$\rho - \gamma$ mixing, the e^+e^- vs τ puzzle and the muon $g-2$

Fred Jegerlehner
HU Berlin/DESY Zeuthen,
fjeger@physik.hu-berlin.de

Theorie Seminar, DESY Hamburg, Mai 2, 2011

Abstract

The energy dependence of the $\rho-\gamma$ mixing in the 2×2 $\gamma-\rho$ propagator matrix, is shown to be able to account for the e^+e^- vs. τ spectral function discrepancy. Consequences for the muon $g-2$ are discussed.

Work in collaboration with Robert Szafron [e-Print: arXiv:1101.2872]

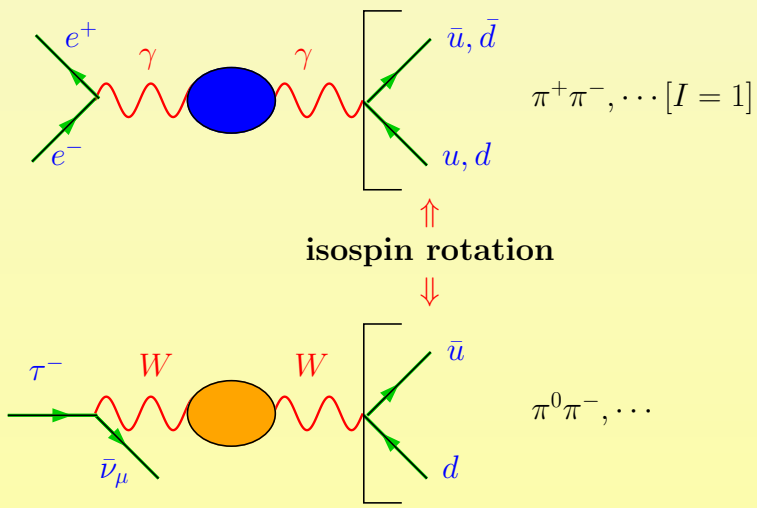
Outline of Talk:

- The τ vs. e^+e^- problem (as known)
- A minimal model: VMD + sQED
- $F_\pi(s)$ with $\rho - \gamma$ mixing at one-loop
- Applications: a_μ and $B^{\rm CVC}_{\pi\pi^0} = \Gamma(\tau \to \nu_\tau \pi\pi^0)/\Gamma_\tau$
- Summary and Conclusions
- Future

❑ The τ vs. e^+e^- problem

Concerns: calculation of hadronic vacuum polarization from appropriate hadron production data.

① A good idea: enhance e^+e^-–data by isospin rotated/corrected τ–data + CVC



ALEPH–Coll., (OPAL, CLEO), Alemany, Davier, Höcker 1996,
Belle–Coll. Fujikawa, Hayashii, Eidelman 2008

$$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+e^- \to X^0$$

where X^- and X^0 are hadronic states related by isospin rotation. The e^+e^- cross–section is then given by

$$\sigma^{I=1}_{e^+e^-\to X^0} = \frac{4\pi\alpha^2}{s}\frac{\beta_0^3(s)}{\beta_-^3(s)} v_{1,X^-} \quad , \quad \sqrt{s} \le M_\tau$$

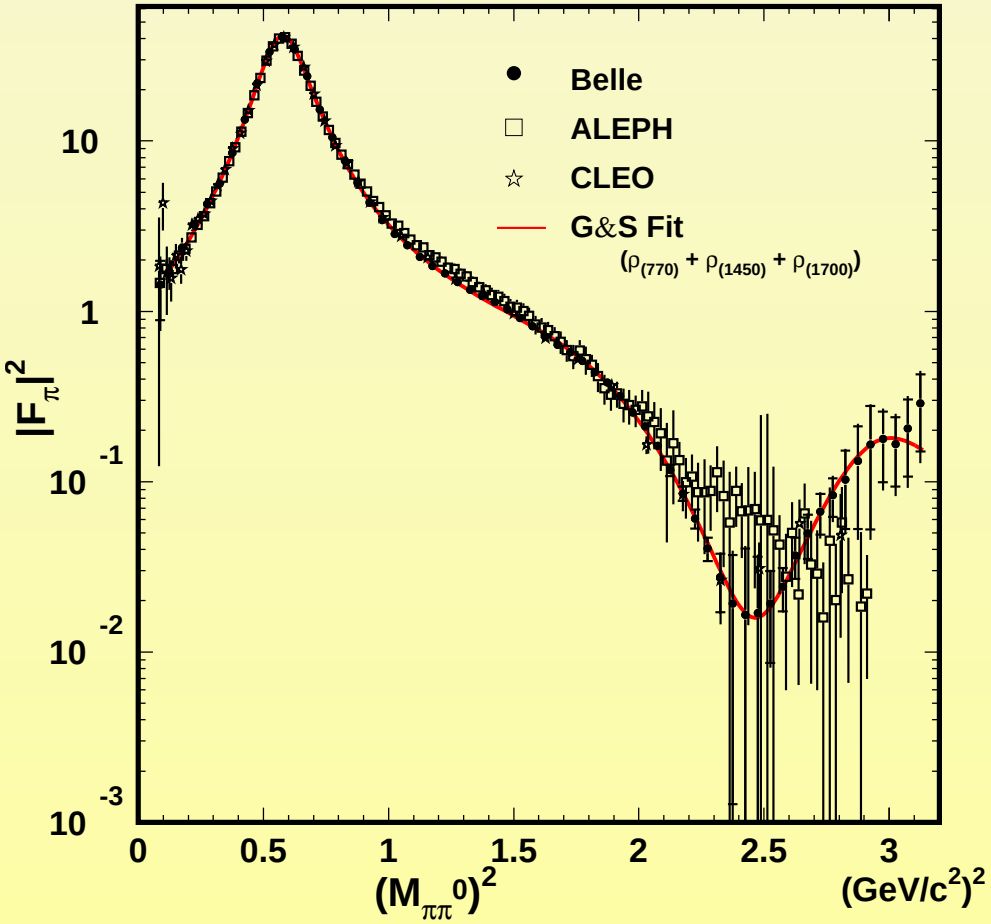
in terms of the τ spectral function v_1.

- mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ–resonance contribution)
- which is dominating in $a_\mu^{\rm had}$ (72%)

$I = 1 \sim 75\%$; $I = 0 \sim 25\%$ τ–data cannot replace e^+e^-–data

$$\begin{array}{lcccl} \delta a_\mu & : & 15.6\times 10^{-10} & \to & 10.2\times 10^{-10} \\ \delta\Delta\alpha & : & 0.00067 & \to & 0.00065 \quad (ADH1997) \end{array}$$

Data: ALEPH 97, ALEPH 05, OPAL, CLEO and
most recent measurement from *Belle* (2008):

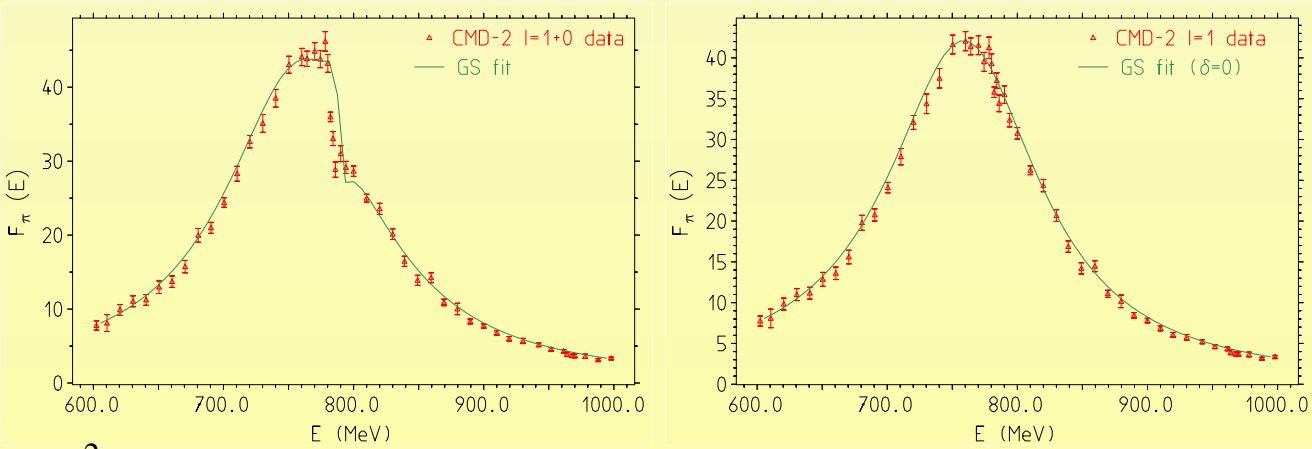


e^+e^-–data*= data corrected for isospin violations: In e^+e^- (neutral channel) $\rho-\omega$ mixing due isospin violation be quark mass difference $m_u \neq m_d$ $\Rightarrow$ I=0 component; to be subtracted for comparison with τ data

❒ Use Gounaris-Sakurai ansatz

$$F_\pi(s) = \frac{\mathrm{BW}^{\mathrm{GS}}_{\rho(770)}(s)\cdot\left(1+\delta\frac{s}{M_\omega^2}\mathrm{BW}_\omega(s)\right)+\beta\,\mathrm{BW}^{\mathrm{GS}}_{\rho(1450)}(s)+\gamma\,\mathrm{BW}^{\mathrm{GS}}_{\rho(1700)}(s)}{1+\beta+\gamma},$$

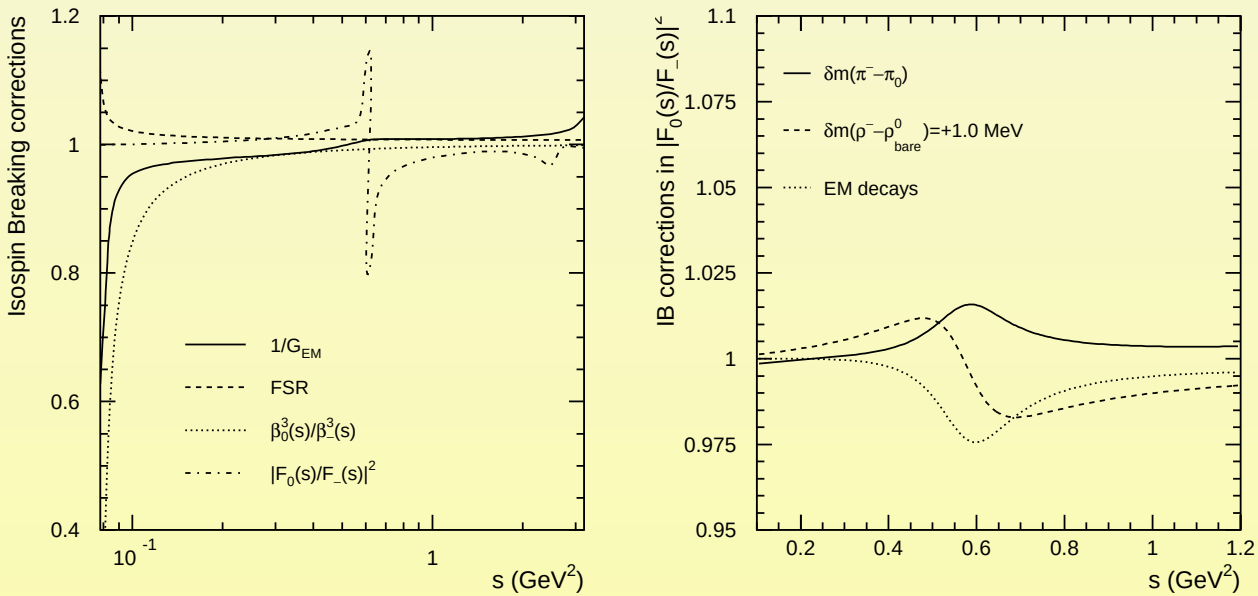
❒ Fit e^+e^--data for $|F_\pi(s)|^2$ ➠ $\delta_{\rho\omega}$ (complex) and set $\delta=0$ to obtain $|F_\pi^{I=1}(s)|^2$



CMD-2 data for $|F_\pi|^2$ in $\rho-\omega$ region together with Gounaris-Sakurai fit. Left before subtraction right after subtraction of the ω.

I=0 component to be added to τ data for calculating a_μ^{had} !

Other isospin-breaking corrections Cirigliano et al. 2002, López Castro el al. 2007

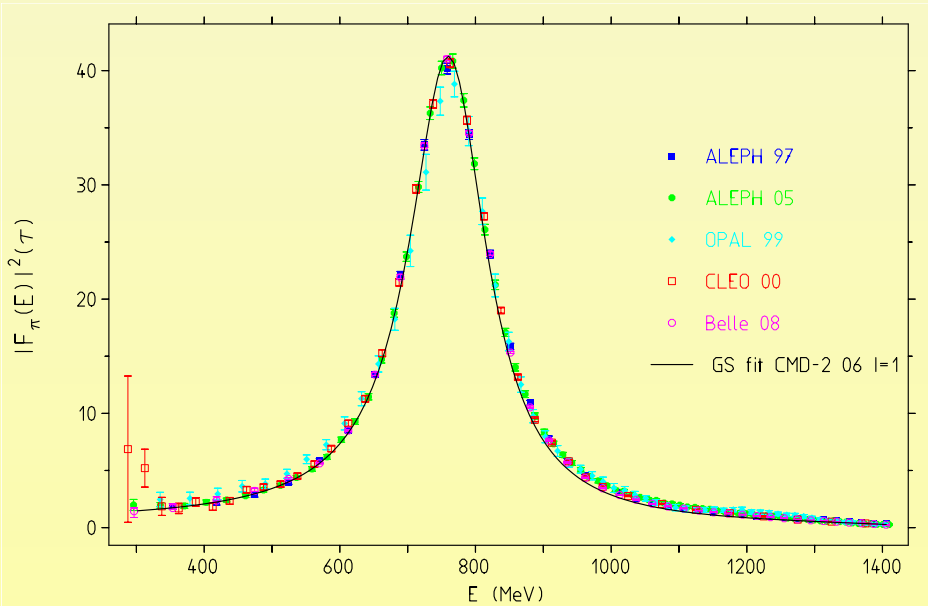


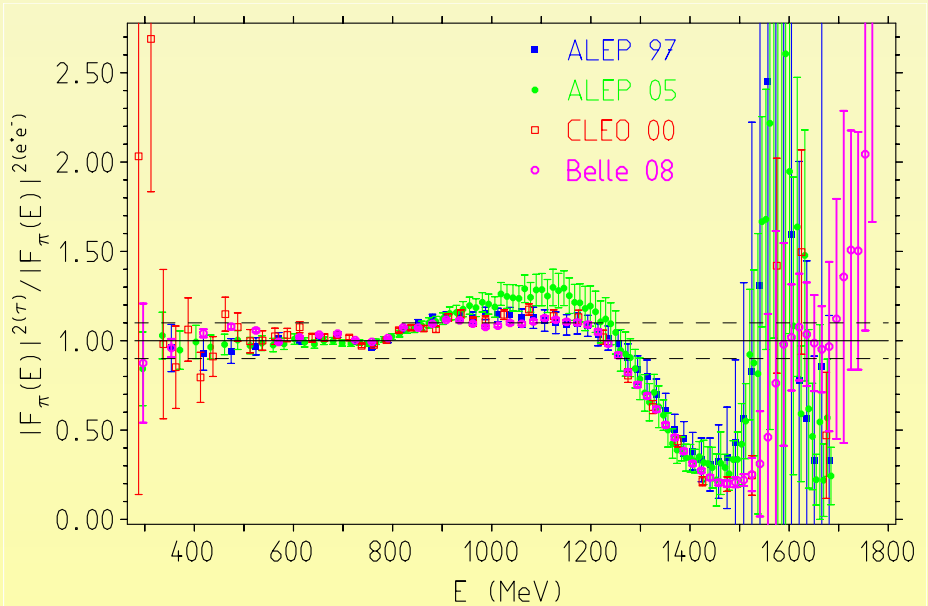
Left: Isospin-breaking corrections $G_{\rm EM}$, FSR, $\beta_0^3(s)/\beta_-^3(s)$ and $|F_0(s)/F_-(s)|^2$.

Right: Isospin-breaking corrections in $I=1$ part of ratio $|F_0(s)/F_-(s)|^2$:

- π mass splitting $\delta m_\pi = m_{\pi^\pm} - m_{\pi^0}$,
- ρ mass splitting $\delta m_\rho = m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$, and
- ρ width splitting $\delta\Gamma_\rho = \Gamma_{\rho^\pm} - \Gamma_{\rho^0}$.

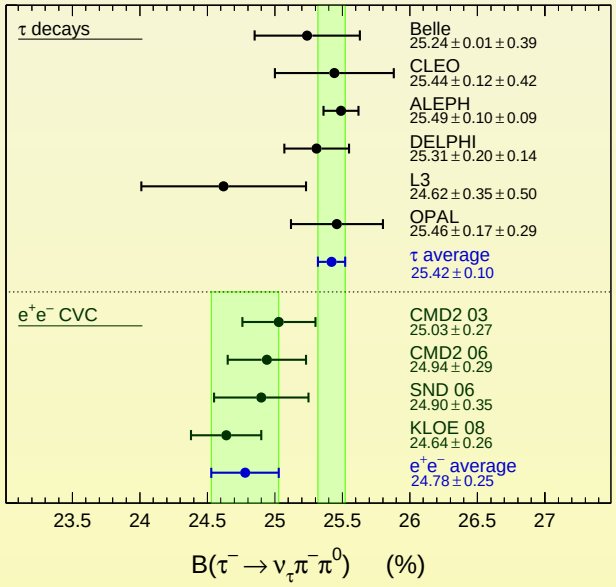
New isospin corrections applied shift in mass and width [as advocated by S. Ghozzi and FJ in 2003!!!] plus changes [López Castro, Toledo Sánchez et al 2007] below the ρ which Davier et al say are not understood! The discrepancy now substantially reduced but with the KLOE data persists. New BaBar radiative return $\pi\pi$ spectrum in much better agreement, in particular with *Belle* τ spectrum!





e^+e^- vs τ spectral functions: $|F_{ee}|^2/|F_\tau|^2 - 1$ as a function of s. Isospin-breaking (IB) corrections are applied to τ data with its uncertainties included in the error band.

CVC prediction of
$\mathcal{B}_{\pi\pi^0}$
normalization of
BELLE, CLEO and OPAL
not fixed
by the experiment itself



The measured branching fractions for $\tau^- \to \pi^-\pi^0\nu_\tau$ compared to the predictions from the $e^+e^- \to \pi^+\pi^-$ spectral functions (after isospin-breaking corrections). (Named e^+e^- results for $0.63 - 0.958$GeV). The long and short vertical error bands correspond to the τ and e^+e^- averages of 25.42 ± 0.10 and 24.76 ± 0.25, respectively.

Possible origin of problems:

- Radiative corrections involving hadrons fully under control?
- IB in parameter shifts: $m_{\rho^+} - m_{\rho^0}$, $\Gamma_{\rho^+} - \Gamma_{\rho^0}$ fully known?

Key problem: on basis of commonly used Gounaris-Sakurai type parametrizations

e^+e^- vs. τ fit with same formula $\Rightarrow$ differ in parameters only: NC vs. CC process
δM_ρ, $\delta\Gamma_\rho$, mixing coefficients etc.

Other possible source: do we really understand quantum interference?

- e^+e^-: $|F_\pi^{(e)}(s)|^2 = |F_\pi^{(e)}(s)[I=1] + F_\pi^{(e)}(s)[I=0]|^2$ what we need and measure
- τ: $|F_\pi^{(\tau)}(s)[I=1]|^2$ measured in τ-decay
- $ee + \tau$: $|F_\pi^{(e)}(s)|^2 \simeq |F_\pi^{(e,\tau)}(s)[I=1]|^2 + |F_\pi^{(e)}(s)[I=0]|^2$??? usual approximation

Need $\boxed{\text{theory}}$ $\rightarrow$ specific model for the complex amplitudes

❑ A minimal model: VMD + sQED

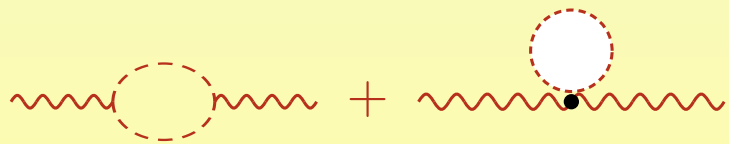
Effective Lagrangian $\mathcal{L} = \mathcal{L}_{\gamma\rho} + \mathcal{L}_{\pi}$

$$\mathcal{L}_{\pi} = D_\mu \pi^+ D^{+\mu} \pi^- - m_\pi^2 \pi^+ \pi^- \; ; \quad D_\mu = \partial_\mu - \mathrm{i}\, e\, A_\mu - \mathrm{i}\, g_{\rho\pi\pi}\, \rho_\mu$$

$$\mathcal{L}_{\gamma\rho} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{M_\rho^2}{2} \rho_\mu \rho^\mu + \frac{e}{2\, g_\rho} \rho_{\mu\nu} F^{\mu\nu}$$

Self-energies: pion loops to photon-rho vacuum polarizations

$$-\mathrm{i}\, \Pi_{\gamma\gamma}^{\mu\nu\,(\pi)}(q) =$$



bare $\gamma - \rho$ transverse self-energy functions

$$\Pi_{\gamma\gamma} = \frac{e^2}{48\pi^2} f(q^2)\,, \quad \Pi_{\gamma\rho} = \frac{e g_{\rho\pi\pi}}{48\pi^2} f(q^2) \quad \text{and} \quad \Pi_{\rho\rho} = \frac{g_{\rho\pi\pi}^2}{48\pi^2} f(q^2)\,,$$

Propagators = inverse of symmetric 2×2 self-energy matrix

$$\hat{D}^{-1} = \begin{pmatrix} q^2 + \Pi_{\gamma\gamma}(q^2) & \Pi_{\gamma\rho}(q^2) \\ \Pi_{\gamma\rho}(q^2) & q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) \end{pmatrix}$$

inverted $\Rightarrow$

$$\begin{aligned} D_{\gamma\gamma} &= \frac{1}{q^2 + \Pi_{\gamma\gamma}(q^2) - \frac{\Pi_{\gamma\rho}^2(q^2)}{q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)}} \\ D_{\gamma\rho} &= \frac{-\Pi_{\gamma\rho}(q^2)}{(q^2 + \Pi_{\gamma\gamma}(q^2))(q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)) - \Pi_{\gamma\rho}^2(q^2)} \\ D_{\rho\rho} &= \frac{1}{q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) - \frac{\Pi_{\gamma\rho}^2(q^2)}{q^2 + \Pi_{\gamma\gamma}(q^2)}} \, . \end{aligned}$$

Resonance parameters $\Leftrightarrow$ location s_P of the pole of the propagator

$$s_P - m^2_{\rho^0} + \Pi_{\rho^0\rho^0}(s_P) - \frac{\Pi^2_{\gamma\rho^0}(s_P)}{s_P + \Pi_{\gamma\gamma}(s_P)} = 0 \ ,$$

with $s_P = \tilde{M}^2_{\rho^0}$ complex.

$$\tilde{M}^2_\rho \equiv \left(q^2\right)_{\text{pole}} = M^2_\rho - \mathrm{i}\, M_\rho\, \Gamma_\rho$$

Diagonalization $\Rightarrow$ physical ρ acquires a direct coupling to the electron

$$\begin{aligned} \mathcal{L}_{\text{QED}} &= \bar{\psi}_e \gamma^\mu (\partial_\mu - \mathrm{i}\, e_b\, A_{b\mu})\, \psi_e \\ &\Downarrow \\ \mathcal{L}_{\text{QED}} &= \bar{\psi}_e \gamma^\mu (\partial_\mu - \mathrm{i}\, e\, A_\mu + \mathrm{i}\, g_{\rho ee} \rho_\mu)\, \psi_e \end{aligned}$$

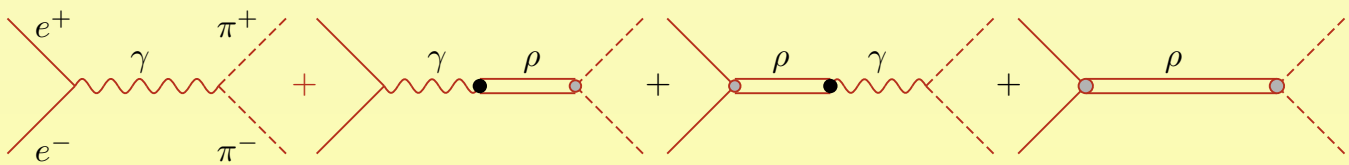
with $g_{\rho ee} = e\,(\Delta_\rho + \Delta_0)$, where in our case $\Delta_0 = 0$.

❑ $F_\pi(s)$ with $\rho - \gamma$ mixing at one-loop

The $e^+e^- \to \pi^+\pi^-$ matrix element in sQED is given by

$$\mathcal{M} = -\mathrm{i}\, e^2\, \bar{v}\gamma^\mu u\, (p_1 - p_2)_\mu\, F_\pi(q^2)$$

with $F_\pi(q^2) = 1$. In our extended VMD model we have the four terms



Diagrams contributing to the process $e^+e^- \to \pi^+\pi^-$.

$$F_\pi(s) \propto e^2\, D_{\gamma\gamma} + e g_{\rho\pi\pi}\, D_{\gamma\rho} - g_{\rho ee} e D_{\rho\gamma} - g_{\rho ee} g_{\rho\pi\pi}\, D_{\rho\rho}\,,$$

Properly normalized (VP subtraction: $e^2(s) \to e^2$):

$$F_\pi(s) = \left[e^2 D_{\gamma\gamma} + e\,(g_{\rho\pi\pi} - g_{\rho ee})\, D_{\gamma\rho} - g_{\rho ee} g_{\rho\pi\pi} D_{\rho\rho}\right] / \left[e^2 D_{\gamma\gamma}\right]$$

Typical couplings

$$g_{\rho\pi\pi\,\mathrm{bare}} = 5.8935,\ g_{\rho\pi\pi\,\mathrm{ren}} = 6.1559,\ g_{\rho ee} = 0.018149,\ x = g_{\rho\pi\pi}/g_\rho = 1.15128.$$

We note that the precise s-dependence of the effective ρ-width is obtained by evaluating the imaginary part of the ρ self-energy:

$$\mathrm{Im}\,\Pi_{\rho\rho} = \frac{g^2_{\rho\pi\pi}}{48\,\pi}\beta^3_\pi\, s \equiv M_\rho\,\Gamma_\rho(s)\ ,$$

which yields

$$\Gamma_\rho(s)/M_\rho = \frac{g^2_{\rho\pi\pi}}{48\,\pi}\beta^3_\pi\,\frac{s}{M^2_\rho}\ ;\quad \Gamma_\rho/M_\rho = \frac{g^2_{\rho\pi\pi}}{48\,\pi}\beta^3_\rho\ ;\quad g_{\rho\pi\pi} = \sqrt{48\,\pi\,\Gamma_\rho/(\beta^3_\rho\, M_\rho)}\ .$$

In our model, in the given approximation, the on ρ-mass-shell form factor reads

$$F_\pi(M_\rho^2) = 1 - \mathrm{i}\,\frac{g_{\rho ee} g_{\rho\pi\pi}}{e^2}\,\frac{M_\rho}{\Gamma_\rho}\ ;\ \ |F_\pi(M_\rho^2)|^2 = 1 + \frac{36}{\alpha^2}\frac{\Gamma_{ee}}{\beta_\rho^3\,\Gamma_\rho}\ ,$$

$$\Gamma_{\rho ee} = \frac{1}{3}\,\frac{g_{\rho ee}^2}{4\pi}\,M_\rho\ ;\ \ g_{\rho ee} = \sqrt{12\pi\,\Gamma_{\rho ee}/M_\rho}\ .$$

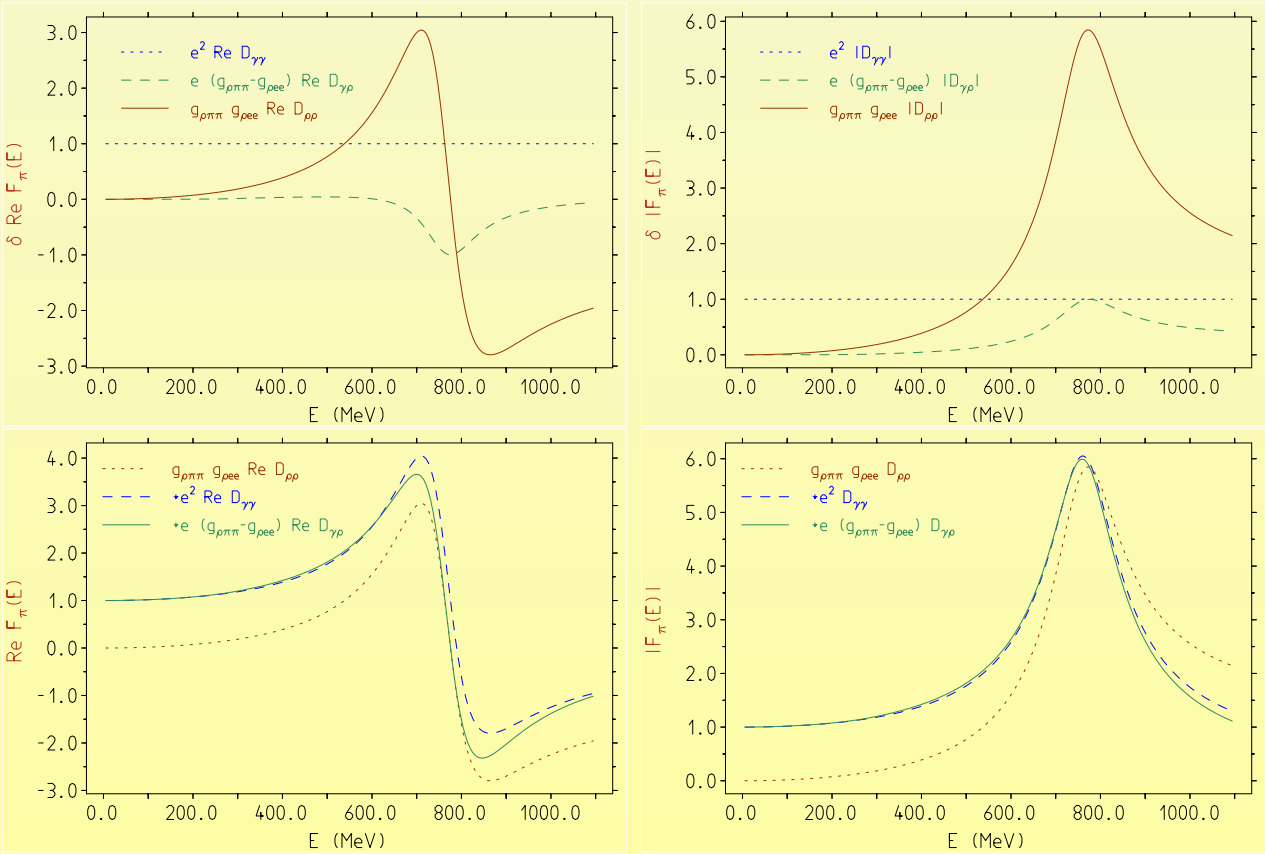
Compare: Gounaris-Sakurai (GS) formula

$$F_\pi^{\mathrm{GS}}(s) = \frac{-M_\rho^2 + \Pi_{\rho\rho}^{\mathrm{ren}}(0)}{s - M_\rho^2 + \Pi_{\rho\rho}^{\mathrm{ren}}(s)}\ ;\ \ \Gamma_{\rho ee}^{\mathrm{GS}} = \frac{2\alpha^2\,\beta_\rho^3 M_\rho^2}{9\,\Gamma_\rho}\,\left(1 + d\,\Gamma_\rho/M_\rho\right)^2\ .$$

GS does not involve $g_{\rho ee}$ resp. $\Gamma_{\rho ee}$ in a direct way, as normalization is fixed by applying an overall factor $1 + d\,\Gamma_\rho/M_\rho \equiv 1 - \Pi_{\rho\rho}^{\mathrm{ren}}(0)/M_\rho^2 \simeq 1.089$ to enforce $F_\pi(0) = 1$ (in our approach "automatic" by gauge invariance).

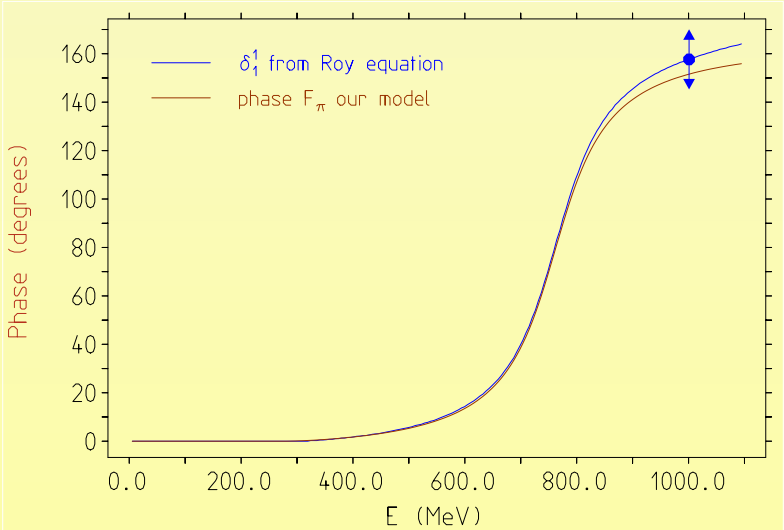
The interference of terms in $F_\pi^{(e)}$

Real parts and moduli of the 3 individual and added terms normalized to the sQED term are displayed:

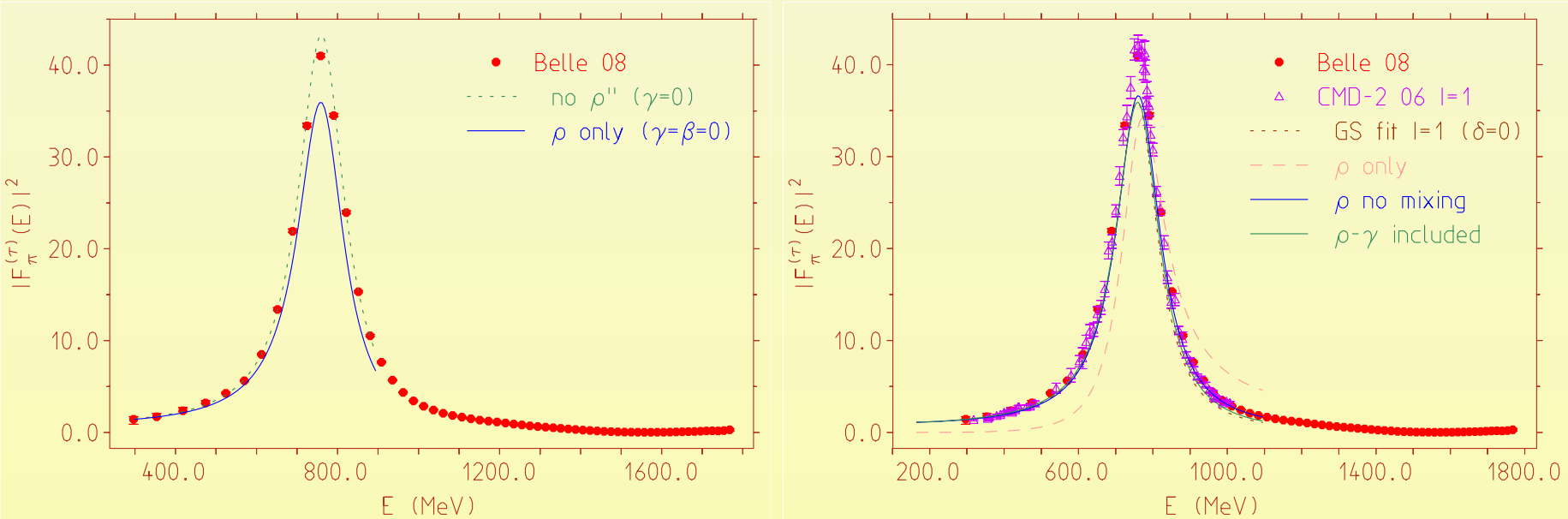


Comparison of $\pi\pi$ rescattering with Colangelo-Leutwyler's first principles approach

One of the key ingredients in this approach is the strong interaction phase shift $\delta_1^1(s)$ of $\pi\pi$ (re)scattering in the final state. We compare the phase of $F_\pi(s)$ in our model with the one obtained by solving the Roy equation with $\pi\pi$-scattering data as input. We notice that the agreement is surprisingly good up to about 1 GeV. It is not difficult to replace our phase by the more precise exact one.



Relation to data:

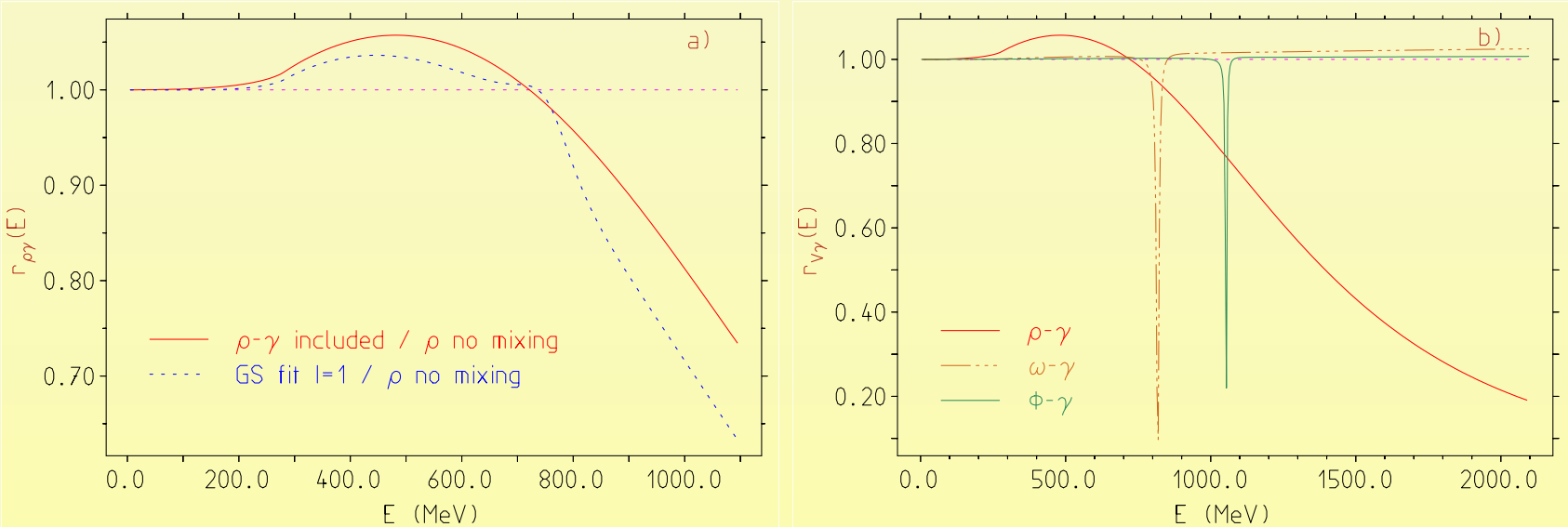


Left: GS fits of the Belle data and the effects of including higher states ρ' and ρ'' at fixed M_ρ and Γ_ρ. Right: Effect of $\gamma - \rho$ mixing in our simple EFT model

Parameters: $M_\rho = 775.5$ MeV, $\Gamma_\rho = 143.85$ MeV,
$\mathcal{B}[(\rho \to ee)/(\rho \to \pi\pi)] = 4.67 \times 10^{-5}$, $e = 0.302822$, $g_{\rho\pi\pi} = 5.92$, $g_{\rho ee} = 0.01826$.

Detailed comparison, in terms of the ratio:

$$r_{\rho\gamma}(s) \equiv \frac{|F_\pi(s)|^2}{|F_\pi(s)|^2_{D_{\gamma\rho}=0}}$$



a) Ratio of $|F_\pi(E)|^2$ with mixing vs. no mixing. Same ratio for GS fit with PDG parameters. b) The same mechanism scaled up by the branching fraction $\Gamma_V/\Gamma(V \to \pi\pi)$ for $V = \omega$ and ϕ. In the $\pi\pi$ channel the effects for resonances $V \neq \rho$ are tiny if not very close to resonance.

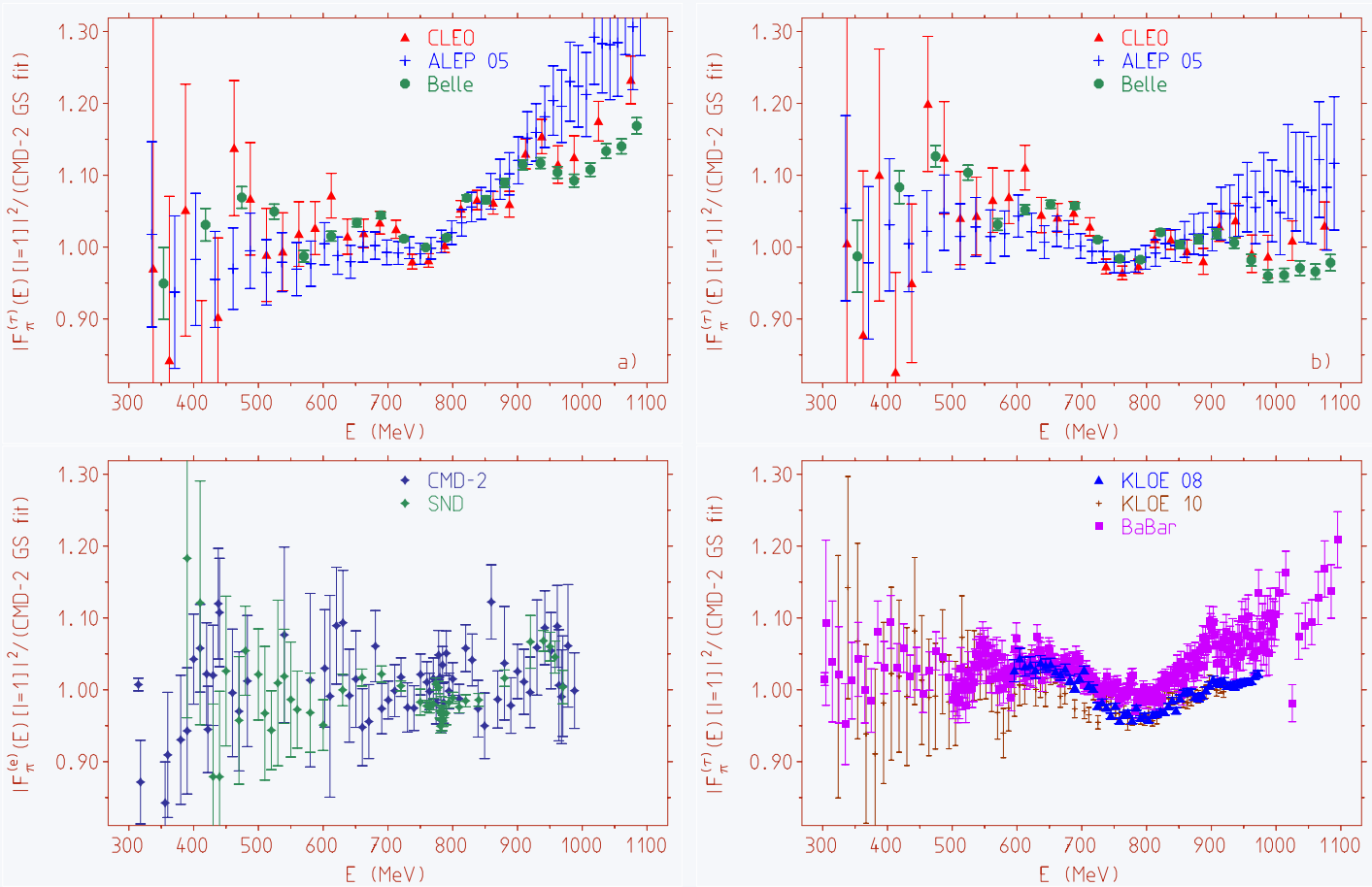
If mixing not included in $F_0(s) \Rightarrow$ total correction formula on spectral functions

$$v_0(s) = r_{\rho\gamma}(s)\, R_{\rm IB}(s)\, v_-(s)$$

$$R_{\rm IB}(s) = \frac{1}{G_{\rm EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

- $G_{\rm EM}(s)$ electromagnetic radiative corrections
- $\beta_0^3(s)/\beta_-^3(s)$ phase space modification by $m_{\pi^0} \neq m_{\pi^\pm}$
- $|F_0(s)/F_-(s)|^2$ incl. shifts in masses, widths etc

Final state radiation correction FSR(s) and vacuum polarization effects $(\alpha/\alpha(s))^2$ and I=0 component ($\rho - \omega$) we have been subtracted from all e^+e^--data.



$|F_\pi(E)|^2$ in units of e^+e^- I=1 (CMD-2 GS fit): a) τ data uncorrected for $\rho-\gamma$ mixing, and b) after correcting for mixing. Lower panel: e^+e^- energy scan data [left] and e^+e^- radiative return data [right]

❑ Applications: a_μ and $B^{\rm CVC}_{\pi\pi^0} = \Gamma(\tau \to \nu_\tau \pi\pi^0)/\Gamma_\tau$

How does the new correction affect the evaluation of the hadronic contribution to a_μ ? To lowest order in terms of e^+e^--data, represented by $R(s)$, we have

$$a_\mu^{\rm had,LO}(\pi\pi) = \frac{\alpha^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} {\rm d}s\, R^{(0)}_{\pi\pi}(s) \frac{K(s)}{s} \ ,$$

with the well-known kernel $K(s)$ and

$$R^{(0)}_{\pi\pi}(s) = (3s\sigma_{\pi\pi})/(4\pi\alpha^2(s)) = 3v_0(s) \ .$$

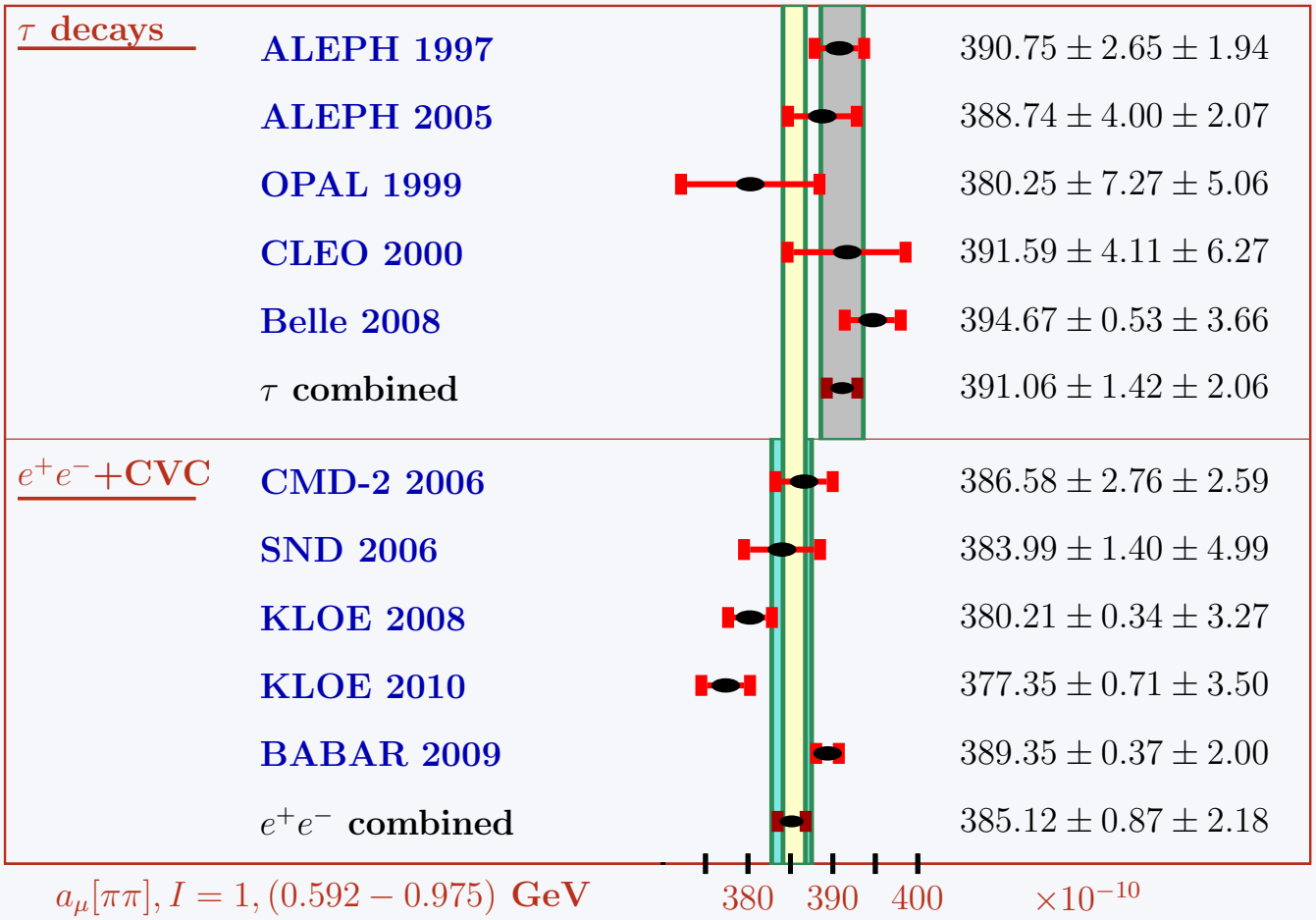
Note that the $\rho - \gamma$ interference is included in the measured e^+e^--data, and so is its contribution to $a_\mu^{\rm had}$. In fact $a_\mu^{\rm had}$ is intrinsic an e^+e^--based "observable" (neutral current channel).

How to utilize τ data: subtract CVC violating corrections

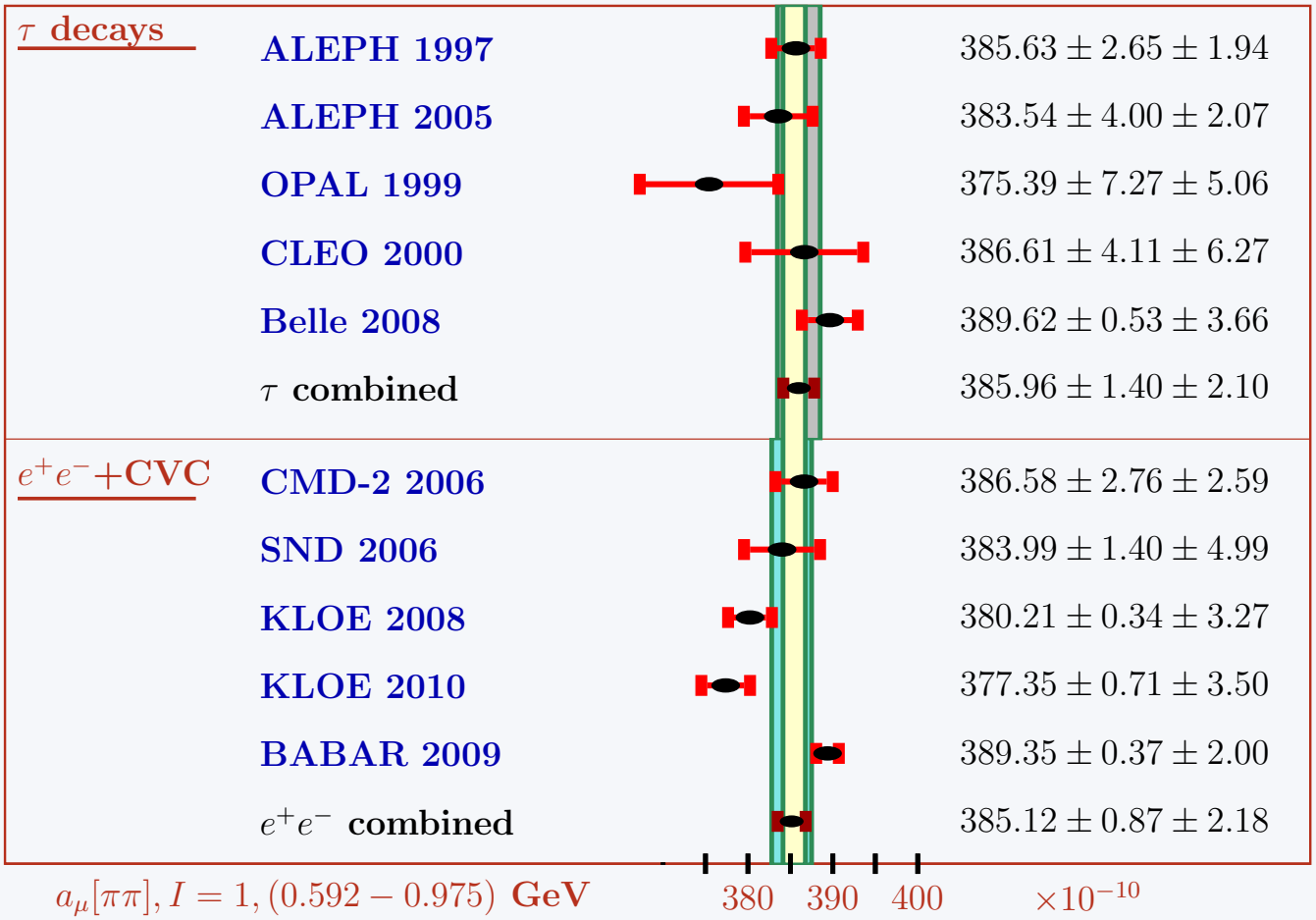
❖traditionally $v_-(s) \to v_0(s) = R_{\rm IB}(s)\, v_-(s)$

❖our correction $v_-(s) \to v_0(s) = r_{\rho\gamma}(s)\; R_{\rm IB}(s)\, v_-(s)$

Result for the I=1 part of $a_\mu^{\rm had}[\pi\pi]$: $\delta a_\mu^{\rm had}[\rho\gamma] \simeq (-5.1 \pm 0.5) \times 10^{-10}$



I=1 part of $a_\mu^{\rm had}[\pi\pi]$

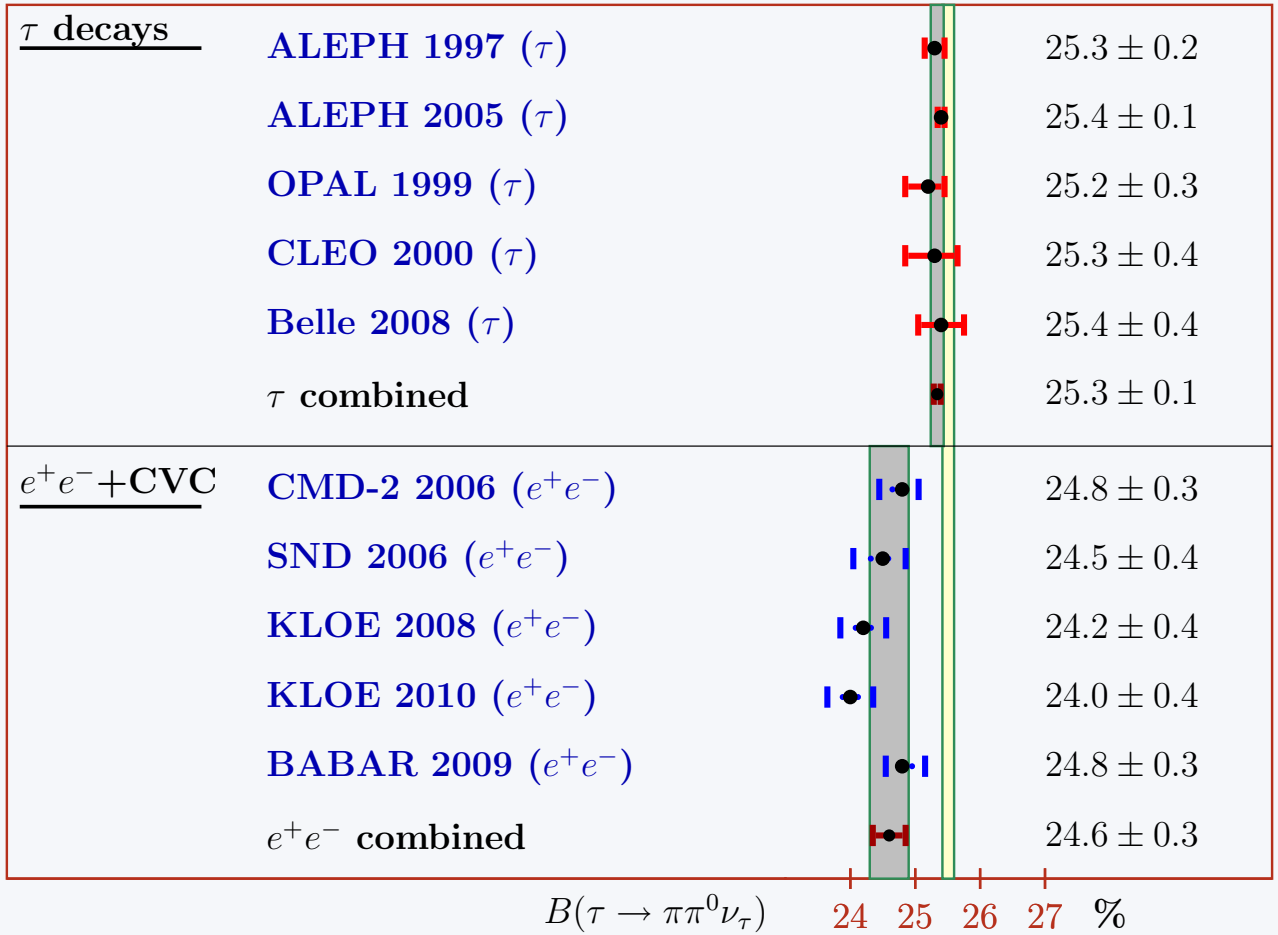


I=1 part of $a_\mu^{\rm had}[\pi\pi]$

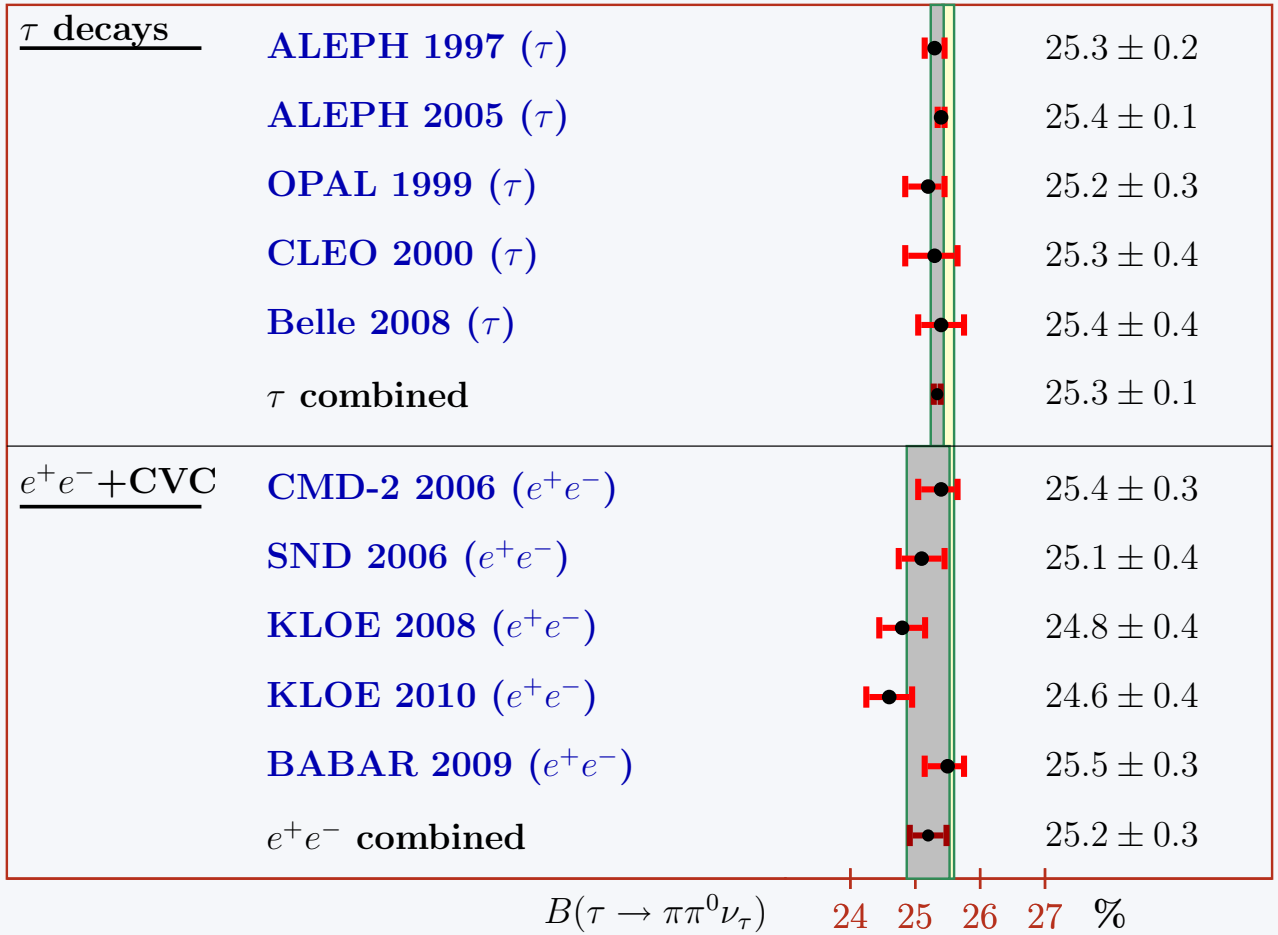
The $\tau \to \pi^0 \pi \nu_\tau$ branching fraction $B_{\pi\pi^0} = \Gamma(\tau \to \nu_\tau \pi\pi^0)/\Gamma_\tau$ is another important quantity which can be directly measured. This "τ-observable" can be evaluated in terms of the I=1 part of the $e^+e^- \to \pi^+\pi^-$ cross section, after taking into account the IB correction $v_0(s) \to v_-(s) = v_0(s)/R_{\rm IB}(s)$ $/r_{\rho\gamma}(s)$,

$$B^{\rm CVC}_{\pi\pi^0} = \frac{2S_{\rm EW} B_e |V_{ud}|^2}{m_\tau^2} \int_{4m_\pi^2}^{m_\tau^2} {\rm d}s\, R^{(0)}_{\pi^+\pi^-}(s) \left(1 - \frac{2}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \frac{1}{r_{\rho\gamma}(s)\, R_{\rm IB}(s)} \,,$$

where here we also have to "undo" the $\rho - \gamma$ mixing which is absent in the charged isovector channel. The shift is $\delta B^{\rm CVC}_{\pi\pi^0}[\rho\gamma] = +0.62 \pm 0.06\ \%$



Branching fractions $B(\tau \to \pi\pi^0\nu_\tau)$

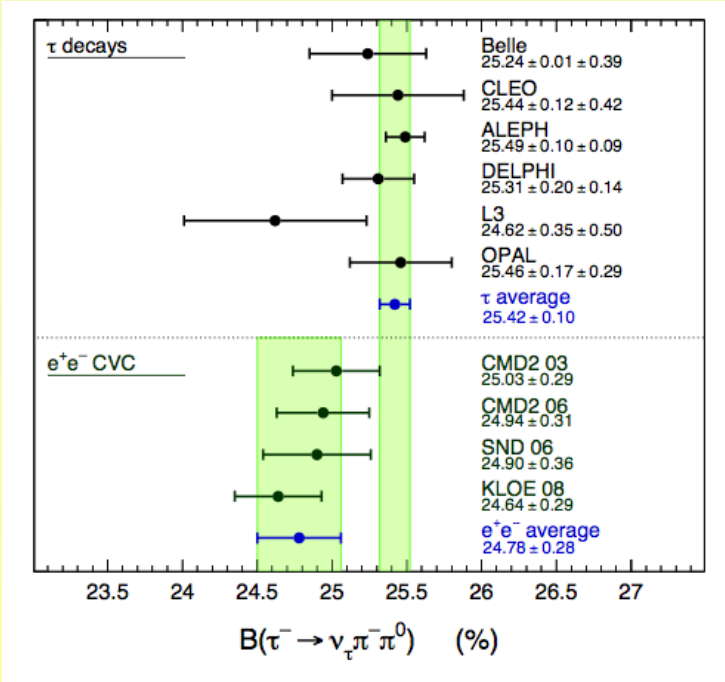


Branching fractions $B(\tau \to \pi\pi^0\nu_\tau)$

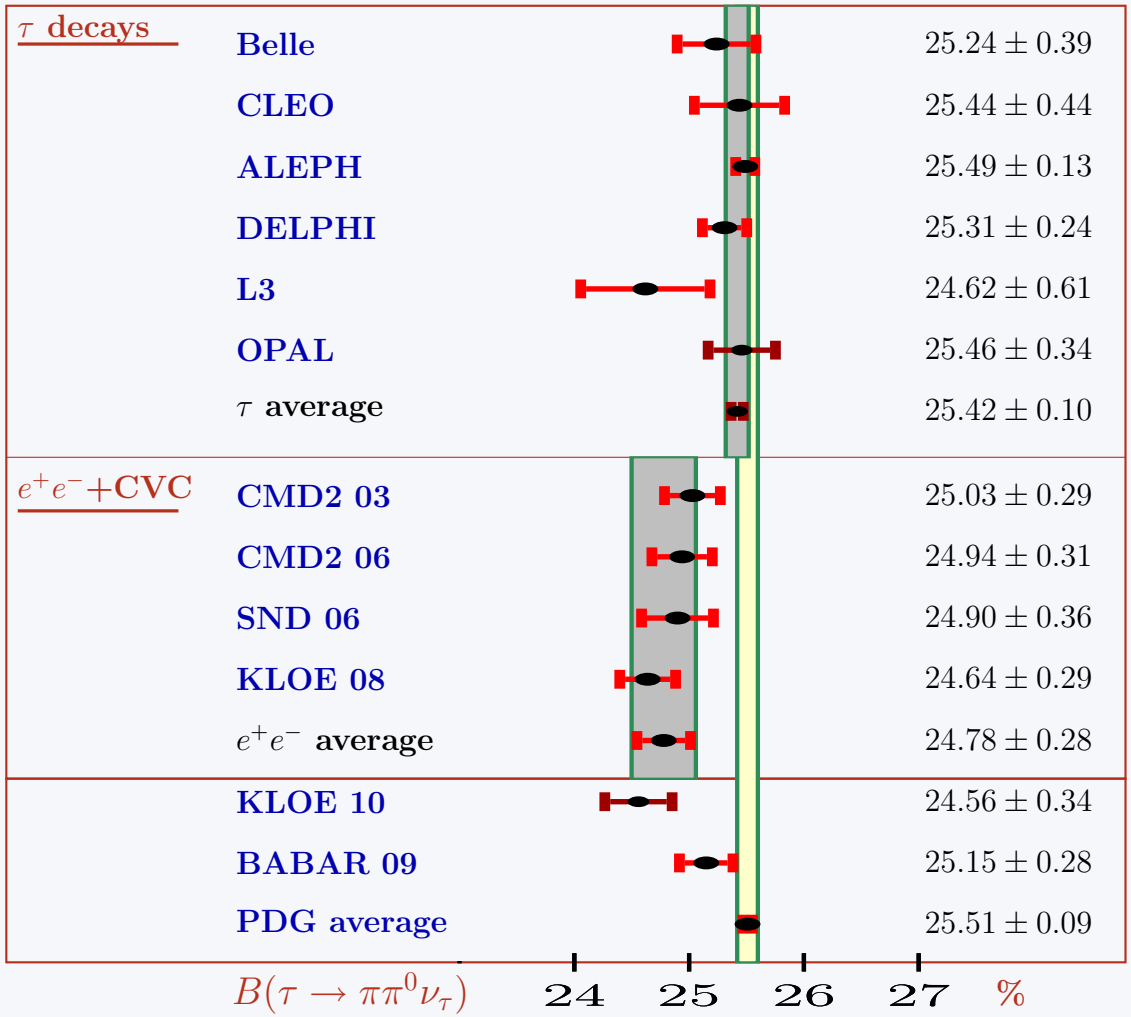
Most recent results of Davier et al:

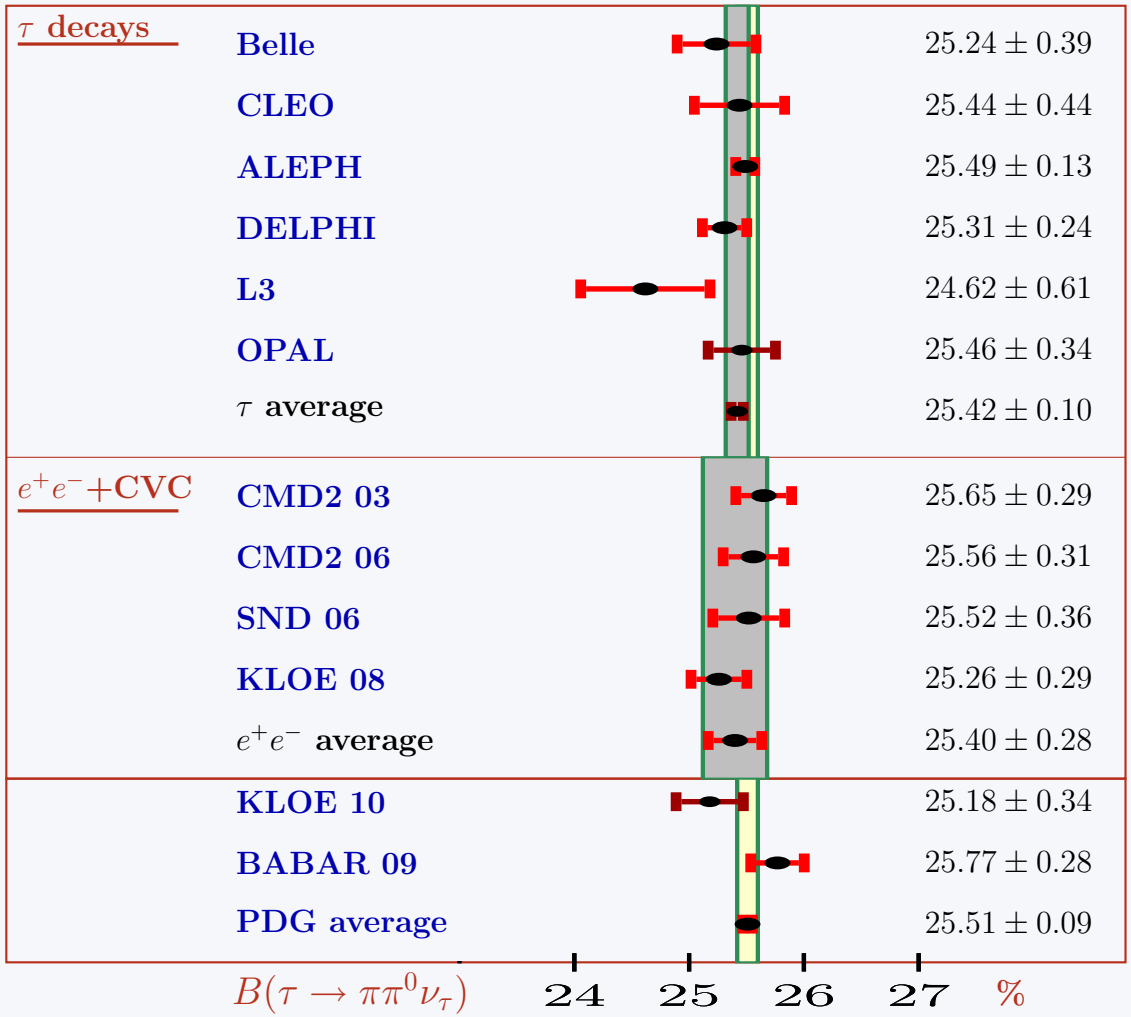
- Pre BaBar: 25.42 ± 0.10 % for τ

  24.78 ± 0.28 % $\overset{+\rho\gamma}{\Rightarrow}$ 25.40 ± 0.28 ± 0.06 % for e^+e^- + CVC
- New BaBar: 25.15 ± 0.28 % $\overset{+\rho\gamma}{\Rightarrow}$ 25.77 ± 0.28 ± 0.06 % for e^+e^- + CVC



shift $\delta B^{\rm CVC}_{\pi\pi^0}[\rho\gamma] = +0.62 \pm 0.06\ \%$


τ decays
Belle
25.24 ± 0.39
CLEO
25.44 ± 0.44
ALEPH
25.49 ± 0.13
DELPHI
25.31 ± 0.24
L3
24.62 ± 0.61
OPAL
25.46 ± 0.34
τ average
25.42 ± 0.10
e+e−+CVC
CMD2 03
25.03 ± 0.29
CMD2 06
24.94 ± 0.31
SND 06
24.90 ± 0.36
KLOE 08
24.64 ± 0.29
e+e− average
24.78 ± 0.28
KLOE 10
24.56 ± 0.34
BABAR 09
25.15 ± 0.28
PDG average
25.51 ± 0.09
B(τ → ππ0ντ)
24
25
26
27
%


τ decays
Belle
25.24 ± 0.39
CLEO
25.44 ± 0.44
ALEPH
25.49 ± 0.13
DELPHI
25.31 ± 0.24
L3
24.62 ± 0.61
OPAL
25.46 ± 0.34
τ average
25.42 ± 0.10
e+e−+CVC
CMD2 03
25.65 ± 0.29
CMD2 06
25.56 ± 0.31
SND 06
25.52 ± 0.36
KLOE 08
25.26 ± 0.29
e+e− average
25.40 ± 0.28
KLOE 10
25.18 ± 0.34
BABAR 09
25.77 ± 0.28
PDG average
25.51 ± 0.09
B(τ → ππ0ντ)
24
25
26
27
%

Summary and Conclusions

➠VMD+sQED EFT understood as the tail of the more appropriate resonance Lagrangian approach (Ecker et al. 1989) in low energy $\pi\pi$ production yields

- proper ρ propagator self-energy effects for GS form factor ($\rho \to \pi\pi$)
- pion-loop effects in $\rho - \gamma$ mixing contributes sizable interferences

Note: so far PDG parameters masses, withs, branching fractions etc. of resonances like ρ^0 all extracted from data assuming GS like form factors (model dependent!)

Pattern:

❒ moderate positive interference (up to +5%) below ρ,
substantial negative interference (-10% and more)
above the ρ (must vanish at $s = 0$ and $s = M_\rho^2$)

- remarkable agreement with pattern of e^+e^- vs τ discrepancy
- shift of the τ data to lie perfectly within the ballpark of the e^+e^- data

Lesson: effective field theory the basic tool (not ad hoc pheno. ansätze)

- $\rho - \gamma$ correction function $r_{\rho\gamma}(s)$ entirely fixed from neutral channel
- τ data provide independent information

What does it mean for the muon $g-2$?

- it looks we have fairly reliable model to include τ data to improve $a_\mu^{\rm had}$
- there is no τ vs. e^+e^- alternative of $a_\mu^{\rm had}$

For the lowest order hadronic vacuum polarization (VP) contribution to a_μ we find

$$a_\mu^{\rm had,LO}[e,\tau] = 690.96(1.06)(4.63)\times 10^{-10} \quad (e+\tau)$$

$$a_\mu^{\rm the} = 116591797(60) \times 10^{-11}$$

$$a_\mu^{\rm exp} = 116592080(54)(33) \times 10^{-11}$$

$$a_\mu^{\rm exp} - a_\mu^{\rm the} = (283 \pm 87) \times 10^{-11}$$

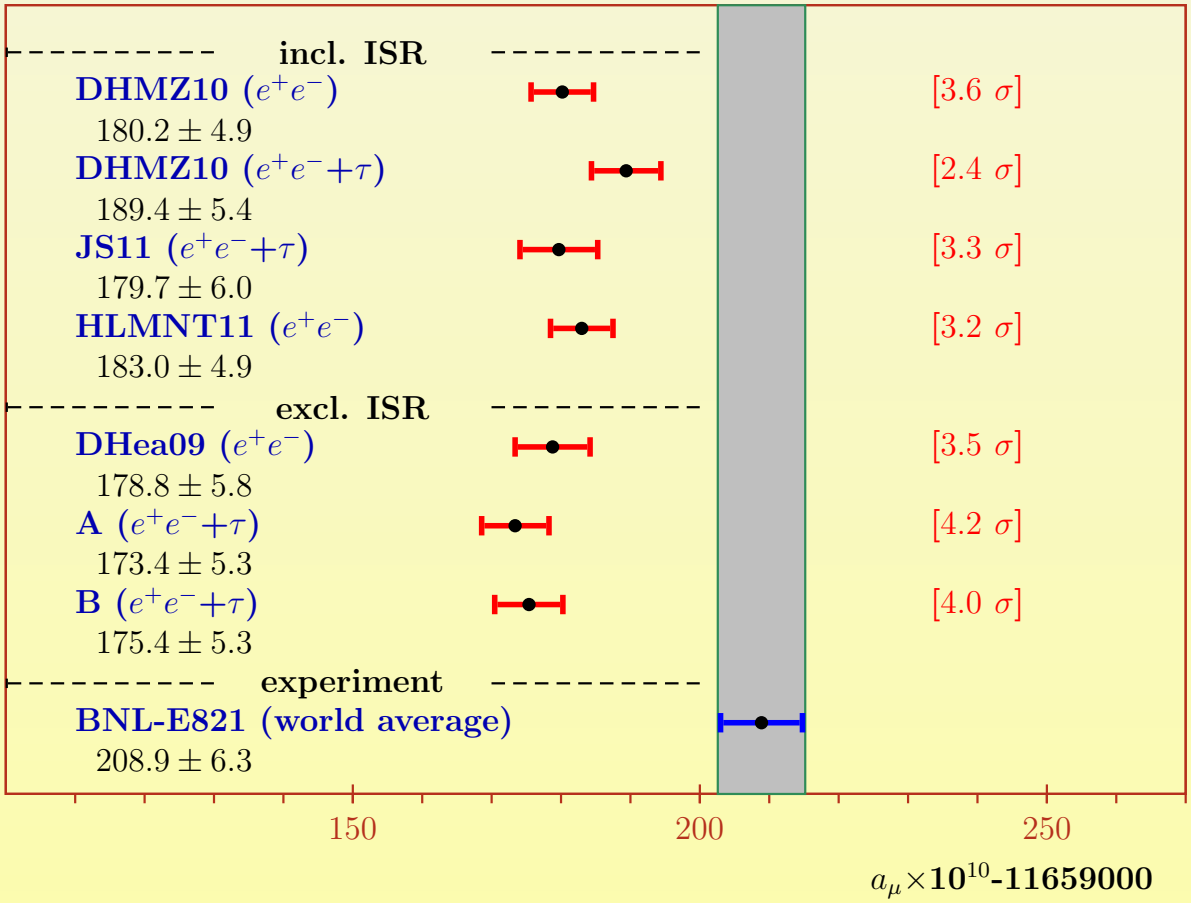
$$3.3\,\sigma$$

Höcker 2010 (theory-driven analysis)

$a_\mu^{\rm had,LO}[e] \quad = (692.3 \pm 1.4 \pm 3.1 \pm 2.4 \pm 0.2 \pm 0.3) \times 10^{-10}$ (e^+e^- based),
$a_\mu^{\rm had,LO}[e,\tau] = (701.5 \pm 3.5 \pm 1.9 \pm 2.4 \pm 0.2 \pm 0.3) \times 10^{-10}$ (e^+e^-+τ based),

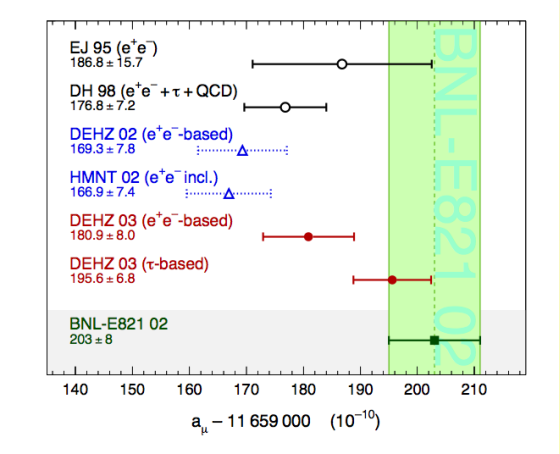
❒ Note: ratio $F_0(s)/F_-(s)$ could be measured within lattice QCD, without reference to sQED or other hadronic models. Do it!

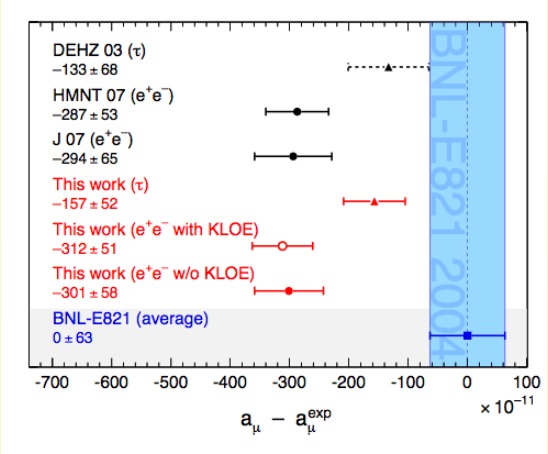
❒ Including $\omega, \phi, \rho', \rho'', \cdots$ requires to go to appropriate Resonance Lagrangian extension (e.g HLS model Benayoun et al.)

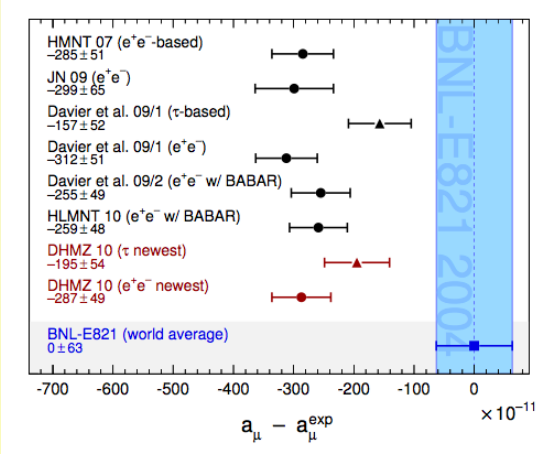


Some recent results: Davier et al., Hagiwara et al., Benayoun et al.

History:


EJ 95 (e+e−)
186.8 ± 15.7
DH 98 (e+e− + τ + QCD)
176.8 ± 7.2
DEHZ 02 (e+e−-based)
169.3 ± 7.8
HMNT 02 (e+e− incl.)
166.9 ± 7.4
DEHZ 03 (e+e−-based)
180.9 ± 8.0
DEHZ 03 (τ-based)
195.6 ± 6.8
BNL-E821 02
203 ± 8
BNL-E821 02
140 150 160 170 180 190 200 210
aμ − 11 659 000 (10−10)


DEHZ 03 (τ)
−133 ± 68
HMNT 07 (e+e−)
−287 ± 53
J 07 (e+e−)
−294 ± 65
This work (τ)
−157 ± 52
This work (e+e− with KLOE)
−312 ± 51
This work (e+e− w/o KLOE)
−301 ± 58
BNL-E821 (average)
0 ± 63
BNL-E821 2004
-700 -600 -500 -400 -300 -200 -100 0 100
× 10−11
aμ − aμexp


HMNT 07 (e+e−-based)
−285 ± 51
JN 09 (e+e−)
−299 ± 65
Davier et al. 09/1 (τ-based)
−157 ± 52
Davier et al. 09/1 (e+e−)
−312 ± 51
Davier et al. 09/2 (e+e− w/ BABAR)
−255 ± 49
HLMNT 10 (e+e− w/ BABAR)
−259 ± 48
DHMZ 10 (τ newest)
−195 ± 54
DHMZ 10 (e+e− newest)
−287 ± 49
BNL-E821 (world average)
0 ± 63
BNL-E821 2004
-700 -600 -500 -400 -300 -200 -100 0
× 10−11
aμ − aμexp

Future

Fermilab E989: Approved January 2011

- Re-locate the $(g - 2)$ storage ring to Fermilab
- Use the many proton storage rings to form the ideal proton beam
- Use one of the antiproton rings as a 900 m decay line to produce a pure muon beam
- Accumulate 21 times the statistics
- Improve the systematic errors
- Final goal: At least a factor of 4 more precise over E821

BOSTON UNIVERSITY

Lee Roberts - INT Workshop on HLBL 28 February 2011

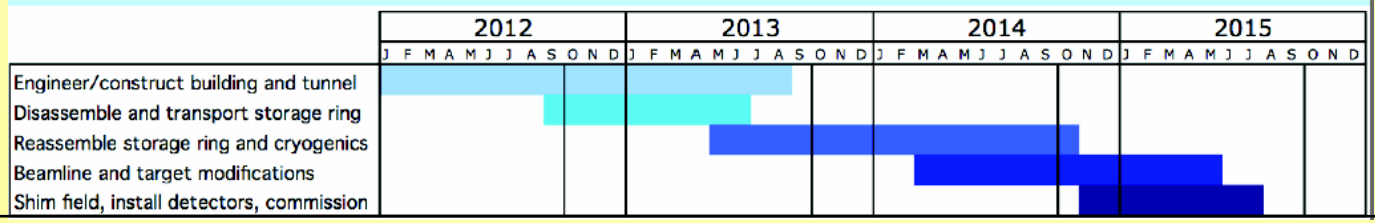
- p. 22/24

The adventure:

Sikorsky S64F 12.5 T hook weight (Outer coil/cryostat 8T)

- Transport coils to and from barge via Sikorsky aircrane
- Ship through St Lawrence –> Great Lakes –> Calumet SAG
- Subsystems can be transported overland, but probably more cost effective to ship steel on barge as well.

Timeline presented to DOE this week



On this timescale it's essential that the theory improve

- Lowest-order hadronic
  - BaBar and Belled have additional unanalyzed data
    - especially important for multihadron channels
  - VEPP2000 at Novosibirsk
    - CMD3
    - SND
- HLBL
  - Agreement among theorists and additional work
  - KLOE 2 photon physics
  - BES, Mainz

BOSTON UNIVERSITY

Lee Roberts - INT Workshop on HLBL 28 February 2011

- p. 25/25

The new muon $g-2$: Fermilab E989

- $\delta a_\mu = 16 \times 10^{-11}$ by 2015
- Magnetic field: $\frac{\delta\langle B\rangle_\mu}{\langle B\rangle_\mu} \leq 2 \times 10^{-8}$
- Requires 10% error on HLbL
- HLbL white paper in progress

Present:

- $a_\mu^{\rm exp} = 116\,592\,089(63) \times 10^{-11}$; $a_\mu^{\rm SM} = 116\,591\,793 \pm 51 \times 10^{-11}$

E989: statistics $21\times$; total error factor 4 more precise

$$\left.\begin{array}{l}\sigma_{\rm stat} = 0.1 \text{ ppm} \\ \sigma_{\rm syst} = 0.1 \text{ ppm}\end{array}\right\} \sigma_{\rm tot} = 0.14 \text{ ppm}$$

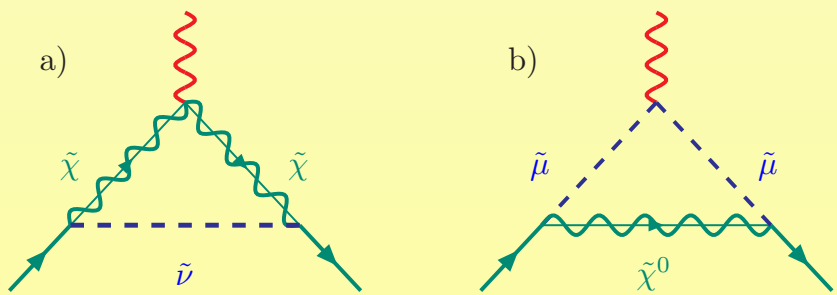
- $a_\mu^{\rm exp} = 116\,59x\,xxx(16) \times 10^{-11}$

Right now: $3.3\,\sigma$ missing!

What is it?

- statistical fluctuation of experimental result
- underestimated systematic error
- missing higher-order SM contributions
- underestimated theory error (incl. possible computational)
- physics beyond SM

Most promising New Physics scenario: SUSY



Leading SUSY contributions to $g-2$ in supersymmetric extension of the SM.

- $\tilde{m}$ lightest SUSY particle; SUSY requires two Higgs doublets

- $\tan\beta = \frac{v_1}{v_2}$, $v_i =< H_i >$; $i = 1, 2$; $\tan\beta \sim m_t/m_b \sim 40$ [4 – 40]

- $$a_\mu^{\rm SUSY} \simeq \frac{{\rm sign}(\mu M_2)\,\alpha(M_Z)}{8\pi\sin^2\Theta_W}\,\frac{\left(5+\tan^2\Theta_W\right)}{6}\,\frac{m_\mu^2}{M_{\rm SUSY}^2}\,\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{M_{\rm SUSY}}{m_\mu}\right)$$

  with $M_{\rm SUSY}$ a typical SUSY loop mass and the sign is determined by the Higgsino mass term μ, RG improved.

There are a lot of “SUSY’s”

- General MSSM has > 100 free parameters

- CMSSM – “constrained” and, related but even more constrained MSUGRA, and others

- These models assume many degeneracies of masses and couplings in order to restrict the number of parameters
- Typically, $m_o, m_{1/2}, \mathrm{sign}(\mu), \tan\beta, A$ (or even more)

- Then there is R–parity – is sparticle number conserved (dark matter candidate!)

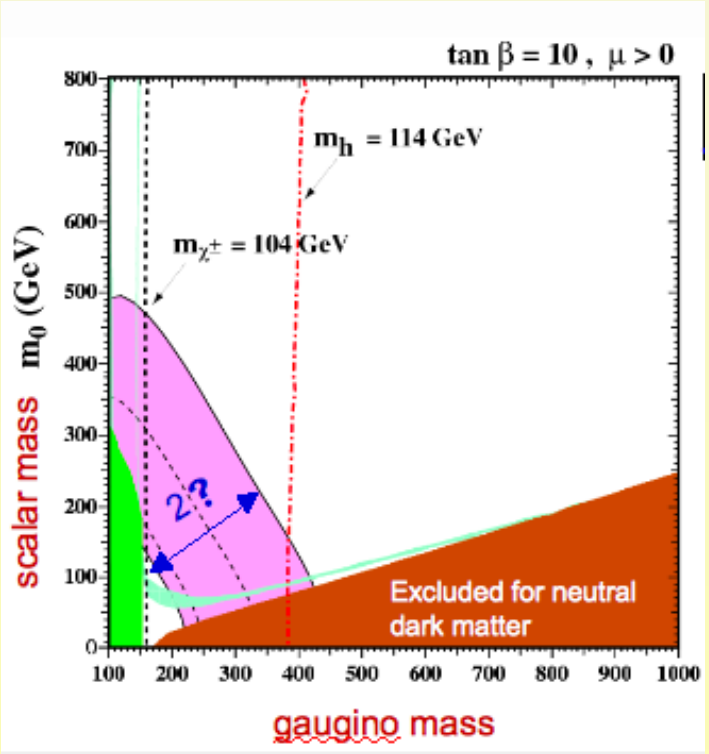
- And, many ways to describe EW symmetry breaking

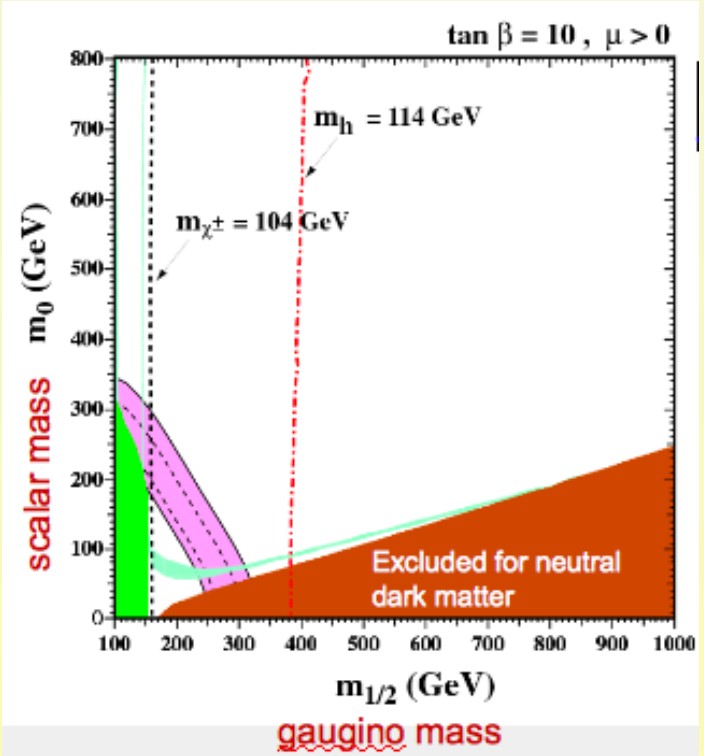
Role for LHC searches:
3 σ deviation in muon g-2 (if real) requires $\mathrm{sign}(\mu)$ positive and $\tan\beta$ preferable large. Note:

- $\mathrm{sign}(\mu)$ cannot be obtained from LHC (hadron collider)

- $\tan\beta$ can't be pinned down by LHC (hadron collider)

so muon g-2 important hint for constraining SUSY parameter space (is SUSY)





LHC soon will provide severe constraints on SUSY mass spectrum [plots Olive 09].

Outlook

LHC the big challenge ahead!

- its running!

Precision experiments remain an important complement of LHC:

a_μ maybe the best!

- this we hope will be realised!

Time horizon for next step in imporvement: 5 years

Will provide important information on Physics Beyond the SM scenarios!

Provided deviation is real $3\sigma \to 9\sigma$ possible?

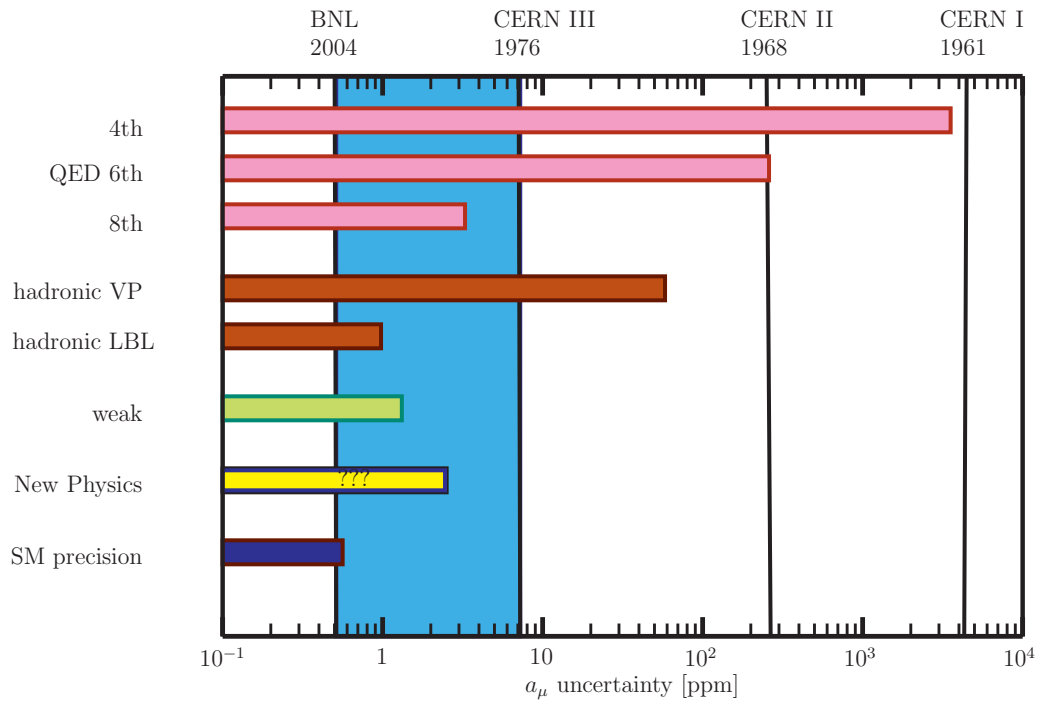
If SUSY:

$$\delta a_\mu \leftrightarrow \mathrm{sign}(\mu) \text{ and } \tan\beta$$

If not SUSY or 2HDM may be even more interesting!

In any case
establishing a new theory replacing SM
likely is a long way to go and requires
efforts on very different levels

The Muon $g-2$
"the closer you look the more there is to see"



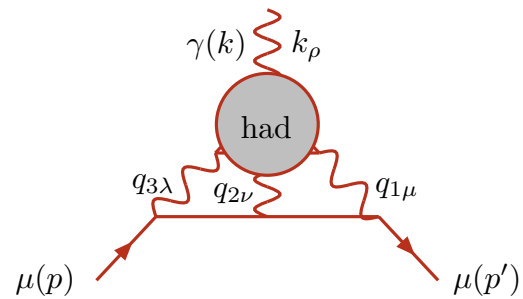
History of sensitivity to various contributions

The anomalous magnetic moment of the muon by itself a tiny 0.116 % effect now measured at 5 $\times 10^{-7}$!

Backup slides on hadronic LbL contribution

The hadronic LbL: setup and problems

Hadrons in $\langle 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0\rangle$



Key object full rank-four hadronic vacuum polarization tensor

$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int \mathrm{d}^4x_1\,\mathrm{d}^4x_2\,\mathrm{d}^4x_3\,\mathrm{e}^{\mathrm{i}\,(q_1x_1+q_2x_2+q_3x_3)} \times\langle\,0\,|\,T\{j_\mu(x_1)j_\nu(x_2)j_\lambda(x_3)j_\rho(0)\}\,|\,0\,\rangle\ .$$

- non-perturbative physics
- general covariant decomposition involves 138 Lorentz structures of which
- 32 can contribute to $g-2$

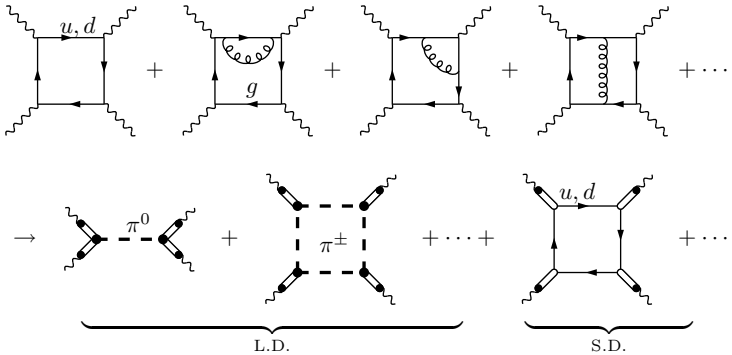
- fortunately, dominated by the pseudoscalar exchanges $\pi^0, \eta, \eta', ...$ described by the effective Wess-Zumino Lagrangian

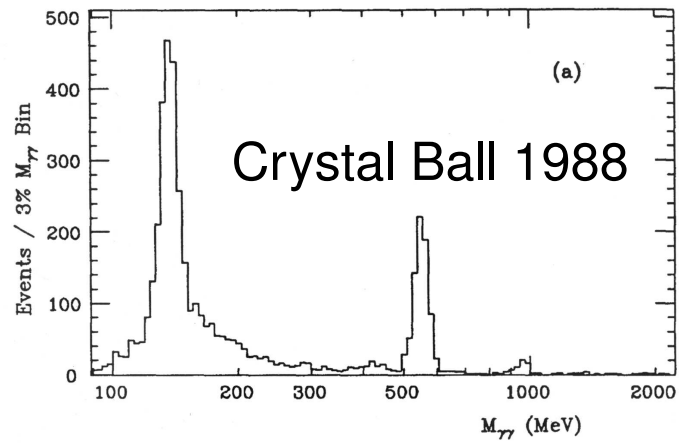
- generally, pQCD useful to evaluate the short distance (S.D.) tail

- off-shell form factors needed not directly accessible to experiment!

- the dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar Goldstone bosons as well as the vector mesons which play a dominant role (vector meson dominance mechanism); HLS, ENJL, general RLA, large N_c inspired ansätze, and others

Need appropriate low energy effective theory $\Rightarrow$amount to calculate the following type diagrams

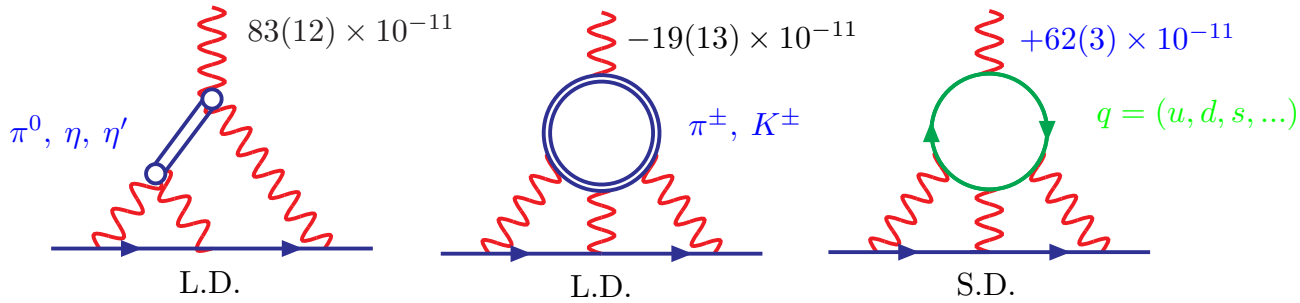




Data show almost background free spikes of the PS mesons! Substantial background form quark loop is absent (seems to contradict large quark-loop contribution as obtained in SDA). Clear message from data: fully non-perturbative, evidence for PS dominance. However, no information about axial mesons (Landau-Yang theorem). Illustrates how data can tell us where we are.

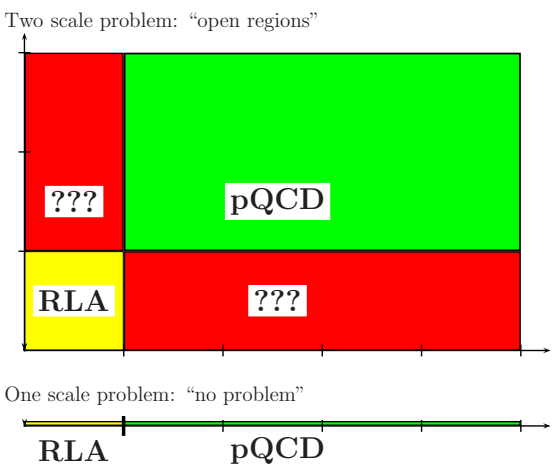
Low energy expansion in terms of hadronic components: theoretical models vs experimental data

➠ KLOE, KEDR, BES, BaBar, Belle, ?



LD requires low energy effective hadronic models: simplest case $\pi^0\gamma\gamma$ vertex

Basic problem: (s, s_1, s_2)–domain of $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$; here $(0, s_1, s_2)$–plane



??? – Data, effective Lagrangians, OPE,
– QCD factorization,
– Brodsky-Lepage approach

Models and Controversies

❒ Low energy effective field theory
Traditional approach: low energy effective Lagrangians: HLS, ENJL (resonance chiral theory) Kinoshita et al., Bijnens et al, ➠matching and double counting problems

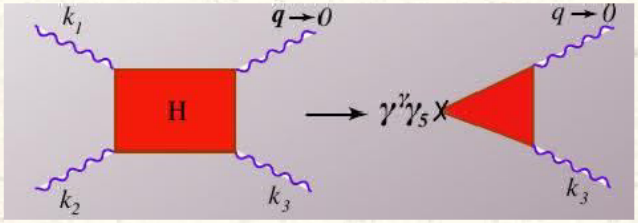
❒ Large N_c QCD inspired approach
Novel approach: refer to quark–hadron duality of large-N_c QCD, hadron spectrum known, infinite series of narrow spin 1 resonances 't Hooft 79 ⇒no matching problem (resonance representation has to match quark level representation)
De Rafael 94, Knecht, Nyffeler 02

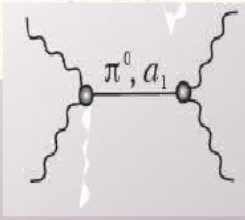
❒ other new approaches:
- HLbL from string theory
  Cappiello, Catá, D' Ambrosio
- QCD based numeric Schwinger-Dyson/Bethe-Salpeter equations approach
  Goecke, Fischer, Williams

❒ The Melnikov-Vainshtein constraint and model

The model that fits the box

- We simplify the problem by picking up a particular part in the phase-space $q_1^2 \gg q_2^2 \gg q_3^2 \gg \Lambda^2_{\rm QCD}$ However, we require that in that part of the phase-space the amplitude is reproduced ``exactly''





$$\mathcal{A}_{\pi^0} = -\frac{N_c W^{(3)}}{2\pi^2 F_\pi^2} \frac{F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2)}{q_3^2 + m_\pi^2} \{f_2 \tilde{f}_1\}\{\tilde{f} f_3\}$$

$$\mathcal{M} = \alpha^2 N_c \, {\rm Tr}[\hat{Q}^4] \, \mathcal{A}$$

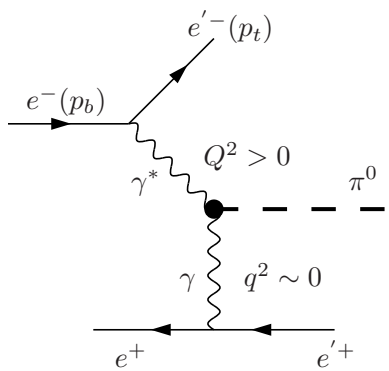
$$\mathcal{A} = \frac{4}{q_3^2 \, \hat{q}^2} \{f_2 \tilde{f}_1\}\{\tilde{f} f_3\}$$

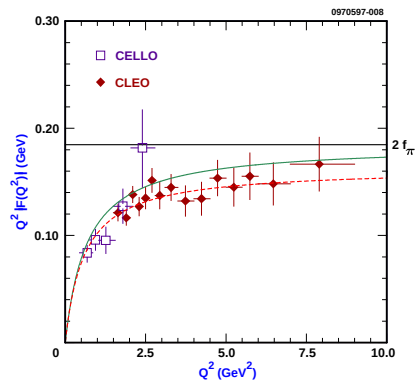
$$- \frac{4}{q_3^2 \, \hat{q}^4} \left(\{q_2 f_2 \tilde{f}_1 \tilde{f} f_3 q_3\} + \{q_1 f_1 \tilde{f}_2 \tilde{f} f_3 q_3\} + \frac{q_1^2 + q_2^2}{4} \{f_2 \tilde{f}_1\}\{\tilde{f} f_3\} \right) + \cdots$$

Constraints for on-shell pions (pion pole approximation)

- The constant $e^2\,\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2,0,0) = \frac{e^2 N_c}{12\pi^2 f_\pi} = \frac{\alpha}{\pi f_\pi} \approx 0.025$ GeV^{-1} well determined by $\pi^0 \to \gamma\gamma$ decay rate (from Wess-Zumino Lagrangian); experimental improvement needed!

- Information on $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ from $e^+e^- \to e^+e^-\pi^0$ experiments





CELLO and CLEO measurement of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ at high space–like Q^2. outdated now by BaBar?

Brodsky–Lepage interpolating formula gives an acceptable fit.

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0) \simeq \frac{1}{4\pi^2 f_\pi} \frac{1}{1 + (Q^2/8\pi^2 f_\pi^2)} \sim \frac{2 f_\pi}{Q^2}$$

Inspired by pion pole dominance idea this FF has been used mostly (HKS,BPP,KN) in the past, but has been criticized recently (MV and FJ07).

❒ Melnikov, Vainshtein: in chiral limit vertex with external photon must be non-dressed! i.e. use $\mathcal{F}_{\pi^0\gamma^*\gamma}(0,0,0)$, which avoids eventual kinematic inconsistency, thus no VMD damping ⇒result increases by 30% !

❒ In $g-2$ external photon at zero momentum ⇒ only $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$ not $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ is consistent with kinematics. Unfortunately, this off–shell form factor is not known and in fact not measurable and CELLO/CLEO constraint does not apply!. Obsolete far off-shell pion (in space-like region).

Can we check such questions experimentally or in lattice QCD?

Present status:

Pseudoscalar exchanges

Model for $\mathcal{F}_{P^{(*)}\gamma^*\gamma^*}$	$a_\mu(\pi^0) \times 10^{11}$	$a_\mu(\pi^0, \eta, \eta') \times 10^{11}$
modified ENJL (off-shell) [BPP]	59(9)	85(13)
VMD / HLS (off-shell) [HKS,HK]	57(4)	83(6)
LMD+V (on-shell, $h_2 = 0$) [KN]	58(10)	83(12)
LMD+V (on-shell, $h_2 = -10$ GeV^2) [KN]	63(10)	88(12)
LMD+V (on-shell, constant FF at ext. vertex) [MV]	77(7)	114(10)
nonlocal χQM (off-shell) [DB]	65(2)	–
LMD+V (off-shell) [N]	72(12)	99(16)
AdS/QCD (off-shell ?) [HoK]	69	107
AdS/QCD/DIP (off-shell) [CCD]	65.4(2.5)	–
DSE (off-shell) [FGW]	58(7)	84(13)
[PdRV]	–	114(13)
[JN]	72(12)	99(16)

BPP = Bijnens, Pallante, Prades '95, '96, '02 (ENJL = Extended Nambu-Jona-Lasinio model); HK(S) = Hayakawa, Kinoshita, Sanda '95, '96; Hayakawa, Kinoshita '98, '02 (HLS = Hidden Local Symmetry model); KN = Knecht, Nyffeler '02; MV = Melnikov, Vainshtein '04; DB = Dorokhov, Broniowski '08 (χQM = Chiral Quark Model); N = Nyffeler '09; HoK = Hong, Kim '09; CCD = Cappiello, Catà, D'Ambrosio '10 (used AdS/QCD to fix parameters in DIP (D'Ambrosio, Isidori, Portolés) ansatz); FGW = Fischer, Goecke, Williams '10, '11 (Dyson-Schwinger equation)

Reviews on LbyL: PdRV = Prades, de Rafael, Vainshtein '09; JN = Jegerlehner, Nyffeler '09

A. Nyffeler, Seattle

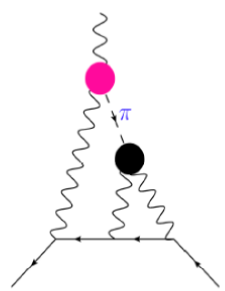
- p. 29

Data constrain Models

Details in Seattle talks:

Dario Morriciani	KLOE small angle tagger (low energy $\pi^0\gamma\gamma$)
Achim Denig	BaBar and BES results and plans
Simon Eidelman	Belle and KEDR results and plans [work in progress]
Henryk Czyż	EKHARA a Monte Carlo for $\gamma^*\gamma^*$ physics

Pion exchange in hadronic LbL



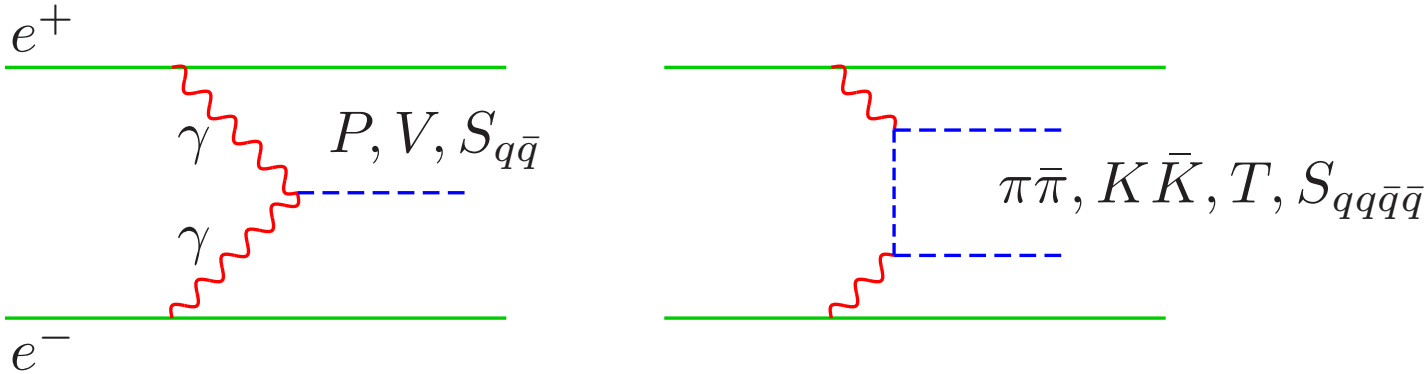
$F_{\pi^*\gamma^*\gamma^*}$ form factors are key objects

- external vertex : $F_{\pi\gamma\gamma}(t_\pi, t_\pi, 0^2)$
  - ✓ far off-shell pion ¿
  - ✓ zero-energy photon ¿
- internal vertex : $F_{\pi\gamma\gamma}(t_\pi, t_1, t_2)$
  - ✓ totally off-shell object

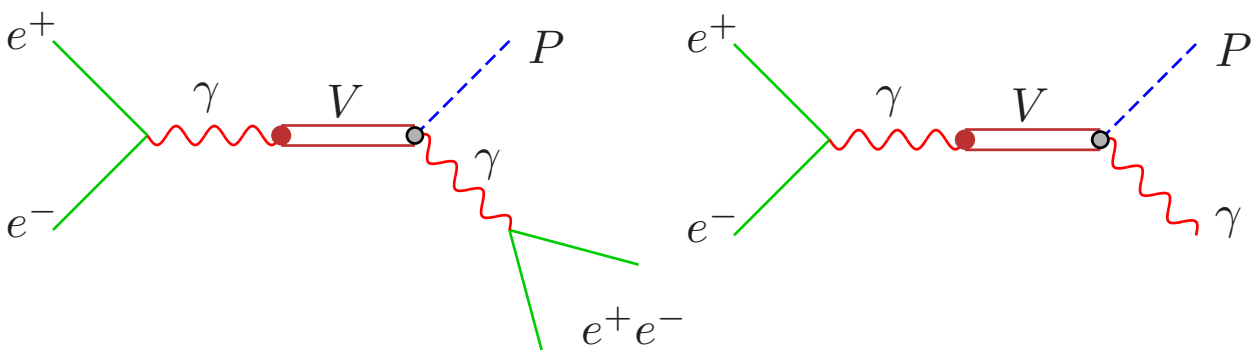
These form factors were never measured

H. Czyż, IF, UŚ, Katowice, EKHARA 2.0+ ... 3

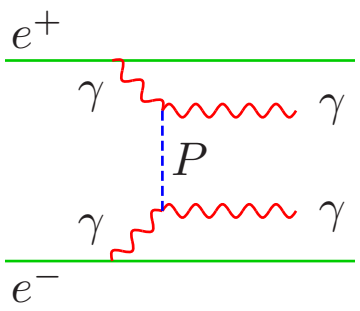
Overview (Eidelman incl progress report for Belle,KEDR):



mostly single-tag events: KLOE, KEDR (taggers), BaBar, Belle, BES III (high luminosity)



Dalitz-decays: $\rho, \omega, \phi \to \pi^0(\eta)e^+e^-$ Novosibirsk, NA60,JLab, Mainz, Bonn, Jülich, BES



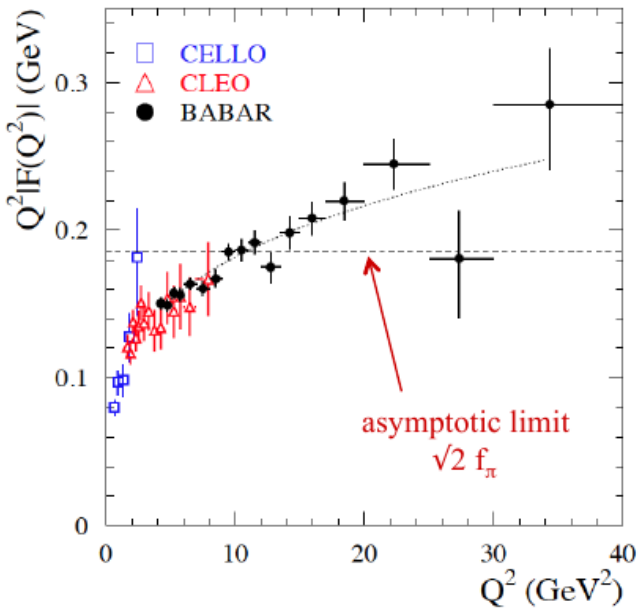
would be interesting, but is buried in the background

BaBar

10

The π^0 Transition Form Factor

Comparison with world data set and QCD



- In Q^2 range 4-9 GeV^2 BaBar results are in a reasonable agreement with CLEO and CELLO
- Expectation: >10 GeV^2 reach asymptotic limit for $Q^2\, F(Q^2) = \sqrt{2} f_\pi = 0.185$ GeV predicted by Brodsky-Lepage in 1979

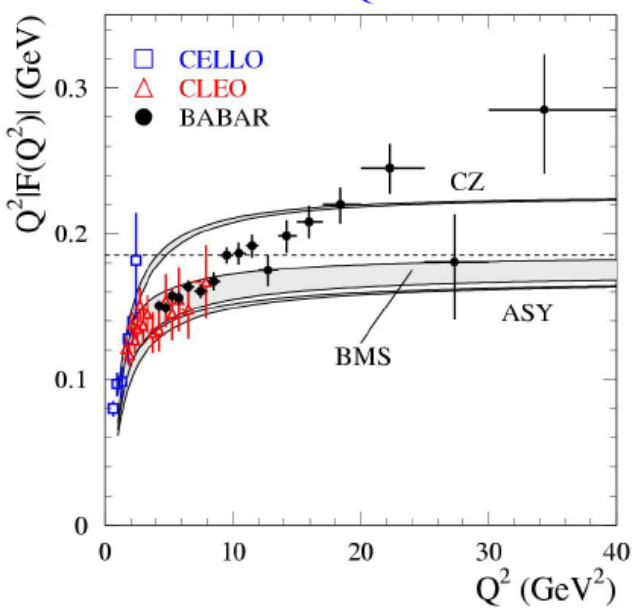
PRD22,2157

Data exceed asymptotic limit $Q^2 > 10$ GeV^2

Achim Denig *Meson Transition FFs at BaBar*

The π^0 Transition Form Factor

Comparison with world data set and QCD



- Several DAs to confront theory vs. experiment:

  CZ: Chernyak-Zhitnitsky DA

  ASY: Asymptotic DA

  BMS: Bakulev-Mikhailov-Stefanis

Models for DA do not describe data
→ Is the DA for pions not known ?

- Use Bakulev-Mikhailov-Stefanis light-cone sum rule theory at NLO pQCD + twist-4 power corrections PRD 67, 074012

Need for higher order pQCD corrections ?!

Present & Future

❒ Role of Melnikov-Vainshtein constraint still under debate (is virtual photon dressed or undressed at external $\pi^0\gamma\gamma$ vertex?)

❒ Role of quark loop: is it an independent contribution? solving Schwinger-Dyson equation approach yields very large value.

❒ Large Q^2 behavior of $\mathcal{F}_{\pi^{0*}\gamma\gamma^*}(m_\pi^2, -Q^2, 0)$ from BaBar shows much weaker fall-off than expected by theory

❒ New muon $g-2$ experiment is on its way !!!

- Need to improve accuracy for the hadronic light-by-light contribution.
- New input form $\gamma\gamma$ physics to constrain theoretical models for HLbL (KLOE-2,BES,MAINZ)
- Challenge for theory: radiative corrections needed

- Question of asymptotic behavior seen by BaBar, will likely be settled by Belle
- Can we check controversial dressed/undressed (i.e damping or not?) at external vertex? Can Primakoff-effect plus DR help?

❒ Lattice QCD makes big progress: we may expect relevant results for constraining models

❒ Not to forget: urgent improvement of VP mandatory [lattice QCD may become competitive] (Novosibirsk: CMD3, SND, unanalised data from BaBar & Belle?)