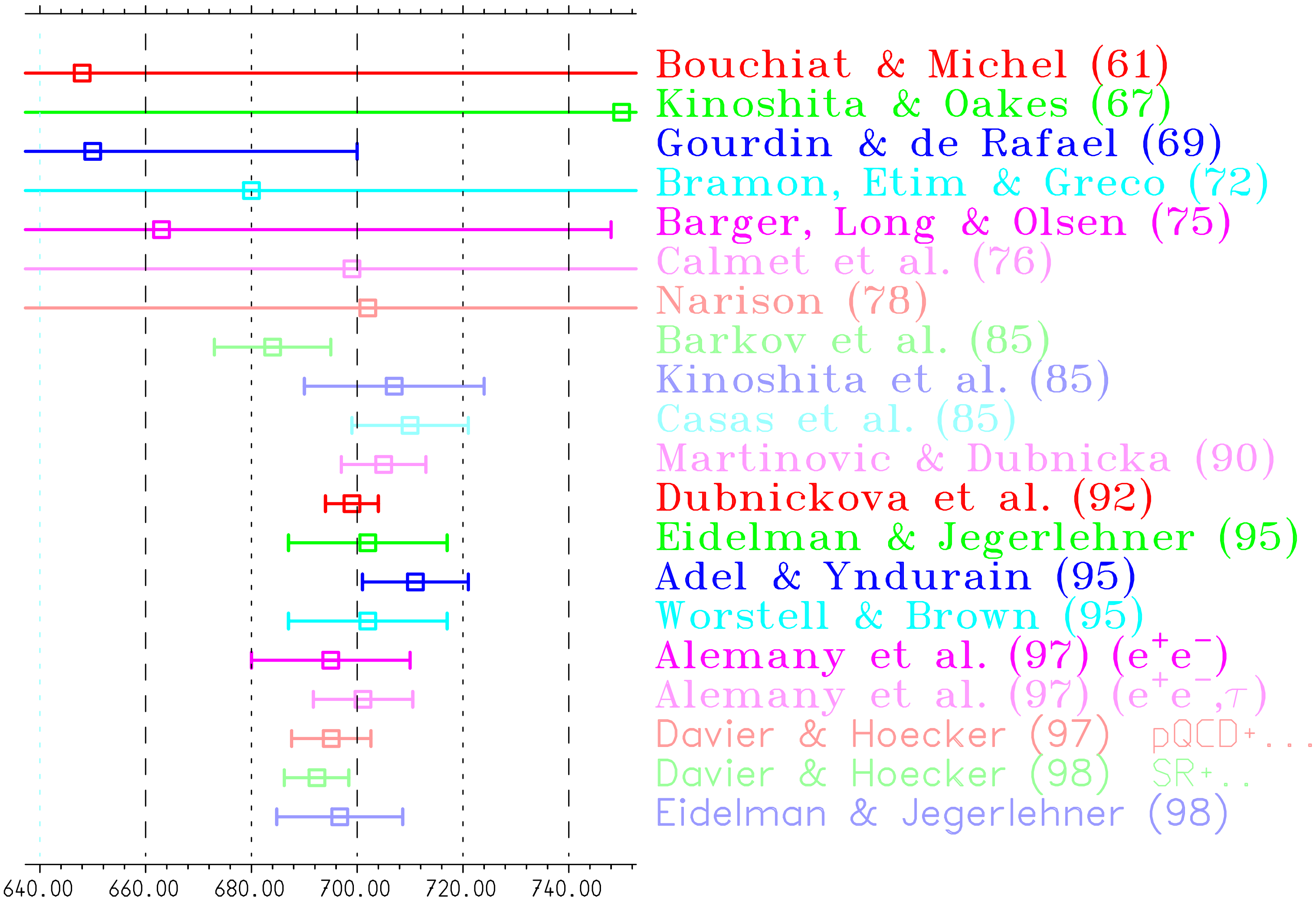
- **Note the extra $1/s$–enhancement of contributions from low energies in a_μ as compared to $\Delta\alpha$.**

$$\begin{aligned} a_\mu^{\rm had} &= (698.75 \pm 11.11) \times 10^{-10} && \textbf{(FJ 00)}^* \\ a_\mu^{\rm had} &= (692.4 \;\pm\; 6.2\;) \times 10^{-10} && \textbf{(Davier, Höcker 98b)} \\ a_\mu^{\rm exp} &= (\;\cdots\; \pm\; 4.0\;) \times 10^{-10} && \textbf{(BNL E821 (0?))} \end{aligned}$$

*[CDM-2 99, BESII 99]

Evaluations of $a_\mu^{\rm hadrons} \times 10^{10}$ by different authors (before 2000)

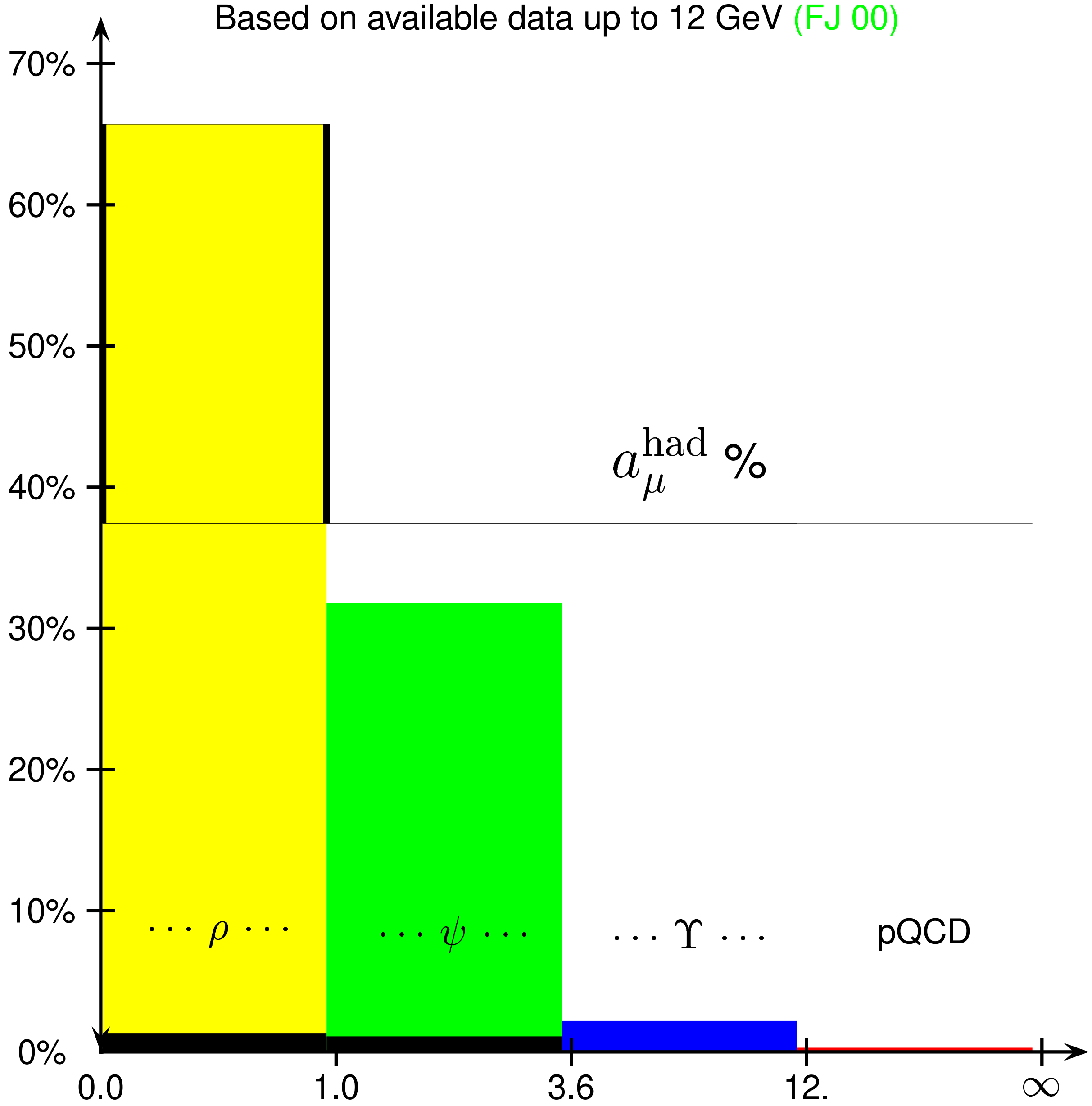
Only the leading hadronic vacuum polarization diagram is included.

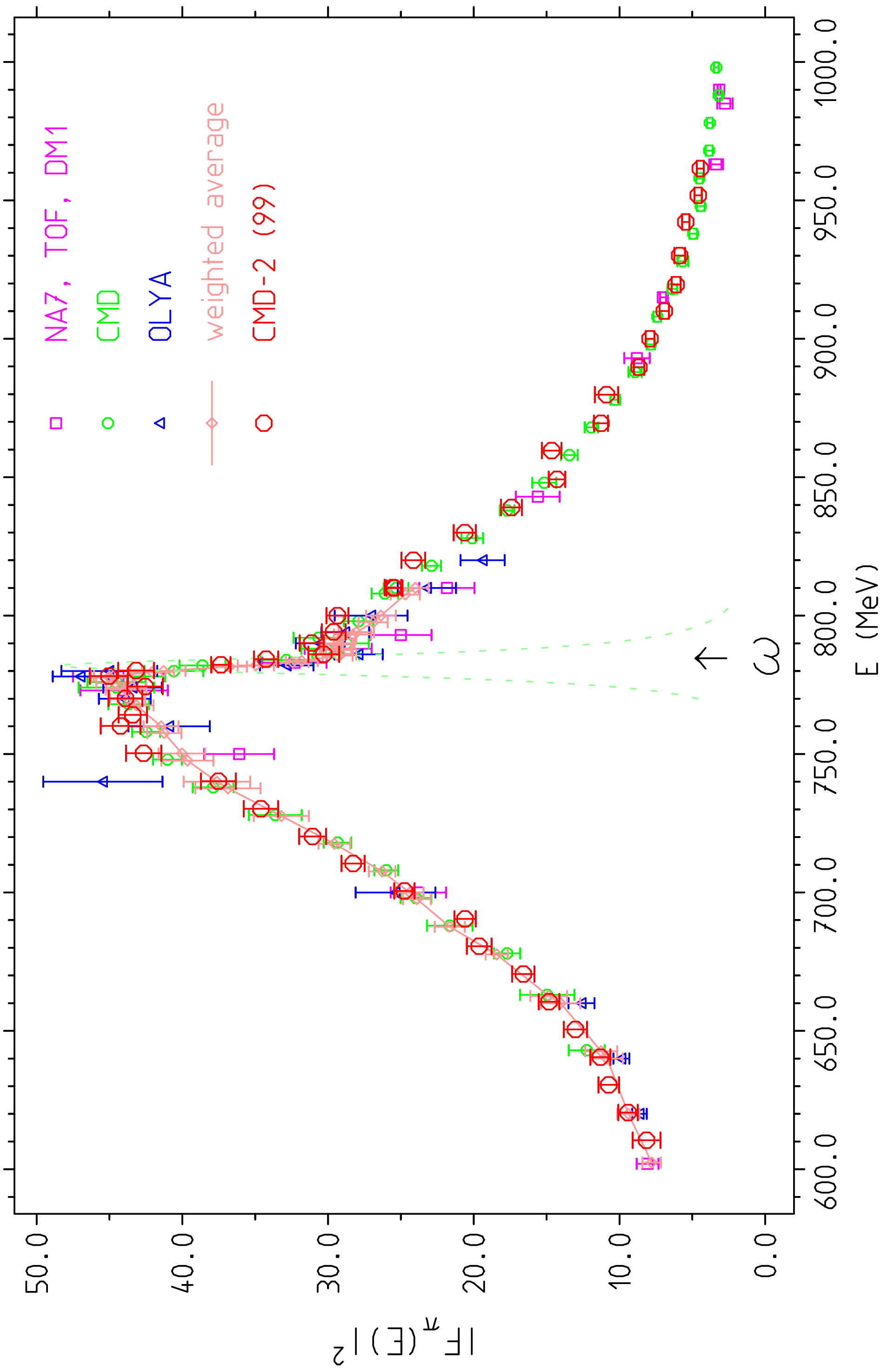


Distribution of hadronic contributions to $a_\mu^{\rm had}$

e^+e^- data based approach:

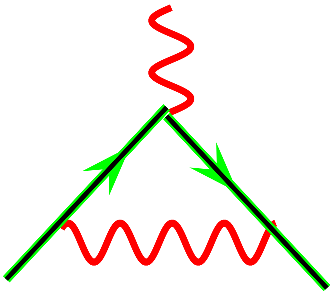
Based on available data up to 12 GeV (FJ 00)




NA7, TOF, DM1
CMD
OLYA
weighted average
CMD-2 (99)
ω
E (MeV)
|F_π(E)|^2

a_μ status:

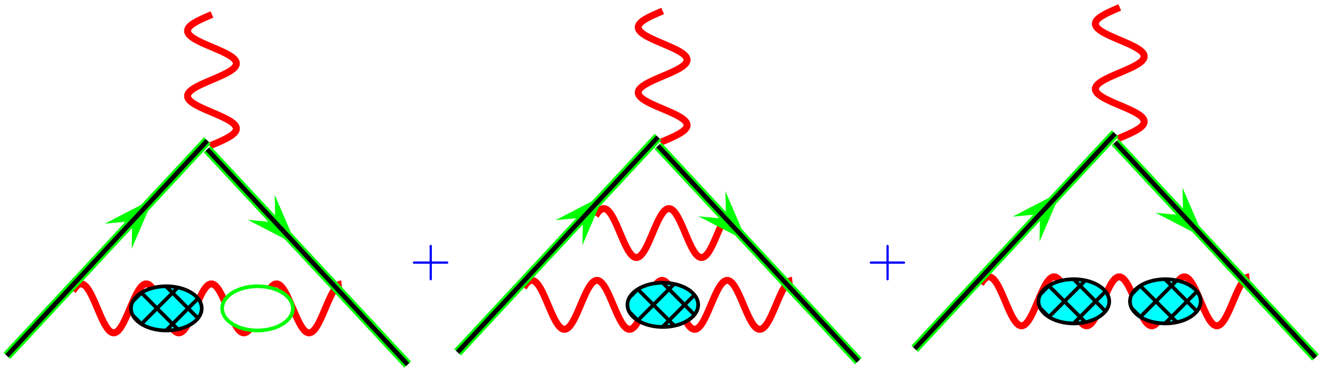
- QED



$$a_\mu^{\mathrm{QED}(1)} = (116140981 \pm 6) \times 10^{-11}$$
$$\vdots$$
$$a_\mu^{\mathrm{QED}(5)} = (116584706 \pm 3) \times 10^{-11}$$

(Schwinger 48, $\cdots$, Kinoshita 93, $\cdots$, Laporta, Remiddi 96)

- Higher order hadronic contributions

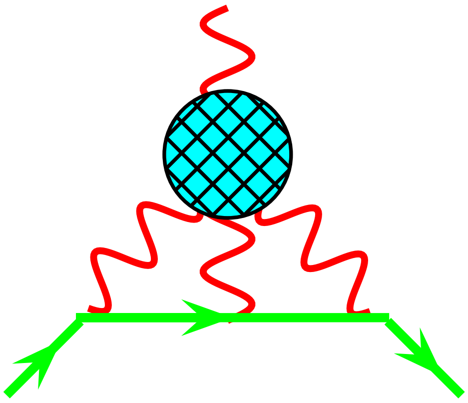


$$a_\mu^{\mathrm{had}(2)} = \frac{1}{4\pi^3} \int\limits_{4m_\pi^2}^{\infty} ds\, K^{(2)}(s)\, \sigma_{\mathrm{had}}(s)$$

$$a_\mu^{\mathrm{had}(2)} = -(100 \pm 6) \times 10^{-11}$$

(Krause 96)

- Hadronic light–by–light scattering contribution

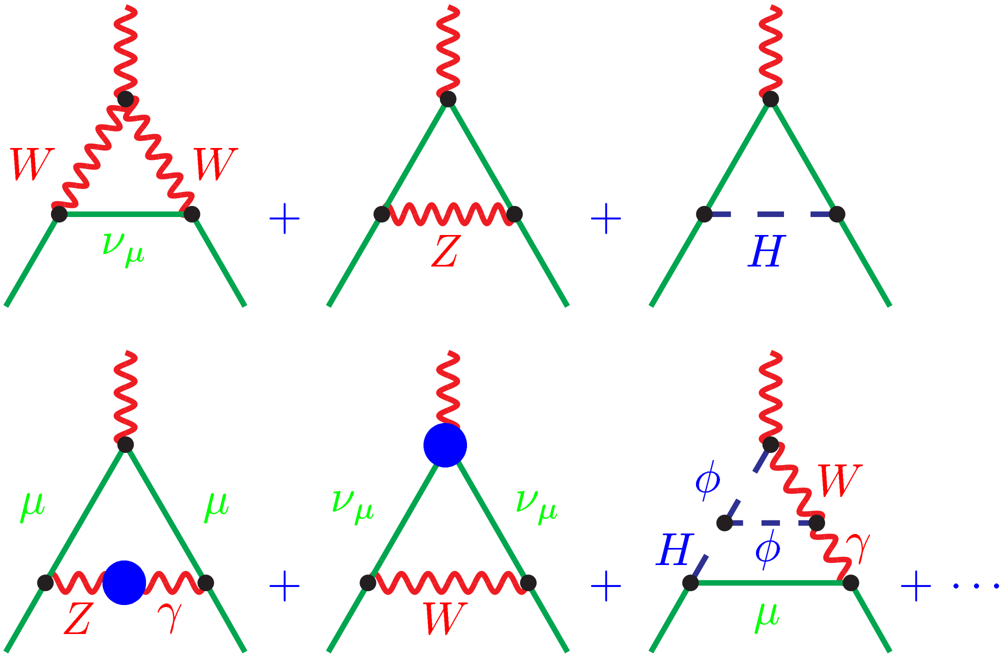


QCD inspired models used: ENJL and HGS models supplemented by VMD

$$a_\mu^{\mathrm{light-by-light}} = \begin{cases} -(92 \pm 32) & \qquad BPP96 \\ -(79 \pm 15) \quad \times 10^{-11} & \qquad HK98 \\ -(85 \pm 25) & \qquad CM98 \end{cases}$$

model ambiguity? (Bijnens, Pallante, Prades 96, Hayakawa, Kinoshita 98)

- Higher order electroweak contributions



$$a_\mu^{\mathrm{weak}(2)} = -(44 \pm 4) \times 10^{-11}$$

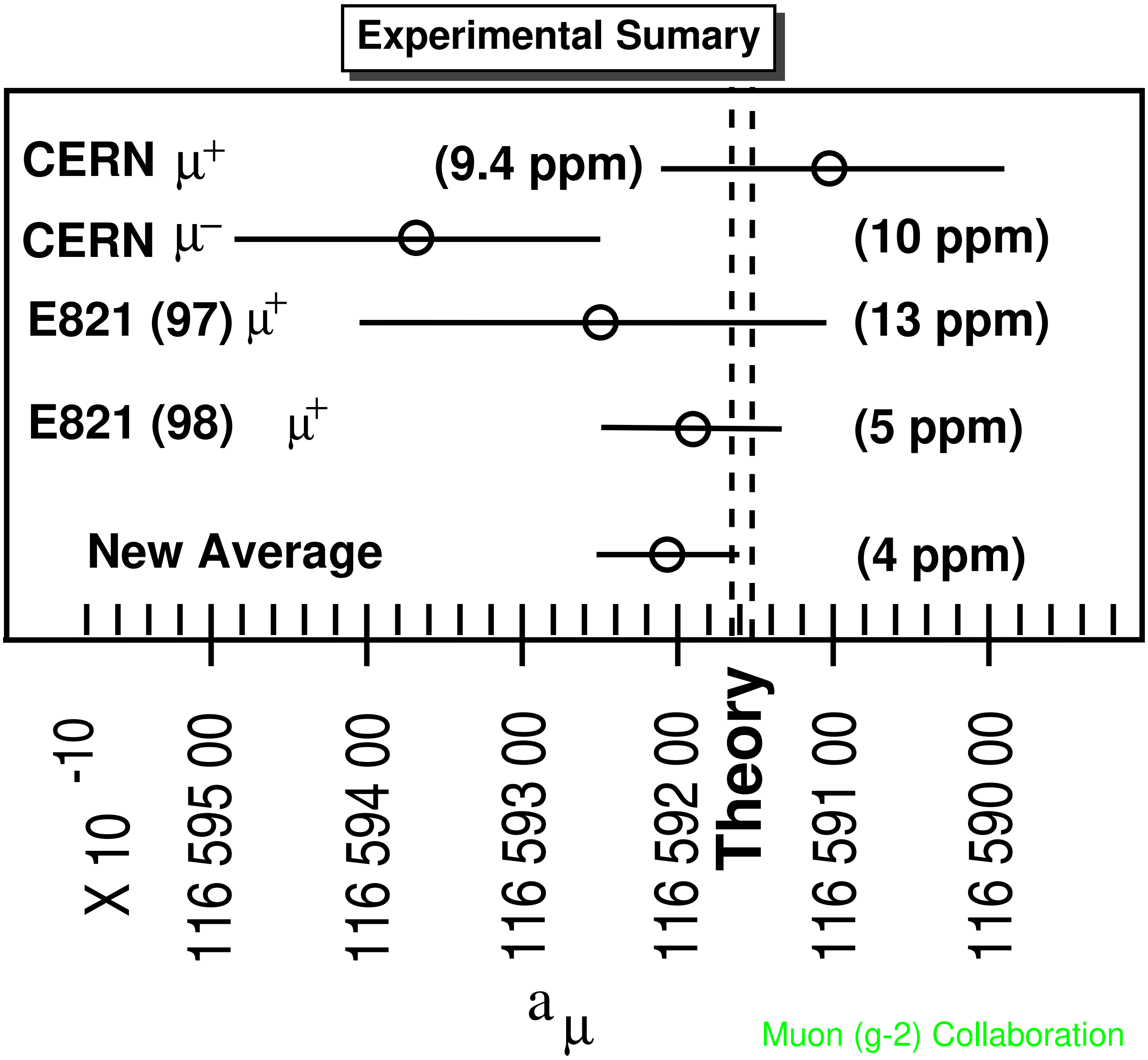
(Czarnecki, Krause, Marciano 98)

a_μ summary:

	$a_\mu \times 10^{11}$		
CERN experiment (79)	116592400	$\pm$	850
BNL E821 exp. (97)	116592500	$\pm$	1520
BNL E821 goal $\rightarrow$	$\cdots$	$\pm$	40
Theory (98)	116591595	$\pm$	67
QED	116584706	$\pm$	3
1–loop EW	195		
2–loop EW	− 44	$\pm$	4
hadronic (1)	6988	$\pm$	111
$\rightarrow$	6925	$\pm$	62[a]
hadronic (2)	− 100	$\pm$	6
light–by–light	− 92	$\pm$	32
$\rightarrow$	− 79	$\pm$	15[b]

[a] Davier, Höcker 98

[b] Hayakawa, Kinoshita 98



Muon (g-2) Collaboration
nucl-ex/0009029

Uncertainties:

expt	4.00ppm
theo	0.66ppm
BNL design	0.35ppm

New physics sensitivity: (example)

$$\Delta a_\mu^{\mathrm{SUSY}}/a_\mu \simeq 1.25\mathrm{ppm}\,\left(\frac{100\mathrm{GeV}}{\tilde{m}}\right)^2\,\tan\beta$$

$\tilde{m}$ lightest SUSY particle

$$\tan\beta = \frac{v_1}{v_2}$$

❹ Comments on recent progress

❶ Additional data: τ–data $+$ CVC

ALEPH–Coll., (OPAL–Coll.,), Alemany, Davier, Höcker 96

$$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+ e \to X^0$$

where X^- and X^0 are hadronic states related by isospin rotation. The e^+e^- cross–section is then given by

$$\sigma^{I=1}_{e^+e^- \to X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \leq M_\tau$$

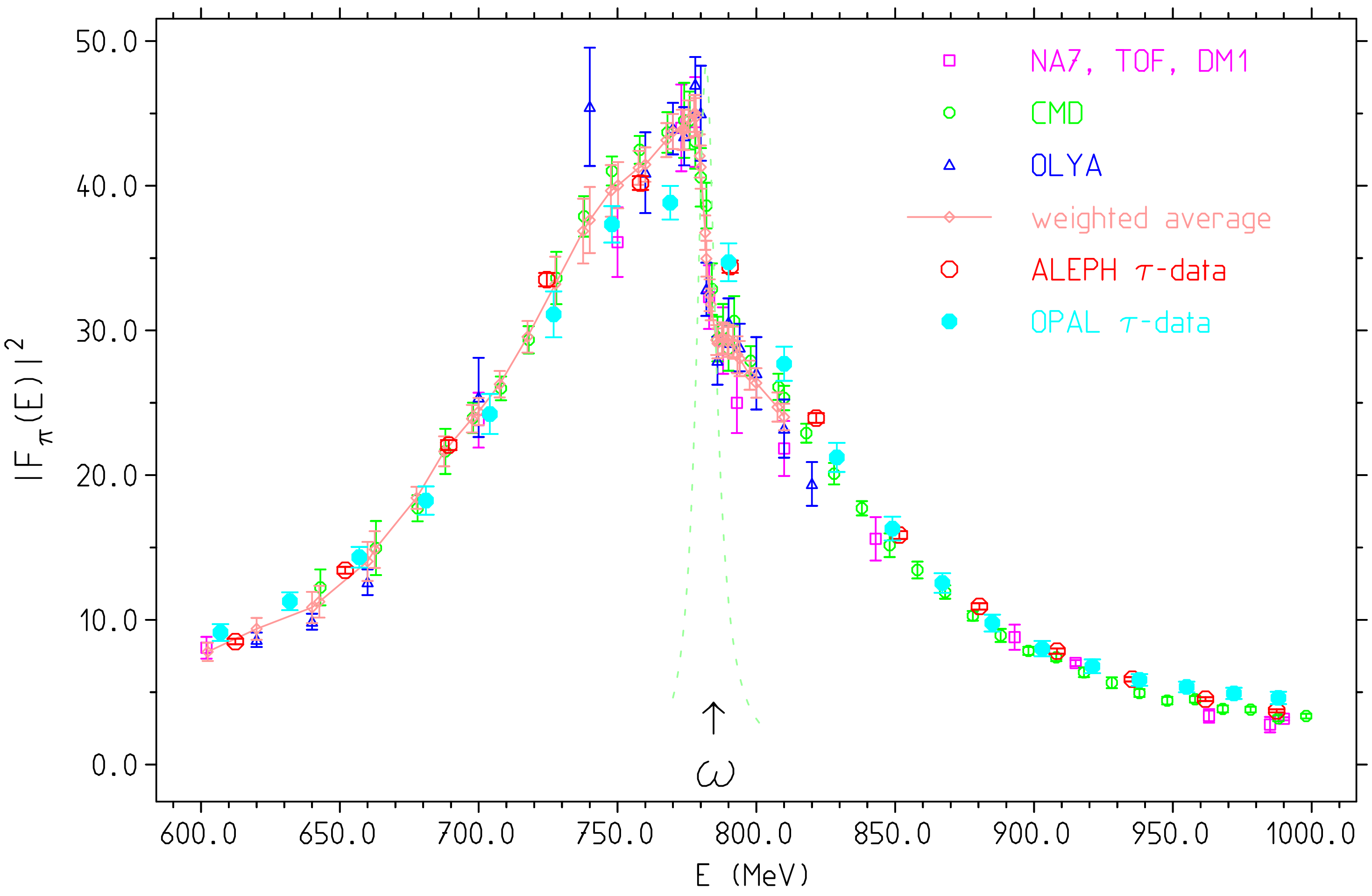
in terms of the τ spectral function v_1.

- mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ–resonance contribution)
- which is dominating in $a_\mu^{\rm had}$ (72%)

$$I = 1 \sim 75\% \, ; \; I = 0 \sim 25\%$$

τ–data cannot replace e^+e^-–data

$$\begin{array}{lcccc} \delta a_\mu & : & 15.6 \times 10^{-10} & \to & 10.2 \times 10^{-10} \\ \delta \Delta\alpha & : & 0.00067 & \to & 0.00065 \end{array}$$



systematic deviations not yet understood!

❷ Progress in pQCD calculations

Assume pQCD to be valid down to M_τ

$$\sigma_{\rm tot}(e^+e^- \to \text{hadrons}) \sim \sigma_{\rm tot}(e^+e^- \to \text{quarks and gluons})$$

The assumption seems to be supported by

- **the apparent applicability of pQCD to τ physics. In fact the running of $\alpha_s(M_\tau) \to \alpha_s(M_Z)$ from the τ mass up to LEP energies agrees well with the LEP value. The estimated uncertainty may be debated, however.**
- **non–perturbative (NP) effects if parametrized as prescribed by the operator product expansion (OPE) of the electromagnetic current correlator are seemingly small.**

For a more conservative approach see below (scheme dependence, mass effects, Euclidean approach)

❸ Non perturbative effects?

Parametrize NP effects at sufficiently large energies and away from resonances as prescribed by the OPE. Non–vanishing gluon and light quark condensates (Shifman, Vainshtein, Zakharov 79) imply the leading power corrections

$$
\begin{aligned}
\Pi'^{\mathrm{NP}}_{\gamma}(Q^2) \;=\; & \frac{4\pi\alpha}{3} \sum_{q=u,d,s} Q_q^2 N_{cq} \cdot \Bigg[\frac{1}{12} \left(1 - \frac{11}{18} a\right) \frac{<\frac{\alpha_s}{\pi} GG>}{Q^4} \\
+ \; & 2 \left(1 + \frac{a}{3} + \left(\frac{11}{2} - \frac{3}{4} l_{q\mu}\right) a^2\right) \frac{<m_q \bar{q} q>}{Q^4} \\
& + \left(\frac{4}{27} a + \left(\frac{4}{3}\zeta_3 - \frac{257}{486} - \frac{1}{3} l_{q\mu}\right) a^2\right) \sum_{q'=u,d,s} \frac{<m_{q'} \bar{q}' q'>}{Q^4} \Bigg] \\
+ \; & \cdots
\end{aligned}
$$

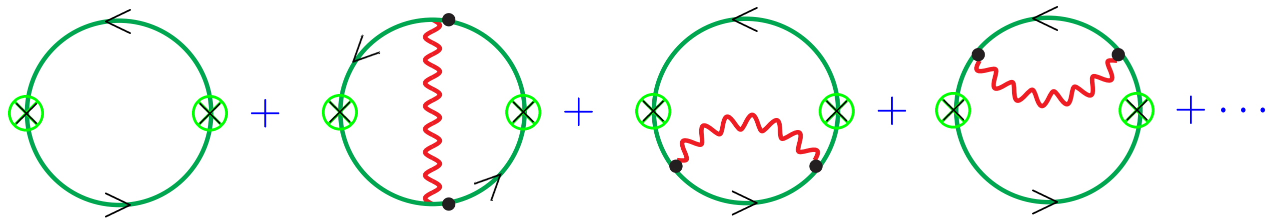
where $a \equiv \alpha_s(\mu^2)/\pi$ and $l_{q\mu} \equiv \ln(Q^2/\mu^2)$. $< \frac{\alpha_s}{\pi} GG >$ and $< m_q \bar{q} q >$ are the scale-invariantly defined condensates. Sum rule estimates of the condensates yield typically (large uncertainties)

$$
\begin{aligned}
< \tfrac{\alpha_s}{\pi} GG > &\sim \;\; (0.389\ \mathbf{GeV})^4 \\
< m_q \bar{q} q > &\sim -(0.098\ \mathbf{GeV})^4 \quad \text{for} \quad q = u, d \\
< m_q \bar{q} q > &\sim -(0.218\ \mathbf{GeV})^4 \quad \text{for} \quad q = s
\end{aligned}
$$

Note that the above expansion is just a parametrization of the high energy tail of NP effects associated with the existence of non–vanishing condensates. There are other kind of NP phenomena like bound states, resonances, instantons and what else. The dilemma with is that it works only for Q^2 large enough and it has been successfully applied in heavy quark physics. It fails do describe NP physics at lower Q^2, once it starts to be numerically relevant pQCD starts to fail because of the growth of the strong coupling constant.

❺ Progress in utilizing pQCD in a safe way

❶ pQCD calculations of vacuum polarization amplitudes



up to 4–loops massless — Groshny, Kataev, Larin 91

up to 3–loops massive — Chetyrkin, Kühn et al. 97

up to 2–loops massive BF–MOM RG — F. J., Tarasov 98

❷ use old idea: testing non–perturbative effects with help of the Adler function

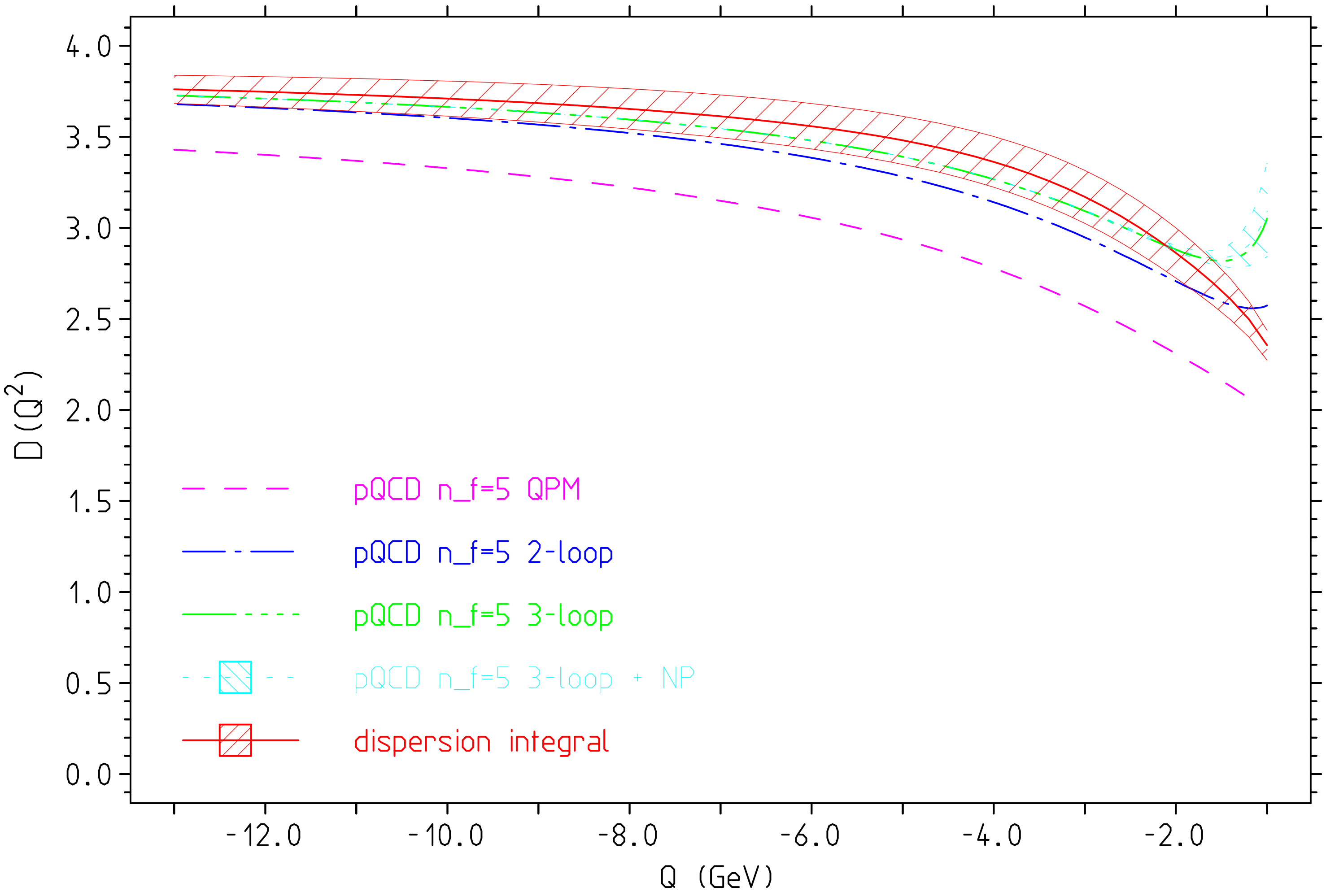
Eidelman, F. J., Kataev, Veretin 98

$$
\begin{aligned}
D(-s) &\doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\mathrm{had}}(s) \\
&= -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}
\end{aligned}
$$

$$
\Rightarrow \quad D(Q^2) = Q^2 \int\limits_{4m_\pi^2}^{\infty} ds\, \frac{R(s)}{(s+Q^2)^2}
$$

pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in <u>Euclidean</u> region

“Experimental” Adler–function versus theory (pQCD + NP)



(Eidelman, F.J., Kataev, Veretin 98)

$\Rightarrow$ pQCD works well to predict $D(Q^2)$ down to $s_0 = (2.5\,\textbf{GeV})^2$ (not down to m_τ ! however); use this to calculate

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int dQ'^2\,\frac{D(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0 = (2.5\,\textbf{GeV})^2$: **(FJ 98)**

$$\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} = 0.007541 \pm 0.000254$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.027737 \pm 0.000254$$

$$\alpha^{-1}(-M_Z^2) = 128.917 \pm 0.035$$

The virtues of this analysis are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.5 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!**

Remaining problems are the following:

a) contributions to the Adler function up to three–loops all have the same sign and are substantial. Four– and higher–orders could still add up to non-negligible contribution. An error for missing higher order terms is not included.

b) the link between space–like and time-like region is the difference

$$\Delta = \Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.000045 \pm 0.000002$$

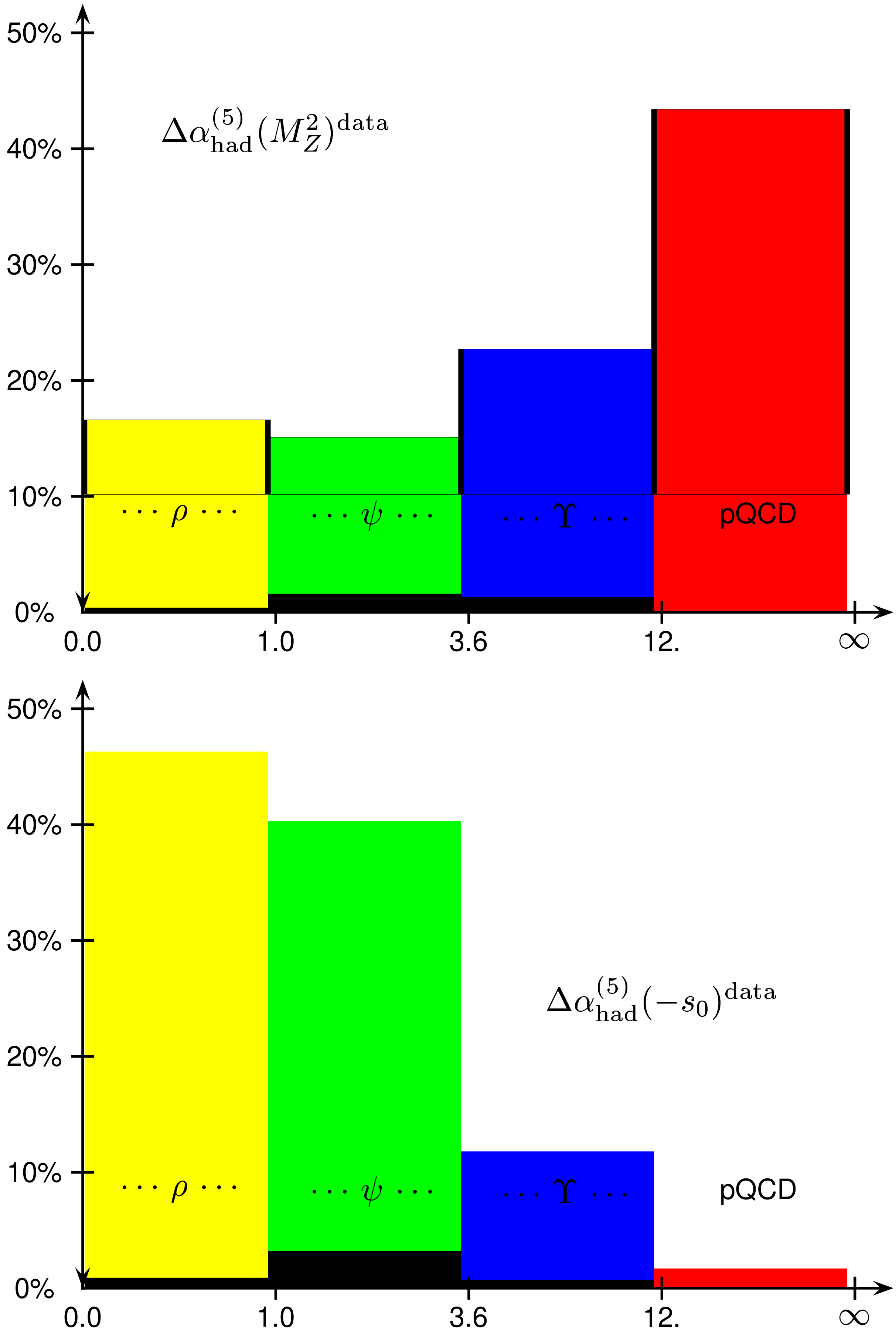
which can be calculated in pQCD. It accounts for the $i\pi$–terms from the logs $\ln(-q^2/\mu^2) = \ln(|q^2/\mu^2|) + i\pi$: Including this term yields

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027782 \pm 0.000254$$

$$\alpha^{-1}(M_Z^2) = 128.913 \pm 0.035$$

One may ask the question whether terms like Δ should be resummed at all, i.e., included in the running coupling. Usually such terms tend to cancel against constant rational terms which are not included in the renormalization group (RG) evolution. Note that the Dyson resummation in general is not a systematic resummation of leading, subleading etc. terms as the RG resummation is.

Distribution of hadronic contributions to $\Delta\alpha$



❻ Precise measurement of $\sigma(e^+e^- \to hadrons)$ at DAΦNE

Aim: Reduce hadronic uncertainty in $(g-2)_\mu, \alpha(E), \cdots$

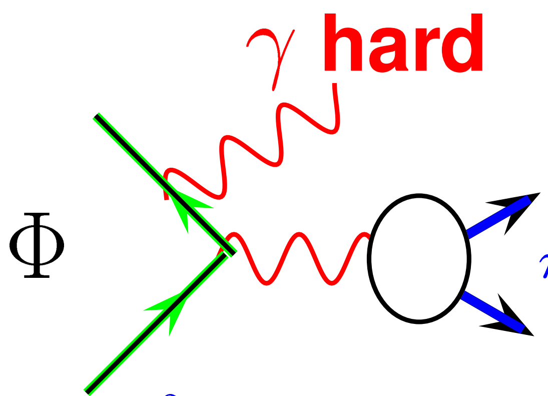
crucial for future of precision physics

Possible at ϕ–factory DAΦNE:

$\sigma(e^+e^- \to hadrons)$ **at** $\sqrt{s} \lesssim 2$ **GeV**

❶ $F_\pi(s) \quad \sqrt{s} \lesssim 1$ **GeV**

❷ $R(s) \quad 1$ **GeV** $\lesssim \sqrt{s} \lesssim 2$ **GeV**

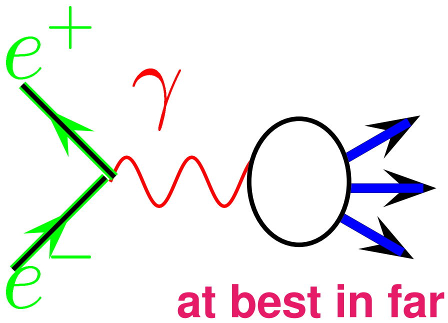


High priority !

- very clean experimentally
- normalization at one energy only

$\pi^+\pi^-,\ \rho_0,\ 3\pi,\ 4\pi, \cdots$

$s = M_\Phi^2 \qquad s' = s\,(1-k) \qquad [k = E_\gamma/E_{\rm beam}]$



hadrons

at best in far future !

requires non trivial upgrade of DAΦNE such that energy scan is possible

$\to F_\pi(s)$, $R(s)$ **direct** $\sqrt{s} \lesssim 2$ **GeV.**

Radiative corrections: $\pi^+\pi^-$–spectrum etc.

- Theoretical Background (Eidelman, Kataev, Jegerlehner, Veretin)

control extended use of perturbative QCD (theory) by applying Euclidean fied theory methods (Adler Function)

- Radiative corrections: $e^+e^- \to \pi^+\pi^-\gamma$ (Hoefer, Gluza, Jegerlehner)
  - $O(\alpha)$: ISR, FSR, Interferrence
  - mass effects investigated
  - $O(\alpha^2)$: ISR to invariant mass spectrum

At DAΦNE : fully inclusive approach to invariant mass spectrum of $\pi^+\pi^-$ pair

$\to$ in this approach best controll of theoretical uncertainties available at .4% in predominant $\pi^+\pi^-$ channel.

Still not fully under control: pion form factor model dependence ?

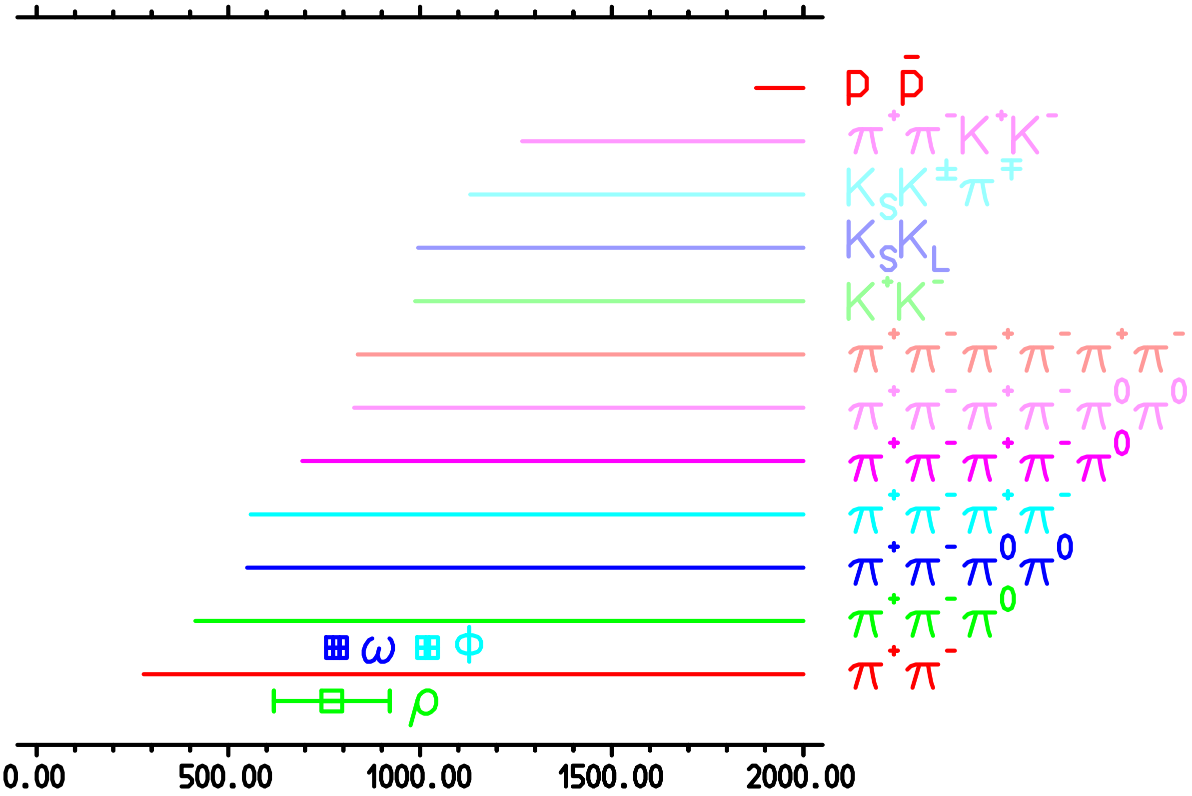
compositness of pion ?

- Multi hadron channel to be considered

Radiative corrections: Photon tagging

- Radiative return $e^+e^- \to \pi^+\pi^-\gamma$
  - Theory: ISR, $O(\alpha)$ corrections, FSR reduction by cuts
    A. Arbuzov et al., V. Khoze et al., S. Binner, J. Kühn, K. Melnikov
  - Monte Carlo simulation program for KLEO detector:
    1) Cataldi, Denig, Kluge, Venenzoni,
    2) Melnikov, Nguyen, Valeriani, Venanzoni
  - Firs experimental results (still low statistics)
- Background study: $\Phi \to \pi^+\pi^-\gamma$ “direct”, $f_0(480)$ contribution, $e^+e^- \to \pi^0\pi^0\gamma$ as a reference process.
- Multi hadron final states:
  $e^+e^- \to$
  $\pi^+\pi^-\pi^+\pi^-,\ \pi^+\pi^-\pi^0\pi^0$ $\nearrow$ Theory: Czyż Kühn
  $\searrow$ MC/Exp: Venanzoni at al. KLOE
- Relation to τ decay investigated: Kühn et al., Eidelman, Ivanchenko

Multi-particle channels in e^+e^-–annihilation at low energy



F_π, R : **inclusive quantities ! How to measure?**

competing two body channels

$$e^+e^-,\ \mu^+\mu^-,\ \pi^+\pi^-,\ K^+K^-,\ \cdots$$

separation requires particle identification

$\longrightarrow$ **exclusive measurements**

requires detailed understanding of individual channels.

Contributions to $\tilde{a}_\mu^{\rm had} = a_\mu^{\rm had} \times 10^{10}$ from exclusive channels.

channel	$\tilde{a}_\mu^{\rm had}$	acc.
$\rho, \omega \to \pi^+\pi^-$	506	0.3%
$\omega \to 3\pi$	47	$\sim$ 1%
ϕ	40	$\downarrow$
$\pi^+\pi^-\pi^0\pi^0$	24	$\cdot$
$\pi^+\pi^-\pi^+\pi^-$	14	$\cdot$
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	5	10%
3π	4	$\downarrow$
K^+K^-	4	$\cdot$
K_SK_L	1	$\cdot$
$\pi^+\pi^-\pi^+\pi^-\pi^0$	1.8	$\cdot$
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	0.5	$\cdot$
$p\bar{p}$	0.2	$\cdot$
$2\ \mathrm{GeV} \leq E \leq M_{J/\psi}$	22	
$M_{J/\psi} \leq E \leq M_\Upsilon$	20	
$M_\Upsilon < E$	$\lesssim 5$	

Theoretical work with the aim to calculate radiative corrections at the level of precision as indicated in the Table is in progress. $\to \delta a_\mu^{\rm had} \lesssim 26 \times 10^{-11}$ **from** $\sqrt{s} \lesssim 2$ **GeV.**

❼ Status and Outlook

$a_\mu^{\rm had}$: uncertainty from $\pi^+\pi^-$, ρ^0 dominates

Uncertainties of $a_\mu^{\rm had}$ in units 10^{-11}.

δa_μ	input
~ 156	e^+e^- data (EJ95, ADH98)
~ 60	DaΦne + additional τ data
~ 62	theory-driven (DH98b)
$\sim ??$	"Adler based" (EJV00) ?
~ 102	e^+e^-, τ data (ADH98)
~ 111	e^+e^- data incl. CMD-2 99, BESII 99 (FJ00)
~ 40	BNL a_μ–experiment (CM98)

Precision measurement of a_μ provides most sensitive test of magnetic helicity flip transition

$$\bar{\psi}_L\,\sigma_{\mu\nu}\,F^{\mu\nu}\,\psi_R \qquad (\,\mathrm{dim}\;5\;\mathrm{operator}\,)$$

such a term must be absent for any fermion in any renormalizable theory at tree level

$\to$ finite prediction

$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$:

$$\frac{\delta M_W}{M_W} \sim \frac{1}{2}\frac{\sin^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 0.23\;\delta\Delta\alpha$$

$\delta\Delta\alpha$	δM_W (MeV)	Ref.	Method
0.00065	12.0	[EJ95]	data $<\ 12.$ GeV
0.00016	3.0	[DH98]	data $<\ 1.7$ GeV
0.00025	4.6	[FJ98]	Euclidean $>\ 2.5$ GeV
0.000197	3.6	[MOR00]	rescaled data, pQCD 2.8-3.7, 5-∞
0.000220	4.0	[MOR00]	same but "exclusive"
0.000460	8.5	[BP00]	data
0.000391	7.2	[FJ00]	as EJ95 + new data (BESII)
0.00007	1.3	$\delta\sigma \lesssim 1\%$ up to J/ψ	
0.00005	0.9	$\delta\sigma \lesssim 1\%$ up to Υ	
WA	22.0	Osaka 2000	

$$\frac{\delta \sin^2\theta}{\sin^2\theta} \sim \frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 1.54\;\delta\Delta\alpha$$

$\delta\Delta\alpha$	$\delta\sin^2\theta$	Ref.	Method
0.00065	0.000232	[EJ95]	data $<\;12.$ GeV
0.00016	0.000057	[DH98]	data $<\;1.7$ GeV
0.00025	0.000089	[FJ98]	Euclidean $>\;2.5$ GeV
0.000197	0.000070	[MOR00]	rescaled data, pQCD 2.8-3.7, 5-∞
0.000220	0.000078	[MOR00]	same but "exclusive"
0.000460	0.000164	[BP00]	data
0.000391	0.000139	[FJ00]	as EJ95 + new data
0.00007	0.000025	$\delta\sigma \lesssim 1\%$ up to J/ψ	
0.00005	0.000018	$\delta\sigma \lesssim 1\%$ up to Υ	
WA	0.000160	LEP+SLD	

Concluding Remarks:

Experimental efforts to measure very precisely the total cross section $\sigma(e^+e^- \to hadrons)$ at low energies are mandatory for the future of electroweak precision physics. Taking into account recent theoretical progress, these "low energy" measurements are not only important for testing a_μ but as well for the effective fine structure constant $\alpha(M_Z)$. A real beakthrough would be possible by measuring $\sigma(e^+e^- \to hadrons)$ at 1% accuracy below the τ–threshold.

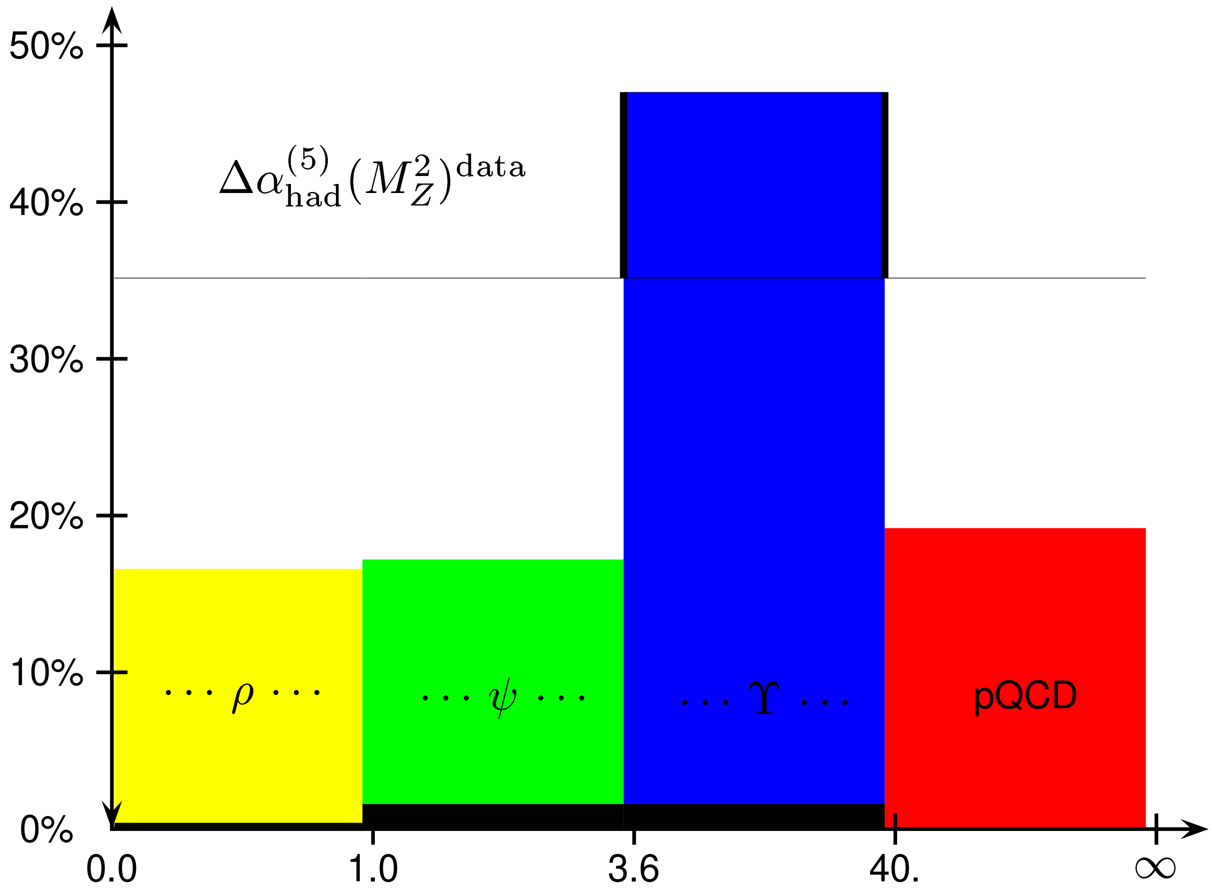
Work in progress:

- **VEPP-2M Novosibirsk (CMD-2, SND): can still improve to 1.5% up to 1.4 GeV (upgrade?)**
- **DAΦNE Frascati (KLOE): upgrade for direct measurement at better than 1% level needed (2004?)**
- **BEPC Beijing (BES): can improve a lot in the important J/Ψ region (upgrade planned)**
- **future "τ–charm facility" tunable between 2 GeV and 3.6 GeV would settle the remaining problems.**

Distribution of hadronic contributions to $\Delta\alpha$

Data based approach (E):

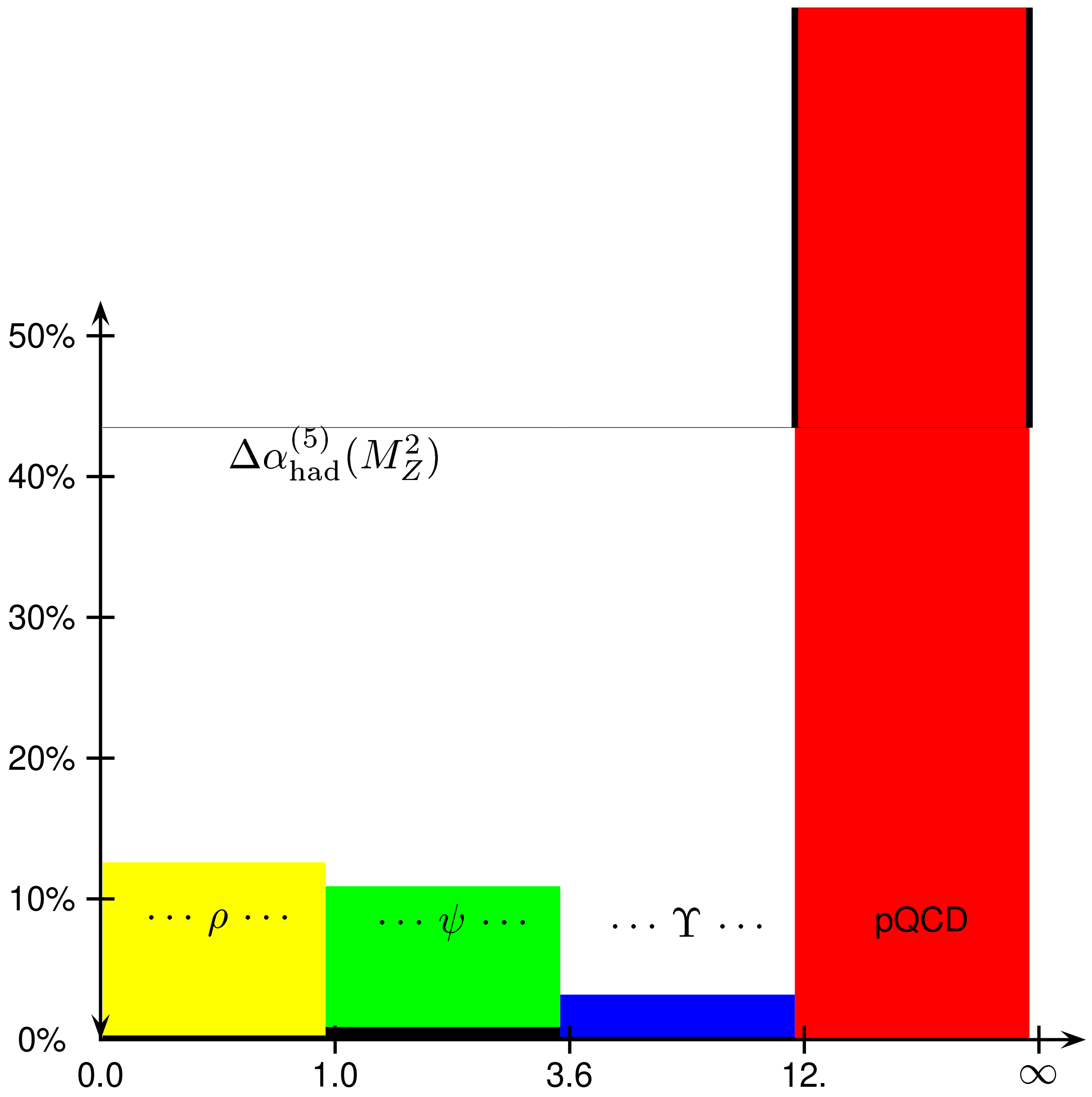
Based on available data up to 40 GeV $(EJ95)$



Distribution of hadronic contributions to $\Delta\alpha$

Euclidean improved approach (T):

Based on available data up to 12 GeV $(FJ98)$



Note: Our Euclidean approach (based on the Adler function $D(Q^2)$) allows us to rely on pQCD in a safe way, this is in contrast to the so called "theory driven" approaches, which try to predict $R(s)$ directly.

Another possibility, partially already adopted by some authors:

① **find the Higgs and determine its mass** m_H

??? $m_H = 115^{+1.3}_{-0.9}$ **LEP Nov. 2000 ???**

② **SM tells you what** $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ **is !**

Summary of my Y2K update: ($M_Z = 91.19$**GeV**)

Update (standard) EJ 95 + BESII 98/99 :

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = 0.027896 \pm 0.000395$$

Update (Euclidean) FJ 98 + BESII 98/99 :

$$\Delta\alpha^{(5)}_{\text{had}}(-s_0)^{\text{data}} = 0.007489 \pm 0.000146$$

$$\Delta\alpha^{(5)}_{\text{had}}(-M_Z^2) = 0.027685 \pm 0.000146 + \text{ pQCD error}$$

$$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) = 0.027730 \pm 0.000148 + \text{ pQCD error}$$

pQCD uncertainty still under investigation !