

Does $\rho - \gamma$ mixing solve the e^+e^- vs. τ spectral function puzzle ?

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Abstract

The energy dependence of the $\rho - \gamma$ mixing in the 2×2 $\gamma - \rho$ propagator matrix, is shown to be able to account for the e^+e^- vs. τ spectral function discrepancy.

Work in collaboration with Robert Szafron [e-Print: [arXiv:1101.2872](https://arxiv.org/abs/1101.2872)]

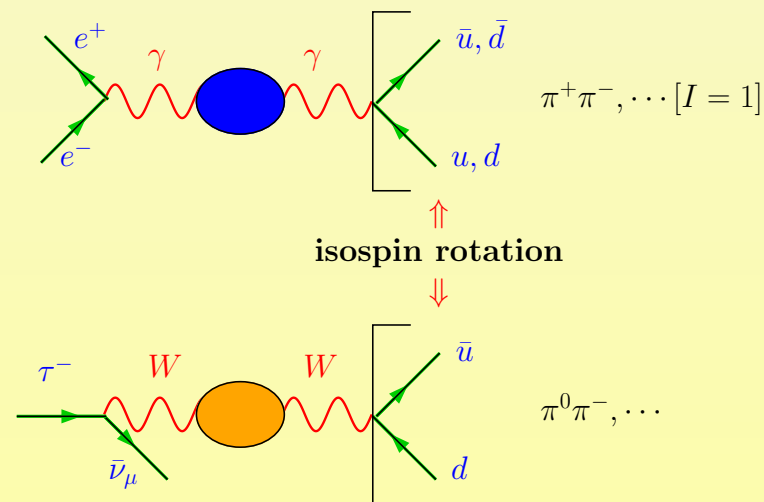
Outline of Talk:

- ❖ The τ vs. e^+e^- problem (as known)
- ❖ A minimal model: VMD + sQED
- ❖ $F_\pi(s)$ with $\rho - \gamma$ mixing at one-loop
- ❖ Applications: a_μ and $B_{\pi\pi^0}^{\text{CVC}} = \Gamma(\tau \rightarrow \nu_\tau \pi\pi^0)/\Gamma_\tau$
- ❖ Summary and Outlook

□ The τ vs. e^+e^- problem

Concerns: calculation of hadronic vacuum polarization from appropriate hadron production data.

① A good idea: enhance e^+e^- -data by isospin rotated/corrected τ -data + CVC



ALEPH-Coll., (OPAL, CLEO), Alemany, Davier, Höcker 1996,
Belle-Coll. Fujikawa, Hayashii, Eidelman 2008

$$\tau^- \rightarrow X^- \nu_\tau \quad \leftrightarrow \quad e^+ e^- \rightarrow X^0$$

where X^- and X^0 are hadronic states related by isospin rotation. The e^+e^- cross-section is then given by

$$\sigma_{e^+e^- \rightarrow X^0}^{I=1} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \leq M_\tau$$

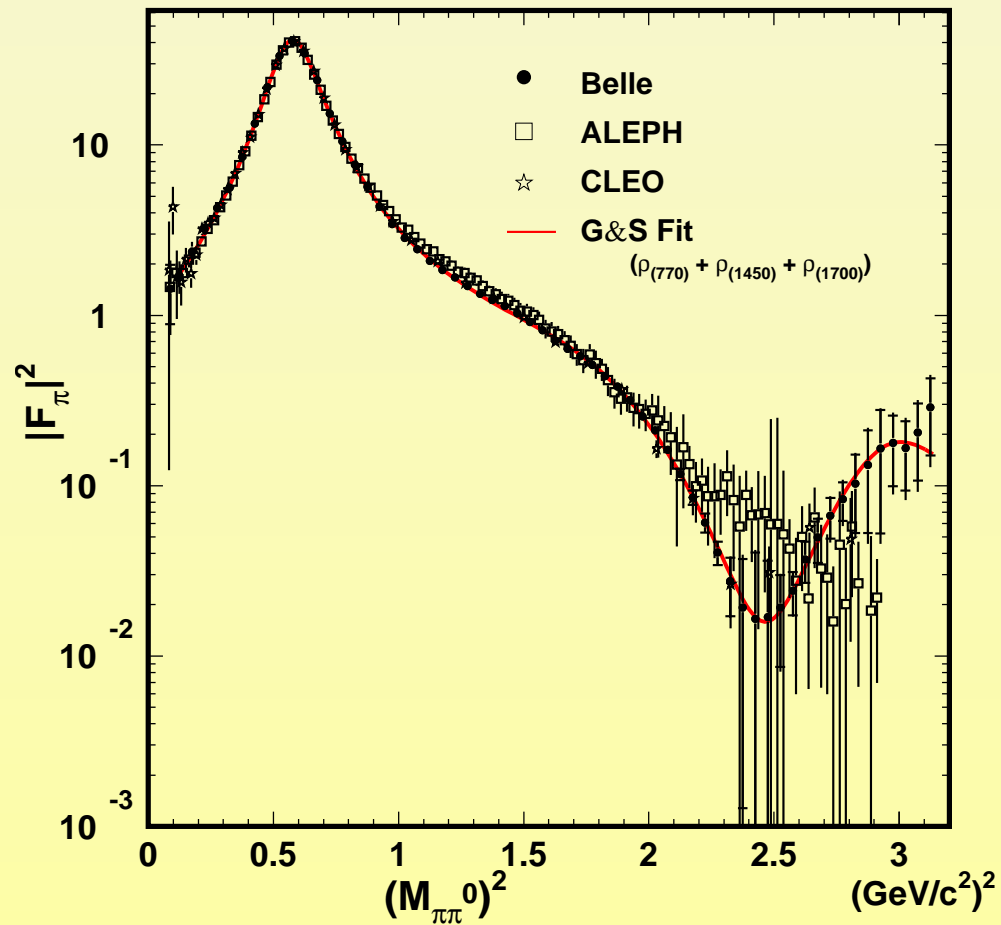
in terms of the τ spectral function v_1 .

- ❖ mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ -resonance contribution)
- ❖ which is dominating in a_μ^{had} (72%)

$I = 1 \sim 75\%$; $I = 0 \sim 25\%$ τ -data cannot replace e^+e^- -data

$$\begin{array}{l} \delta a_\mu \quad : \quad 15.6 \times 10^{-10} \quad \rightarrow \quad 10.2 \times 10^{-10} \\ \delta \Delta\alpha \quad : \quad 0.00067 \quad \rightarrow \quad 0.00065 \quad (\text{ADH1997}) \end{array}$$

Data: ALEPH 97, ALEPH 05, OPAL, CLEO and
most recent measurement from *Belle* (2008):



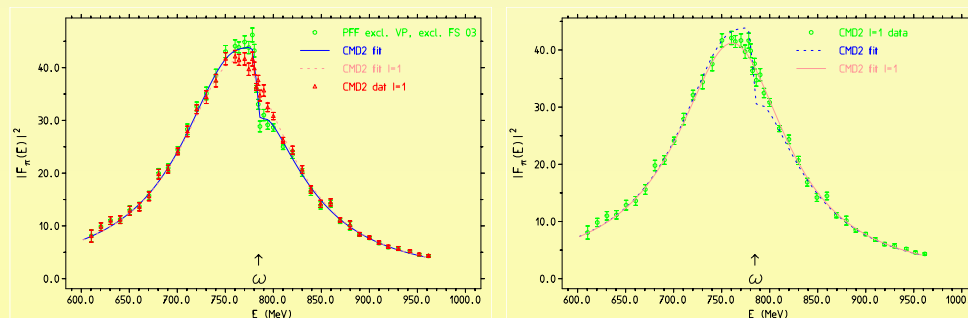
e^+e^- -data* = data corrected for isospin violations:

In e^+e^- (neutral channel) $\rho - \omega$ mixing due isospin violation be quark mass difference $m_u \neq m_d \Rightarrow$

$I=0$ component; to be subtracted for comparison with τ data

$$|F(s)|^2 = (|F(s)|^2\text{-data}) / \left| \left(1 + \frac{\epsilon s}{s_\omega - s} \right) \right|^2 \quad \text{with } s_\omega = (M_\omega - \frac{i}{2}\Gamma_\omega)^2$$

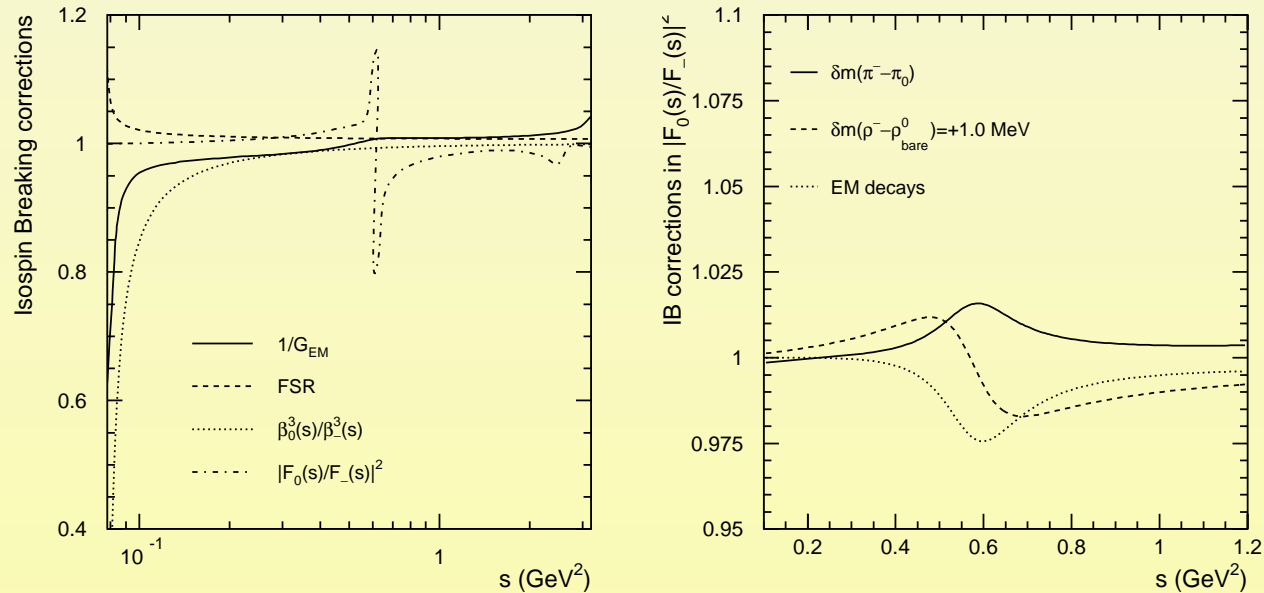
ϵ determined by fit to the data: $\epsilon = 0.00172$



CMD-2 data for $|F_\pi|^2$ in $\rho - \omega$ region together with Gounaris-Sakurai fit. Left before subtraction right after subtraction of the ω .

$I=0$ component to be added to τ data for calculating a_μ^{had} !

Other isospin-breaking corrections Cirigliano et al. 2002, López Castro et al. 2007

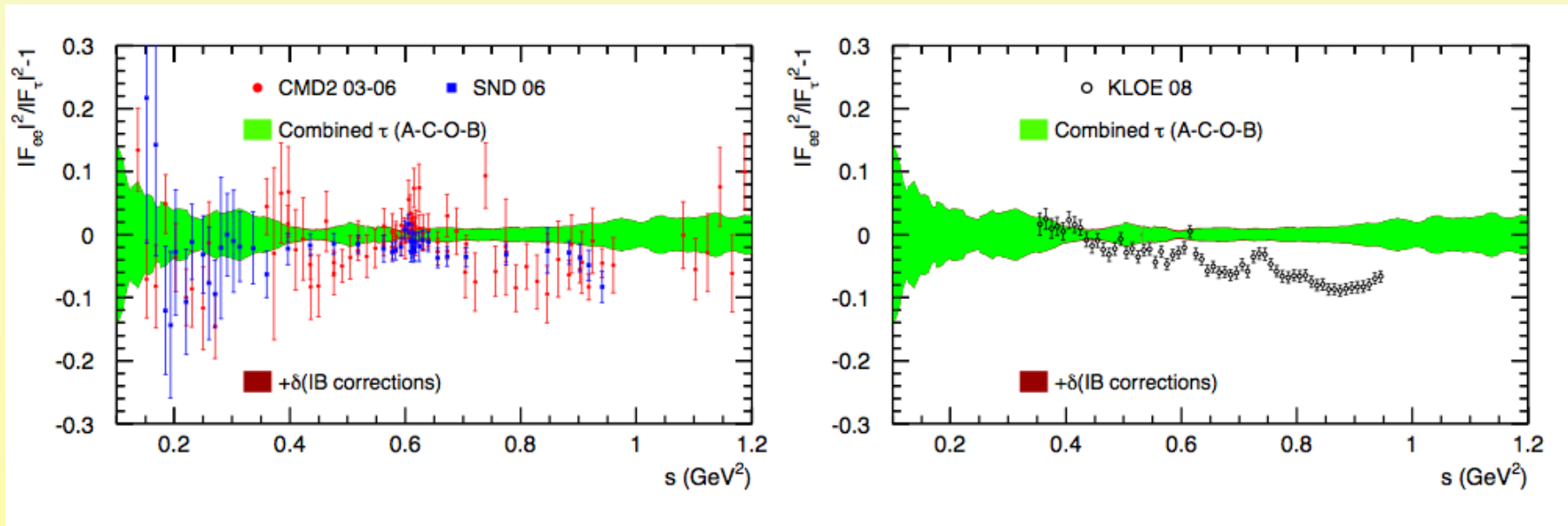


Left: Isospin-breaking corrections G_{EM} , FSR , $\beta_0^3(s)/\beta_-^3(s)$ and $|F_0(s)/F_-(s)|^2$.

Right: Isospin-breaking corrections in $I = 1$ part of ratio $|F_0(s)/F_-(s)|^2$:

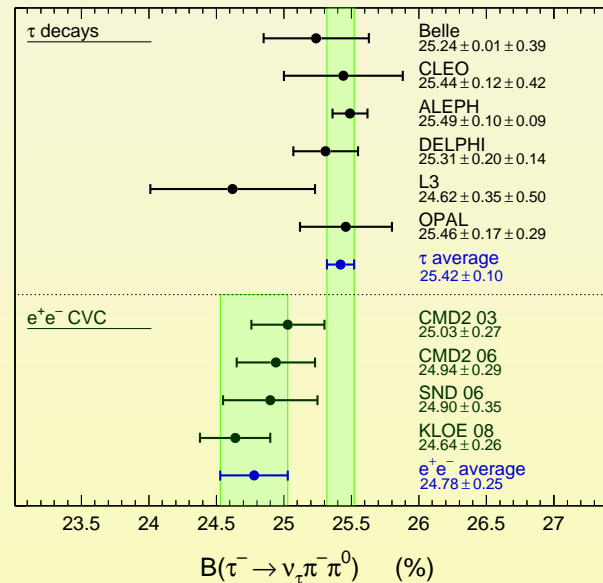
- π mass splitting $\delta m_\pi = m_{\pi^\pm} - m_{\pi^0}$,
- ρ mass splitting $\delta m_\rho = m_{\rho^\pm} - m_{\rho_{bare}^0}$, and
- ρ width splitting $\delta \Gamma_\rho = \Gamma_{\rho^\pm} - \Gamma_{\rho^0}$.

New isospin corrections applied shift in mass and width [as advocated by S. Ghozzi and FJ in 2003!!!] plus changes [López Castro, Toledo Sánchez et al 2007] below the ρ which Davier et al say are not understood! The discrepancy now substantially reduced but with the KLOE data persists. New BABAR radiative return $\pi\pi$ spectrum in much better agreement, in particular with *Belle* τ spectrum!



e^+e^- vs τ spectral functions: $|F_{ee}|^2/|F_{\tau}|^2 - 1$ as a function of s . Isospin-breaking (IB) corrections are applied to τ data with its uncertainties included in the error band.

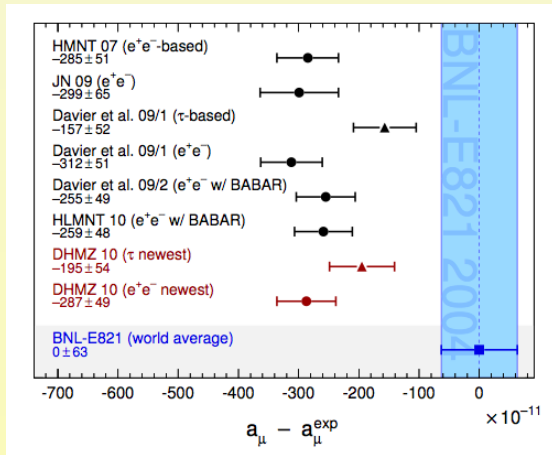
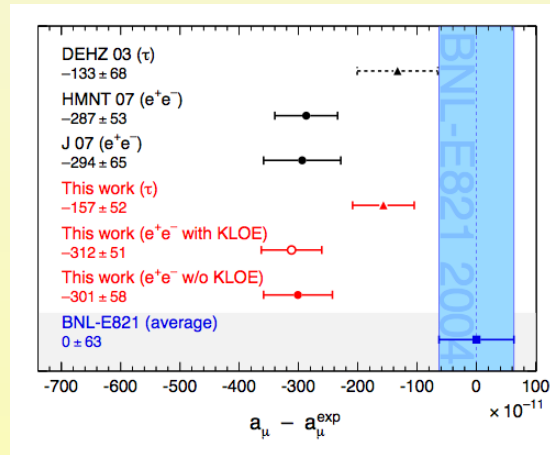
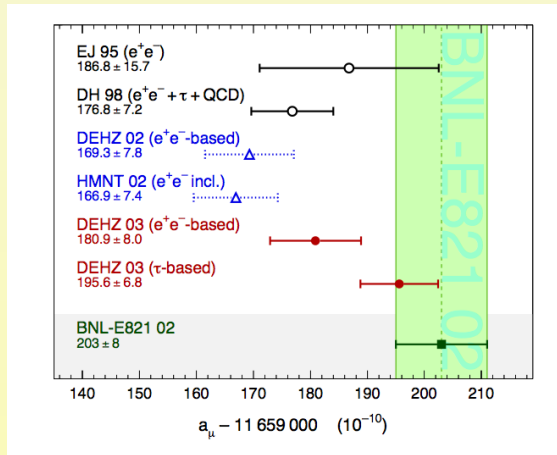
CVC prediction of
 $B_{\pi\pi^0}$
normalization of
BELLE, CLEO and OPAL
not fixed
by the experiment itself



The measured branching fractions for $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ compared to the predictions from the $e^+e^- \rightarrow \pi^+\pi^-$ spectral functions (after isospin-breaking corrections). (Named e^+e^- results for 0.63 – 0.958GeV). The long and short vertical error bands correspond to the τ and e^+e^- averages of 25.42 ± 0.10 and 24.76 ± 0.25 , respectively.

Note -2% in *Belle* τ data means $25.42 \rightarrow 24.91$ in agreement with e^+e^-
 $[|F_\tau(0)|^2 = 1.02 \rightarrow |F_\tau(0)|^2 = 1]$

History:



Possible origin of problems:

□ Radiative corrections involving hadrons fully under control?

□ IB in parameter shifts: $m_{\rho^+} - m_{\rho^0}$, $\Gamma_{\rho^+} - \Gamma_{\rho^0}$ fully known?

Key problem: on basis of commonly used Gounaris-Sakurai type parametrizations

e^+e^- vs. τ fit with same formula \Rightarrow differ in parameters only: NC vs. CC process
 δM_ρ , $\delta \Gamma_\rho$, mixing coefficients etc.

Other possible source: do we really understand quantum interference?

● e^+e^- : $|F_\pi^{(e)}(s)|^2 = |F_\pi^{(e)}(s)[I = 1] + F_\pi^{(e)}(s)[I = 0]|^2$ what we need and measure

● τ : $|F_\pi^{(\tau)}(s)[I = 1]|^2$ measured in τ -decay

● $ee + \tau$: $|F_\pi^{(e)}(s)|^2 \simeq |F_\pi^{(e,\tau)}(s)[I = 1]|^2 + |F_\pi^{(e)}(s)[I = 0]|^2$??? usual approximation

Need theory \rightarrow specific model for the complex amplitudes

□ A minimal model: VMD + sQED

Effective Lagrangian $\mathcal{L} = \mathcal{L}_{\gamma\rho} + \mathcal{L}_{\pi}$

$$\mathcal{L}_{\pi} = D_{\mu}\pi^{+}D^{+\mu}\pi^{-} - m_{\pi}^2\pi^{+}\pi^{-}; \quad D_{\mu} = \partial_{\mu} - ieA_{\mu} - ig_{\rho\pi\pi}\rho_{\mu}$$

$$\mathcal{L}_{\gamma\rho} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{M_{\rho}^2}{2}\rho_{\mu}\rho^{\mu} + \frac{e}{2g_{\rho}}\rho_{\mu\nu}F^{\mu\nu}$$

Self-energies: pion loops to photon-rho vacuum polarizations

$$-i\Pi_{\gamma\gamma}^{\mu\nu}(\pi)(q) = \text{diagram 1} + \text{diagram 2}$$

bare $\gamma - \rho$ transverse self-energy functions

$$\Pi_{\gamma\gamma} = \frac{e^2}{48\pi^2}f(q^2), \quad \Pi_{\gamma\rho} = \frac{eg_{\rho\pi\pi}}{48\pi^2}f(q^2) \quad \text{and} \quad \Pi_{\rho\rho} = \frac{g_{\rho\pi\pi}^2}{48\pi^2}f(q^2),$$

Explicitly, in the $\overline{\text{MS}}$ scheme (μ the $\overline{\text{MS}}$ renormalization scale)

$$h(q^2) \equiv f(q^2)/q^2 = 2/3 + 2(1-y) - 2(1-y)^2 G(y) + \ln \frac{\mu^2}{m_\pi^2},$$

where $y = 4m_\pi^2/s$ and $G(y) = \frac{1}{2\beta_\pi} (\ln \frac{1+\beta_\pi}{1-\beta_\pi} - i\pi)$, for $q^2 > 4m_\pi^2$.

Mass eigenstates, diagonalization: renormalization conditions are such that the matrix is diagonal and of residue unity at the photon pole $q^2 = 0$ and at the ρ resonance $s = M_\rho^2$, [$\Pi_{\gamma\gamma}(0) = 0$, $\Pi'_{\gamma\gamma}(q^2) = \Pi_{\gamma\gamma}(q^2)/q^2$]

$$\Pi_{\gamma\gamma}^{\text{ren}}(q^2) = \Pi_{\gamma\gamma}(q^2) - q^2 \Pi'_{\gamma\gamma}(0) \doteq q^2 \Pi_{\gamma\gamma}^{\text{ren}'}(q^2)$$

$$\Pi_{\gamma\rho}^{\text{ren}}(q^2) = \Pi_{\gamma\rho}(q^2) - \frac{q^2}{M_\rho^2} \text{Re} \Pi_{\gamma\rho}(M_\rho^2)$$

$$\Pi_{\rho\rho}^{\text{ren}}(q^2) = \Pi_{\rho\rho}(q^2) - \text{Re} \Pi_{\rho\rho}(M_\rho^2) - (q^2 - M_\rho^2) \text{Re} \frac{d\Pi_{\rho\rho}}{ds}(M_\rho^2)$$

Propagators = inverse of symmetric 2×2 self-energy matrix

$$\hat{D}^{-1} = \begin{pmatrix} q^2 + \Pi_{\gamma\gamma}(q^2) & \Pi_{\gamma\rho}(q^2) \\ \Pi_{\gamma\rho}(q^2) & q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) \end{pmatrix}$$

inverted \Rightarrow

$$D_{\gamma\gamma} = \frac{1}{q^2 + \Pi_{\gamma\gamma}(q^2) - \frac{\Pi_{\gamma\rho}^2(q^2)}{q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)}}$$

$$D_{\gamma\rho} = \frac{-\Pi_{\gamma\rho}(q^2)}{(q^2 + \Pi_{\gamma\gamma}(q^2))(q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)) - \Pi_{\gamma\rho}^2(q^2)}$$

$$D_{\rho\rho} = \frac{1}{q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) - \frac{\Pi_{\gamma\rho}^2(q^2)}{q^2 + \Pi_{\gamma\gamma}(q^2)}} .$$

Resonance parameters \Leftrightarrow location s_P of the pole of the propagator

$$s_P - m_{\rho^0}^2 + \Pi_{\rho^0\rho^0}(s_P) - \frac{\Pi_{\gamma\rho^0}^2(s_P)}{s_P - \Pi_{\gamma\gamma}(s_P)} = 0 ,$$

with $s_P = \tilde{M}_{\rho^0}^2$ complex.

$$\tilde{M}_{\rho}^2 \equiv (q^2)_{\text{pole}} = M_{\rho}^2 - i M_{\rho} \Gamma_{\rho}$$

Diagonalization \Rightarrow physical ρ acquires a direct coupling to the electron

$$\mathcal{L}_{\text{QED}} = \bar{\psi}_e \gamma^{\mu} (\partial_{\mu} - i e_b A_{b\mu}) \psi_e$$

\Downarrow

$$\mathcal{L}_{\text{QED}} = \bar{\psi}_e \gamma^{\mu} (\partial_{\mu} - i e A_{\mu} + i g_{\rho ee} \rho_{\mu}) \psi_e$$

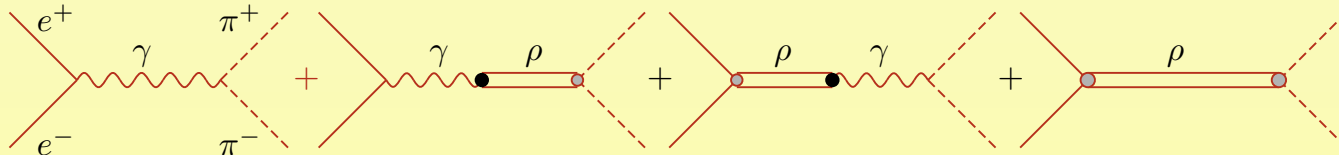
with $g_{\rho ee} = e (\Delta_{\rho} + \Delta_0)$, where in our case $\Delta_0 = 0$.

□ $F_\pi(s)$ with $\rho - \gamma$ mixing at one-loop

The $e^+e^- \rightarrow \pi^+\pi^-$ matrix element in sQED is given by

$$\mathcal{M} = -i e^2 \bar{v} \gamma^\mu u (p_1 - p_2)_\mu F_\pi(q^2)$$

with $F_\pi(q^2) = 1$. In our extended VMD model we have the four terms



Diagrams contributing to the process $e^+e^- \rightarrow \pi^+\pi^-$.

$$F_\pi(s) \propto e^2 D_{\gamma\gamma} + e g_{\rho\pi\pi} D_{\gamma\rho} - g_{\rho ee} e D_{\rho\gamma} - g_{\rho ee} g_{\rho\pi\pi} D_{\rho\rho},$$

Properly normalized (VP subtraction: $e^2(s) \rightarrow e^2$):

$$F_\pi(s) = \left[e^2 D_{\gamma\gamma} + e (g_{\rho\pi\pi} - g_{\rho ee}) D_{\gamma\rho} - g_{\rho ee} g_{\rho\pi\pi} D_{\rho\rho} \right] / \left[e^2 D_{\gamma\gamma} \right]$$

Typical couplings

$$g_{\rho\pi\pi \text{ bare}} = 5.8935, \quad g_{\rho\pi\pi \text{ ren}} = 6.1559, \quad g_{\rho ee} = 0.018149, \quad x = g_{\rho\pi\pi}/g_\rho = 1.15128.$$

We note that the precise s -dependence of the effective ρ -width is obtained by evaluating the imaginary part of the ρ self-energy:

$$\text{Im } \Pi_{\rho\rho} = \frac{g_{\rho\pi\pi}^2}{48 \pi} \beta_\pi^3 s \equiv M_\rho \Gamma_\rho(s),$$

which yields

$$\Gamma_\rho(s)/M_\rho = \frac{g_{\rho\pi\pi}^2}{48 \pi} \beta_\pi^3 \frac{s}{M_\rho^2}; \quad \Gamma_\rho/M_\rho = \frac{g_{\rho\pi\pi}^2}{48 \pi} \beta_\rho^3; \quad g_{\rho\pi\pi} = \sqrt{48 \pi \Gamma_\rho / (\beta_\rho^3 M_\rho)}.$$

In our model, in the given approximation, the on ρ -mass-shell form factor reads

$$F_\pi(M_\rho^2) = 1 - i \frac{g_{\rho ee} g_{\rho\pi\pi} M_\rho}{e^2 \Gamma_\rho} ; \quad |F_\pi(M_\rho^2)|^2 = 1 + \frac{36}{\alpha^2} \frac{\Gamma_{ee}}{\beta_\rho^3 \Gamma_\rho} ,$$

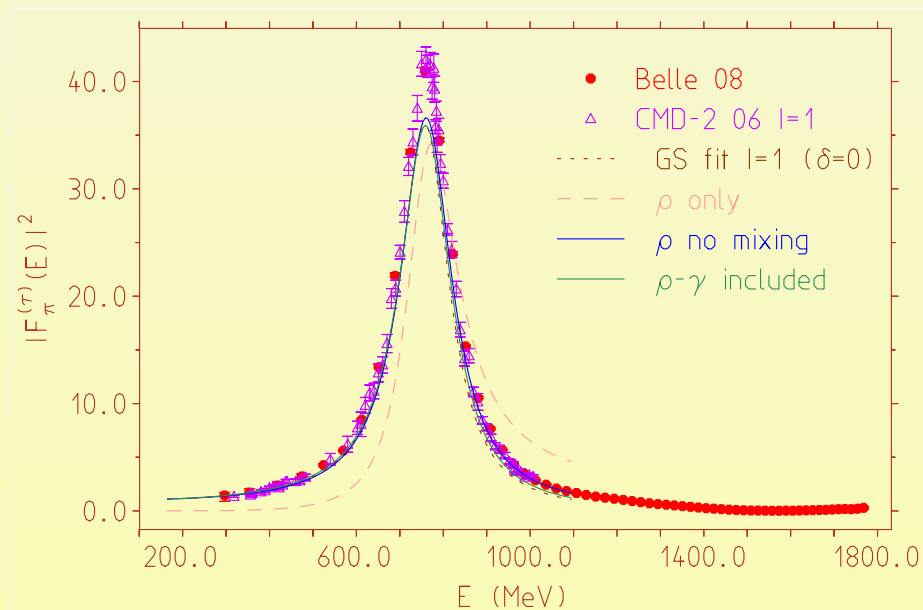
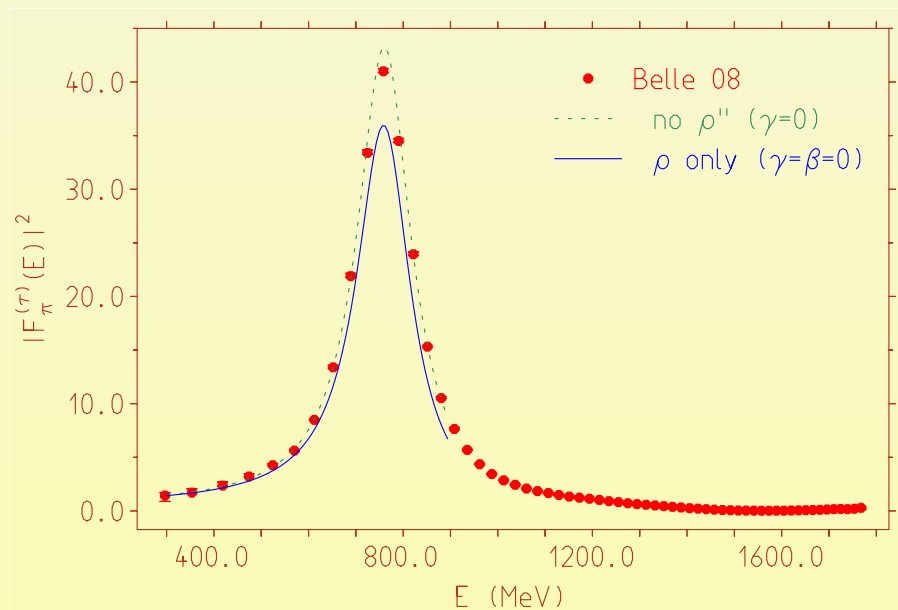
$$\Gamma_{\rho ee} = \frac{1}{3} \frac{g_{\rho ee}^2}{4\pi} M_\rho ; \quad g_{\rho ee} = \sqrt{12\pi \Gamma_{\rho ee} / M_\rho} .$$

Compare: Gounaris-Sakurai (GS) formula

$$F_\pi^{\text{GS}}(s) = \frac{-M_\rho^2 + \Pi_{\rho\rho}^{\text{ren}}(0)}{s - M_\rho^2 + \Pi_{\rho\rho}^{\text{ren}}(s)} ; \quad \Gamma_{\rho ee}^{\text{GS}} = \frac{2\alpha^2 \beta_\rho^3 M_\rho^2}{9 \Gamma_\rho} \left(1 + d \Gamma_\rho / M_\rho\right)^2 .$$

GS does not involve $g_{\rho ee}$ resp. $\Gamma_{\rho ee}$ in a direct way, as normalization is fixed by applying an overall factor $1 + d \Gamma_\rho / M_\rho \equiv 1 - \Pi_{\rho\rho}^{\text{ren}}(0) / M_\rho^2 \simeq 1.089$ to enforce $F_\pi(0) = 1$ (in our approach “automatic” by gauge invariance).

Relation to data:

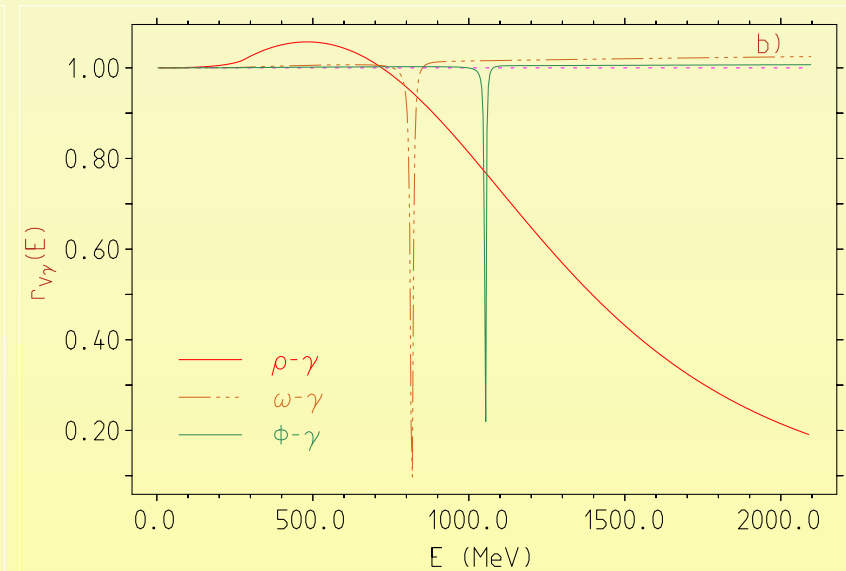
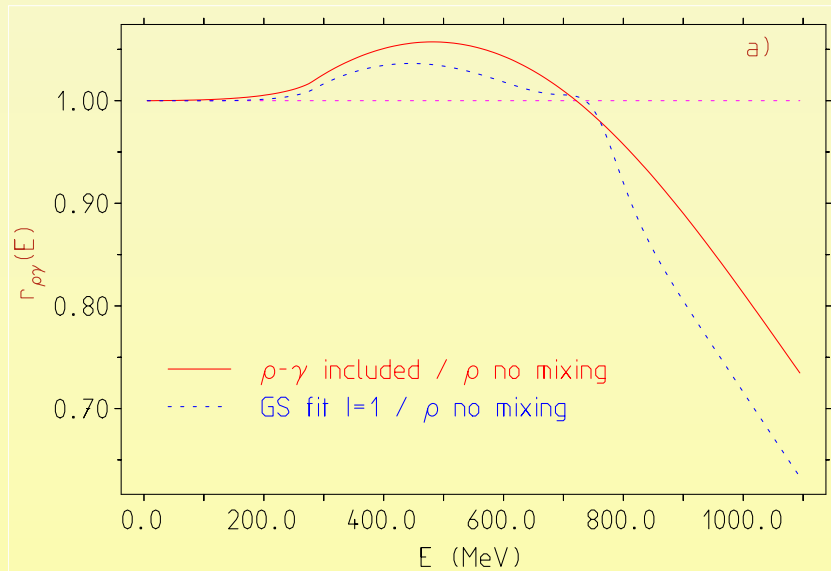


Left: GS fits of the Belle data and the effects of including higher states ρ' and ρ'' at fixed M_{ρ} and Γ_{ρ} . Right: Effect of $\gamma - \rho$ mixing in our simple EFT model

Parameters: $M_{\rho} = 775.5$ MeV, $\Gamma_{\rho} = 143.85$ MeV,
 $\mathcal{B}[(\rho \rightarrow ee)/(\rho \rightarrow \pi\pi)] = 4.67 \times 10^{-5}$, $e = 0.302822$, $g_{\rho\pi\pi} = 5.92$, $g_{\rho ee} = 0.01826$.

Detailed comparison, in terms of the ratio:

$$r_{\rho\gamma}(s) \equiv \frac{|F_{\pi}(s)|^2}{|F_{\pi}(s)|_{D\gamma\rho=0}^2}$$



a) Ratio of $|F_{\pi}(E)|^2$ with mixing vs. no mixing. Same ratio for GS fit with PDG parameters. b) The same mechanism scaled up by the branching fraction $\Gamma_V/\Gamma(V \rightarrow \pi\pi)$ for $V = \omega$ and ϕ . In the $\pi\pi$ channel the effects for resonances $V \neq \rho$ are tiny if not very close to resonance.

If mixing not included in $F_0(s) \Rightarrow$ total correction formula on spectral functions

$$v_0(s) = r_{\rho\gamma}(s) R_{\text{IB}}(s) v_-(s)$$

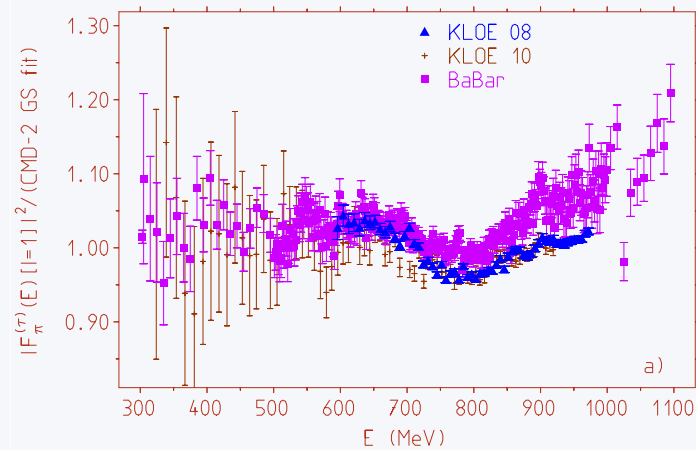
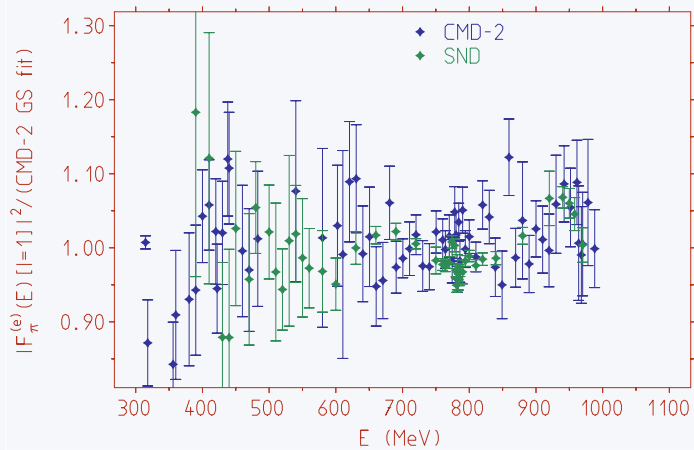
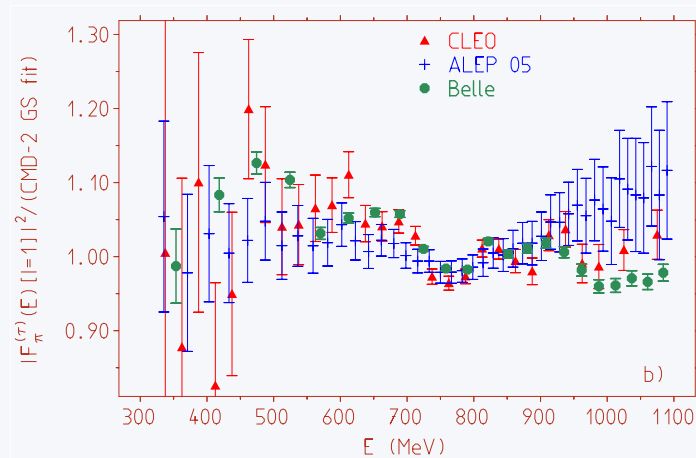
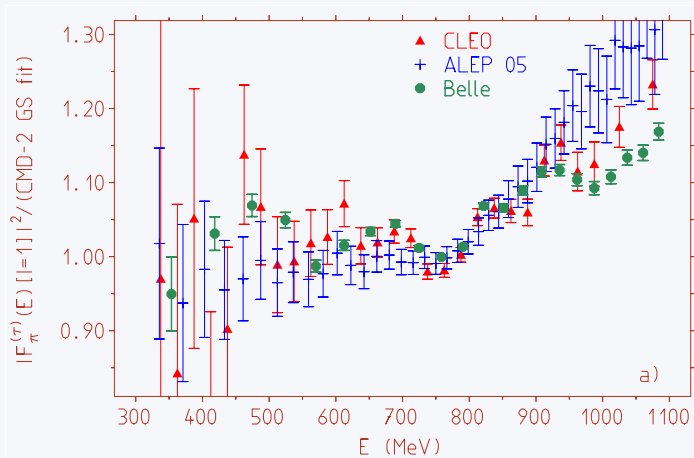
$$R_{\text{IB}}(s) = \frac{1}{G_{\text{EM}}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

□ $G_{\text{EM}}(s)$ electromagnetic radiative corrections

□ $\beta_0^3(s)/\beta_-^3(s)$ phase space modification by $m_{\pi^0} \neq m_{\pi^\pm}$

□ $|F_0(s)/F_-(s)|^2$ incl. shifts in masses, widths etc

Final state radiation correction $\text{FSR}(s)$ and vacuum polarization effects $(\alpha/\alpha(s))^2$ and $I=0$ component $(\rho - \omega)$ we have been subtracted from all e^+e^- -data.



$|F_\pi(E)|^2$ in units of $e^+e^- |_{=1}$ (CMD-2 GS fit): a) τ data uncorrected for $\rho - \gamma$ mixing, and b) after correcting for mixing. Lower panel: e^+e^- energy scan data [left] and e^+e^- radiative return data [right]

□ Applications: a_μ and $B_{\pi\pi^0}^{\text{CVC}} = \Gamma(\tau \rightarrow \nu_\tau \pi\pi^0)/\Gamma_\tau$

How does the new correction affect the evaluation of the hadronic contribution to a_μ ? To lowest order in terms of e^+e^- -data, represented by $R(s)$, we have

$$a_\mu^{\text{had,LO}}(\pi\pi) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds R_{\pi\pi}^{(0)}(s) \frac{K(s)}{s},$$

with the well-known kernel $K(s)$ and

$$R_{\pi\pi}^{(0)}(s) = (3s\sigma_{\pi\pi})/4\pi\alpha^2(s) = 3v_0(s).$$

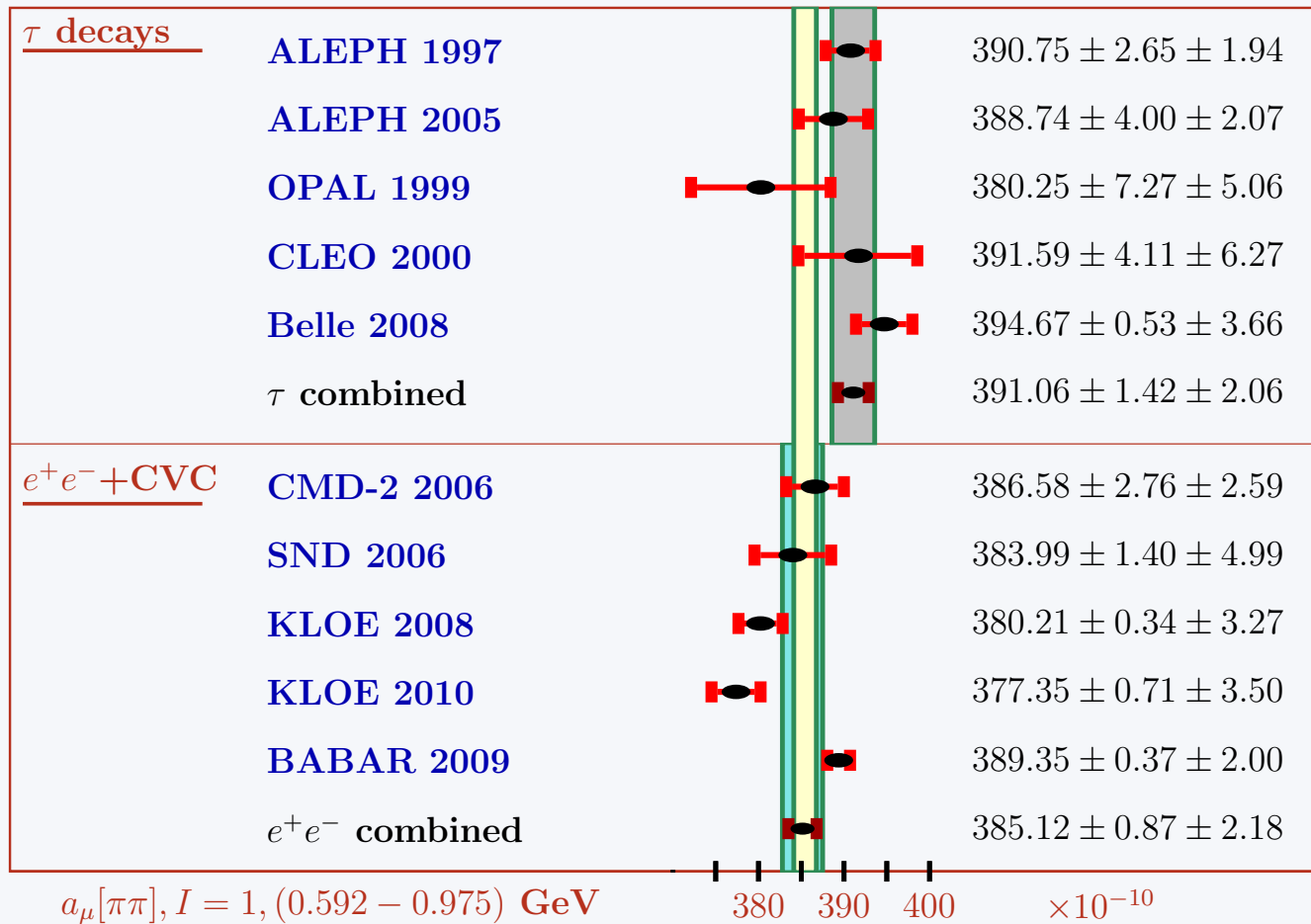
Note that the $\rho - \gamma$ interference is included in the measured e^+e^- -data, and so is its contribution to a_μ^{had} . In fact a_μ^{had} is intrinsic an e^+e^- -based “observable” (neutral current channel).

How to utilize τ data: subtract CVC violating corrections

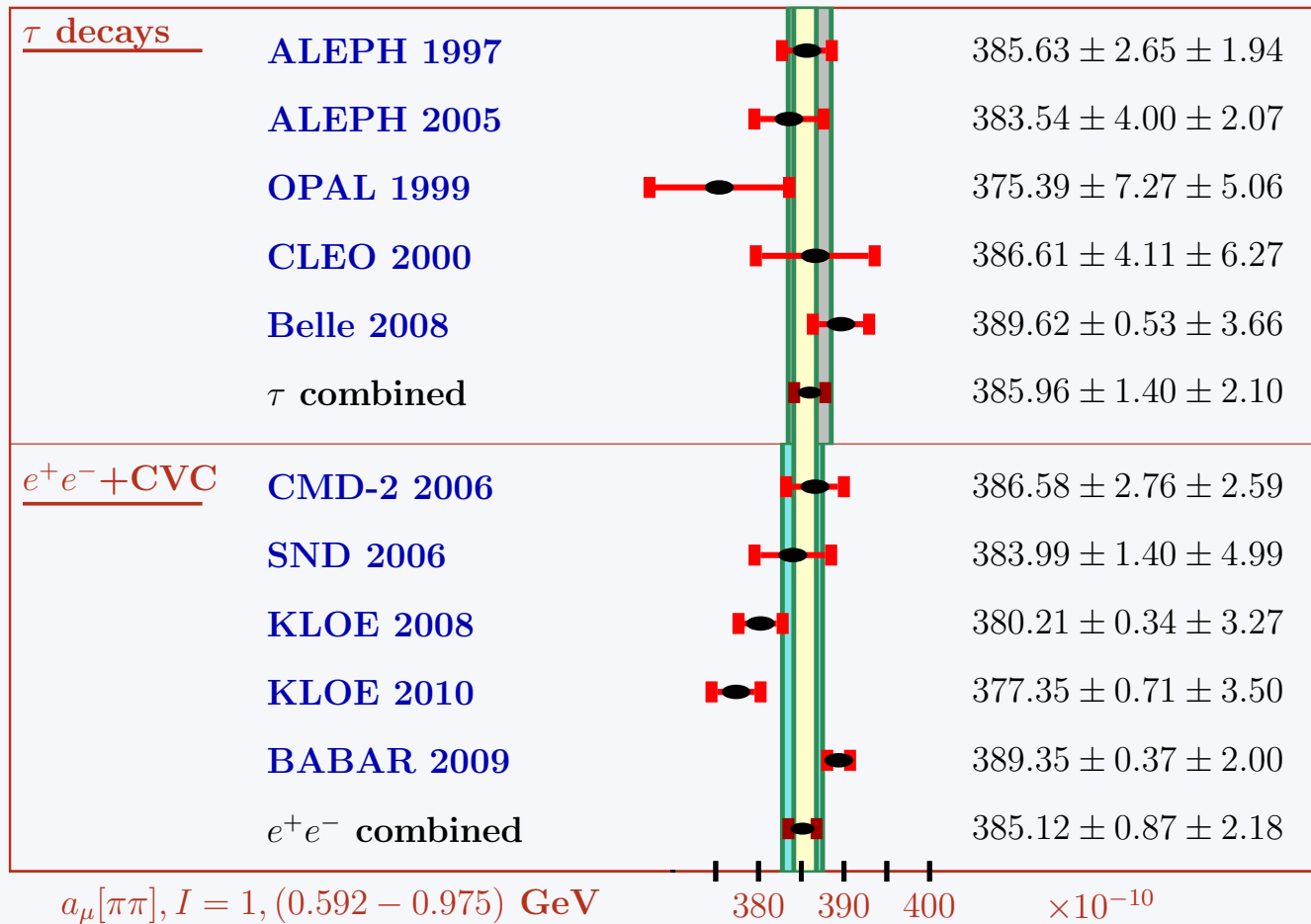
❖ traditionally $v_-(s) \rightarrow v_0(s) = R_{\text{IB}}(s) v_-(s)$

❖ our correction $v_-(s) \rightarrow v_0(s) = r_{\rho\gamma}(s) R_{\text{IB}}(s) v_-(s)$

Result for the $l=1$ part of $a_\mu^{\text{had}}[\pi\pi]$: $\delta a_\mu^{\text{had}}[\rho\gamma] \simeq (-5.1 \pm 0.5) \times 10^{-10}$



$I=1$ part of $a_\mu^{\text{had}}[\pi\pi]$

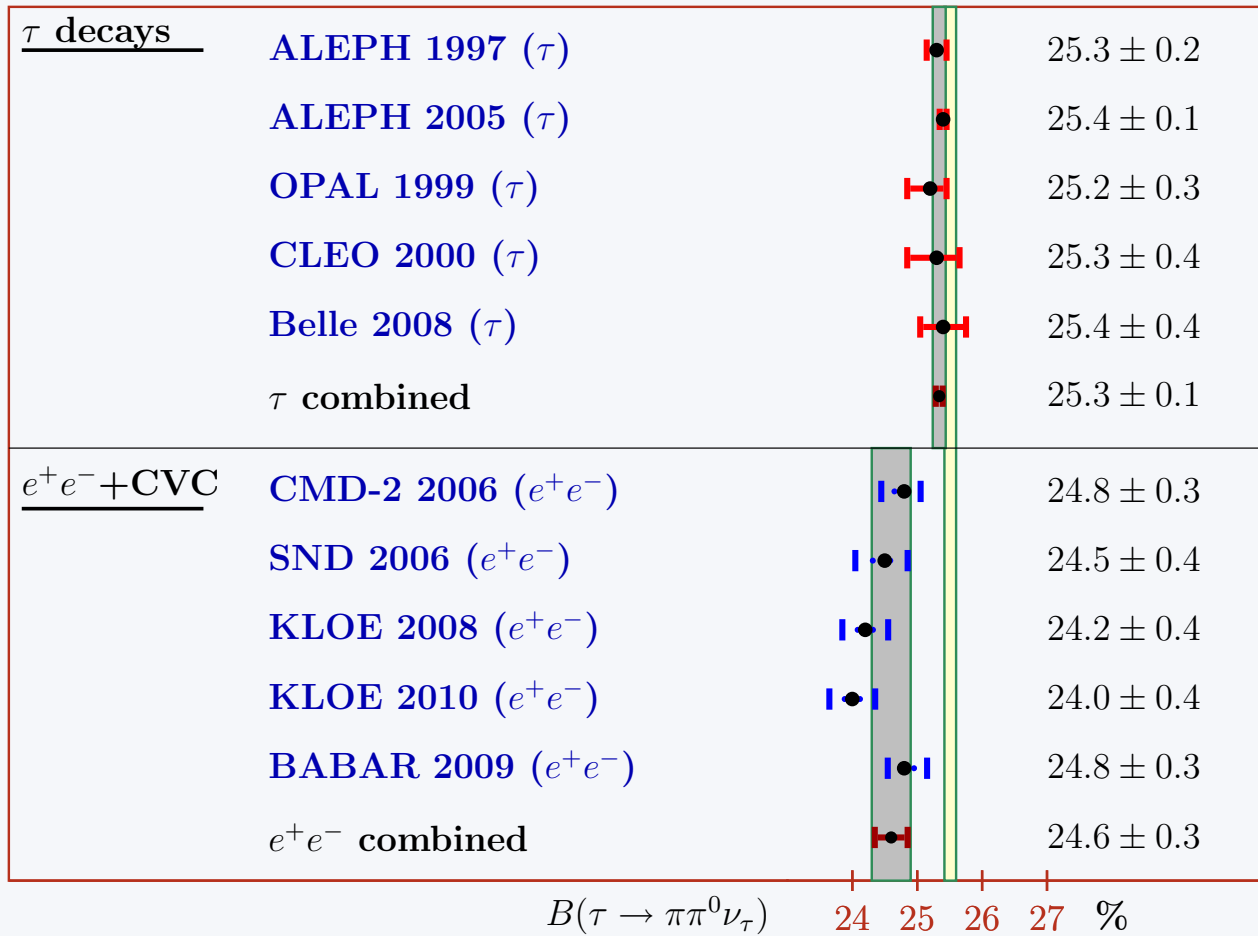


$I=1$ part of $a_\mu^{\text{had}}[\pi\pi]$

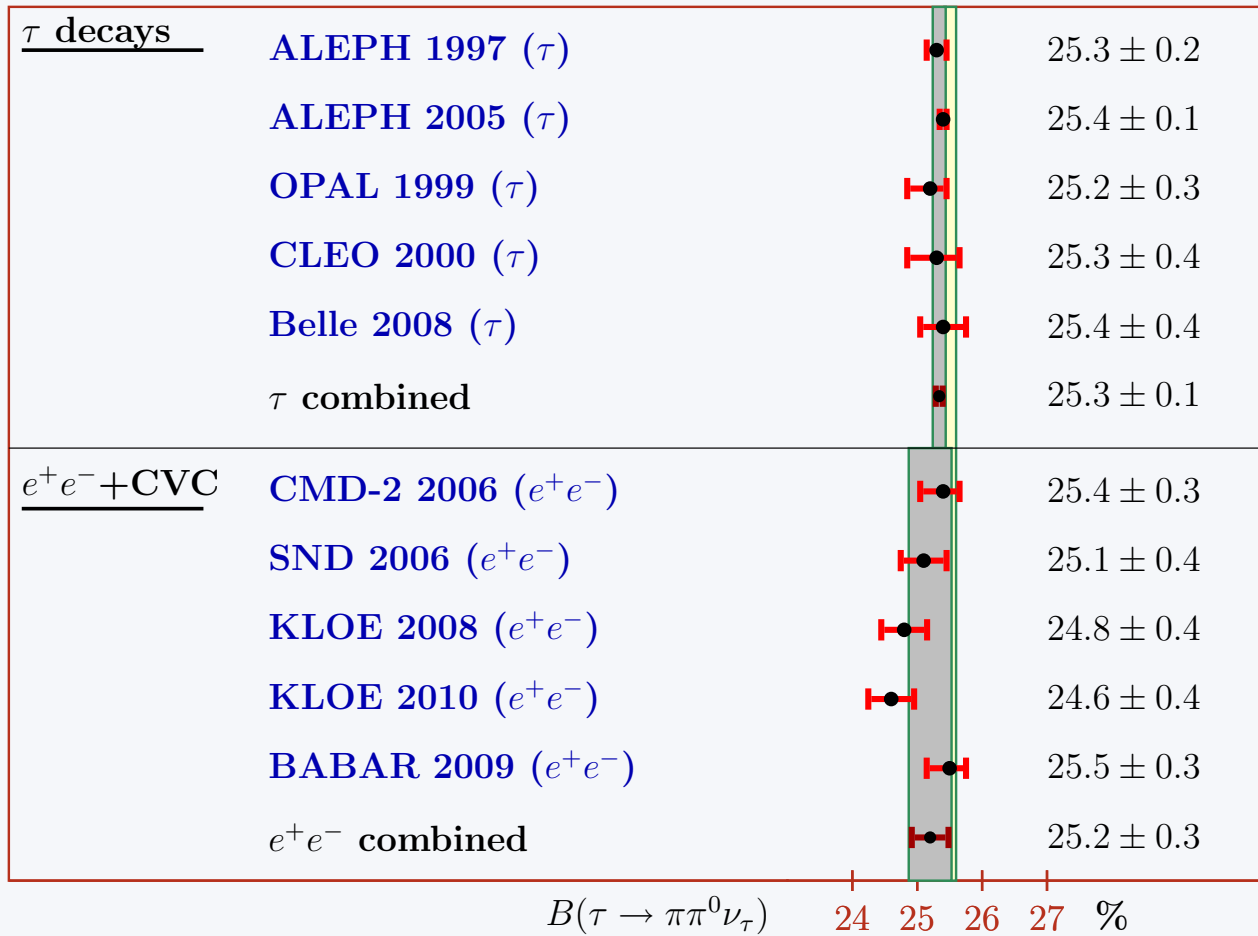
The $\tau \rightarrow \pi^0 \pi \nu_\tau$ branching fraction $B_{\pi\pi^0} = \Gamma(\tau \rightarrow \nu_\tau \pi\pi^0)/\Gamma_\tau$ is another important quantity which can be directly measured. This “ τ -observable” can be evaluated in terms of the $l=1$ part of the $e^+e^- \rightarrow \pi^+\pi^-$ cross section, after taking into account the IB correction $v_0(s) \rightarrow v_-(s) = v_0(s)/R_{\text{IB}}(s) / r_{\rho\gamma}(s)$,

$$B_{\pi\pi^0}^{\text{CVC}} = \frac{2S_{\text{EW}}B_e|V_{ud}|^2}{m_\tau^2} \int_{4m_\pi^2}^{m_\tau^2} ds R_{\pi^+\pi^-}^{(0)}(s) \left(1 - \frac{2}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \frac{1}{r_{\rho\gamma}(s) R_{\text{IB}}(s)},$$

where here we also have to “undo” the $\rho - \gamma$ mixing which is absent in the charged isovector channel. The shift is $\delta B_{\pi\pi^0}^{\text{CVC}}[\rho\gamma] = +0.62 \pm 0.06 \%$



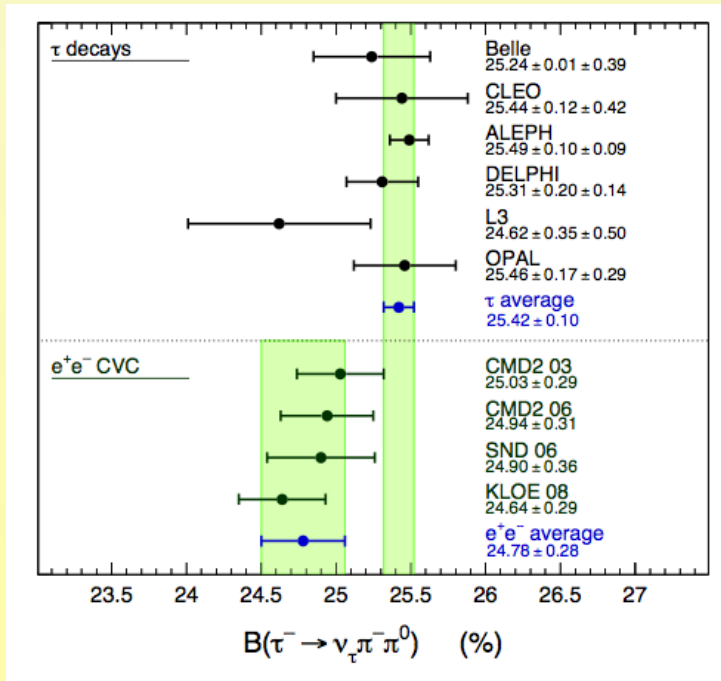
Branching fractions $B(\tau \rightarrow \pi\pi^0\nu_\tau)$



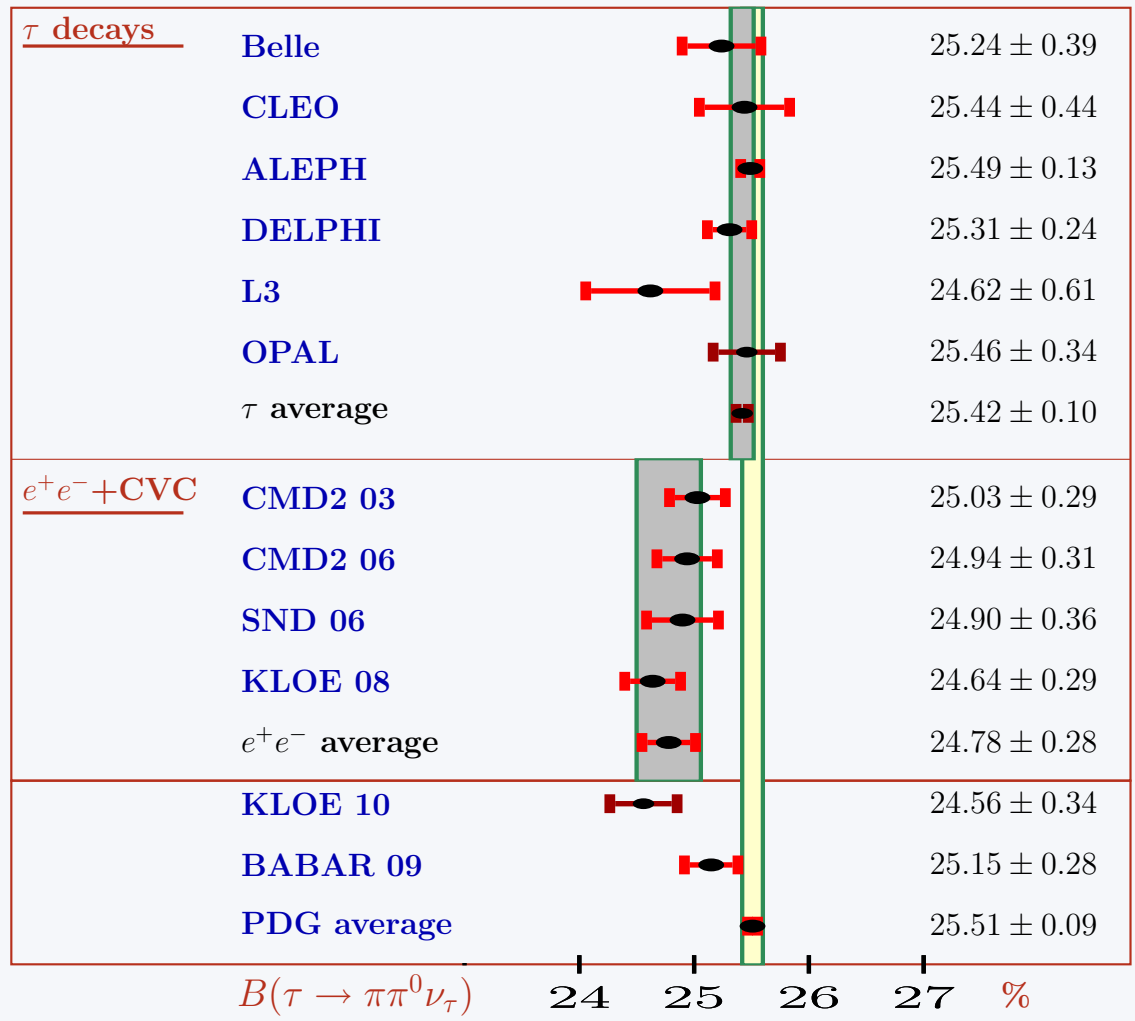
Branching fractions $B(\tau \rightarrow \pi\pi^0\nu_\tau)$

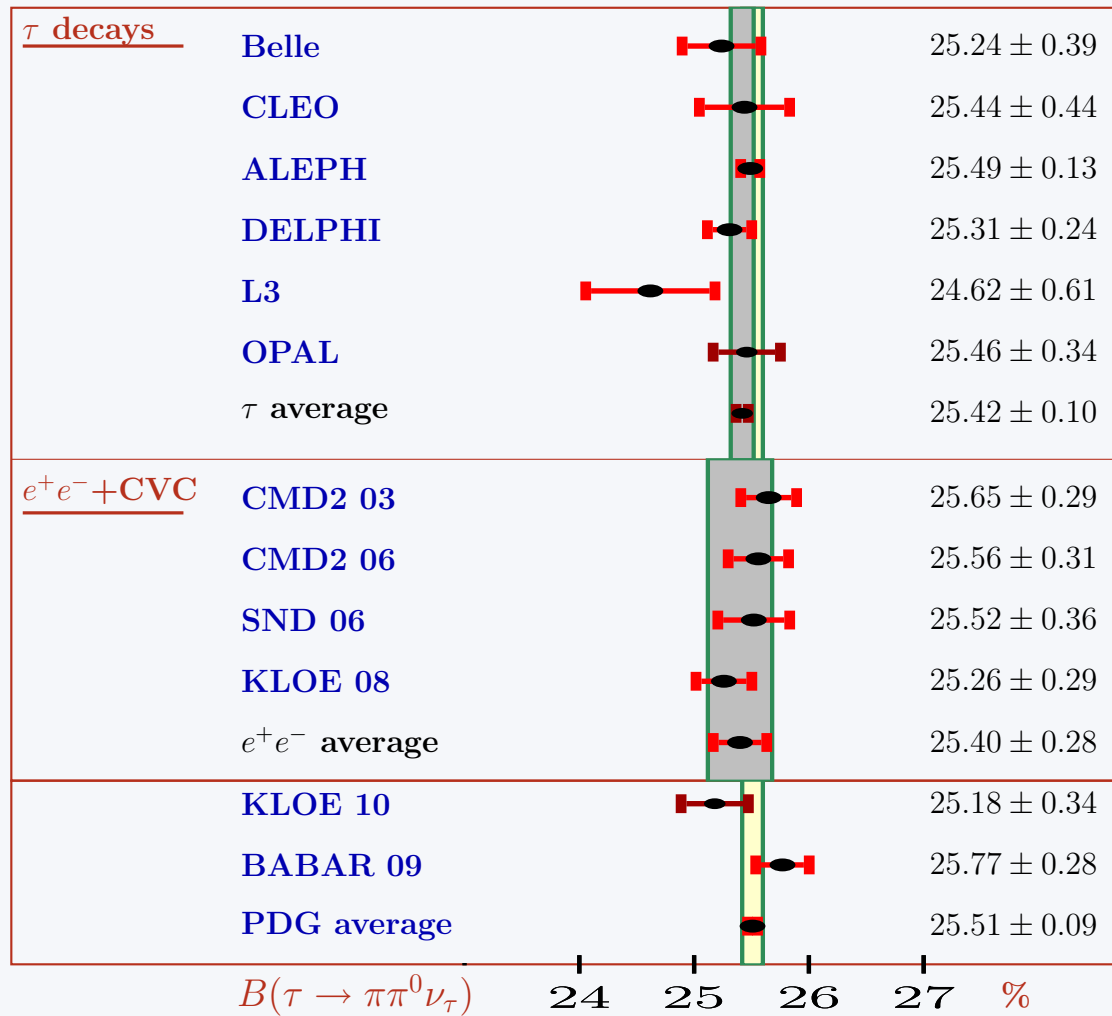
Most recent results of Davier et al:

- Pre BaBar: 25.42 ± 0.10 % for τ
- 24.78 ± 0.28 % \Rightarrow $25.40 \pm 0.28 \pm 0.06$ % for $e^+e^- + \text{CVC}$
- New BaBar: 25.15 ± 0.28 % \Rightarrow $25.77 \pm 0.28 \pm 0.06$ % for $e^+e^- + \text{CVC}$



shift $\delta B_{\pi\pi^0}^{\text{CVC}}[\rho\gamma] = +0.62 \pm 0.06$ %





Summary and Conclusions

⇒ VMD+sQED EFT understood as the tail of the more appropriate resonance Lagrangian approach (Ecker et al. 1989) in low energy $\pi\pi$ production yields

- proper ρ propagator self-energy effects for GS form factor ($\rho \rightarrow \pi\pi$)
- pion-loop effects in $\rho - \gamma$ mixing contributes sizable interferences

Note: so far PDG parameters masses, widths, branching fractions etc. of resonances like ρ^0 all extracted from data assuming GS like form factors (model dependent!)

Pattern:

- moderate positive interference (up to +5%) below ρ , substantial negative interference (-10% and more) above the ρ (must vanish at $s = 0$ and $s = M_\rho^2$)

- remarkable agreement with pattern of e^+e^- vs τ discrepancy
- shift of the τ data to lie perfectly within the ballpark of the e^+e^- data

Lesson: effective field theory the basic tool (not ad hoc pheno. ansätze)

- ❖ $\rho - \gamma$ correction function $r_{\rho\gamma}(s)$ entirely fixed from neutral channel
- ❖ τ data provide independent information

What does it mean for the muon $g - 2$?

- it looks we have fairly reliable model to include τ data to improve a_μ^{had}
- there is no τ vs. e^+e^- alternative of a_μ^{had}

For the lowest order hadronic vacuum polarization (VP) contribution to a_μ we find

$$a_\mu^{\text{had,LO}}[e, \tau] = 690.96(1.06)(4.63) \times 10^{-10} \quad (e + \tau)$$

$$a_{\mu}^{\text{the}} = 116591797(60) \times 10^{-11}$$

$$a_{\mu}^{\text{exp}} = 116592080(54)(33) \times 10^{-11}$$

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{the}} = (283 \pm 87) \times 10^{-11}$$

3.3 σ

Höcker 2010 (theory-driven analysis)

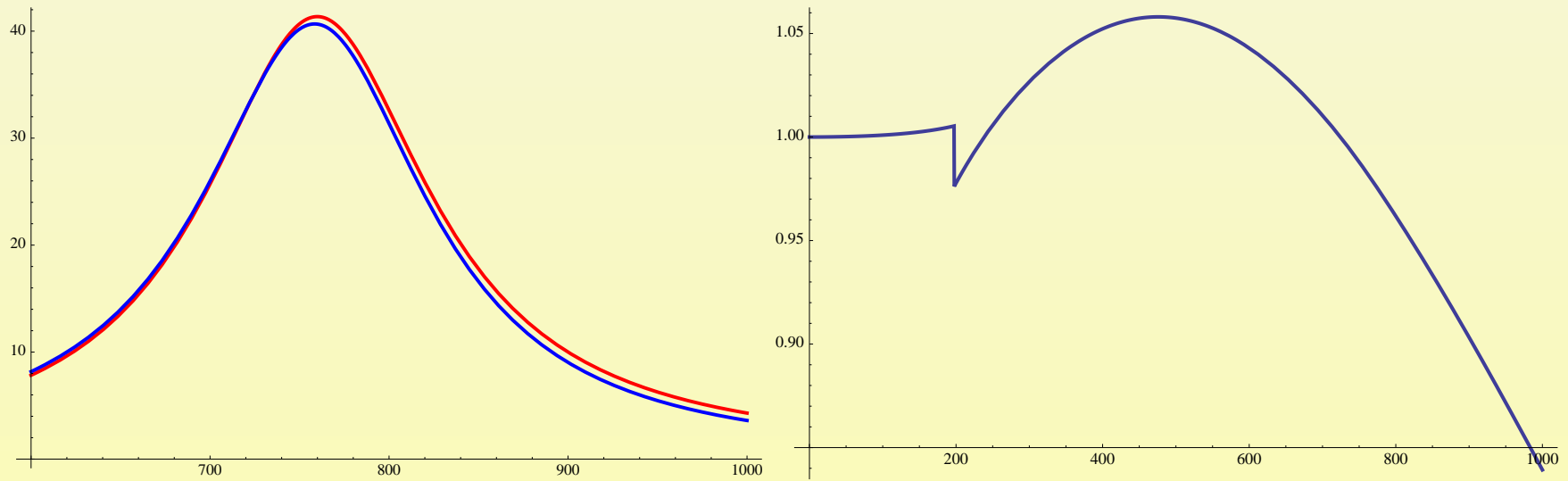
$$a_{\mu}^{\text{had,LO}}[e] = (692.3 \pm 1.4 \pm 3.1 \pm 2.4 \pm 0.2 \pm 0.3) \times 10^{-10} \text{ (} e^+e^- \text{ based),}$$

$$a_{\mu}^{\text{had,LO}}[e, \tau] = (701.5 \pm 3.5 \pm 1.9 \pm 2.4 \pm 0.2 \pm 0.3) \times 10^{-10} \text{ (} e^+e^- + \tau \text{ based),}$$

□ Note: ratio $F_0(s)/F_-(s)$ could be measured within lattice QCD, without reference to sQED or other hadronic models. Do it!

□ Including $\omega, \phi, \rho', \rho'', \dots$ requires to go to appropriate Resonance Lagrangian extension (e.g HLS model [Benayoun et al.](#))

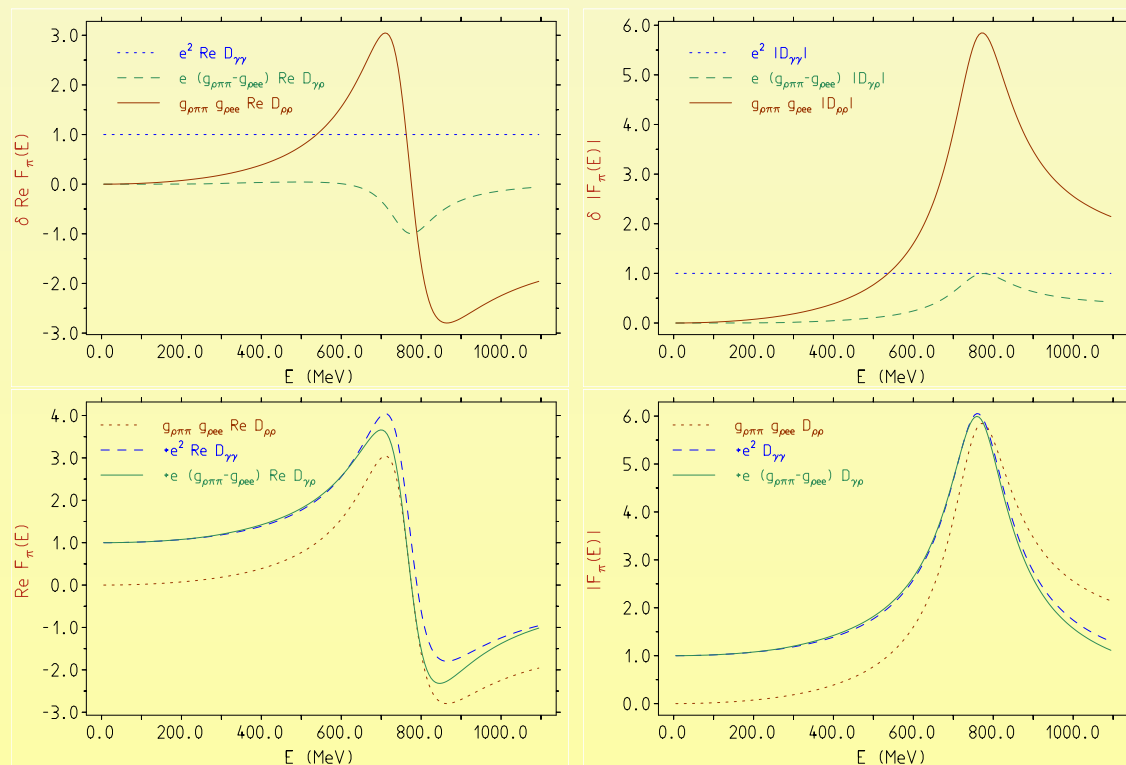
Backup slides



Rober Szafrons first attempt to $\rho - \gamma$ mixing (based on my QCD lectures at Katowice (see: <http://www-com.physik.hu-berlin.de/~fjeger/books.html>)).

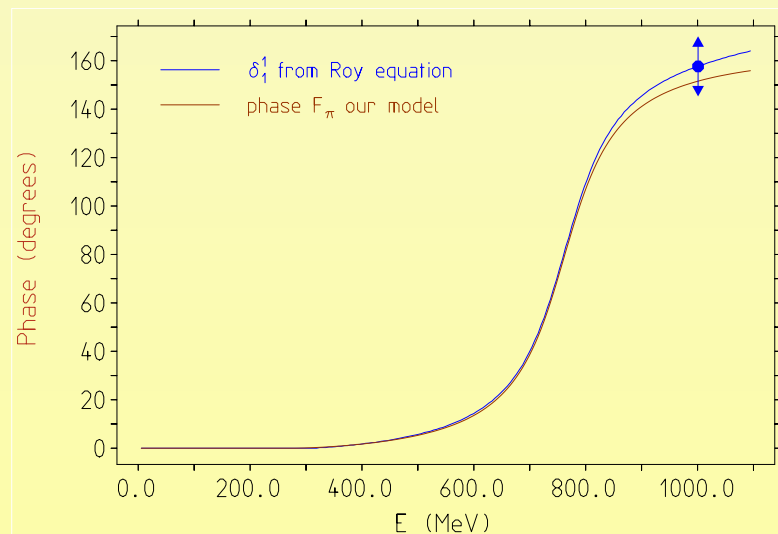
The interference of terms in $F_{\pi}^{(e)}$

Real parts and moduli of the 3 individual and added terms normalized to the sQED term are displayed:



Comparison of $\pi\pi$ rescattering with Colangelo-Leutwyler's from first principles approach

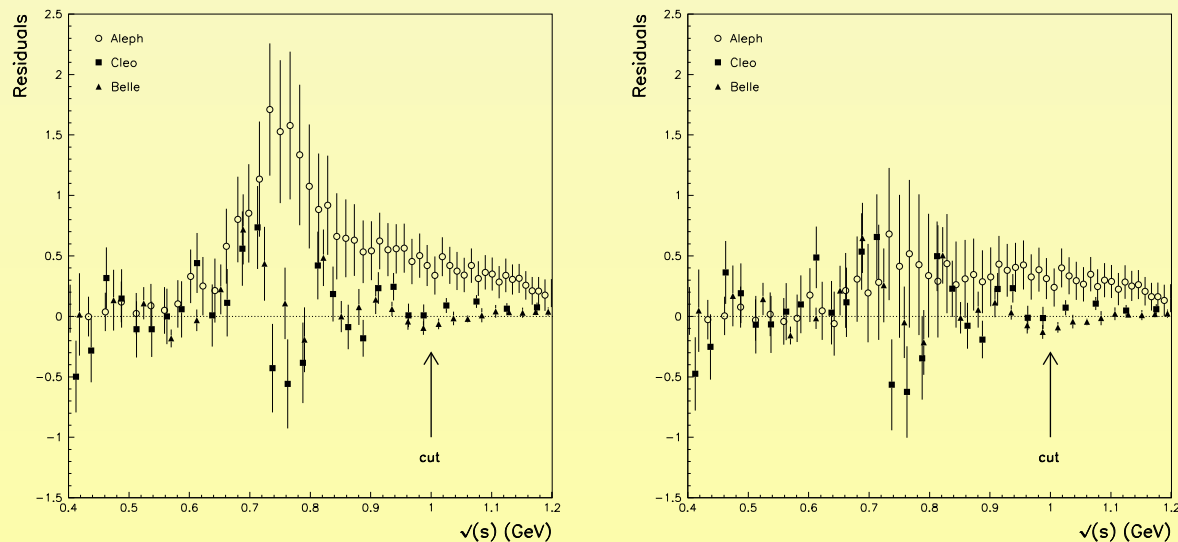
One of the key ingredients in this approach is the strong interaction phase shift $\delta_1^1(s)$ of $\pi\pi$ (re)scattering in the final state. We compare the phase of $F_\pi(s)$ in our model with the one obtained by solving the Roy equation with $\pi\pi$ -scattering data as input. We notice that the agreement is surprisingly good up to about 1 GeV. It is not difficult to replace our phase by the more precise exact one.



The HLS model calculation of $F_\pi(e)$ and $F_\pi(\tau)$

Benayoun et al 09

includes $\rho - \gamma$ mixing as well. The Figure shows τ data vs. residual distribution in the fit of τ data: Left: BELLE+CLEO, Right: ALEPH+BELLE+CLEO (from Benayoun et al 09))



The model yields good simultaneous fits e^+e^- and τ -data.

Our results in Tables:

Isovector ($I=1$) contribution to $a_{\mu}^{\text{had}} \times 10^{10}$ from the range [0.592 - 0.975] GeV from selected experiments. First entry: results from τ -data after standard isospin breaking (IB) corrections. Second entry: results from τ -data after applying in addition the $\rho - \gamma$ mixing corrections $r_{\rho\gamma}(s)$, with fitted values for M_{ρ} , Γ_{ρ} and $\Gamma_{\rho ee}$ [$M_{\rho} = 775.65$ MeV, $\Gamma_{\rho} = 149.99$ MeV, $\mathcal{B}(\rho \rightarrow ee)/(\rho \rightarrow \pi\pi) = 4.10 \times 10^{-5}$]. For the $\rho - \omega$ mixing we subtracted 2.67×10^{-10} . Errors are statistical, systematic, isospin breaking and $\rho - \gamma$ mixing, assuming a 10% uncertainty for the latter. Final state radiation is not included.

Data	standard IB corrections	incl. $\rho - \gamma$ mixing
ALEPH 1997	390.75(2.69)(1.97)(1.45)	385.63(2.65)(1.94)(1.43)(0.50)
ALEPH 2005	388.74(4.05)(2.10)(1.45)	383.54(4.00)(2.07)(1.43)(0.50)
OPAL 1999	380.25(7.36)(5.13)(1.45)	375.39(7.27)(5.06)(1.43)(0.50)
CLEO 2000	391.59(4.16)(6.81)(1.45)	386.61(4.11)(6.72)(1.43)(0.50)
BELLE 2008	394.67(0.53)(3.66)(1.45)	389.62(0.53)(3.66)(1.43)(0.50)
average	391.06(1.42)(1.47)(1.45)	385.96(1.40)(1.45)(1.43)(0.50)
CMD-2 2006		386.34(2.26)(2.65)
SND 2006		383.99(1.40)(4.99)
KLOE 2008		380.24(0.34)(3.27)
KLOE 2010		377.35(0.71)(3.50)
BABAR 2009		389.35(0.37)(2.00)
average		385.12(0.87)(2.18)
all e^+e^- data		385.21(0.18)(1.54)
$e^+e^- + \tau$		385.42 (0.53)(1.21)

Calculated branching fractions in % from selected experiments. Experimental data completed down to threshold and up to m_τ by corresponding world averages where necessary. The experimental world average of direct branching fractions is

$$B_{\pi\pi^0}^{\text{CVC}} = 25.51 \pm 0.09 \% .$$

τ data	$B_{\pi\pi^0}[\%]$	e^+e^- data	$B_{\pi\pi^0}^{\text{CVC}}[\%]$
ALEPH 97	$25.27 \pm 0.17 \pm 0.13$	CMD-2 06	$25.40 \pm 0.21 \pm 0.28$
ALEPH 05	$25.40 \pm 0.10 \pm 0.09$	SND 06	$25.09 \pm 0.30 \pm 0.28$
OPAL 99	$25.17 \pm 0.17 \pm 0.29$	KLOE 08	$24.82 \pm 0.29 \pm 0.28$
CLEO 00	$25.28 \pm 0.12 \pm 0.42$	KLOE 10	$24.65 \pm 0.29 \pm 0.28$
Belle 08	$25.40 \pm 0.01 \pm 0.39$	BaBar 09	$25.45 \pm 0.18 \pm 0.28$
combined	$25.34 \pm 0.06 \pm 0.08$	combined	$25.20 \pm 0.17 \pm 0.28$

For the direct τ branching fractions the first error is statistical the second systematic. For e^+e^- +CVC the first error is experimental the second error includes uncertainties of the IB correction +0.06 from the new mixing effect. Remaining problems seem to be experimental.

$\rho - \omega$ mixing

see our paper e-Print: [arXiv:1101.2872](https://arxiv.org/abs/1101.2872) for an first attempt in the field theory approach. A complete treatment requires an extension of the model to include the $\omega \rightarrow \pi\pi\pi^0$ and $\omega \rightarrow \pi^0\gamma$ channel. (see [from Benayoun et al 09](#))