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*Talk given at the Nato Advanced Research Workshop "Radiative Corrections: Results and Perspectives", Juli 9 - 14, 1989, Brighton, Great Britain

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RENORMALIZATION SCHEME DEPENDENCE OF

ELECTROWEAK RADIATIVE CORRECTIONS

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INTRODUCTION

Precision tests of the electroweak Standard Model will be a major goal at LEP. The effects we want to establish are the *genuine* weak corrections, the self-interactions of gauge fields, Higgs boson and top quark interactions or similar effects from *new physics*, expected to be of the order of about 1%. Their detection requires both experimental and theoretical uncertainties to be not more than 0.1%. Typically the expected level of accuracy for various observables at LEP [1] is $\delta M_Z \lesssim 50$ MeV from the Z line-shape, $\delta \sin^2 \Theta_W \lesssim 0.0015$ from the forward-backward asymmetry A_{FB}, $\delta \sin^2 \Theta_W \lesssim 0.0004$ from left-right asymmetry A_{LR} and $\delta M_W \sim 100$ MeV from the W-mass measurement at LEP2.

In order to be able to establish clearly small effects an estimate of the theoretical uncertainties is necessary. The study of the scheme dependence (SD) of predictions is a tool to get an estimate for the quality of perturbative approximations used in standard model calculations.

In perturbative quantum field theory models, like the standard model of electroweak interactions, the starting point is a *classical* Lagrangian with some free mass and coupling parameters. The classical Lagrangian is suitable only for tree level calculations of limited accuracy. If we aim to make precise predictions, we have to take into account higher order corrections. Their calculation is possible only if we properly quantize the Lagrangian. The theory needs be regularized and a gauge fixing prescription is necessary. The validity of the Slavnov-Taylor identities infers the gauge invariance of the S-matrix elements. The parameters of the Lagrangian now become the bare parameters. The relation between the bare and the physical (renormalized) parameters takes the form

$$e_b = e + \delta e, \quad M_b = M + \delta M, \quad \cdots \tag{1}$$

where the shifts δe, δM, $\cdots$ between the bare (b) and the renormalized quantities are the *counter terms*. The latter are fixed either by some formal prescription or by defining how a particular physical parameter is measured. We will not dwell on "intermediate" renormalization schemes, like the minimal subtraction scheme familiar from QCD, which work with unphysical auxiliary quantities at intermediate steps. We are interested to discuss cases where physical quantities are calculated in terms of physical quantities.

- Before we can make predictions, a set of independent parameters must be determined from experiment.
- A specific choice of experimental data points used as an input parameter set defines a renormalization scheme (RS).

Parametrizations frequently used are the following:

1) A natural choice of "basic" parameters is the QED-like parametrization in terms of the fine structure constant α and the physical particle masses

$$\alpha, M_W, M_Z, m_f, m_H \qquad (I)$$

often referred to as the "on-shell scheme". We shall refer to it as the α-scheme. It allows for a natural separation of the QED part of the electroweak radiative corrections which is dominated often by large soft photon effects accompanying external charged particles.

2) In the Standard Model , which unifies weak and electromagnetic interactions, we can use as a coupling parameter as well the Fermi constant G_μ instead of α. We then have

$$G_\mu, M_W, M_Z, m_f, m_H \qquad (II)$$

as an independent set of parameters. This set is suitable for processes which are dominated by neutral (NC) or charged (CC) current transitions. An important property of G_μ is that it is not running from low energy up to the vector boson mass scale $M_W(M_Z)$. This G_μ-scheme thus is a genuine high energy scheme in the sense that no large logarithms show up in the calculation of vector boson processes in the LEP energy region (Z and W-pair production).

We know that the parameters of the two schemes are related by [2]

$$\sqrt{2}G_\mu = \frac{\pi\alpha}{M_W^2 \sin^2\Theta_W}\frac{1}{1-\Delta r}, \tag{2}$$

where Δr is the non-QED correction to μ-decay calculated in the α-scheme. If not stated otherwise, we use the definition

$$\sin^2\Theta_W = s_W^2 = 1 - \frac{M_W^2}{M_Z^2} \tag{3}$$

for the weak mixing angle.

A disadvantage of the parametrizations (I) and (II) is that they require a precise knowledge of M_W which will be measured precisely at LEP2 only. In order to keep the input parameter errors as small as possible we have to replace M_W by G_μ in (I).

3) The scheme to be used as a starting point for precise calculations of radiative corrections uses

$$\alpha, G_\mu, M_Z, m_f, m_H \qquad (III)$$

as input parameters, with M_Z measured from the Z line-shape at LEP1.

4) Another interesting possibility would be to predict quantities in terms of the low energy parameters

$$\alpha, G_\mu, \sin^2\Theta_{\nu_\mu e}, m_f, m_H \qquad (IV)$$

where $\sin^2\Theta_{\nu_\mu e}$ is determined from neutrino-electron scattering (by CHARM II for example).

Scheme-dependence can be investigated by predicting an observable in terms of different input parameter sets. Since not all the parameters are known to the same precision we proceede as follows: We first predict M_W and $\sin^2\Theta_{\nu_\mu e}$ in the scheme (III) and then take any 3 parameters which are independent at tree level to calculate quantities like the vector boson widths $\Gamma_{Zf\bar{f}}$, $\Gamma_{Wf\bar{f}'}$, or the cross-sections $\sigma(e^+e^- \to f\bar{f})$, $\sigma(e^+e^- \to W^+W^-)$ e.t.c.

Predictions of physical quantities of course should not depend on the specific choice of the input parameters and they if fact do not if we include all orders of the perturbation expansion. Actually, the reparametrization invariance is infered by renormalization group invariance. However, practical perturbative calculations are *approximations* obtained by truncation of the perturbation series. The accuracy of the finite order approximations depends on the choice of the input parameters i.e. finite order results are scheme dependent.

Let us illustrate this point by an example: Suppose we compute a matrix element M in the α-scheme (I) to one-loop order yielding a result

$$M^{(1)} = \alpha^n C[1 + b\alpha].$$

Now, suppose we calculate the same quantity in the G_μ-scheme (II) which amounts to a replacement of $\alpha \simeq 137^{-1}$ by $\alpha' = \frac{\alpha}{1-\Delta r} \simeq 127^{-1}$ i.e. to one-loop order $\alpha' = \alpha[1 + a\alpha]$ and

$$M'^{(1)} = \alpha'^n C[1 + b'\alpha'].$$

Inserting α' we get

$$M'^{(1)} = M^{(1)} + \delta M$$

with $b' = b - na$ and

$$\delta M = \alpha^n C \left[(\frac{n(n-1)}{2} a^2 + (n+1)ab')\alpha^2 + \cdots + a^{n+1} b' \alpha^{n+2} \right].$$

Thus the result differs by δM. If we do not actually calculate the higher orders

$$\delta M = M'^{(1)} - M^{(1)} \qquad (4)$$

must be considered as an uncertainty due to unknown higher order effects.

For LEP experiments one-loop calculations are insufficient to get the precision of 0.1% and one has to go to resummation improved calculations by including leading higher order effects. The study of the scheme dependence of resummation improved results is a way to estimate missing higher order contributions (*educated guess*). Of course only an actual n-loop calculation can tell us what the full n-loop answer is.

SCHEME DEPENDENCE OF $\sigma(e^+e^- \to W^+W^-)$

Some important points concerning the SD of higher order predictions can be well illustrated by a calculation of $\sigma(e^+e^- \to W^+W^-)$ in the α- and G_μ-scheme respectively. For a more detailed discussion of W-pair production we refer to A. Denner's contribution [3].

In Fig. 1, the total cross-section is shown as a function of the c.m. energy $\sqrt{s}$ for both the G_μ and the α scheme [4]. At the Born, level the cross-sections differ by

$$\delta\sigma = \frac{\sigma(\alpha) - \sigma(G_\mu)}{\sigma(G_\mu)} \simeq -13.7\%$$

in the two schemes. This difference, as expected, gets substantially smaller with

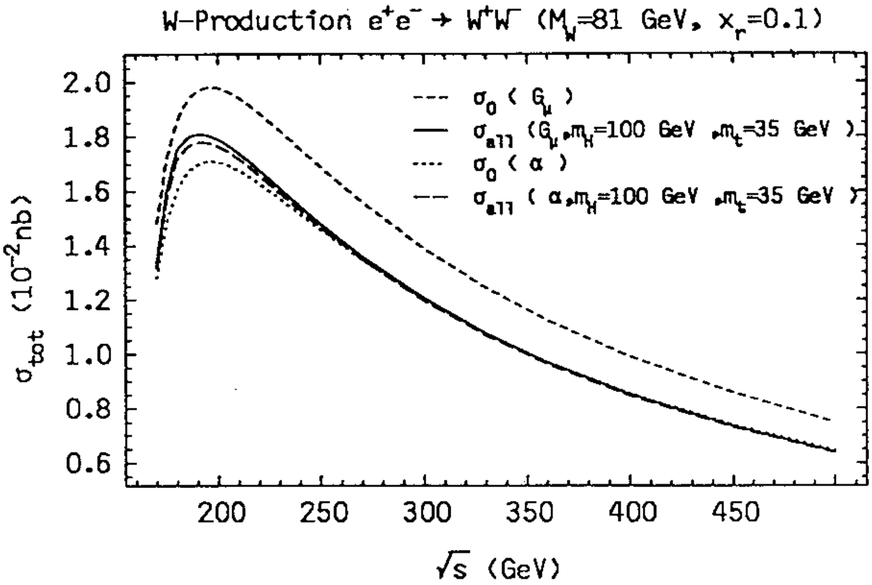

Figure 1: Total cross-section in lowest order(σ_0) and including radiative corrections (σ_{all}) in the G_μ- and α-scheme respectively

the inclusion of the one-loop corrections. Nevertheless, there remains a non-negligible difference which signals missing higher order effects. The difference is largest, $\simeq$ -1.7%, at about 180 GeV ($\simeq$ 12 GeV below the peak), -1.1% at 170 GeV and -1.2% at 200 GeV. For a required precision of better than 1% the missing leading higher order effects must be included. Since we know that the parameters of the two schemes are related by Eq. (2) we have to linear order ($O(\alpha)$) in the cross-section

$$\sigma_0(G_\mu) \simeq \sigma_0(\alpha)(1 + 2\Delta r).$$

This is what is usually included at $O(\alpha)$ in the α-scheme. To second order we get

$$\sigma_0(G_\mu) \simeq \sigma_0(\alpha)(1 + 2\Delta r + 3\Delta r^2).$$

and the additional term $3\Delta r^2$ is the missing next leading term. Typically for $\Delta r \simeq$ 7% we get $3\Delta r^2 \simeq 1.5\%$ which essentially accounts for the above mentioned difference.

This example shows *one* important aspect of the dependence of a prediction on the choice of parameters.

- in the α-scheme large leading logarithms are found which must be resummed using renormalization group arguments in order to get a prediction of acceptable accuracy, whereas

- in the G_μ-scheme no large logs show up and one gets a good approximation without any resummation of higher order effects.
- Even after summing leading logs in the α-scheme the results differ and the difference actually is energy-dependent and hence cannot be solely a matter of change of parameters. The remaining difference is merely due to the omission of higher order effects and unless we perform the next order calculation must be taken as the theoretical uncertainty of the prediction.

Perhaps an even more important drawback of a particular choice of parametrization is the different sensitivity to unknown parameters like m_t and m_H.

The sources of m_H- and m_t- dependence are:

- contributions from γ, Z and W self-energy diagrams
- contributions from ZWW and γWW form factors

Using the G_μ-scheme, with G_μ, M_Z and M_W as input parameters, the dependence of the cross-section on m_H and m_t is very weak, as can be seen from Tab. 1 [4].

Table 1. σ_{tot} (pb) for some values of m_t and m_H. Energies and masses in GeV

	(m_H, m_t)	$E_{c.m.}$= 163	170	180	200
G_μ-scheme	(100,30)	5.042	13.392	17.566	18.005
	(100,200)	4.998	13.297	17.461	17.908
	(100,300)	5.001	13.314	17.496	17.965
	variation	0.85 %	0.71 %	0.60 %	0.54 %
α-scheme	(100,30)	5.039	13.240	17.268	17.793
	(100,200)	4.489	11.908	15.613	16.041
	(100,300)	3.938	10.576	13.959	14.293
	$(E_{c.m.}, m_t)$	m_H= 10	100	500	1000
G_μ-scheme	(195,60)	17.861	18.000	17.988	17.986

Of particular importance is the possible top-mass dependence of the W-mass measurement [5]. A crucial fact is that the threshold region is dominated by the t-channel exchange terms, where only the renormalization effects (counter terms) depend on unknown physics, mainly showing up in the vector boson self-energies. Only the t-channel amplitude $T_1^{(-)}$, which is proportional to the ν-exchange diagram at the tree level, exhibits such terms:

$$\begin{aligned} T_1^{(-)} &= -\sqrt{2}G_\mu \frac{2M_W^2}{t}\{1+2\Sigma_r^{\nu_e\nu_e}(t)+2A_{1r}^{We\nu_e}(t)\}+\cdots \\ &= -\sqrt{2}G_\mu \frac{2M_W^2}{t}\{1+\Delta C^W+\cdots\}+\cdots \end{aligned} \tag{5}$$

where the m_H and m_t dependence of ΔC^W has been analyzed in Ref. 6. Formally,

$$\begin{aligned} \Delta C^W &= \frac{Re\Pi^{WW}(M_W^2)}{M_W^2} - \frac{\Pi^{WW}(0)}{M_W^2} - Re\frac{d\Pi^{WW}}{dq^2}(M_W^2) \\ &= -M_W^2\frac{d\pi^{WW}}{dq^2}(M_W^2) \end{aligned}$$

is determined solely by the (twice subtracted) W self-energy function

$$\Pi^{WW}(q^2) = \Pi^{WW}(0) + q^2\pi^{WW}(q^2).$$

One finds for the top contribution $\Delta C^{Wt} = -2K$ for $m_t \gg M_W$ and $\Delta C^{Wt} = 2K$ for $m_t \ll M_W$ with $K = \frac{\sqrt{2}G_\mu M_W^2}{16\pi^2}$, i.e. not even a logarithmic dependence on m_t is present!

Similarly, for the Higgs dependence, we find $\Delta C^{WH} = 0$ for $m_H \gg M_W$ and $\Delta C^{WH} = 2K(\ln\frac{m_H^2}{M_W^2} + \frac{47}{12})$ for $m_H \ll M_W$. The potentially interesting infrared log for a light Higgs disappears if Higgs Bremsstrahlung off the final state W's is taken into account. Actually, one finds $\delta\Gamma_{W\to Hff'}/\Gamma_{W\to ff'} = -\Delta C^{WH}$ such that $\Delta C^{WH,virtual+soft} = 0$ for $m_H \ll M_W$. Hence there is a very weak dependence only on both m_t and m_H near threshold. The situation is quite different if we use α, M_Z and M_W as input parameters. Using Eq. (2) we get

$$T_1^{(-)} = -\frac{\pi\alpha}{M_W^2(1-M_W^2/M_Z^2)}\frac{2M_W^2}{t}\{1+\Delta r+\Delta C^W+\cdots\}+\cdots \tag{6}$$

instead of Eq. (5), the following quadratic m_t- and logarithmic m_H-dependence results [6]

$$\begin{aligned} \Delta r^{top} &= \frac{\sqrt{2}G_\mu M_W^2}{16\pi^2}\left\{-3\frac{c_W^2}{s_W^2}\frac{m_t^2}{M_W^2} + 2\left(\frac{c_W^2}{s_W^2}-\frac{1}{3}\right)\ln\frac{m_t^2}{M_W^2}+\cdots\right\} \\ \Delta r^{Higgs} &\simeq \frac{\sqrt{2}G_\mu M_W^2}{16\pi^2}\left\{\frac{11}{3}(\ln\frac{m_H^2}{M_W^2}-\frac{5}{6})\right\} \quad (m_H \gg M_W). \end{aligned}$$

In Figs. 2 and 3 we illustrate the m_t-dependence of the W-production production cross-section in the threshold region (see also Ref. 5 for plots of results which include finite widths effects).

The α-scheme is therefore completely inadequate for a model independent determination of the W-mass since the cross-section for given α and M_Z depends in an essential way on *two parameters*, M_W and m_t. In contrast for given G_μ and M_Z the cross-section is a function of M_W only to high accuracy i.e. this scheme is very good for a *model independent* fit of M_W (independent on M_Z, m_t, m_H and possible new physics)!

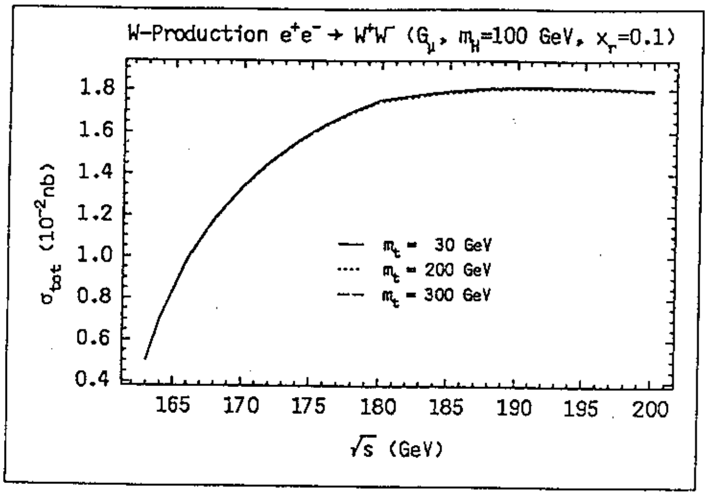

Figure 2: Total cross-section including radiative corrections in the G_μ-scheme for various values of m_t

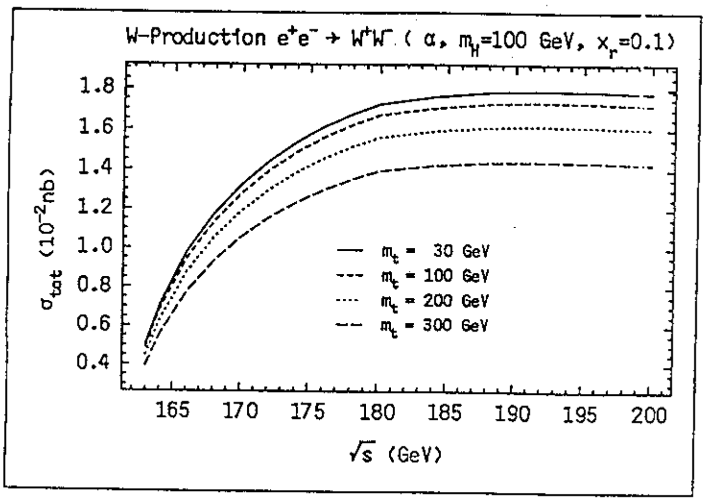

Figure 3: Total cross-section including radiative corrections in the α-scheme for various values of m_t

SCHEME DEPENDENCE OF W-MASS PREDICTION

Of particular interest is the precise relationship between different parameter sets. The relationship between the schemes (I), (II) and (III) is determined by the quantity Δr as introduced in Eq. (2). For example, once Δr is given, the W mass can be predicted by using the values of α, G_μ and M_Z from LEP1. According to Eqs. (2) and (3) we obtain

$$M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{4A_0^2}{M_Z^2}\frac{1}{1-\Delta r}}\right) \qquad (7)$$

with $A_0 = (\frac{\pi\alpha}{\sqrt{2}G_\mu})^{1/2} = 37.2802(3)\ GeV$. $\sin^2\Theta_W$ is given then by Eq. (3).

We have mentioned earlier that a straight forward one-loop calculation would not be sufficient to obtain the precision we need. Indeed, in Eqs. (2) and (7) we have resummed the one-loop result as prescribed by the renormalization group (RG)

$$1+\Delta r^{(1)} \to \frac{1}{1-\Delta r^{(1)}} = 1+\Delta r^{(1)} + (\Delta r^{(1)})^2 + \cdots . \tag{8}$$

Since $\Delta r^{(1)} \simeq 0.07$ we have $(\Delta r^{(1)})^2 \simeq 0.005$ which is not negligible i.e. omission of such higher order terms would show up as a substantial SD. Since we know how to include leading higher order terms we can substancially reduce the SD which would result from a straight forward one-loop calculation. Before we discuss the SD of Δr we therefore have to say how large higher order terms have to be taken into account. To this end we exhibit the large and potentially large terms in Δr by writing

$$\Delta r \;=\; \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{rem} \tag{9}$$

with

$$\begin{aligned} \Delta\alpha &= \Pi^{\gamma\gamma}(0) - \Pi^{\gamma\gamma}(M_Z^2) \\ \Delta\rho &= \frac{\Sigma^Z(0)}{M_Z^2} - \frac{\Sigma^W(0)}{M_W^2}. \end{aligned} \tag{10}$$

and a small but non-negligible remainder Δr_{rem}. By Σ^i ($i = W, Z, \gamma$, or γZ) we denote the unrenormalized vector-boson self-energies, $\Pi^\gamma = \Sigma^\gamma/s$.
$\Delta\alpha$ is the photon vacuum polarization contribution, large due to the large change in scale from zero momentum (Thomson limit) to the Z-mass scale:

$$\Delta\alpha = \frac{\alpha}{3\pi}\Sigma_f N_{cf} Q_f^2 (\ln\frac{M_Z^2}{m_f^2} - \frac{5}{3}\,) \tag{11}$$

where the sum extends over the light fermions. N_{cf} is the color factor. In contrast to $\Delta\alpha$, $\Delta\rho$ is minuscule for light fermions but large for heavy fermions with a light iso-doublet partner [7]. A typical large term is the heavy top contribution

$$\Delta\rho^{top} = \frac{\sqrt{2}G_\mu}{16\pi^2} 3m_t^2. \tag{12}$$

1. Leading log summation

The leading logarithms (11) may be resummed according to the renormalization group. The result may be cast into the form

$$\Delta\alpha^{-1} = \frac{1}{\alpha} - \frac{1}{\alpha(M_Z)} \simeq \frac{1}{3\pi}\Sigma_f N_{cf} Q_f^2 \ln\frac{M_Z^2}{m_f^2} \tag{13}$$

which shows that the leading log summation is scheme independent because the right hand side does not depend on the choice of the parametrization. Beyond the leading log approximation, assuming $\Delta\rho$ to be small,

$$\Delta\alpha^{-1} = \frac{1}{\alpha} - \frac{1}{\alpha(M_Z)} = \frac{\Delta r^{(1)}(\alpha,\cdots)}{\alpha} \tag{14}$$

is"weakly scheme dependent" because only subleading terms depend on the actual parametrization. The RG solution precisely leads to the substitution (8) and thus

$$\alpha(M_Z) = \frac{\alpha}{1-\Delta r^{(1)}}. \tag{15}$$

This expression includes all terms of the form $\alpha^n \ln^n \frac{M_Z^2}{m_f^2}$ in a scheme independent way. The two-loop irreducible contributions are known and may be included by substituting

$$\alpha \ln(\cdot) \rightarrow \alpha(1 + \frac{3\alpha}{4\pi}) \ln(\cdot)$$

in $\Delta\alpha^{(1)}$. With $\frac{3\alpha}{4\pi} \simeq 1.7 \times 10^{-3}$, this yields a negligible effect only.

2. Summation of heavy particle effects

In contrast to $\Delta\alpha$ the summation of large $\Delta\rho$ contributions does not follow from the RG. According to Ref. 8, we have to modify Eq. (2) by replacing

$$\frac{1}{1-\Delta r} \rightarrow \frac{1}{1-\Delta\alpha} \frac{1}{1 + \frac{c_W^2}{s_W^2}\Delta\rho_G} + \Delta r_{rem} \tag{16}$$

where $\Delta\rho_G$ represents the leading irreducible contribution to the ρ parameter defined by the ratio of neutral to charged current amplitudes at low energy, i.e.

$$\frac{G_{NC}}{G_\mu}(0) = \rho_G = \frac{1}{1-\Delta\rho_G}. \tag{17}$$

By $\Delta\rho_G$ we specifically denote $\Delta\rho$ calculated in the G_μ-scheme. It is important to notice that, in contrast to $\Delta\alpha$, which is not significantly modified by the inclusion of two-loop irreducible contributions, ρ as defined in Eq. (17), can differ sizeably from the one-loop result. In fact as shown in Ref. 8, by including the two-loop irreducible terms calculated in Ref. 9, one finds

$$\Delta\rho_G = N_{cf} x_f [1 - (2\pi^2 - 19) x_f + \cdots] \tag{18}$$

with

$$x_f = \frac{\sqrt{2} G_\mu \Delta m_f^2}{16\pi^2} \tag{19}$$

and $N_{cf} = 1$ for leptons, $N_{cf} = 3$ for quarks. Notice that the negative sign of the

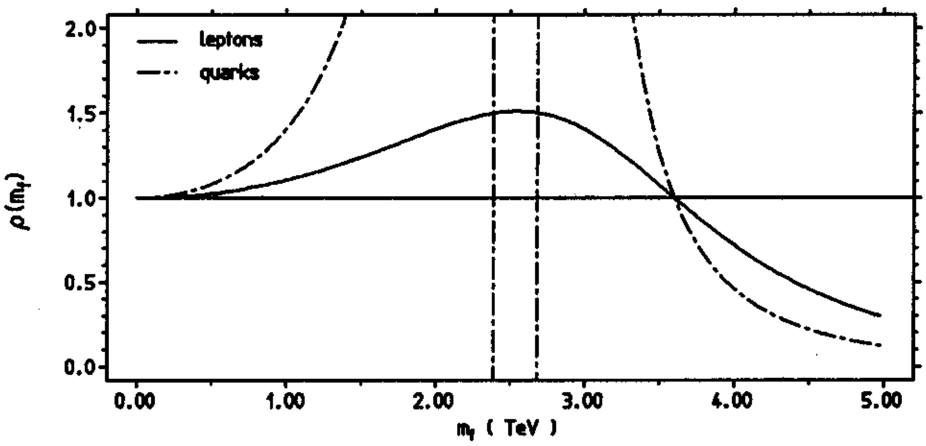

Figure 4: The ρ-parameter as a function of a heavy fermion mass at two-loop order

two-loop contribution ($2\pi^2 - 19 \simeq 0.739$) leads to a screening of heavy particle effects and signals a possible restoration of decoupling (see Fig. 4). This effect, if confirmed by higher order calculations would have serious drawbacks for our understanding of the Standard Model .

Using Eqs. (2) and (16) the W mass is determined by expression

$$M_W^2 = \frac{\rho_G M_Z^2}{2}\,(1+\sqrt{1-\frac{4A_0^2}{\rho_G M_Z^2}\,(\frac{1}{1-\Delta\alpha}+\Delta r_{rem})}\;). \tag{20}$$

This relation has been obtained in Ref. 10 for the case of a more general Higgs structure with $\rho \neq 1$ at the tree level. The resummation makes sense for the leading fermion contributions, which form a gauge invariant subset. Since terms like the two-loop irreducible contribution proportional to $\frac{\alpha}{4\pi}\sqrt{2}G_\mu m_t^2 \ln(m_t^2/M_Z^2)$ are unknown, non leading fermionic contributions and the bosonic contributions , including the vertex and box corrections, should be added perturbatively, i.e. included in the term Δr_{rem}.

It is important to stress that Eqs. (16) and (20) properly include the higher order terms only if $\Delta\alpha = \Delta\alpha(\alpha, M_W, M_Z, m_f, m_H)$ is calculated in the α-scheme and $\Delta\rho_G = \Delta\rho(G_\mu, M_W, M_Z, m_f, m_H)$ in the G_μ-scheme!

By defining $\tilde{s}^2$ through the relation

$$\tilde{s}^2\tilde{c}^2 = \frac{A_0^2}{M_Z^2}$$

($\tilde{c}^2 = 1-\tilde{s}^2$) and using Eqs. (2) and (16) we can obtain the $M_W - M_Z$ relation to second order in the large terms

$$\begin{aligned} M_W^2 = \tilde{c}^2 M_Z^2\, \{1 - \frac{\tilde{s}^2}{\tilde{c}^2-\tilde{s}^2}\,(\Delta\alpha - \frac{\tilde{c}^2}{\tilde{s}^2}\Delta\rho_\alpha + \Delta r_{rem}\,) - \frac{\tilde{s}^2(1-3\tilde{s}^2+3\tilde{s}^4)}{(\tilde{c}^2-\tilde{s}^2)^3}\Delta\alpha^2 \\ + \frac{2\tilde{c}^2\tilde{s}^4}{(\tilde{c}^2-\tilde{s}^2)^3}\Delta\alpha\Delta\rho_\alpha + \frac{\tilde{c}^4(1-3\tilde{s}^2)}{(\tilde{c}^2-\tilde{s}^2)^3}(\Delta\rho_\alpha)^2\} \end{aligned} \tag{21}$$

derived in Ref. 11. Here

$$\Delta\rho_\alpha = N_{cf}y_f[1-(2\pi^2-19)y_f+\cdots] \tag{22}$$

where

$$y_f = \frac{\alpha}{16\pi s_W^2 c_W^2}\frac{\Delta m_f^2}{M_Z^2} \tag{23}$$

must be parametrized in the α-scheme with an implicit m_f dependence through $\sin^2\Theta_W$. Another version of Eq. (21) has been presented recently in Ref. 12.

Since the parametrizations of $\Delta\alpha$ and $\Delta\rho_G$ are fixed in order Eqs. (16) and (20) to be valid, the remaining SD is due to Δr_{rem}. Numerical results illustrating the remaining SD are given in Tab. 2. We may summarize the result of Tab. 2 by giving the maximal deviations from the mean value

$$\Delta r = 0.07028 \pm \begin{matrix} 0.00121 \\ 0.00131 \end{matrix}\,.$$

The SD thus suggests a theoretical uncertainty of about [13]

$$\delta(\Delta r)^{higher-order} \simeq 0.0013(0.0009) \tag{24}$$

Table 2: Scheme dependence ($M_Z = 91.17$ GeV, $m_H = 100$ GeV, $m_t = 60$ GeV)

Input Parameters	Δr	$\sin^2\Theta_W$	M_W
α, M_W, M_Z	0.06963	0.23490	79.747
α, G_μ, M_Z	0.07056	0.23524	79.729
α, G_μ, M_W	0.07149	0.23558	79.711
G_μ, M_W, M_Z	0.06897	0.23466	79.759
$\alpha, G_\mu, \sin^2\Theta_{\nu_\mu e}$	0.07074	0.23530	79.725

leading to $\delta M_W \simeq 25(17)\ MeV$ and $\delta \sin^2\Theta_W \simeq 0.0005(0.0003)$. In brackets we have given as a reference the hadronic uncertainty $\delta(\Delta\alpha)^{hadronic} \simeq 0.0009$ of the light quark contributions to the photon vacuum polarization [14].

SCHEME DEPENDENCE OF NC COUPLINGS AT THE Z RESONANCE

We now consider LEP1 observables directly related to the NC process $e^+e^- \to f\bar{f}$ near the Z peak. The tree level widths and cross-sections are given by

$$\begin{aligned} \Gamma_{Zf\bar{f}} &= \frac{\sqrt{2}G_\mu M_Z^3}{3\pi}(v_f^2 + a_f^2)N_{cf} \\ \sigma_{peak}^{f\bar{f}} &\simeq \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2} \end{aligned} \tag{25}$$

where

$$v_f = \frac{I_3^f}{2} - Q_f \sin^2\Theta_W, \quad a_f = \frac{I_3^f}{2} \tag{26}$$

are the neutral current (NC) couplings for fermions with flavor f. All the asymmetries are functions of the NC couplings ratios

$$A_f = \frac{2v_f a_f}{v_f^2 + a_f^2} \tag{27}$$

and thus provide accurate determinations of the weak mixing angle $\sin^2\Theta_W$. At tree level the on-resonance asymmetries are given by

$$A_{FB}^{f\bar{f}} = \frac{3}{4}A_e A_f, \quad A_{LR} = A_{pol}^{\tau} = A_e, A_{FB,pol}^{f\bar{f}} = \frac{3}{4}A_f. \tag{28}$$

Because of the factorization of the non-QED corrections at the resonance, the *weak* corrections of the $Zf\bar{f}$ vertex

$$(\sqrt{2}G_\mu)^{1/2} 2M_Z\gamma^\mu(-Q_f \sin^2\Theta_W + (1-\gamma_5)\frac{I_{3f}}{2})$$

may be included by finite renormalizations [15]

$$\begin{aligned} G_\mu &\to \rho_f G_\mu \\ \sin^2\Theta_W &\to \kappa_f \sin^2\Theta_W, \end{aligned} \tag{29}$$

where $\rho_f = 1 + \Delta\rho_{se} + \Delta\rho_{f,vertex}$ and $\kappa_f = 1 + \Delta\kappa_{se} + \Delta\kappa_{f,vertex}$. The potentially large self-energy contributions (se) are universal. The analogues of Eq. (9) for $\Delta\rho$ and $\Delta\kappa$ read

$$\begin{aligned} \Delta\rho_{se} &= \Delta\rho + \Delta\rho_{se,rem} \\ \Delta\kappa_{se} &= \frac{c_W^2}{s_W^2}\Delta\rho + \Delta\kappa_{se,rem} \end{aligned} \tag{30}$$

with $\Delta\rho$ defined in Eq. (10). The vertex contributions are (if $f \neq b$) relatively small (but not negligible) and flavor dependent. We may define effective $\sin^2\Theta$'s by

$$\begin{aligned} \sin^2\Theta_f &= \kappa_f \sin^2\Theta_W \\ &\simeq (1 + \Delta\kappa_{f,vertex}) \sin^2\bar{\Theta} \\ \sin^2\bar{\Theta} &= (1 + \Delta\kappa_{se}) \sin^2\Theta_W \end{aligned} \tag{31}$$

where $\sin^2\bar{\Theta}$ is a flavor independent auxiliary quantity. The $\sin^2\Theta$'s can be defined directly in terms of the input parameters α, G_μ and M_Z by generalizing

$$\sqrt{2}G_\mu M_Z^2 \cos^2\Theta_W \sin^2\Theta_W = \frac{\pi\alpha}{(1-\Delta r)} \tag{32}$$

to

$$\sqrt{2}G_\mu M_Z^2 \cos^2\bar{\Theta} \sin^2\bar{\Theta} = \frac{\pi\alpha}{(1-\Delta\bar{r})} \tag{33}$$

if we include the self-energy contributions only, or,

$$\sqrt{2}G_\mu M_Z^2 \cos^2\Theta_f \sin^2\Theta_f = \frac{\pi\alpha}{(1-\Delta r_f)} \tag{34}$$

if we include the vertex corrections as well. For the Δr's we find

$$\begin{aligned} \Delta\bar{r} &= \Delta r + \frac{\bar{c}^2 - \bar{s}^2}{\bar{c}^2}\Delta\kappa_{se} \\ \Delta r_f &= \Delta\bar{r} + \frac{\bar{c}^2 - \bar{s}^2}{\bar{c}^2}\Delta\kappa_{f,vertex}. \end{aligned} \tag{35}$$

Using Eq. (30) we obtain

$$\Delta\bar{r} = \Delta\alpha - \Delta\rho + \Delta\bar{r}_{rem}. \tag{36}$$

In all cases the $\Delta\alpha$ term is identical. The leading heavy top dependence is given by

$$\begin{aligned} \Delta\bar{r}^{top} &= \frac{\sqrt{2}G_\mu M_W^2}{16\pi^2}\left\{-3\frac{m_t^2}{M_W^2}\right\} \\ \Delta r_b^{top} &= \frac{\sqrt{2}G_\mu M_W^2}{16\pi^2}\left\{-\frac{1+s_W^2}{c_W^2}\frac{m_t^2}{M_W^2}\right\}. \end{aligned} \tag{37}$$

Except from extra top contributions from the $Zb\bar{b}$-vertex all heavy particle effects are universal i.e. $\Delta r_{f\neq b}^{top} = \Delta\bar{r}^{top}$ and $\Delta r_f^{Higgs} = \Delta\bar{r}^{Higgs}$.

Figure 5 exhibits the different behavior as a function of m_t of the various Δr's.

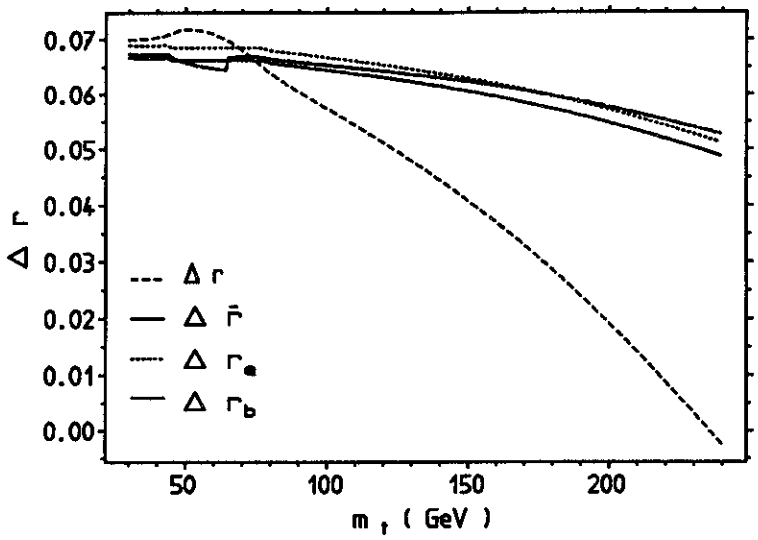

Figure 5: Δr's defined in Eqs. (32-34) as functions of the top mass ($M_Z = 91\ GeV$, $m_H = 100\ GeV$)

For the proper resummation of the large higher terms we obtain

$$\frac{1}{1-\Delta r_f} \rightarrow \frac{1-\Delta\rho_G}{1-\Delta\alpha} + \Delta r_{f,rem}.$$

Again we may estimate the theoretical uncertainty by evaluating the remainder $\Delta r_{f,rem}$ in different parametrizations. Numerical results are given in Tab. 3.

Table 3: Scheme dependence ($M_Z = 91.17$ GeV, $m_H = 100$ GeV, $m_t = 60$ GeV)

Input Parameters	Δr_e	$\sin^2\Theta_e$	ρ_e
α, M_W, M_Z	0.06743	0.23410	0.99749
α, G_μ, M_Z	0.06789	0.23427	0.99805
α, G_μ, M_W	0.06840	0.23445	0.99796
G_μ, M_W, M_Z	0.06696	0.23393	0.99795
$\alpha, G_\mu, \sin^2\Theta_{\nu_\mu e}$	0.06795	0.23429	0.99775

Input Parameters	Δr_b	$\sin^2\Theta_b$	ρ_b
α, M_W, M_Z	0.06409	0.23290	1.01844
α, G_μ, M_Z	0.06455	0.23306	1.01749
α, G_μ, M_W	0.06508	0.23325	1.01744
G_μ, M_W, M_Z	0.06360	0.23272	1.01745
$\alpha, G_\mu, \sin^2\Theta_{\nu_\mu e}$	0.06461	0.23308	1.01731

From Tab. 3 we obtain

$$\Delta r_e = 0.06773 \pm \begin{matrix} 0.00067 \\ 0.00077 \end{matrix} \qquad \Delta r_b = 0.06439 \pm \begin{matrix} 0.00069 \\ 0.00079 \end{matrix}$$

or $\sin^2\Theta_f \simeq 0.0003$. This uncertainty is smaller than the one obtained in Eq. (24) for the W mass prediction.

The corrected widths and asymmetries can be obtained using Eqs. (25) and (28) together with the $\sin^2\Theta_f$'s given in the Tab. 3. In case of hadronic final states a QCD correction factor must be added.

CONCLUSIONS

One-loop predictions of measurable quantities are approximations which depend on the choice of the physical input parameters (to be chosen in a self-consistent way). The differences obtained signal missing higher order terms which must be interpreted as a theoretical uncertainty.

In order to reach the precision required at LEP/SLC, leading higher order effects must be included, thereby the scheme dependence is substantially reduced (by about a factor 5). The remaining uncertainties turn out to be small enough such that they do not obscure measurable effects.

A conservative estimate of the higher order uncertainties obtained by studying the scheme dependence of typical observables yields

$$\delta(\Delta r)^{higher-order} \simeq \delta(\Delta r)^{hadronic} \simeq 0.001 \tag{38}$$

which amounts to a uncertainty of $\delta M_W \simeq 20$ MeV in the W mass and $\delta\sin^2\Theta_W \simeq 0.0004$ in $\sin^2\Theta_W$.

Table 4: Sensitivity of Δr, Eqs. (32-34), to top and Higgs effects versus experimental precision for various measurements. The top contribution $\delta(\Delta r)^{top}$ is given for m_t = 200 GeV, M_Z = 92 GeV and $\sin^2\Theta_W$ = 0.23. For $\delta(\Delta r)^{Higgs}$ we give the shift obtained if we change m_H from 0 to 1 TeV.

	Error	$\delta\Delta r^{exp}$	$\delta\Delta r^{theo}$	$\delta\Delta r^{top}$	$\delta\Delta r^{Higgs}$
M_W	100 MeV	0.0056	0.0017	-0.0494	0.0156
$A_{FB}^{\mu\mu}$	0.025	0.0043	0.0012	-0.0136	0.0067
A_{LR}	0.003	0.0013	0.0012	-0.0136	0.0067
$A_{FB,pol}^{b\bar{b}}$	0.027	0.0028	0.0012	-0.0028	0.0067

We have not considered the scheme dependence of the QCD corrections to the heavy top contribution. We refer to J. Kühn's contribution [16] for a discussion of the QCD corrections. The uncertainties from this source are $\delta(\Delta r)^{top} = 0.0005$ and

$\delta(\Delta r_e)^{top} = 0.00015$ for $m_t < 150\ GeV$.

Table 4 summarizes expected experimental uncertainies compared with effects obtained from virtual top quark or Higgs boson effects. The theoretical uncertainty includes the hadronic error, the higher-order uncertainty and the uncertainty in the perturbative QCD correction of the top contribution, added in quadrature.

In should be kept in mind that full two-loop calculations still could yield unexpected answers. From experience with QCD, we know that surprises concerning the size of next order results are possible in non-abelian gauge theories.

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