$\alpha_{\rm QED}(M_{\rm Z})$ and future prospects with low energy e^+e^- collider data

Fred Jegerlehner*, DESY Zeuthen/HU Berlin,
fjeger@physik.hu-berlin.de

INFN LNF - Laboratori Nazionali di Frascati, Seminar, March 23, 2016

Outline of Talk:

- Motivation
- The Role of $\alpha_{\rm QED,eff}$ in Precision Physics
- Evaluation of $\alpha_{\rm QED}(M_Z^2)$
- Reducing uncertainties via the Euclidean split trick:
  Adler function controlled pQCD
- Prospects for future improvements
- Addendum 1: Using τ–decay spectra + isospin breakings
- Addendum 2: The coupling α_2, M_W and $\sin^2\Theta_f$

1. Motivation

The Parameters of the Standard Model

– in four fermion and vector boson processes –

in addition QCD coupling α_s, y_t vs. M_t, λ_H vs. M_H etc.
unlike in QED and QCD in SM (SBGT)
parameter interdependence

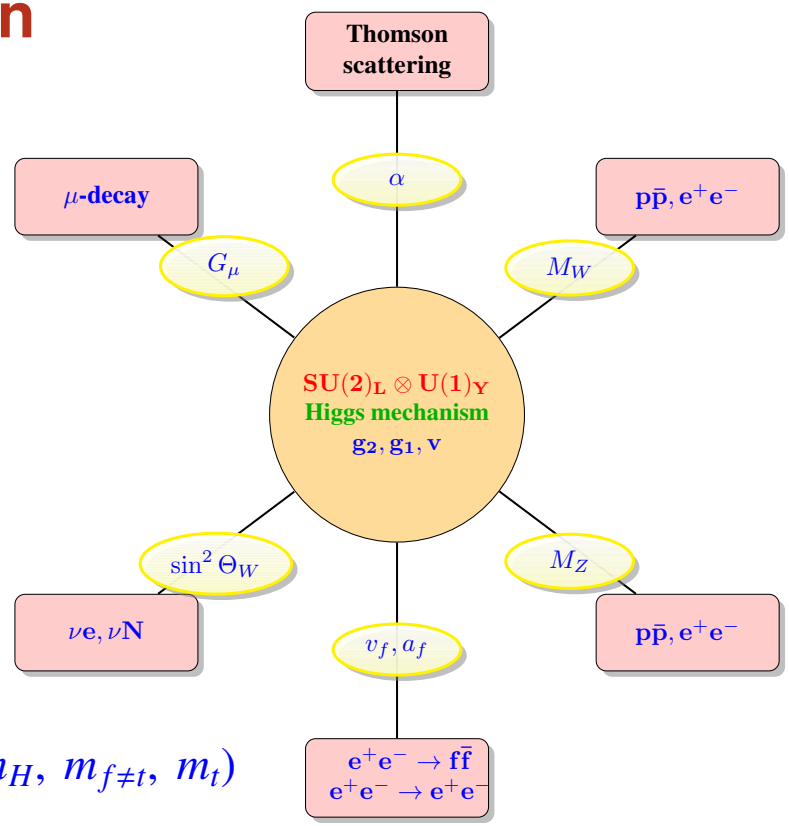
➠

only 3 independent quantities
(besides fermion masses and mixing parameters)

α, G_μ, M_Z $\Rightarrow$ $\alpha_{\rm eff}(M_Z^2)$ $\Rightarrow$ large hadronic correction

$\Downarrow$

$$\sin^2\Theta_i\ \cos^2\Theta_i\ =\ \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i}\quad;\quad \Delta r_i = \Delta r_i(\alpha,\ G_\mu,\ M_Z, m_H,\ m_{f\neq t},\ m_t)$$



parameter relationships between very precisely measurable quantities
precision tests, possible sign of new physics
non-perturbative $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ is limiting precision predictions

Note: 30 SD disagreement between SM prediction and experiment when subleading corrections are dropped!

Gauge coupling unification?

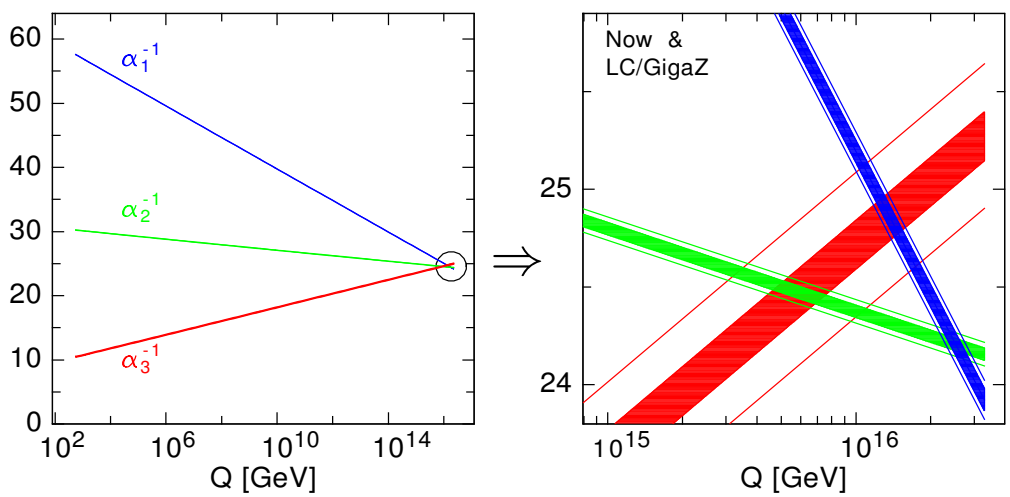
Precise SM predictions require to determine the $U(1)_Y \otimes SU(2)_L \otimes SU(3)_c$ SM gauge couplings $\alpha_{\rm em}$, α_2 and $\alpha_s \equiv \alpha_3$ (QCD) as accurately as possible

**** a theory can not be better than its input parameters ****

$\Rightarrow$ precision limitations due to non-perturbative hadronic contributions $\Leftarrow$

❖beyond SM physics?

from Zerwas

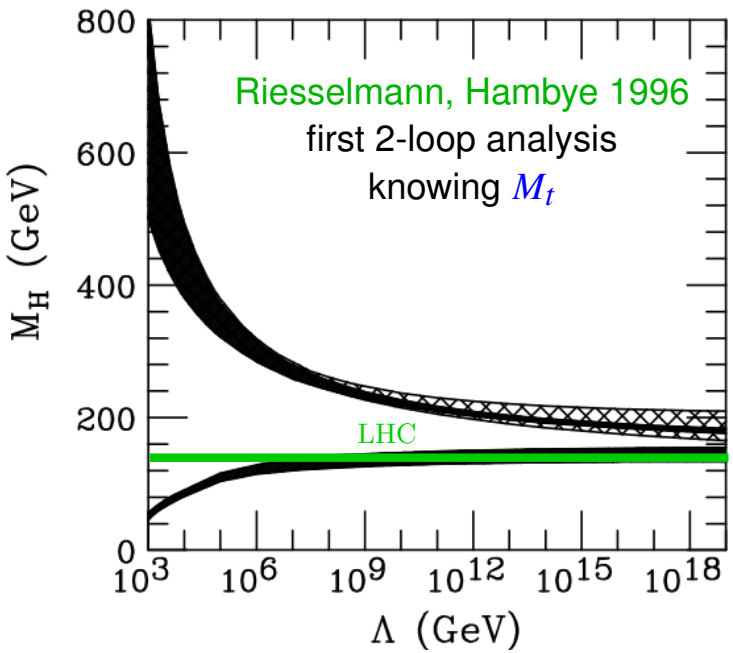


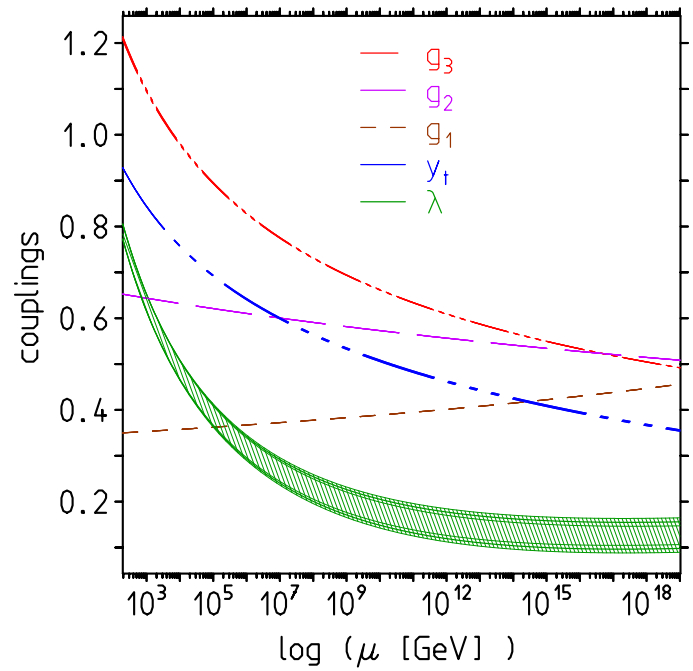
$\alpha_s = 0.1183 \pm 0.0027$ vs ± 0.0009

SM extrapolation up to Planck scale?

After Higgs discovery: Higgs vacuum stability issue!

⇒Need very precise SM parameters: g', g, g_s, y_t, λ





The SM dimensionless couplings in the $\overline{\text{MS}}$ scheme as a function of the renormalization scale for $M_H = 124 - 127$ GeV.

- perturbation expansion works up to the Planck scale!
  no Landau pole or other singularities, Higgs potential likely remains stable!
- ❒ $U(1)_Y$ screening (IR free), $SU(2)_L$, $SU(3)_c$ antiscreening (UV free): g_1, g_2, g_3

as expected (standard wisdom)

- ❒ Top Yukawa y_t and Higgs λ: screening if standalone (IR free, like QED)

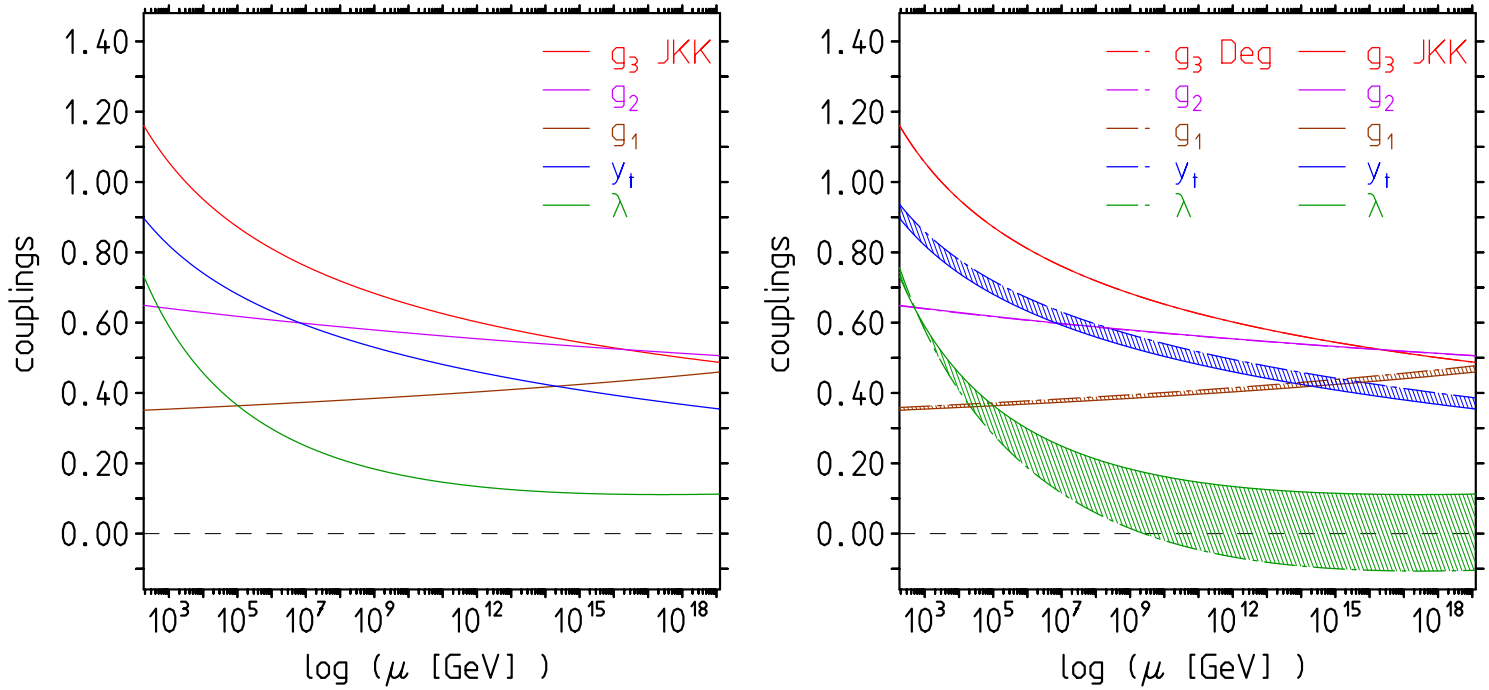
as part of SM, transmutation from IR free to UV free

As SM couplings are as they are: QCD dominance in top Yukawa RG requires $g_3 > \frac{3}{4} y_t$, top Yukawa dominance in Higgs RG requires $\lambda < \frac{3(\sqrt{5}-1)}{2} y_t^2$ in the gaugeless ($g_1, g_2 = 0$) limit.

In the focus:

- ❒ does Higgs self-coupling stay positive $\lambda > 0$ up to $\Lambda_{\rm Pl}$?
- ❒ the key question/problem concerns the size of the top Yukawa coupling y_t
  decides about stability of our world! — [$\lambda = 0$ would be essential singularity!]

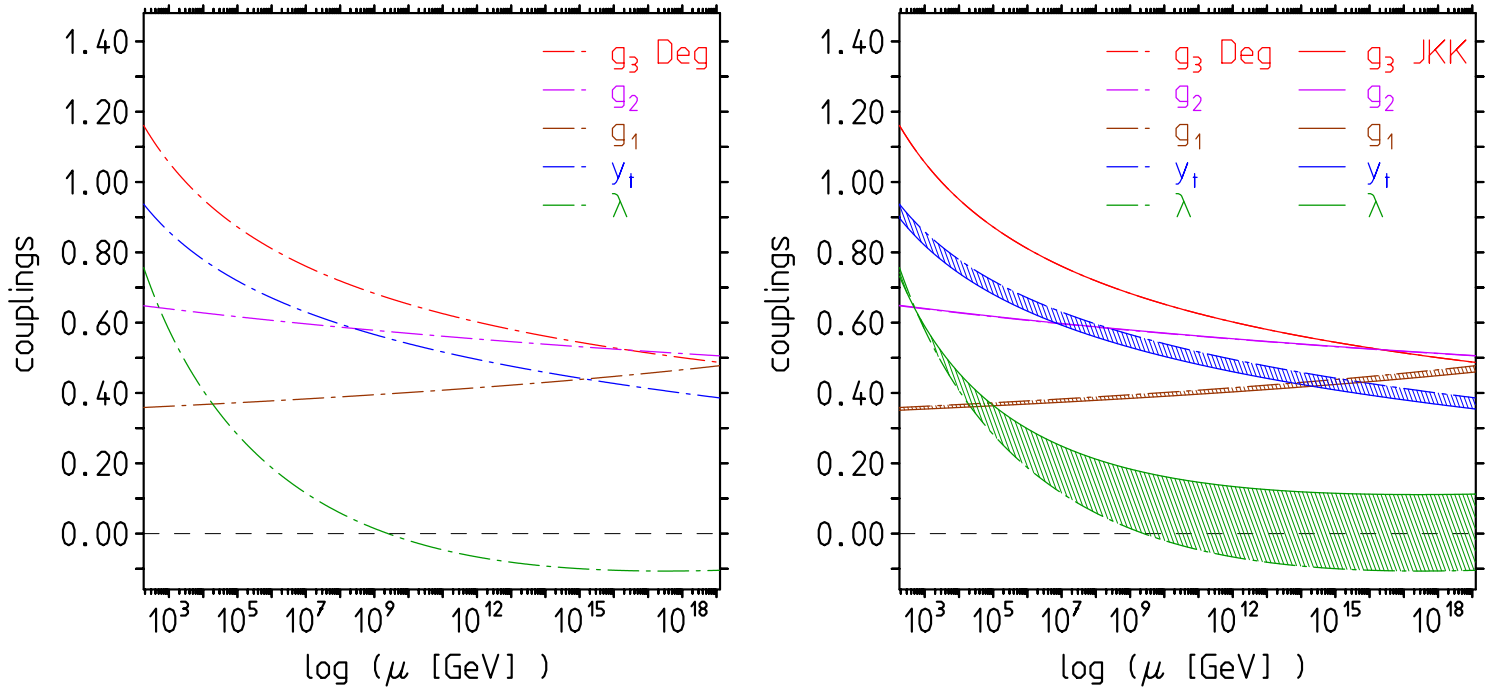
Will be decided by:
- more precise input parameters
- better established EW matching conditions



F.J.,Kalmykow,Kniehl, On-Shell vs $\overline{\mathrm{MS}}$ parameter matching

- the big issue is the very delicate conspiracy between SM couplings: precision determination of parameters more important than ever $\Rightarrow$ the challenge for LHC and ILC/FCC: precision values for λ, y_t and α_s, and for low energy hadron facilities: more precise hadronic cross sections to reduce hadronic uncertainties in $\alpha(M_Z)$ and $\alpha_2(M_Z)$

New gate to precision cosmology of the early universe!



Shaposnikov et al., Degrassi et al. matching

- the big issue is the very delicate conspiracy between SM couplings: precision determination of parameters more important than ever $\Rightarrow$ the challenge for LHC and ILC/FCC: precision values for λ, y_t and α_s, and for low energy hadron facilities: more precise hadronic cross sections to reduce hadronic uncertainties in $\alpha(M_Z)$ and $\alpha_2(M_Z)$

New gate to precision cosmology of the early universe!

2. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

α , G_μ, M_Z **most precise input parameters** $\Rightarrow$ **precision predictions**

50% non-perturbative

$\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$\frac{\delta\alpha}{\alpha}$	$\sim$	3.6	$\times$	10^{-9}	
$\frac{\delta G_\mu}{G_\mu}$	$\sim$	8.6	$\times$	10^{-6}	
$\frac{\delta M_Z}{M_Z}$	$\sim$	2.4	$\times$	10^{-5}	
$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)}$	$\sim$	$0.9 \div 1.6$	$\times$	10^{-4}	(present : lost 10^5 in precision!)
$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)}$	$\sim$	5.3	$\times$	10^{-5}	(ILC requirement)

LEP/SLD: $\sin^2\Theta_{\rm eff} = (1 - g_{Vl}/g_{Al})/4 = 0.23148 \pm 0.00017$

$$\delta\Delta\alpha(M_Z) = 0.00020 \qquad \Rightarrow \qquad \delta\sin^2\Theta_{\rm eff} = 0.00007$$

affects Higgs mass bounds, precision tests and new physics searches!!!

For pQCD contributions very crucial: precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t \Rightarrow$ Lattice-QCD

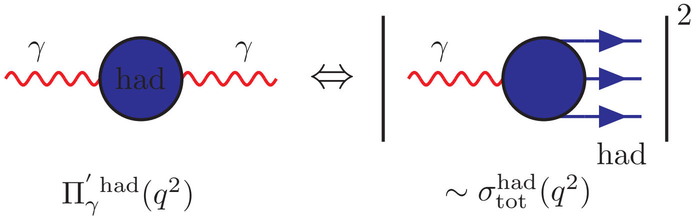
3. Evaluation of $\alpha(M_Z^2)$

Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s) = -\left(\Pi'_\gamma(s) - \Pi'_\gamma(0)\right)$ can be evaluated in terms of $\sigma(e^+e^- \to \text{hadrons})$ data via dispersion integral:

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha\, s}{3\pi}\left(\oint\limits_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \oint\limits_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

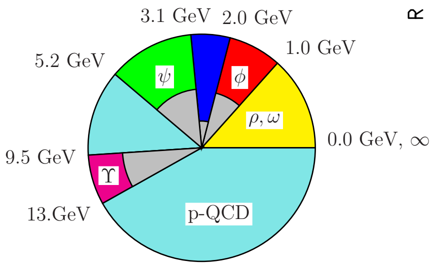
where $R_\gamma(s) \equiv \dfrac{\sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})}{\frac{4\pi\alpha^2}{3s}}$

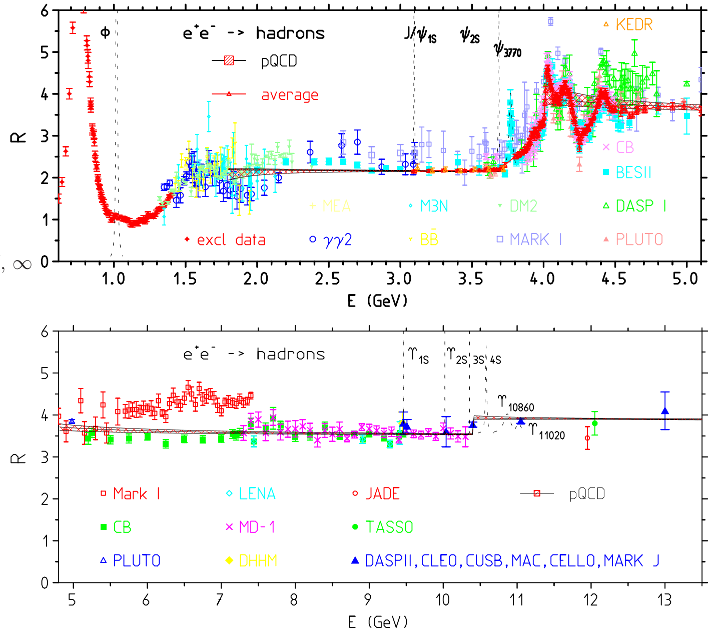
Compilation: FJ 15
Theory = pQCD: Gorishny et al. 91, Chetyrkin et al. 97...09



$\Pi'^{\rm \ had}_\gamma(q^2)$ $\sim \sigma^{\rm had}_{\rm tot}(q^2)$

hadronic vacuum polarization

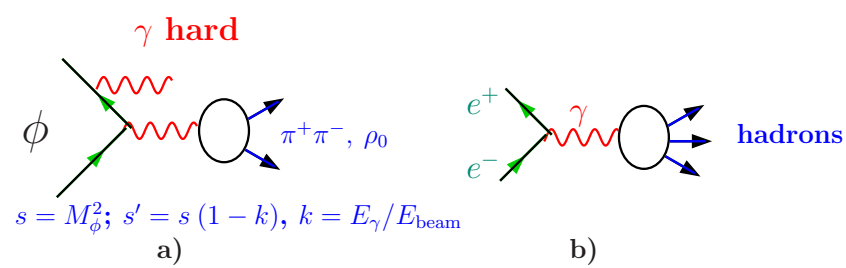




$$\boxed{\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\ ; \quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)}$$

Present situation: (after KLOE, BaBar and first BESIII results)

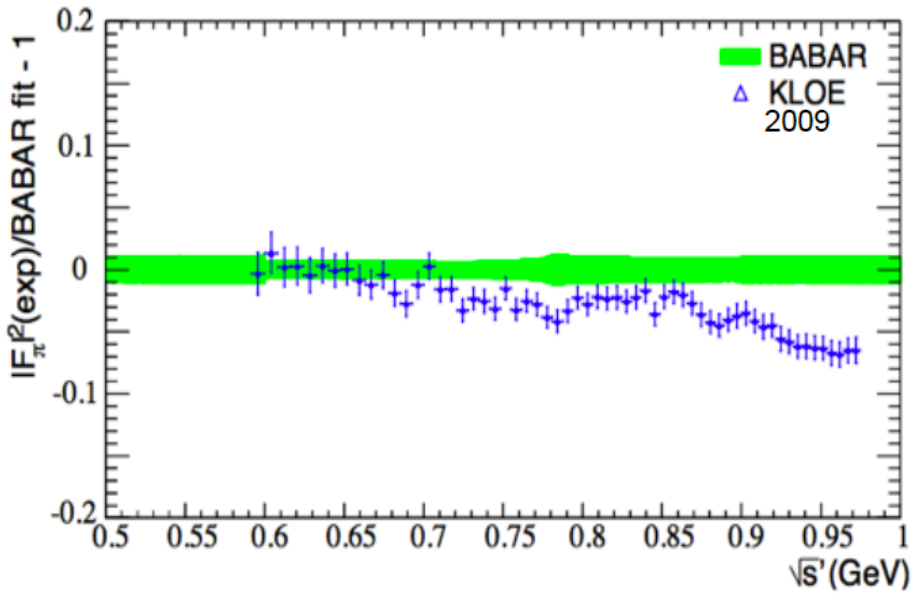
$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2)$	=	0.027648 ± 0.000176	
		0.027504 ± 0.000118	Adler
$\alpha^{-1}(M_Z^2)$	=	128.942 ± 0.021	
		128.961 ± 0.011	Adler

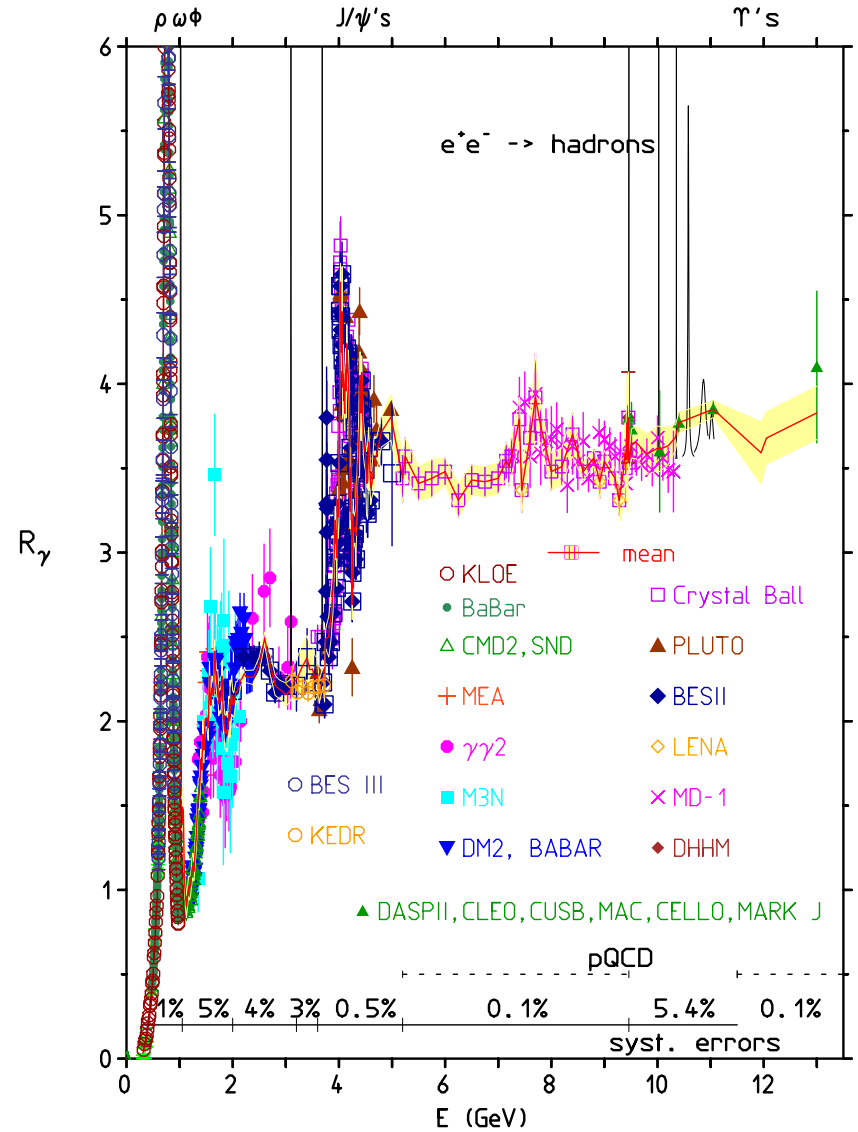


a) Initial state radiation (ISR), b) Standard energy scan.

SCAN: CMD-2, SND (NSK); ISR: KLOE (pioneered the method), BaBar, BESIII

New experimental input for HVP: BESIII-ISR,VEPP-2000



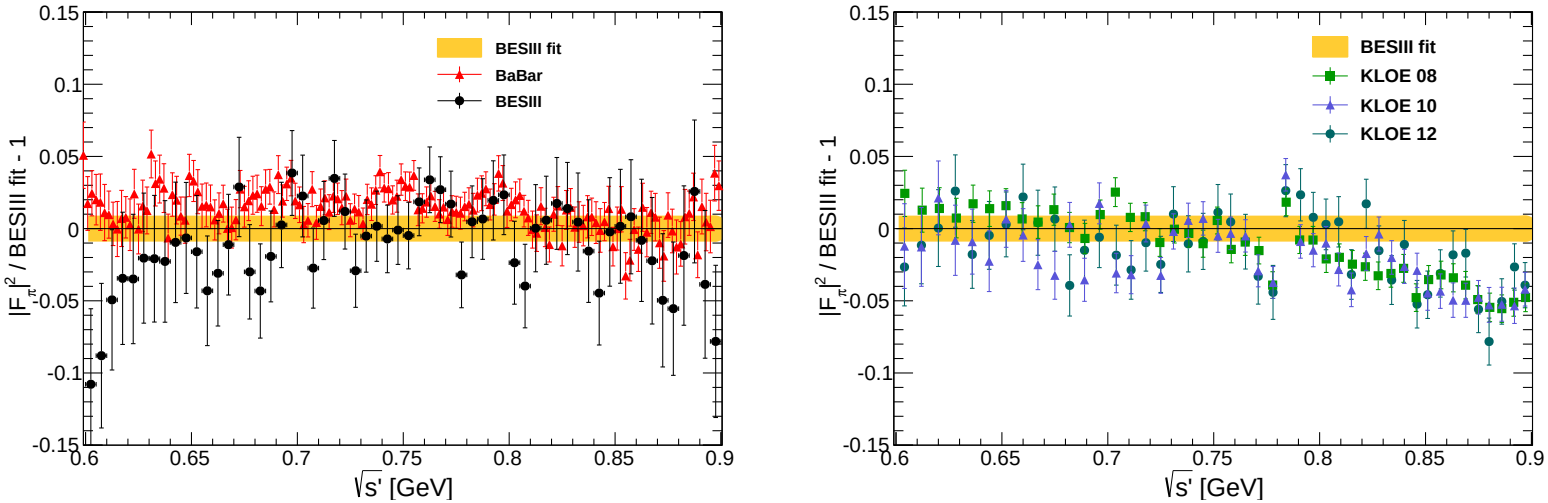


Still: most precise ISR measurements in conflict. BESIII steps to resolve this

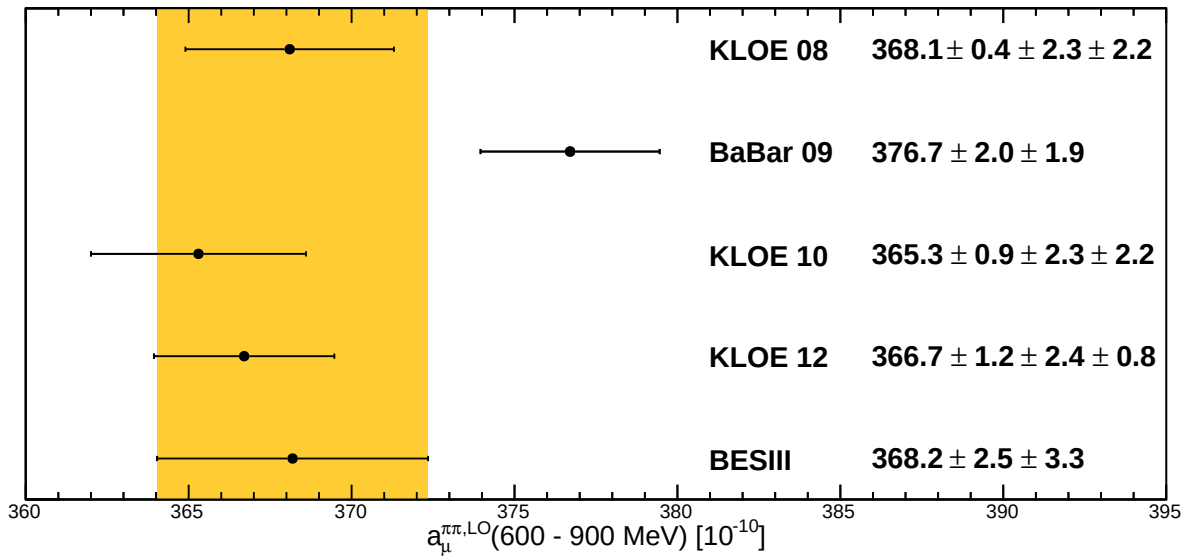
Recent/preliminary results:

- $e^+e^- \to \pi^+\pi^-$ from CMD-3
- $e^+e^- \to \pi^+\pi^-\pi^0$ from Belle
- $e^+e^- \to K^+K^-$ from CMD-3
- $e^+e^- \to K^+K^-$ from SND
- $e^+e^- \to \omega\pi^0 \to \pi^0\pi^0\gamma$ from SND
- $e^+e^- \to \pi^+\pi^-$ from BESIII recent, most important
- R_{uds} and R from 3.21 GeV to 3.72 GeV from KEDR

New from BESIII



BESIII vs BaBar and KLOE



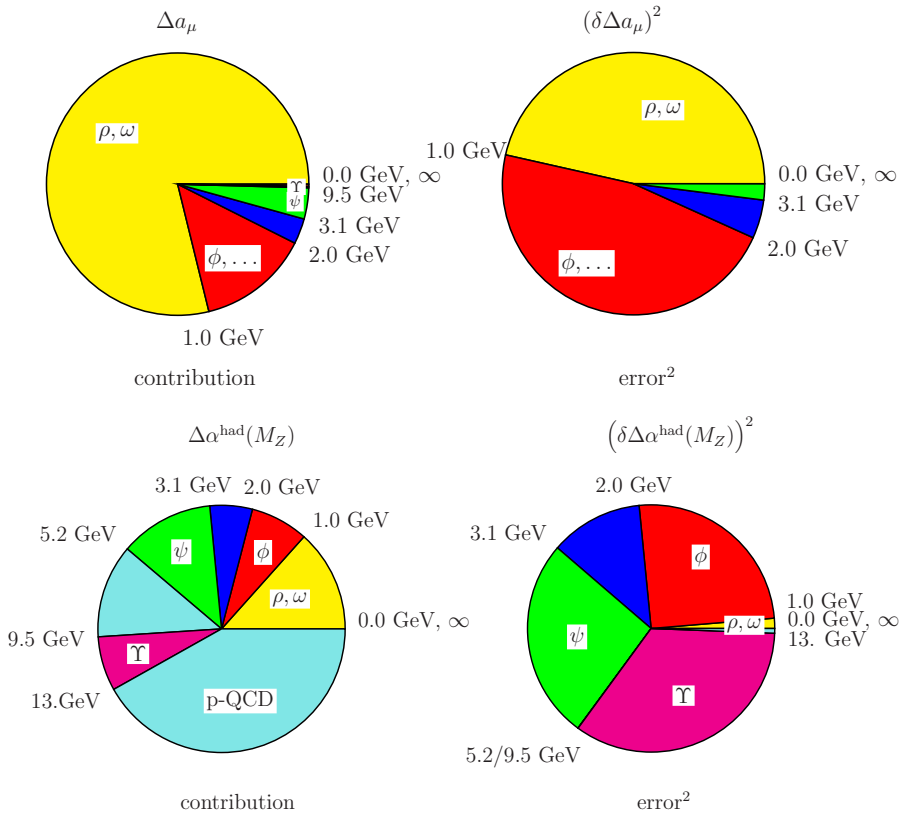
BESIII Collab. arXiv:1507.08188v3: 1.9 σ below BaBar in agreement with KLOE

$\Delta\alpha_{\rm had}(M_Z^2)$ results from ranges:

for $M_Z = 91.1876$ GeV in units 10^{-4}. 2015 update in terms of e^+e^--data and pQCD. 46% data, 54% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 13.0 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had} \times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	33.91 (0.05) (0.18)[0.18]	0.5%	1.1%
ω	(0.42, 0.81)	3.10 (0.04) (0.08)[0.09]	3.0%	0.3%
ϕ	(1.00, 1.04)	4.76 (0.07) (0.11)[0.13]	2.7%	0.5%
J/ψ		12.38 (0.60) (0.67)[0.90]	7.2%	25.5%
Υ		1.30 (0.05) (0.07)[0.09]	6.9%	0.3%
had	(1.05, 2.00)	16.22 (0.07) (0.89)[0.89]	5.5%	25.1%
had	(2.00, 3.10)	15.34 (0.08) (0.61)[0.62]	4.0%	12.1%
had	(3.10, 3.60)	4.93 (0.03) (0.13)[0.14]	2.8%	0.6%
had	(3.60, 5.20)	16.62 (0.11) (0.05)[0.12]	0.3%	0.6%
pQCD	(5.20, 9.46)	33.84 (0.00) (0.03)[0.03]	0.0%	0.1%
had	(9.46,13.00)	18.32 (0.24) (1.01)[1.04]	5.7%	34.0%
pQCD	(13.0,∞)	115.73 (0.00) (0.04)[0.04]	0.0%	0.1%
data	(0.28,13.00)	126.86 (0.67) (1.64)[1.78]	1.4%	100.0%
total		276.43 (0.67) (1.64)[1.78]	0.6%	100.0%

Correlation between different contributions to $a_\mu^{\rm had}$ and $\Delta\alpha^{{\rm had}\,(5)}$



Contributions from e^+e^- data ranges and form pQCD to $a_\mu^{\rm had}$ and $\Delta\alpha^{{\rm had}\,(5)}$.

4. Reducing uncertainties via the Euclidean split trick: Adler function controlled pQCD

- experiment side: new more precise measurements of $R(s)$
- future direct measurements Patrick Janot
- theory side: $\alpha_{\rm em}(M_Z^2)$ by the "Adler function controlled" approach

$$\alpha(M_Z^2) \quad = \quad \alpha^{\rm data}(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD} + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$$

where the space-like $-s_0$ is chosen such that pQCD is well under control for $-s < -s_0$. The monitor to control the applicability of pQCD is the Adler function

$$D(Q^2 = -s) = \frac{3\pi}{\alpha} s \frac{d}{{\rm d}s} \Delta\alpha_{\rm had}(s) = -(12\pi^2)\, s\, \frac{{\rm d}\Pi'_\gamma(s)}{{\rm d}s} = Q^2 \int_{4m_\pi^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}$$

which also is determined by $R(s)$ and can be evaluated in terms of experimental e^+e^-–data. Perturbative QCD tail: $D(Q^2) \to N_c \sum_f Q_f^2\, (1 + O(\alpha_s))$ as $Q^2 \to \infty$.

S. Eidelman, F. J., A. Kataev, O. Veretin, Phys. Lett. B **454** (1999) 369

$\Delta\alpha^{\rm had}$ Adler function controlled

✓ **use old idea: Adler function: Monitor for comparing theory and data**

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

$$\Rightarrow \quad D(Q^2) = Q^2\left(\int_{4m_\pi^2}^{E^2_{\rm cut}} {\rm d}s\, \frac{R(s)^{\rm data}}{(s+Q^2)^2} + \int_{E^2_{\rm cut}}^{\infty} \frac{R^{\rm pQCD}(s)}{(s+Q^2)^2}\, {\rm d}s\right).$$

pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

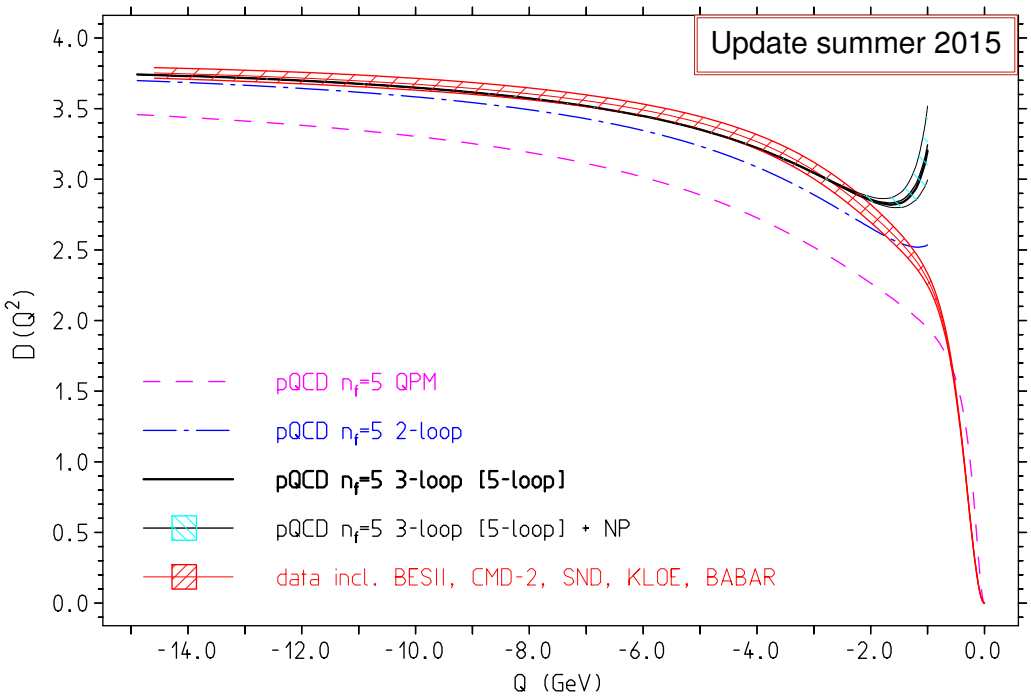
Conclusion:

❖**time-like approach: pQCD works well in "perturbative windows"**
3.00 - 3.73 GeV, 5.00 - 10.52 GeV and 11.50 - ∞ Kühn,Steinhauser

❖**space-like approach: pQCD works well for $\sqrt{Q^2 = -q^2} > 2.0$ GeV (see plot)**

“Experimental” Adler–function versus theory (pQCD + NP)

Error includes statistical + systematic here (in contrast to most R-plots showing statistical errors only)!



(Eidelman, F. J., Kataev, Veretin 98, FJ 08/15 updates)

theory based on results by Chetyrkin, Kühn et al.

⇒ **pQCD works well controlled to predict $D(Q^2)$ down to $s_0 = (2.0\,\textbf{GeV})^2$; use this to calculate**

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int {\rm d}Q'^2 \frac{D(Q'^2)}{Q'^2}$$

$$\boxed{\begin{aligned}\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) \;=\; & \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} \\ & + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}\end{aligned}}$$

and obtain, for $s_0 = (2.0\,\textbf{GeV})^2$: **(FJ 98/15)**

$$\begin{aligned}\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} &= 0.006392 \pm 0.000064 \\ \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) &= 0.027466 \pm 0.000118 \\ \Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027504 \pm 0.000119\end{aligned}$$

❖**shift** $+0.000008$ **from the 5-loop contribution**

❖**error** ± 0.000100 **added in quadrature form perturbative part**

QCD parameters: ● $\alpha_s(M_Z) = 0.1189(20)$**,**

● $m_c(m_c) = 1.286(13)$ $[M_c = 1.666(17)]$ **GeV** , ● $m_b(m_c) = 4.164(25)$ $[M_b = 4.800(29)]$ **GeV**

based on a complete 3–loop massive QCD analysis Kühn et al 2007

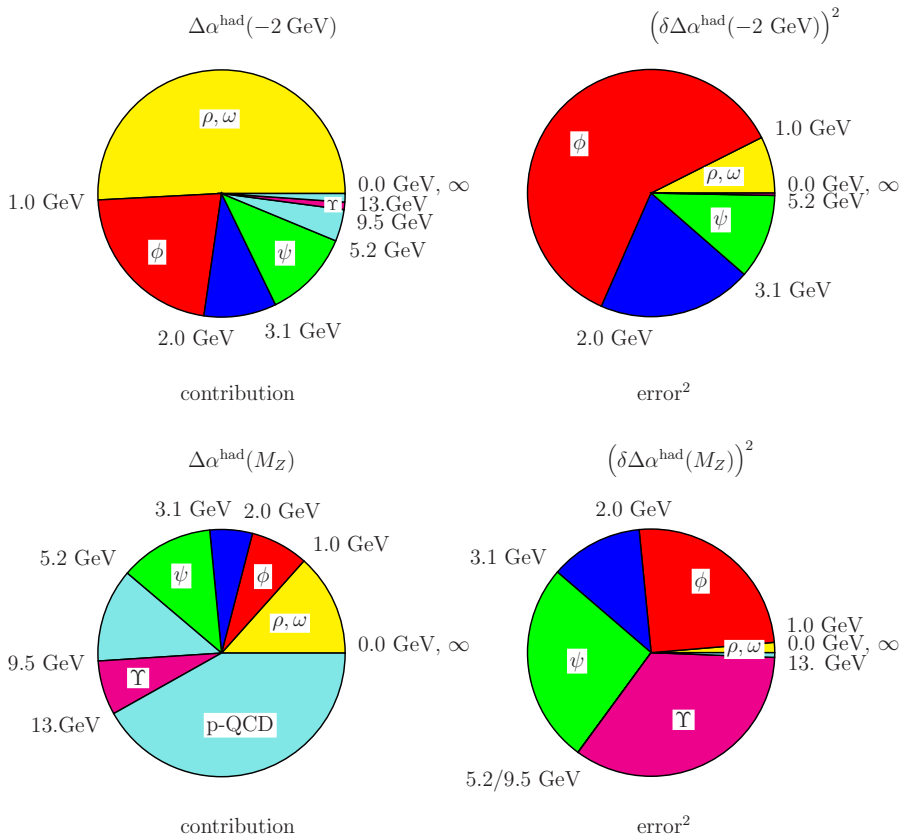
F. J., Nucl. Phys. Proc. Suppl. **181-182** (2008) 135

$\Delta\alpha_{\rm had}(-M_0^2)$ results from ranges:

for $M_0 = 2$ GeV in units 10^{-4}. 2015 update in terms of e^+e^--data and pQCD. 95% data, 5% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 13.0 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had}(-M_0^2)\times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	29.77 (0.04) (0.15)[0.16]	0.5%	5.8%
ω	(0.42, 0.81)	2.69 (0.03) (0.07)[0.08]	3.0%	1.5%
ϕ	(1.00, 1.04)	3.78 (0.05) (0.09)[0.10]	2.7%	2.4%
J/ψ		3.21 (0.15) (0.15)[0.21]	6.7%	10.6%
Υ		0.05 (0.00) (0.00)[0.00]	6.8%	0.0%
had	(1.05, 2.50)	13.76 (0.05) (0.56)[0.56]	4.1%	72.0%
had	(2.50, 3.10)	2.49 (0.01) (0.18)[0.18]	7.2%	7.4%
had	(3.10, 3.60)	1.30 (0.01) (0.03)[0.04]	2.8%	0.3%
had	(3.60, 5.20)	2.87 (0.02) (0.00)[0.02]	0.0%	0.1%
pQCD	(5.20, 9.46)	2.66 (0.00) (0.00)[0.00]	0.0%	0.0%
had	(9.46,13.00)	0.57 (0.01) (0.03)[0.03]	5.5%	0.2%
pQCD	(13.00, 0.00)	0.70 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,13.00)	60.49 (0.18) (0.63)[0.66]	1.0%	0.0%
total		63.85 (0.18) (0.63)[0.66]	1.0%	100.0%

Of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ 22% data, 78% pQCD!



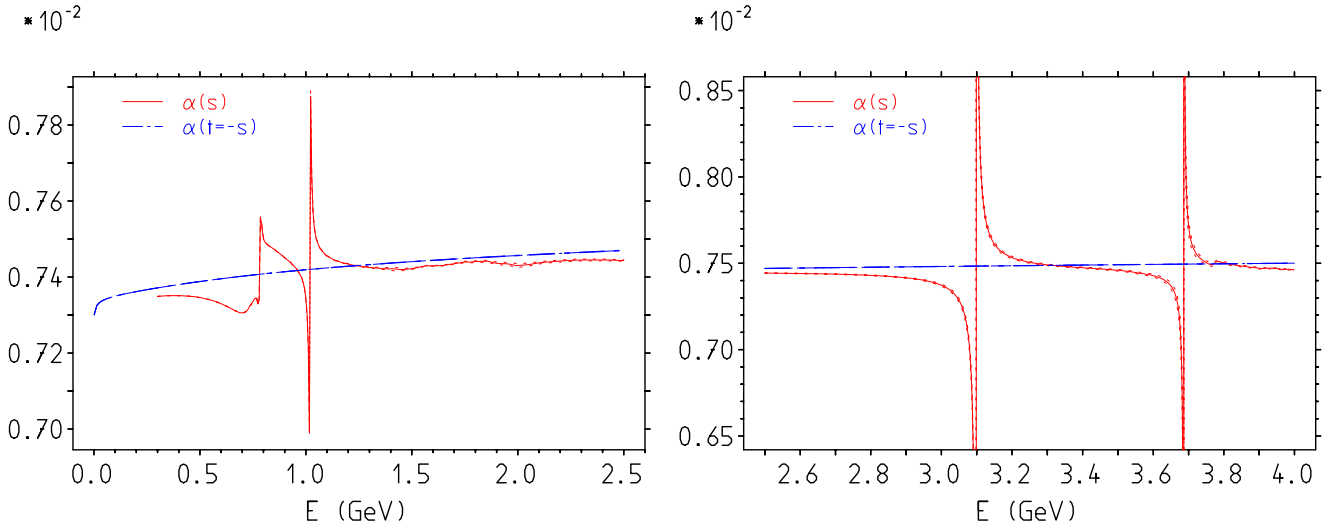
Contributions from e^+e^- data ranges and form pQCD to $\Delta\alpha^{(5)}_{\rm had}(-M_0^2)$ vs. $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$.

How much pQCD?

Method	range [GeV]	pQCD		
Standard approach:	5.2 - 9.5	33.84(0.03)		
My choice	13.0 - ∞	115.73(0.04)	$\rightarrow$	149.57 (0.05)
Standard approach:	2.0 - 9.5	72.09(0.07)		
Davier et al.	11.5 - ∞	123.24(0.05)	$\rightarrow$	195.33 (0.09)
Adler function controlled:	5.2 - 9.5	2.66(0.00)		
	13.0 - ∞	0.70(0.00)		
	$-\infty$ – -2.0	210.68(0.99)		
	$-M_Z \rightarrow M_Z$	0.38(0.00)	$\rightarrow$	214.42 (0.99)

Note: the Adler function monitored Euclidean data vs pQCD split approach is only moderately more pQCD-driven, than the time-like approach adopted by Davier et al. and others.

The time-like vs space-like effective charge

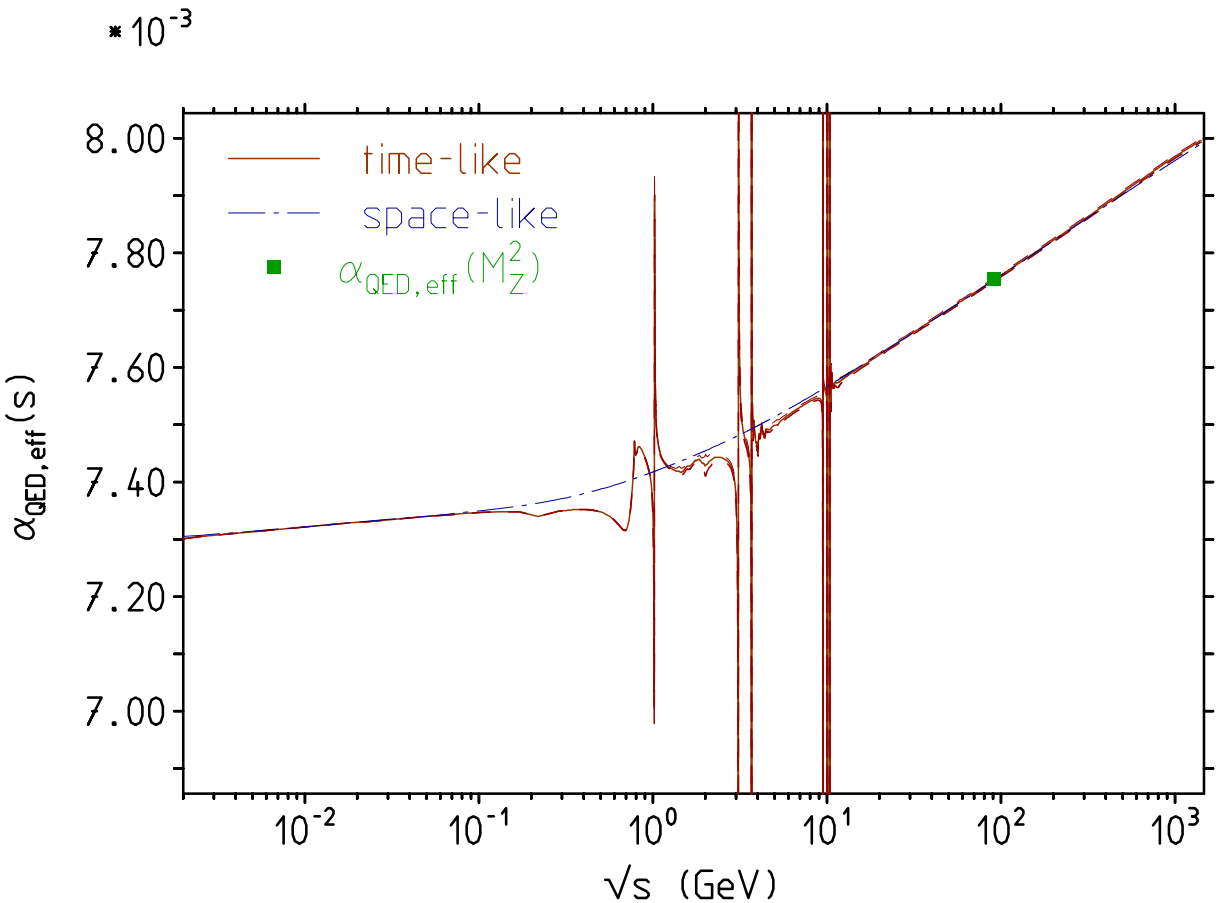


Note that the smooth space-like effective charge agrees rather well with the non-resonant "background" above the Φ (kind of duality)

No proof that this cannot produce non-negligible shifts!

Time-like VP-subtraction cannot be implemented locally near OZI suppressed resonances: $J/\psi, \psi'$ and $\Upsilon_1, \Upsilon_2, \Upsilon_3$

$\alpha_{\rm QED,eff}$: time-like vs. space-like



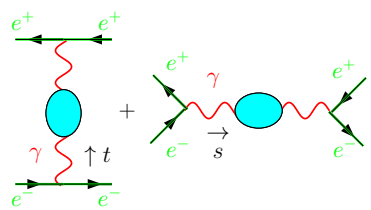
$\alpha_{\rm QED,eff}$ duality: $\alpha_{\rm QED,eff}(s)$ is varying dramatically near resonances, but agrees quite well in average with space-like version

An alternative method: measure space-like $\alpha_{\rm QED,eff}(t)$

Newly proposed recently: [arXiv:1504.02228]
"A new approach to evaluate the leading hadronic corrections to the muon g-2"
Carloni Calame, Passera, Trentadue, Venenzoni 2015

❒ measuring directly low energy running $\alpha_{\rm QED}(s)$ in space-like region via

Bhabha scattering $\quad e^+(p_+)\ e^-(p_-) \to e^+(p'_+)\ e^-(p'_-)$



VP dressed tree level Bhabha scattering in QED

has two tree level diagrams the t– and the s–channel. With the positive c.m. energy

square $s = (p^+ + p^-)^2$ and the negative momentum transfer square

$$t = (p_- - p'_-)^2 = -\frac{1}{2}\,(s - 4m_e^2)\,(1 - \cos\theta)\,,$$

θ the e^- scattering angle, there are two very different scales involved

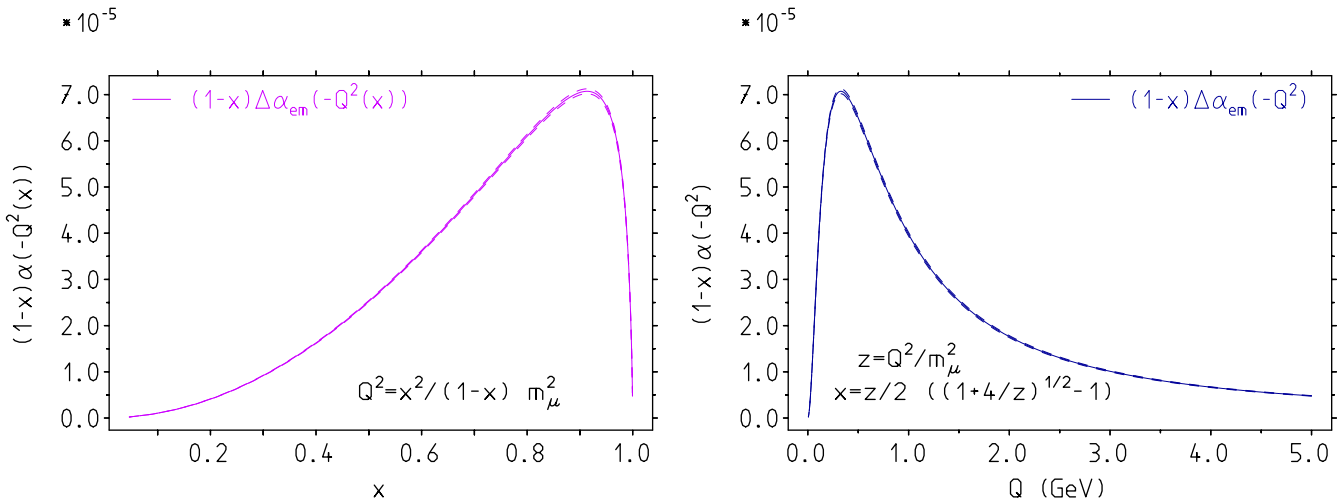
❐ space-like $\alpha_{\rm QED,eff}(-Q^2)$ determines $a_\mu^{\rm had}$ via

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int\limits_0^1 {\rm d}x\,(1-x)\,\Delta\alpha_{\rm had}\Big(-Q^2(x)\Big)$$

where $Q^2(x) \equiv \frac{x^2}{1-x} m_\mu^2$ is the space–like square momentum–transfer or

$$x = \frac{Q^2}{2m_\mu^2}\left(\sqrt{1+\frac{4m_\mu^2}{Q^2}} - 1\right)\quad .$$

Also in the Euclidean region the integrand is highly peaked, now around half of the ρ meson mass scale.



The integrand of $a_\mu^{\rm had}$ integral as functions of x and Q. Strongly peaked at about 330 MeV.

The VP dressed lowest order cross–section is

$$\frac{d\sigma}{d\cos\Theta} = \frac{s}{48\pi}\sum_{ik}|A_{ik}|^2$$

where A_{ik} tree level helicity amplitudes, i, k =L,R left– and right–handed electrons.

Dressed transition amplitudes: ($m_e \approx 0$)

$$
\begin{aligned}
|A_{\mathrm{LL,RR}}|^2 &= \frac{3}{8}(1+\cos\theta)^2 \left|\frac{e^2(s)}{s}+\frac{e^2(t)}{t}\right|^2 \\
|A_{\mathrm{LR,RL}}|^2 &= \frac{3}{8}(1-\cos\theta)^2 \left|\frac{e^2(s)}{s}+\frac{e^2(t)}{t}\right|^2 .
\end{aligned}
$$

Preferably one uses small angle Bhabha scattering (small $|t|$) as a normalizing process which is dominated by the t–channel $\sim 1/t$, however, detecting electrons and positrons along the beam axis often has its technical limitations.

Care also is needed concerning the ISR corrections because cuts for the Bhabha process ($e^+e^- \to e^+e^-$) typically are different from the ones applied to $e^+e^- \to$ hadrons. Usually, experiments have included corresponding uncertainties in their systematic errors, if they not have explicitly accounted for all appropriate radiative corrections.

The primary goal of [arXiv:1504.02228]: determining $a_\mu^{\rm had}$ in an alternative way

My proposal here: determine very accurately

$$\boxed{\Delta\alpha_{\rm had}\left(-Q^2\right) \text{ at } Q \approx 2 \text{ GeV}}$$

by this method (one single number!) as the non-perturbative part of $\Delta\alpha_{\rm had}\left(M_Z^2\right)$ as in "Adler function" approach.

● extremely important for the FCC-ee/ILC physics program

× t–channel dominance requires $|t|/s \ll 1 \Longrightarrow$ better for high energy machine?

Best: can be done at future e^+e^- machine at Z peak:

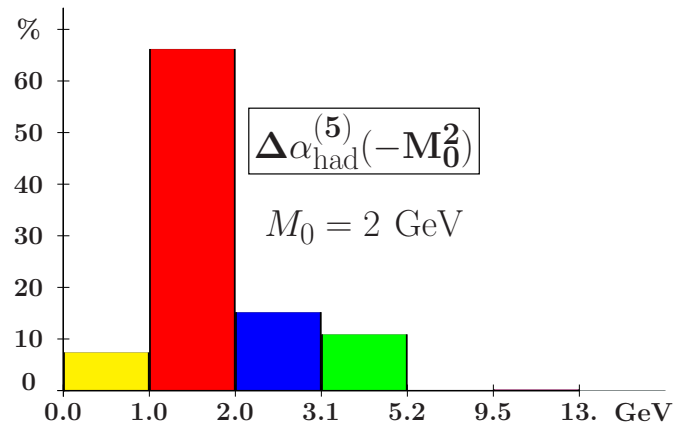
$$\boxed{\cos\theta \approx 1 - 2|t|/s}$$

$$\boxed{\sqrt{|t|} = 2 \text{ GeV} \; ; \quad \sqrt{s} = M_Z \;\Rightarrow\; \theta \sim 2.5^\circ}$$

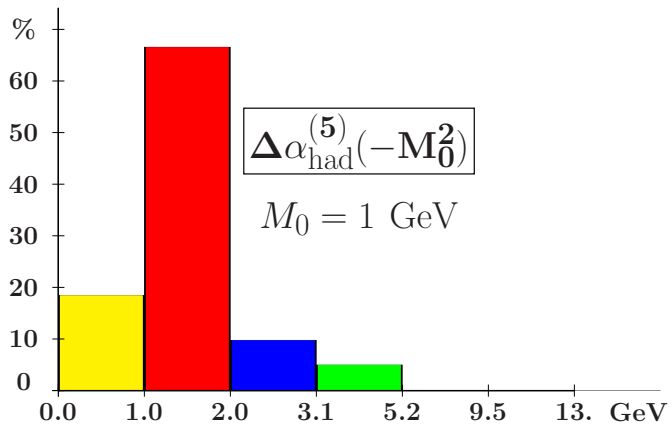
Offset problem: need very accurate Luminosity $\Longrightarrow$ actually one obtains $\Delta\alpha_{\rm had}(t) - \Delta\alpha_{\rm had}(t_0)$ where t_0 corresponds to the minimum reference angle at which the Luminosity is determined. How to get $\Delta\alpha_{\rm had}(t_0)$? In any case we would require $|t_0| \ll |t|$

Note: different for $a_\mu^{\rm had}$ integrand peaks at 0.33 GeV i.e. dominant contribution from $|t|/s \simeq 0.1$ at Dafne, corresponding to an angle $37°$, however, s over t channel at 10% level only.

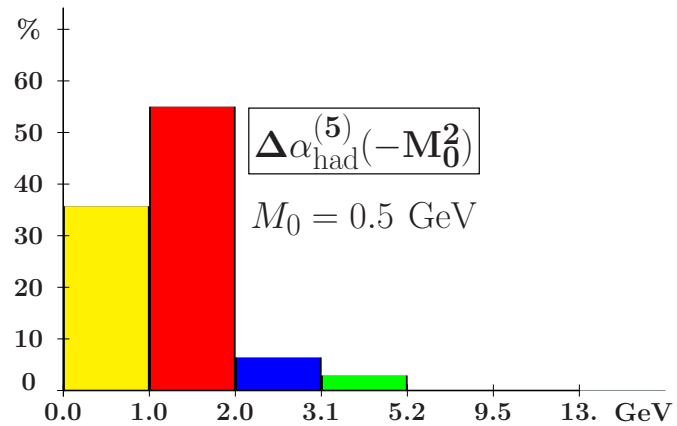
How to get the OFFSET?



$$\Delta\alpha_{\rm had}(-(2.0\ {\rm GeV})^2) = 63.86(65)[40]$$



$$\Delta\alpha_{\rm had}(-(1.0\ {\rm GeV})^2) = 36.67(30)[20]$$



$$\Delta\alpha_{\rm had}(-(0.5\ {\rm GeV})^2) = 15.93(10)[7] \times 10^{-4}$$

Question: can we reduce uncertainty by changing

from

$$\boxed{\Delta\alpha_{\rm had}(-M_0^2)^{\rm direct}}$$

to

$$\boxed{\left\{\Delta\alpha_{\rm had}(-M_0^2) - \Delta\alpha_{\rm had}(-M_{\rm lumi}^2)\right\}^{\rm Bhabha} + \Delta\alpha_{\rm had}(-M_{\rm lumi}^2)^{\rm direct}}$$

What will be the offset scale $M_{\rm lumi}$?

5. Prospects for future improvements

Mandatory pQCD improvements required are:

- **4–loop massive pQCD calculation of Adler function;**
  required are a number of terms in the low and high momentum series expansions which allow for the appropriate Padé improvements
  [essentially equivalent to a massive 4–loop calculation of $R(s)$];
- **m_c, m_b improvements by sum rule and/or lattice QCD evaluations;**
- **improved α_s in low Q^2 region.**

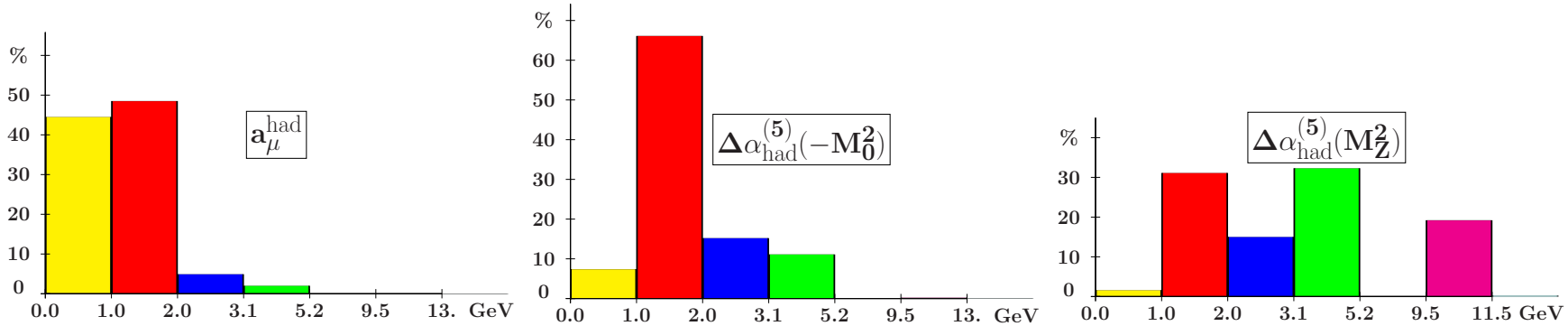
Theory: (QCD parameters) has to improve by factor 10 ! $\rightarrow \pm 0.20$

Requirement may be realistic:

- **pin down experimental errors to 1% level in all non-perturbative regions up to 2.5 GeV**
- **switch to Euclidean approach, monitored by the Adler function**
- **improve on QCD parameters, mainly on m_c and m_b**

Settling the HVP issue for a_μ settles it largely for $\Delta\alpha(-M_0^2)$

Error profiles (standard approach):



Contributions to the total error from different energy regions to the hadronic lowest order vacuum polarization contribution to a_μ, $\Delta\alpha(M_Z^2)$ and $\Delta\alpha(-M_0^2)$ for $M_0 = 2$ GeV in percent. These errors are to be added in quadrature to get the total uncertainty. The graph illustrates where experimental effort is needed in order to get a better precision.

The virtues of Adler function approach are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.0 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder" $\Delta\alpha_{had}^{(5)}(-s_0)$ is mainly sensitive to low energy data !!!**

❖ $\Delta\alpha(-M_0^2)$ **directly accessible in Bhabha Scattering! modulo offset!**

- **To be analyzed in detail**
- **Maybe most promising option**

Future: ILC/FCC-ee requirement: improve by factor 10 in accuracy

- direct integration of data: 46% from data 54% p-QCD

$$\Delta\alpha_{\rm had}^{(5)\,\rm data} \times 10^4 = 126.86 \pm 1.78 \text{ (1.4\%)}$$

1% overall accuracy ± 1.27

1% accuracy for each region (divided up as in table)

added in quadrature: ± 0.40

Data: [1.78] vs. [0.40] $\Rightarrow$ improvement factor 4.5

$$\Delta\alpha_{\rm had}^{(5)\,\rm pQCD} \times 10^4 = 149.57 \pm 0.05 \text{ (0.0\%)}$$

Theory: no improvement needed !

- integration via Adler function: 22% from data 78% p-QCD

$$\Delta\alpha_{\rm had}^{(5)\,\rm data} \times 10^4 = 063.87 \pm 0.66 \text{ (1.1\%)}$$

1% overall accuracy ± 0.60

1% accuracy in region 1.0 to 2.5 GeV

added in quadrature: ± 0.28

Data: [1.19] vs. [1.03,0.57,0.37] $\Rightarrow$ improvement factor 2.1-3.2 (Adler vs Adler)

[1.78] vs. [1.03,0.57,0.37] $\Rightarrow$ improvement factor 3.1-4.8 (Standard vs Adler)

$$\Delta\alpha_{\rm had}^{(5)\,\rm pQCD} \times 10^4 = 214.48 \pm 1.00 \text{ (0.05\%)}$$

Theory: massive 4-loop needed and more accurate m_c, m_b and α_s!

- direct measurement (near/off Z peak) **Patrick Janot's approach**

$\Delta\alpha_{\rm had}(s_0) \times 10^4$: **present**

s_0	M_Z^2	$-(2.5\ \text{GeV})^2$	$-(2.0\ \text{GeV})^2$
data	126.86 ± 1.78 **[1.4%]**	73.72 ± 0.79 **[1.1%]**	63.87 ± 0.66 **[1.1%]**
pQCD	149.57 ± 0.05 **[0.0%]**	201.23 ± 0.99 **[0.5%]**	210.74 ± 1.04 **[0.5%]**
$\Delta\alpha_{\rm had}(M_Z^2)$	0.027643 ± 0.000178	0.027535 ± 0.000127	0.027501 ± 0.000124
$\alpha^{-1}(M_Z^2)$	128.953 ± 0.024	128.968 ± 0.017	128.972 ± 0.017
accuracy in 10^{-5}	**18.92**	**13.52**	**13.15**
improvement	**1.0**	**1.0**	**1.0**

$\Delta\alpha_{\rm had}(s_0)\times 10^4$. **Future I: improving data only a) direct $\delta\sigma\lesssim$ 1% above ϕ to 11.5 GeV, b) low energy space-like cut at $\sqrt{s_0}$, $\delta\sigma\lesssim$ 1% above ϕ to 2.5 GeV**

s_0	M_Z^2	$-(2.5\ \text{GeV})^2$	$-(2.0\ \text{GeV})^2$
data	126.86 ± 0.41 **[0.3%]**	73.72 ± 0.33 **[0.4%]**	63.87 ± 0.28 **[0.4%]**
pQCD	149.57 ± 0.05 **[0.0%]**	201.23 ± 1.03 **[0.5%]**	210.74 ± 1.04 **[0.5%]**
$\Delta\alpha_{\rm had}(M_Z^2)$	0.027643 ± 0.000041	0.027535 ± 0.000105	0.027501 ± 0.000112
$\alpha^{-1}(M_Z^2)$	128.953 ± 0.006	128.968 ± 0.014	128.972 ± 0.015
accuracy in 10^{-5}	**4.39**	**11.16**	**11.95**
improvement	**4.3**	**1.2**	**1.1**

$\Delta\alpha_{\rm had}(s_0)\times 10^4$. **Future II: improving data as above plus pQCD to 0.2% in case b)**

s_0	M_Z^2	$-(2.5\ \text{GeV})^2$	$-(2.0\ \text{GeV})^2$
data	126.86 ± 0.41 **[0.3%]**	73.72 ± 0.33 **[0.4%]**	63.87 ± 0.28 **[0.4%]**
pQCD	149.57 ± 0.05 **[0.0%]**	201.23 ± 0.40 **[0.2%]**	210.74 ± 0.42 **[0.2%]**
$\Delta\alpha_{\rm had}(M_Z^2)$	0.027643 ± 0.000041	0.027535 ± 0.000053	0.027501 ± 0.00060
$\alpha^{-1}(M_Z^2)$	128.953 ± 0.006	128.968 ± 0.007	128.972 ± 0.008
accuracy in 10^{-5}	**4.39**	**5.66**	**6.37**
improvement	**4.3**	**2.4**	**2.1**

$\Delta\alpha_{\rm had}(s_0)\times 10^4$. **Future III: improving data as above plus pQCD to 0.1% in case b)**

s_0	M_Z^2	$-(2.5\ \text{GeV})^2$	$-(2.0\ \text{GeV})^2$
data	126.86 ± 0.41 **[0.3%]**	73.72 ± 0.33 **[0.4%]**	63.87 ± 0.28 **[0.4%]**
pQCD	149.57 ± 0.05 **[0.0%]**	201.23 ± 0.20 **[0.1%]**	210.74 ± 0.21 **[0.1%]**
$\Delta\alpha_{\rm had}(M_Z^2)$	0.027643 ± 0.000041	0.027535 ± 0.000040	0.027501 ± 0.00048
$\alpha^{-1}(M_Z^2)$	128.953 ± 0.006	128.968 ± 0.006	128.972 ± 0.007
accuracy in 10^{-5}	**4.39**	**4.29**	**5.06**
improvement	**4.3**	**3.1**	**2.6**

Other issues:

$$\Delta\alpha_{\rm lep}(M_Z^2) \simeq 0.031419187418$$

A very heavy top decouples like

$$\Delta\alpha_{top} \simeq -\frac{\alpha}{3\pi}\frac{4}{15}\frac{s}{m_t^2} \to 0$$

when $m_t \gg s$. Thus top quark contribution at $s = M_Z^2$:

$$\Delta\alpha_{\rm top}(M_Z^2) = -0.76 \times 10^{-4} \quad .$$

Remaining problems are the following:

a) contributions to the Adler function up to three–loops all have the same sign and are substantial. Four– and higher–orders could still add up to non-negligible contribution. An

error for missing higher order terms is not included.

b) the link between space–like and time-like region is the difference

$$\Delta = \Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.000045 \pm 0.000002$$

which can be calculated in pQCD. It accounts for the $i\pi$–terms from the logs $\ln(-q^2/\mu^2) = \ln(|q^2/\mu^2|) + i\pi$**.**

Since the term is small we can get it as well from direct data integration

$\Delta\alpha_{\rm had}(-M_Z^2) = 276.44 \pm 0.64 \pm 1.78$

$\Delta\alpha_{\rm had}(M_Z^2) = 276.84 \pm 0.64 \pm 1.90$

and taking into account that errors are almost 100% correlated we have

$\Delta\alpha_{\rm had}(M_Z^2) - \Delta\alpha_{\rm had}(-M_Z^2) = 0.40 \pm 0.12$

In $\alpha^{-1}(M_Z^2)$ we included the latter term as well as the above top contribution.

Comment:

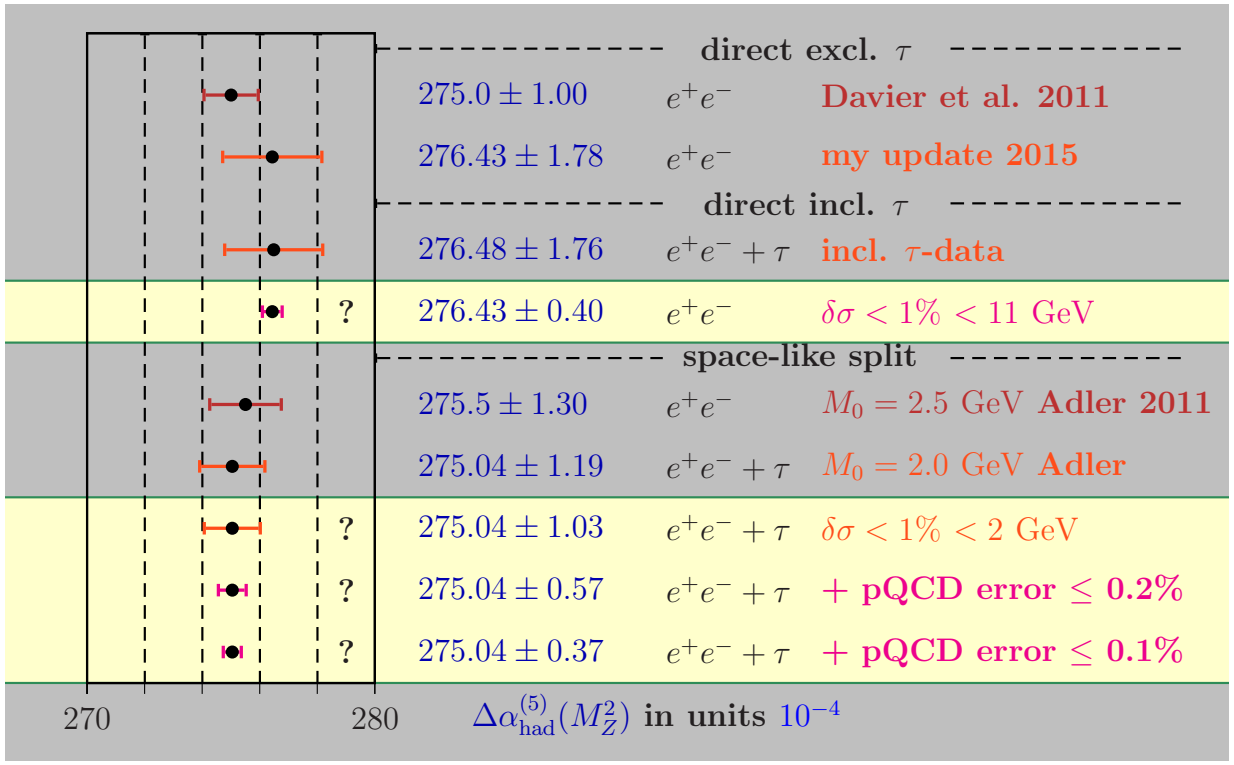
- **since 2009 the error of $\Delta\alpha_{\rm had}(-s_0)$ went down by 40% (more KLOE, BaBar, BESIII, especially BaBar exclusive channels up to 2.3 GeV)**

- **the error budget 50% : 50% we had in 2009 now changed to 39% : 61% for $\sqrt{s_0} = 2.5$ GeV and 29% : 71% for $\sqrt{s_0} = 2.0$ GeV**

- **therefore the improvement obtained by reducing the experimental error to 1% in the range from ϕ to 2.5 GeV gets much less weight, but still is important**

- **by choosing a higher cut point e.g. for $\sqrt{s_0} = 3.0$ GeV one can balance the importance of data vs pQCD providing an important crosscheck. One gets $\Delta\alpha_{\rm had}(-s_0) = 82.21 \pm 0.88[0.38]$ in 10^{-4}. The QCD contribution is then smaller as well as saver, because the mass effects which are responsible for the larger uncertainty of the pQCD prediction also gets substantially reduced.**

❒ **in view that a massive 4-loop QCD calculation is very difficult and better cross section in the range up to 2.5 GeV provides the possibility to optimize the cut scale s_0.**



Davier et al. 2011: use pQCD above 1.8 GeV

- **no improvement by remeasuring cross sections above 1.8 GeV**
- **no proof that pQCD works at 0.04% precision as adopted**

My analysis is data driven: pQCD $5.2 - 9.5$ and > 11.5 GeV

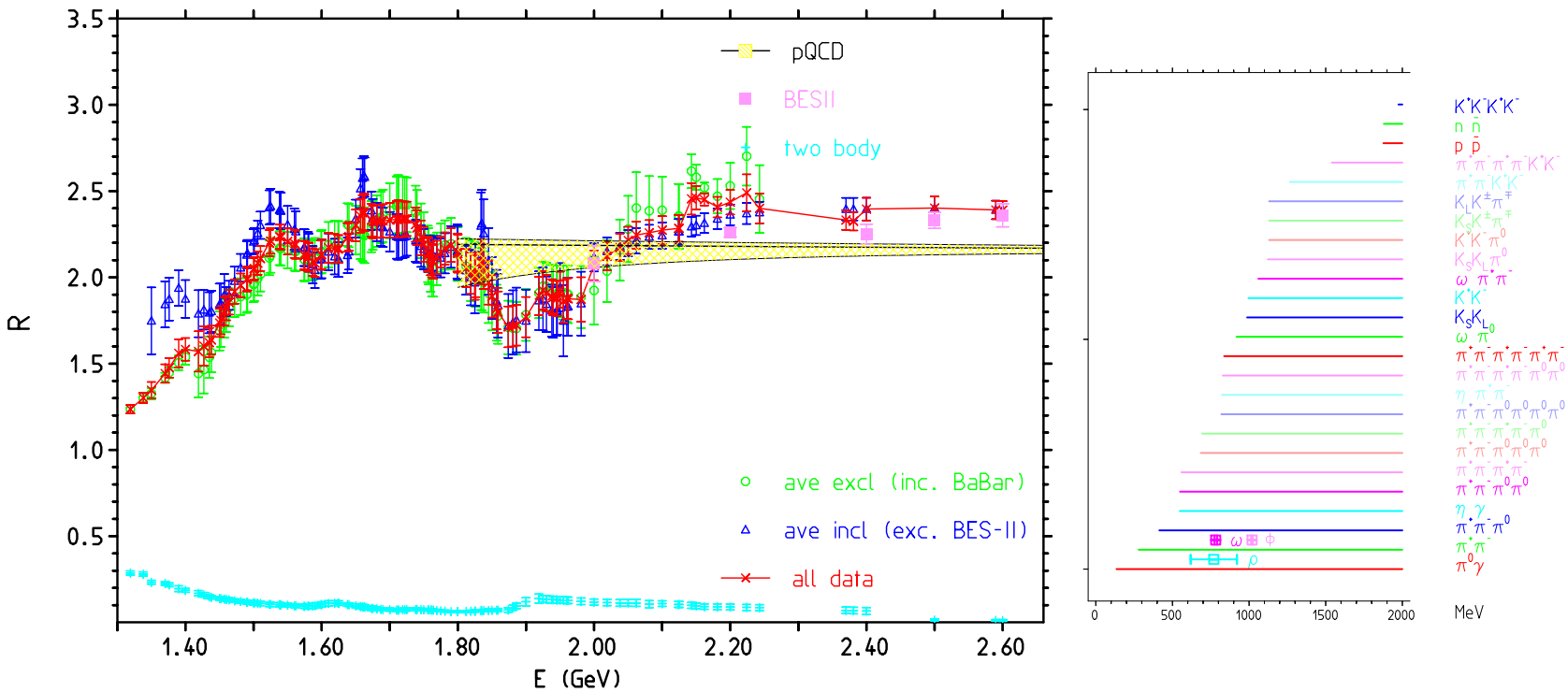
× **pQCD at 0.2% Adler function: pQCD error = ½ × present error**

× **pQCD at 0.1% Adler function: pQCD error = data error ± 0.28**

Note: theory-driven analyses using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

Better data needed in any case for range 1.2 - 2.4 GeV

- one of the main issue in HVP is $R_\gamma(s)$ from 1.2 GeV to 2.4 GeV
- has been improved dramatically by the exclusive channel measurements by BaBar
- 20 out of more than 30 channels are measured, many known at the 10 to 15% level
- now exclusive channel data much better quality than the very old inclusive data from Frascati



ILC/FCC-ee community should actively support these activities as integral part of e^+e^--collider precision physics!!!

Remember: tremendous progress since middle of 90's

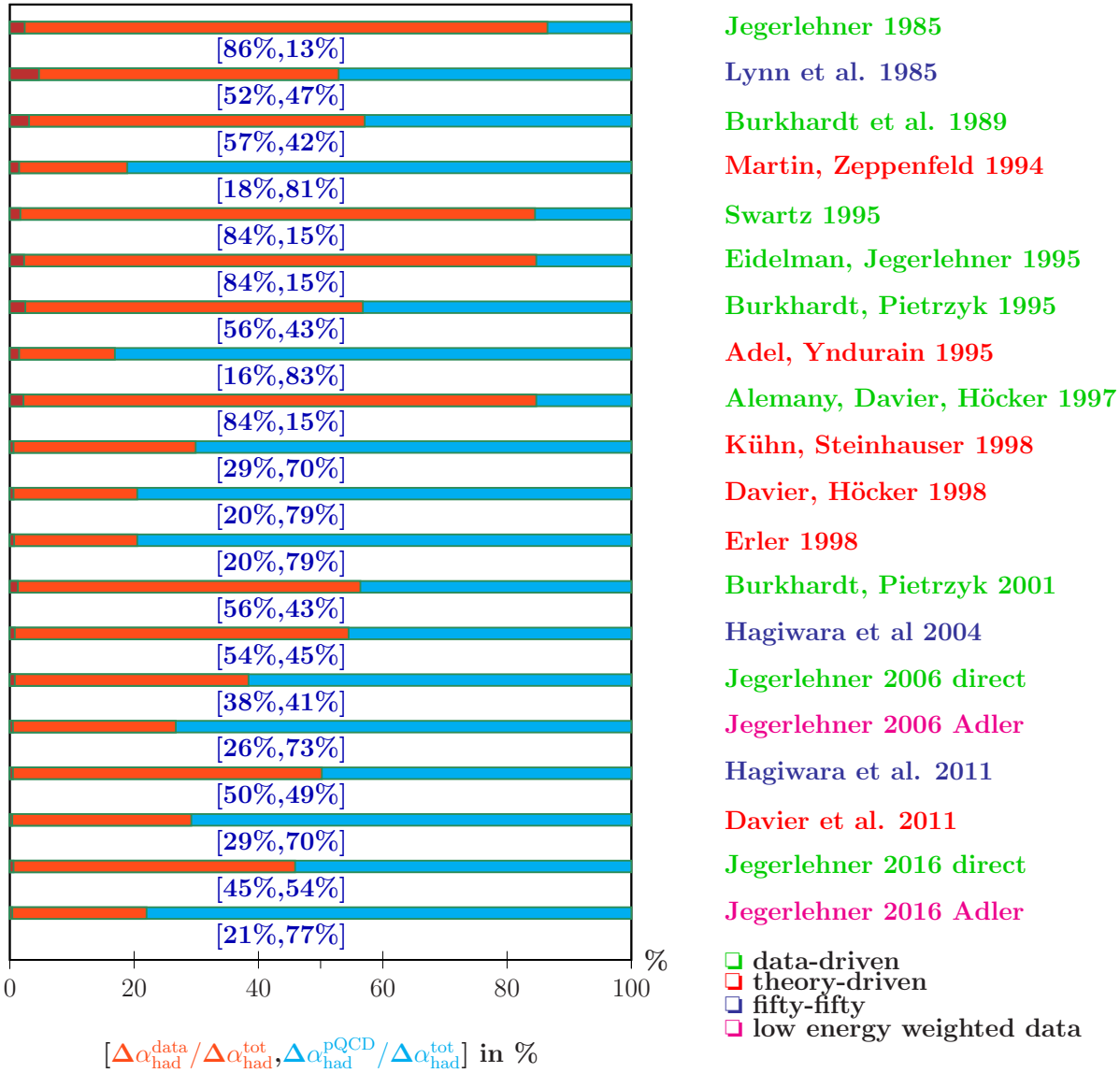
- Novosibirsk VEPP-2M: MD-1, CMD2, SND, KEDR; VEPP-2000: CMD3,SND
- Frascati DAFNE: KLOE
- Beijing BEPC: BES II, BESIII
- Cornell CESR: CLEO
- Stanford SLAC PEP-II: BaBar; KEK Tsukuba: Belle

Many analyzes exploiting these results: Davier et al, Hagiwara et al., Burkhardt, Pietrzyk, Yndurain et al....

Indispensable for Muon $g-2$, indirect vs direct LEP Higgs mass etc. and future precision test at ILC/FCC-ee and new physics signals in precision observables. Impact for cosmology!

History and recent

- big progress in data CMD-2, SND, BESII, KLOE, BaBar, $\cdots$
- progress in pQCD [..., Chetyrkin, Kühn et al. ...], by far more progressive use of pQCD



How much pQCD?

To discover "Physics Behind Precision" at future linear or circular e^+e^-–colliders requires improved SM predictions based more precise input parameters: $\Delta\alpha_{\rm had}(M_Z^2)$ thereby plays a key role.
A factor 5 improvement seem feasible, but is by far not for free.
High precision input parameters crucial to open a new window for precison cosmology of the early universe.

Thanks

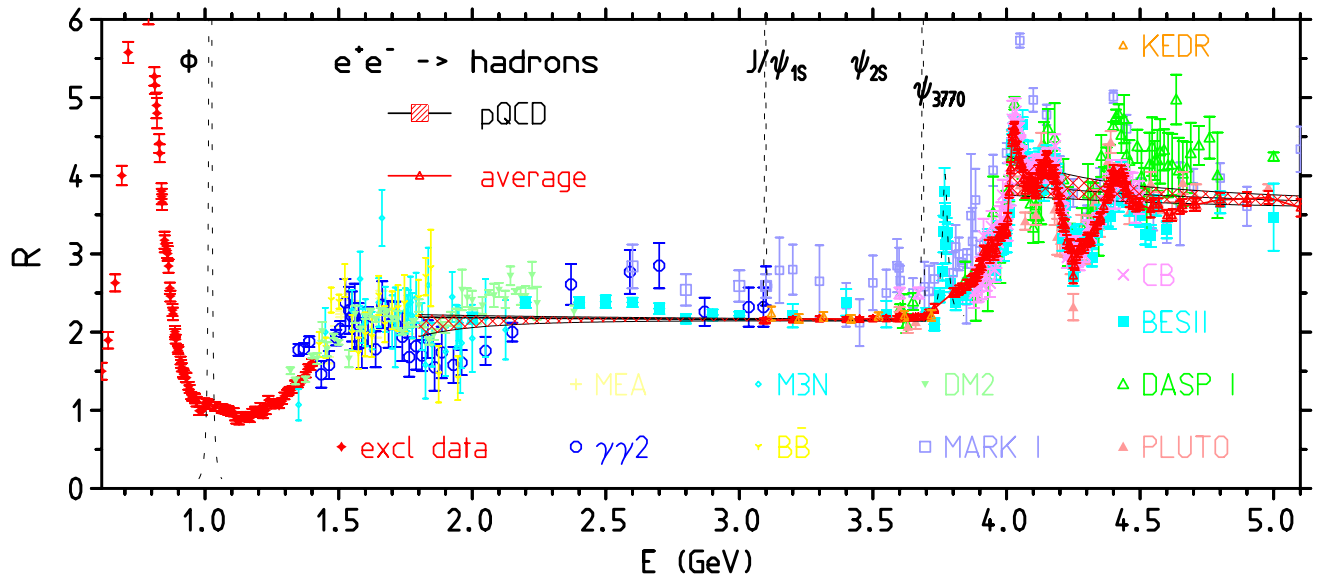
Thanks for your attention!

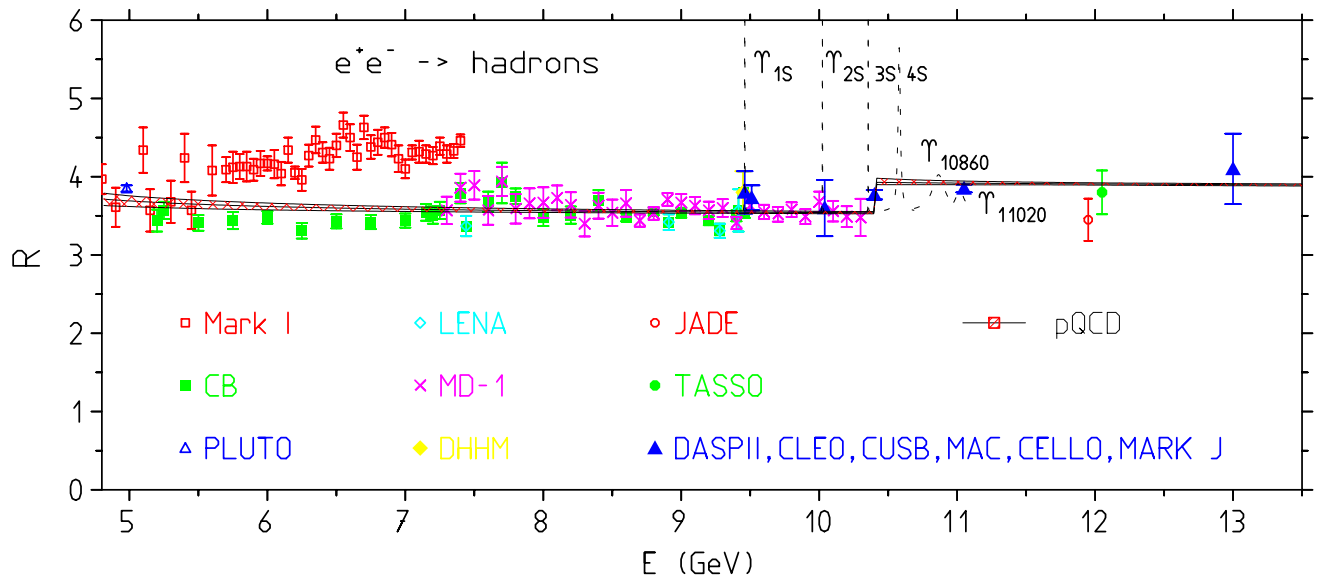
Extras

Authors	data	pQCD	sum
Jegerlehner 1985:	247.37± 7.	38.63±0.37	286. ± 7.
Lynn et al. 1985:	145. ±13.	129. ±1.	274. ±13.
Burkhardt et al. 1989:	164.32± 8.2	123.68±3.71	288. ± 9.
Martin, Zeppenfeld 1994:	51.5 ± 1.1	221.7 ±4.1	273.2 ± 4.2
Swartz 1995:	232.56± 4.6	42.64±0.10	275.2 ± 4.6
Eidelman, Jegerlehner 1995:	237.55± 6.43	42.82±0.10	280.37± 6.43
Burkhardt, Pietrzyk 1995:	159. ± 7.	121. ±0.2	280. ± 7.
Adel, Yndurain 1995:	45.99± 0.85	226.6 ±4.0	272.59± 4.09
Alemany, Davier, Höcker 1997:	238.01± 6.3	42.82±0.10	280.9 ± 6.3
Kühn, Steinhauser 1998:	82.9 ± 1.40	194.45±0.96	277.43± 1.70
Davier, Höcker 1998:	56.53± 0.83	219.77±1.40	276.3 ± 1.6
Erler 1998 :	56.9 ± 1.1	220.8 ±1.5	277.7 ± 1.9
Burkhardt, Pietrzyk 2001:	155.8 ± 3.6	120.3 ±0.2	276.1 ± 3.6
Hagiwara et al 2004:	150.18± 2.3	125.32±0.15	275.5 ± 2.3
Jegerlehner 2006 direct:	106.07± 2.24	115.66±0.11	276.07± 2.25
Jegerlehner 2006 Adler :	73.69± 0.98	201.83±1.03	275.52± 1.42
Hagiwara et al 2011:	138.70± 1.37	137.56±0.16	276.26± 1.38
Davier et al 2011:	80.57± 1.00	195.33±0.09	275.90± 1.00
Jegerlehner 2016 direct:	126.86± 1.78	149.57±0.05	276.43± 1.78
Jegerlehner 2016 Adler:	60.49± 0.66	214.48±1.00	275.04± 1.19

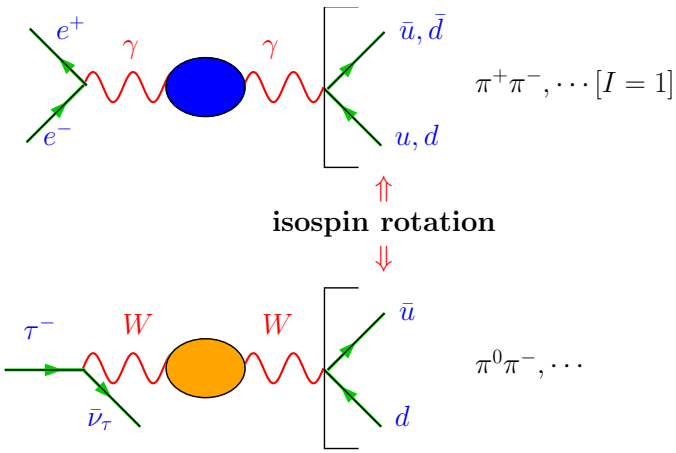
- red theory-driven still can be improved by more precise cross sections below 2 GeV
- green data-driven require better cross section up to above the Υ region
- magenta Adler function based requires 1% cross section from 1.4 to 2.5 GeV and progress in pQCD [massive PQCD to 4-loops, Euclidean regime]
- advantage of euclidean approach can be supplemented directly by lattice QCD data

Data vs pQCD


e+e- -> hadrons
pQCD
average
Φ
J/ψ1S
ψ2S
ψ3770
KEDR
CB
BESII
DASP I
PLUTO
MARK I
DM2
M3N
MEA
BB̄
γγ2
excl data
R
E (GeV)


e+e- -> hadrons
ϒ1S
ϒ2S
3S
4S
ϒ10860
ϒ11020
Mark I
LENA
JADE
pQCD
CB
MD-1
TASSO
PLUTO
DHHM
DASPII, CLEO, CUSB, MAC, CELLO, MARK J
R
E (GeV)

Addendum 1: Using τ–decay spectra + isospin breakings



$e^+e^- \to \pi^+\pi^-, \cdots$ vs $\tau^- \to \bar{\nu}_\tau \pi^0 \pi^-, \cdots$

τ–spectra: ALEPH, OPAL, CLEO, Belle

$a_\mu^{\rm had}$ incl. $I = 1$ $\tau \to \pi\pi\nu_\tau$ in range [0.63-0.96] GeV:

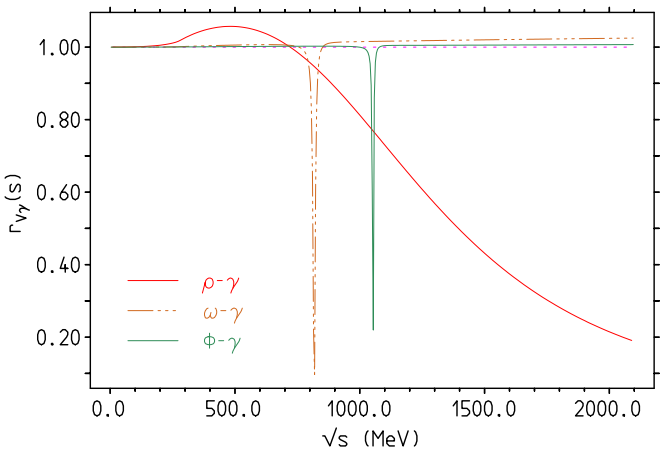
e^+e^-	:	353.82(0.88)(2.17)[2.34]
τ	:	354.25(1.24)(0.61)[1.38]
$e^+e^- + \tau$	:	354.14(0.82)(0.86)[1.19]

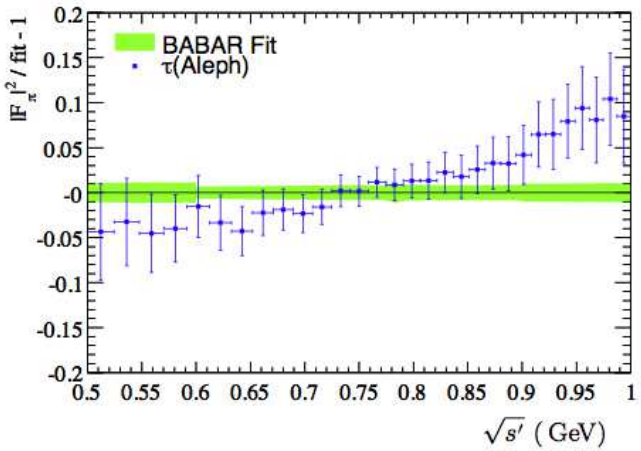
The τ vs e^+e^- puzzle of $\pi^+\pi^-$ data: $\rho^0 - \gamma$ mixing

$-\mathrm{i}\,\Pi^{\mu\nu\,(\pi)}_{\gamma\rho}(q) =$  .

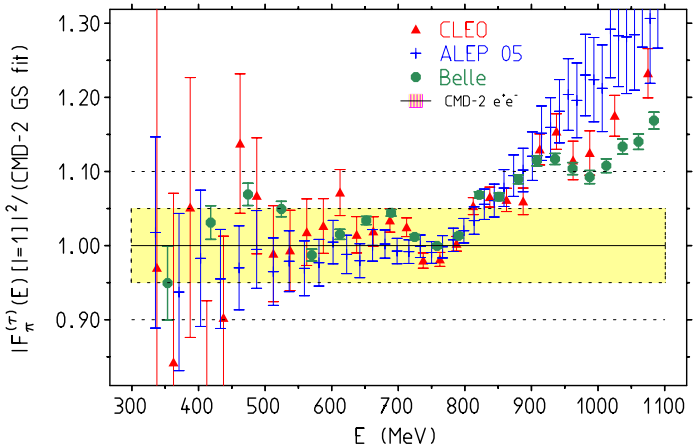
absent in charged (τ) channel

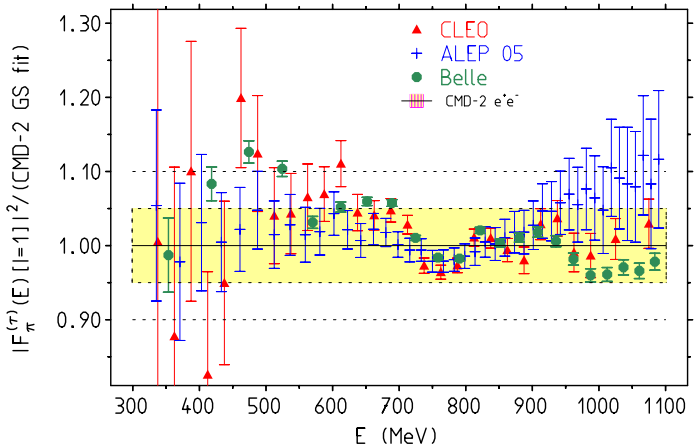
F. J., R. Szafron, Eur. Phys. J. C **71** (2011) 1632



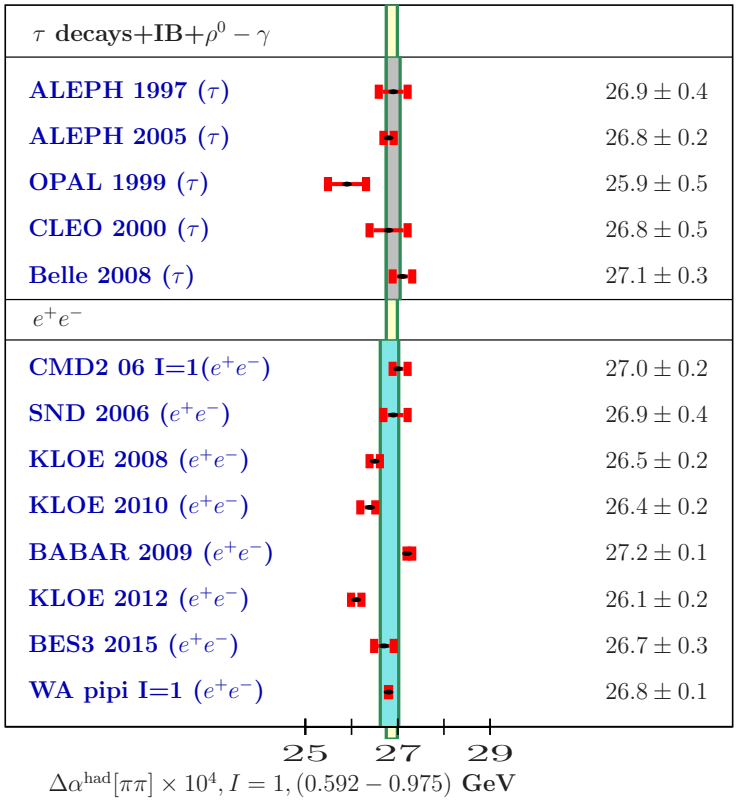


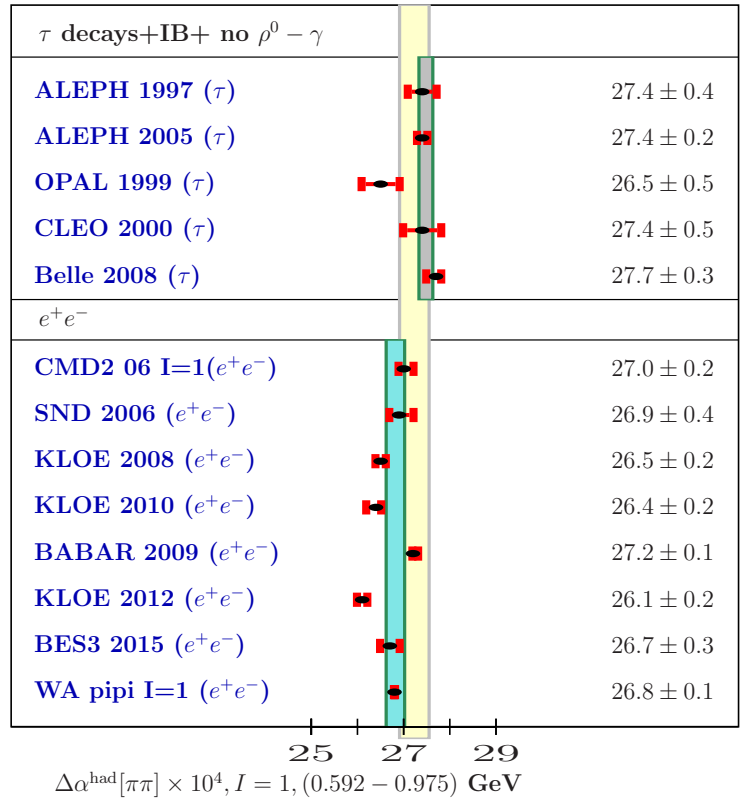
$\rho^0 - \gamma$ mixing correction to be applied to τ data [l], ALEPH versus BaBar plot from Davier et al.[r]





$|F_\pi(E)|^2$ in units of e^+e^- I=1 (CMD-2 GS fit): left no mixing correction, right: after mixing correction





$\Delta\alpha(M_Z)$ contributions form I=1 $\pi\pi$ channel in range [0.63, 0.96] GeV in units 10^{-4}. τ data corrected for isospin breaking and [left] for missing $\rho^0-\gamma$ mixing [right] $\rho^0-\gamma$ mixing is not applied.

$\Delta\alpha(M_Z^2)$ contributions form I=1 $\pi\pi$ channel in range [0.63, 0.96] GeV in units 10^{-4}

data	incl. $\rho^0 - \gamma$ mixing correction	
τ data, IB corrected	26.90 (0.06) (0.14)	27.48 (0.06) (0.14)
e^+e^- data, I=1 $\pi\pi$	26.82 (0.08) (0.18)	26.82 (0.08) (0.18)
Combined	26.87 (0.05) (0.11)	27.23 (0.12) (0.30)
$\Delta\alpha^{\rm had}[ee < 2\ {\rm GeV}] \times 10^4$	57.99(0.12)(0.92)[0.93]	
$\Delta\alpha^{\rm had}[ee + \tau < 2\ {\rm GeV}] \times 10^4$	58.04(0.12)(0.91)[0.92]	

data	incl. $\rho^0 - \gamma$ mixing correction
$\Delta\alpha^{\rm had}[ee]$	$(276.43 \pm 0.67({\rm stat}) \pm 1.64({\rm syst})[\pm 1.78({\rm tot})]) \times 10^{-4}$
$\Delta\alpha^{\rm had}[ee + \tau]$	$(276.48 \pm 0.67({\rm stat}) \pm 1,63({\rm syst})[\pm 1.76({\rm tot})]) \times 10^{-4}$

Central values of pQCD contributions to $\Delta\alpha^{{\rm had}\,(5)}$:

range	$\alpha_s = 0.1184$	$\alpha_s = 0.12$
5.2 GeV - 9.46 GeV	33.84	33.91
13 GeV -∞	115.73	115.82

$\Delta\alpha(-M_0^2)$ contributions form I=1 $\pi\pi$ channel in range [0.63, 0.96] GeV in units 10^{-4}

data	incl. $\rho^0 - \gamma$ mixing correction
τ data, IB corrected	23.46 (0.05) (0.12)
e^+e^- data, I=1 $\pi\pi$	23.39 (0.07) (0.15)
Combined	23.43 (0.04) (0.09)
$\Delta\alpha^{\rm had}[ee < 2\ {\rm GeV}] \times 10^4$	46.43(0.08)(0.56)[0.57]
$\Delta\alpha^{\rm had}[ee + \tau < 2\ {\rm GeV}] \times 10^4$	46.50(0.06)(0.55)[0.56]

data	incl. $\rho^0 - \gamma$ mixing correction
$\Delta\alpha^{\rm had}[ee]$	$(63.85 \pm 0.18({\rm stat}) \pm 0.63({\rm syst})[\pm 0.66({\rm tot})]) \times 10^{-4}$
$\Delta\alpha^{\rm had}[ee + \tau]$	$(63.92 \pm 0.17({\rm stat}) \pm 0.62({\rm syst})[\pm 0.64({\rm tot})]) \times 10^{-4}$

In both cases for $\Delta\alpha_{\rm had}(-M_0^2)$ and $\Delta\alpha_{\rm had}(M_Z^2)$ improvement marginal because $\pi\pi$ channel alone does not give the dominant contribution!

Addendum 2: The coupling α_2, M_W and $\sin^2\Theta_f$

How to measure α_2:

❖charged current channel M_W ($g \equiv g_2$):

$$M_W^2 = \frac{g^2 v^2}{4} = \frac{\pi\,\alpha_2}{\sqrt{2}\,G_\mu}$$

❖neutral current channel $\sin^2\Theta_f$

In fact here running $\sin^2\Theta_f(E)$: LEP scale $\Longleftrightarrow$ low energy $\nu_e e$ scattering

$$\sin^2\Theta_e = \left\{\frac{1-\Delta\alpha_2}{1-\Delta\alpha} + \Delta_{\nu_\mu e,\text{vertex+box}} + \Delta\kappa_{e,\text{vertex}}\right\}\sin^2\Theta_{\nu_\mu e}$$

The first correction from the running coupling ratio is largely compensated by the ν_μ charge radius which dominates the second term. The ratio $\sin^2\Theta_{\nu_\mu e}/\sin^2\Theta_e$ is close to 1.002, independent of top and Higgs mass. Note that errors in the ratio $\frac{1-\Delta\alpha_2}{1-\Delta\alpha}$ can be taken to be 100% correlated and thus largely cancel.

Above result allow us to calculate non-perturbative hadronic correction in $\gamma\gamma$, γZ, ZZ and WW self energies, as

$$\begin{aligned}
\Pi^{\gamma\gamma} &= e^2 \quad \hat{\Pi}^{\gamma\gamma} \\
\Pi^{Z\gamma} &= \frac{eg}{c_\Theta} \quad \hat{\Pi}_V^{3\gamma} \quad - \quad \frac{e^2 s_\Theta}{c_\Theta} \quad \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{ZZ} &= \frac{g^2}{c_\Theta^2} \quad \hat{\Pi}_{V-A}^{33} \quad - \quad 2\,\frac{e^2}{c_\Theta^2} \quad \hat{\Pi}_V^{3\gamma} \quad + \quad \frac{e^2 s_\Theta^2}{c_\Theta^2} \quad \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{WW} &= g^2 \quad \hat{\Pi}_{V-A}^{+-}
\end{aligned}$$

with $\hat{\Pi}(s) = \hat{\Pi}(0) + s\hat{\pi}(s)$.
Leading hadronic contributions:

$$\begin{aligned}
\Delta\alpha_{\rm had}^{(5)}(s) &= -e^2 \left[\mathrm{Re}\,\hat{\pi}^{\gamma\gamma}(s) - \hat{\pi}^{\gamma\gamma}(0)\right] \\
\Delta\alpha_{2\,\rm had}^{(5)}(s) &= -\frac{e^2}{s_\Theta^2}\left[\mathrm{Re}\,\hat{\pi}^{3\gamma}(s) - \hat{\pi}^{3\gamma}(0)\right]
\end{aligned}$$

which exhibit the leading hadronic non-perturbative parts, i.e. the ones involving

the photon field via mixing. $\Delta\alpha^{(5)}_{\rm had}(s)$ and $\Delta\alpha^{(5)}_{2\,\rm had}(s)$ via e^+e^--data and isospin arguments [(u,d), s flavor separation]:

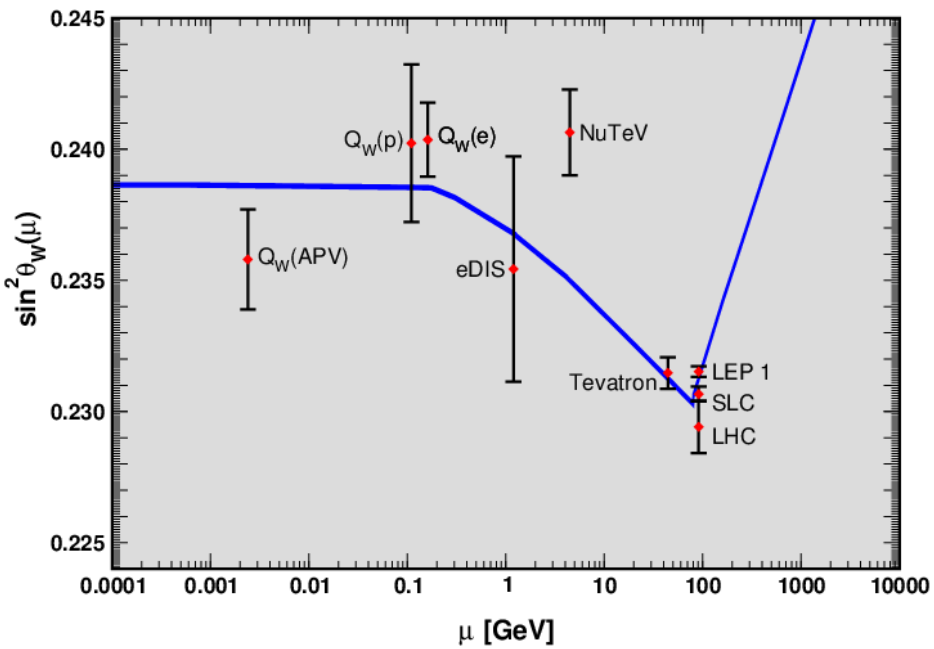
$$\Pi^{3\gamma}_{ud} = \frac{1}{2}\Pi^{\gamma\gamma}_{ud}\,; \quad \Pi^{3\gamma}_{s} = \frac{3}{4}\Pi^{\gamma\gamma}_{s}$$

$$\Pi^{\gamma\gamma} = \Pi^{(\rho)} + \Pi^{(\omega)} + \Pi^{(\phi)} + \cdots \quad \Rightarrow \quad \Pi^{3\gamma} = \tfrac{1}{2}\Pi^{(\rho)} + \tfrac{3}{4}\Pi^{(\phi)} + \cdots$$

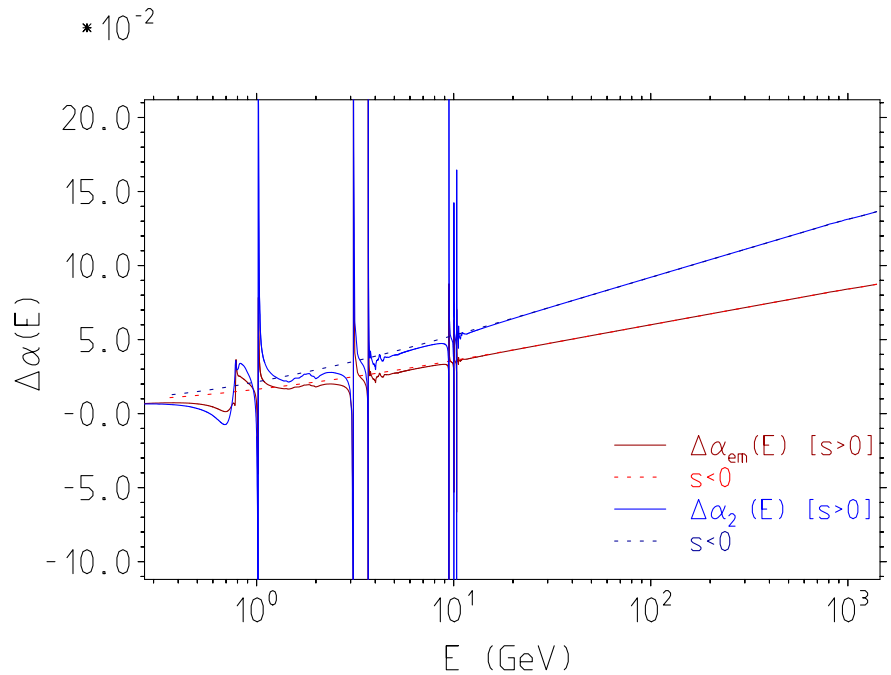
F. J., Z. Phys. C **32** (1986) 195, Nuovo Cim. C **034S1** (2011) 31

Note: gauge boson SE potentially very sensitive to New Physics (oblique corrections)

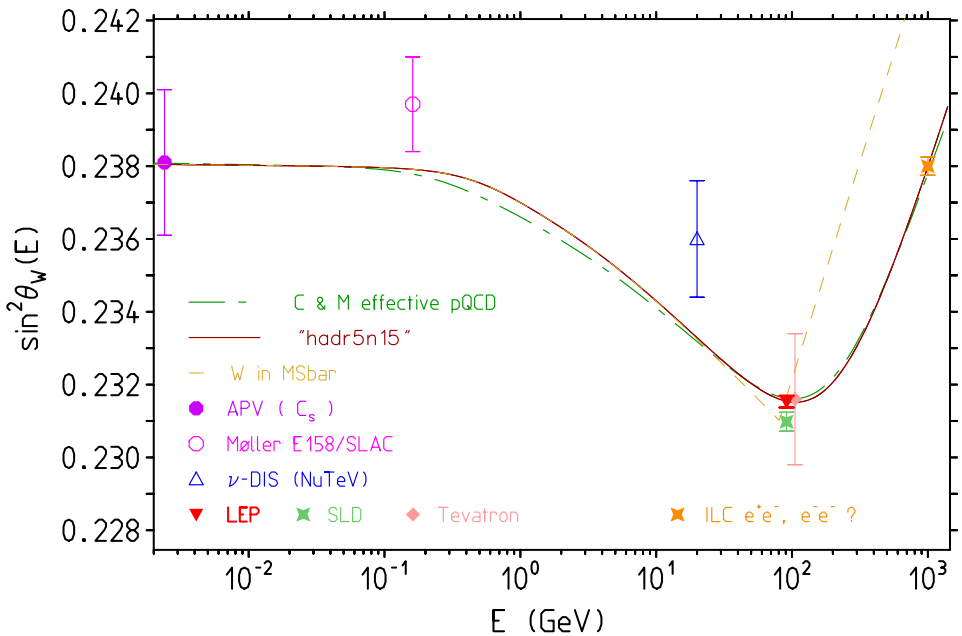
➠new physics may be obscured by non-perturbative hadronic effects; need to fix this!


0.245
0.240
0.235
0.230
0.225
$\sin^2\theta_W(\mu)$
0.0001
0.001
0.01
0.1
1
10
100
1000
10000
μ [GeV]
$Q_W(p)$
$Q_W(e)$
NuTeV
$Q_W(APV)$
eDIS
Tevatron
LEP 1
SLC
LHC

PDG version

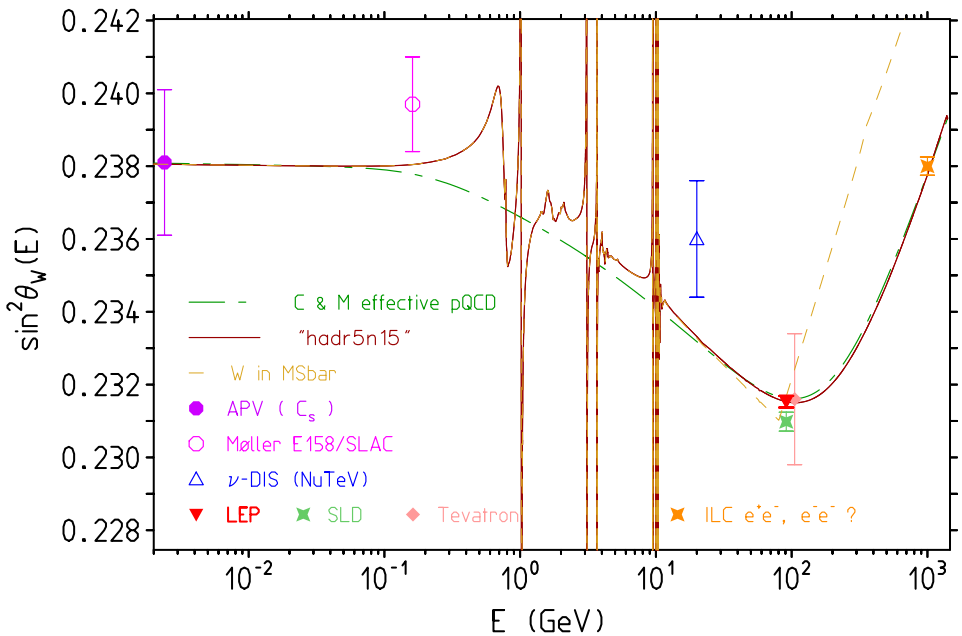


$\Delta\alpha_{\rm em}(E)$ and $\Delta\alpha_2(E)$ as functions of energy E in the time-like and space-like domain. The smooth space-like correction (dashed line) agrees rather well with the non-resonant "background" above the ϕ-resonance (kind of duality). In resonance regions as expected "agreement" is observed in the mean, with huge local deviations.



$\sin^2\Theta_W(Q)$ as a function of Q in the space-like region. Hadronic uncertainties are included but barely visible. Uncertainties from the input parameter $\sin^2\theta_W(0) = 0.23822(100)$ or $\sin^2\theta_W(M_Z^2) = 0.23153(16)$ are not shown. Future ILC/FCC measurements at 1 TeV would be sensitive to Z', H^{--} etc.

Except from the LEP and SLD points (which deviate by 1.8 σ), all existing measurements are of rather limited accuracy unfortunately!



$\sin^2\Theta_W(E)$ as a function of E in the time-like region. Note that $\sin^2\theta_W(0)/\sin^2\theta_W(M_Z^2) = 1.02876$ a 3% correction establised at $6.5\ \sigma$.

$\sin^2\Theta_{\rm eff}$

exhibiting a specific dependence on the gauge boson SEs

is an excellent monitor for New Physics