An improved evaluation of the running $\sin^2\Theta_{\rm eff}$ and some applications

Fred Jegerlehner*
HU Berlin/DESY Zeuthen,
fjeger@physik.hu-berlin.de

Working Group on Radiative Corrections
and Generators for Low Energy Hadronic Cross Section and Luminosity
Frascati, April 16-17, 2012

Abstract

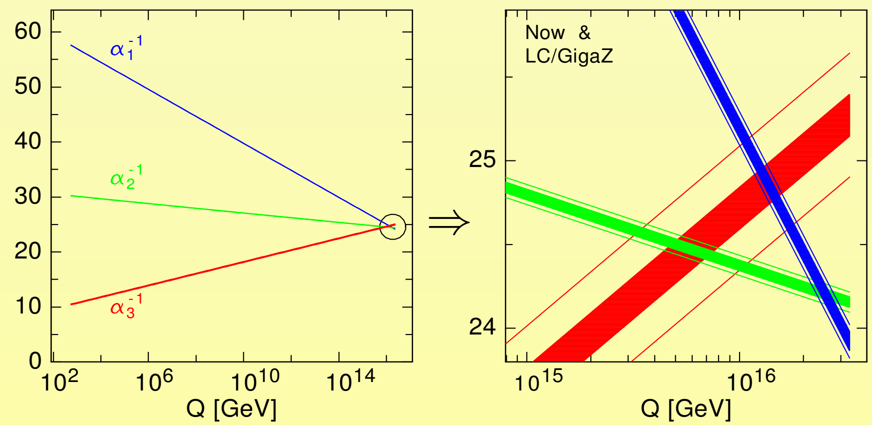
While the hadronic component of the effective fine structure constant $\alpha_{\rm em}(s)$ can be evaluated directly in terms of experimental hadron production cross section $e^+e^- \to$ hadrons, similar parameter shifts of other SM parameters like the $SU(2)_L$ effective coupling $\alpha_2(s)$ or the weak mixing parameter $\sin^2\Theta_{\rm eff}(s) = \alpha(s)/\alpha_2(s)$ require appropriate reweighting of the flavor composition measured in e^+e^-–annihilation. A new evaluation is presented and compared with previous estimates.

History: the hadronic shift $\Delta\alpha^{(5)}_{2\,\rm had}$ has been calculated earlier based on e^+e^--annihilation data using the flavor separation procedure proposed and investigated in F. J., *Hadronic Contributions to Electroweak Parameter Shifts* Z. Phys. C **32** (1986) 195.

Motivation

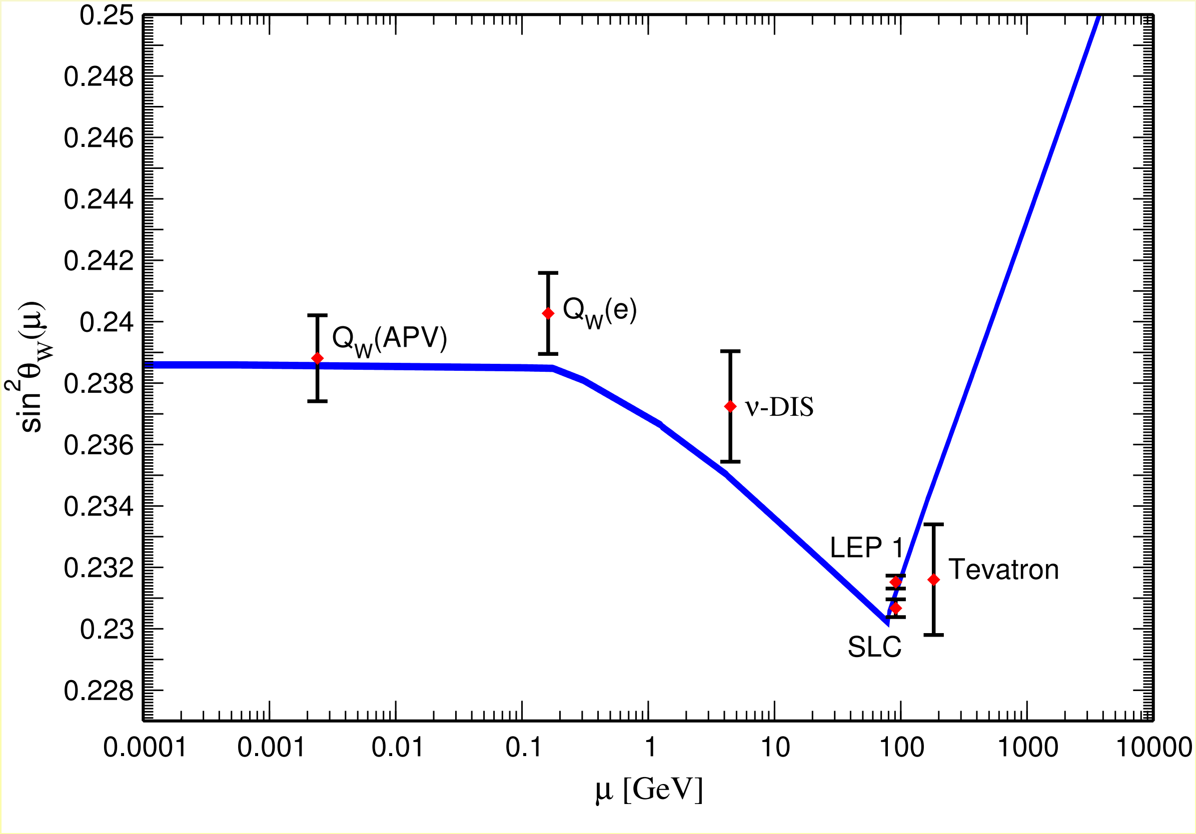
SM two independent gauge couplings: $g, g' \to e, g \to \alpha, \sin^2\Theta_W$. Role of running $\alpha_{\rm eff}(s)$ for precision tests well known, what about $\sin^2\Theta_{\rm eff}(s)$? Neutral Current processes mediated by Z exchange. Role in Z physics ($A_{\rm FB}, A_{\rm LR}$ in $e^+e^- \to e^+e^-$ etc.), neutrino scattering ($\nu_\mu e^- \to \nu_\mu e^-$ etc.), polarized Møller scattering asymmetries ($e^-e^- \to e^-e^-$) (Czarnecki & Marciano) etc. Possible GUT coupling unification of α, α_2 with QCD coupling $\alpha_3 = \alpha_s$. s-channel vs. t-channel processes.

from Zerwas



$\alpha_s = 0.1183 \pm 0.0027$ vs ± 0.0009

See e.g. 2011/12 Review of Particle Properties: Sec. 10. ELECTROWEAK MODEL AND CONSTRAINTS ON NEW PHYSICS (Erler & Langacker)



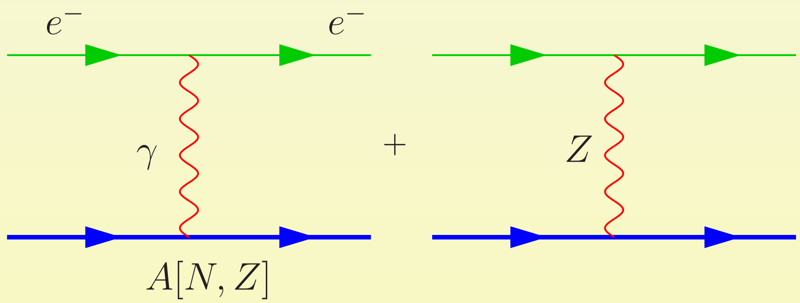
In the following: reference coupling is $\alpha_2 = \alpha / \sin^2 \Theta_e$ with $\sin^2 \Theta_e \equiv \sin^2 \Theta_{\mathrm{eff}}^{\mathrm{lep}} = 0.23135$ from the LEPEEWG. The is a pretty good approximation for a low energy effective weak mixing angle, since $\sin^2 \Theta_{\mathrm{eff}}$ is weakly running only between the Z mass scale M_Z down to low energies. Below gauge boson thresholds $\Delta\alpha = \Delta\alpha_{\mathrm{fermions}}$ [6%] and $\Delta\alpha_2 = \Delta\alpha_{2\mathrm{fermions}}$ are of the same sign and cancel substantially in $\sin^2 \Theta_{\mathrm{eff}} \sim \alpha/\alpha_2$ [-2%].

A typical application is $\sin^2 \Theta$ as measured in neutrino scattering:

$$\sin^2 \Theta_e = \left\{ \frac{1 - \Delta\alpha_2}{1 - \Delta\alpha} + \Delta_{\nu_\mu e,\mathrm{vertex+box}} + \Delta\kappa_{e,\mathrm{vertex}} \right\} \sin^2 \Theta_{\nu_\mu e}$$

The first correction from the running coupling ratio is largely compensated by the ν_μ charge radius which dominates the second term. The ratio $\sin^2 \Theta_{\nu_\mu e} / \sin^2 \Theta_e$ is close to 1.002, independent of top and Higgs mass. Note that errors in the ratio $\frac{1-\Delta\alpha_2}{1-\Delta\alpha}$ can be taken to be 100% correlated and thus largely cancel.

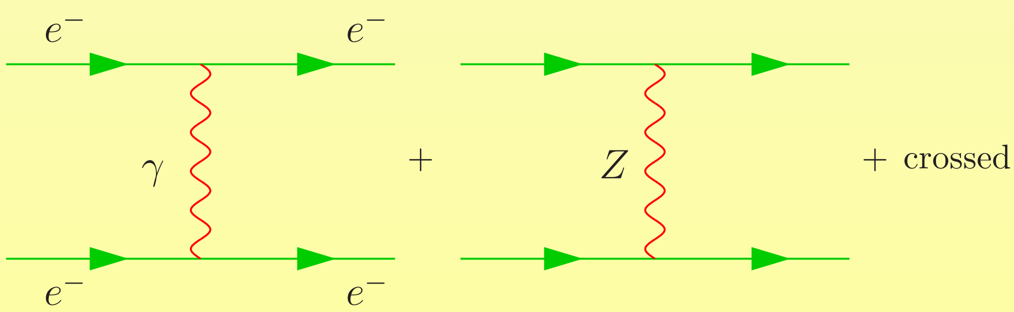
Parity violation in atoms with Z protons and N neutrons:

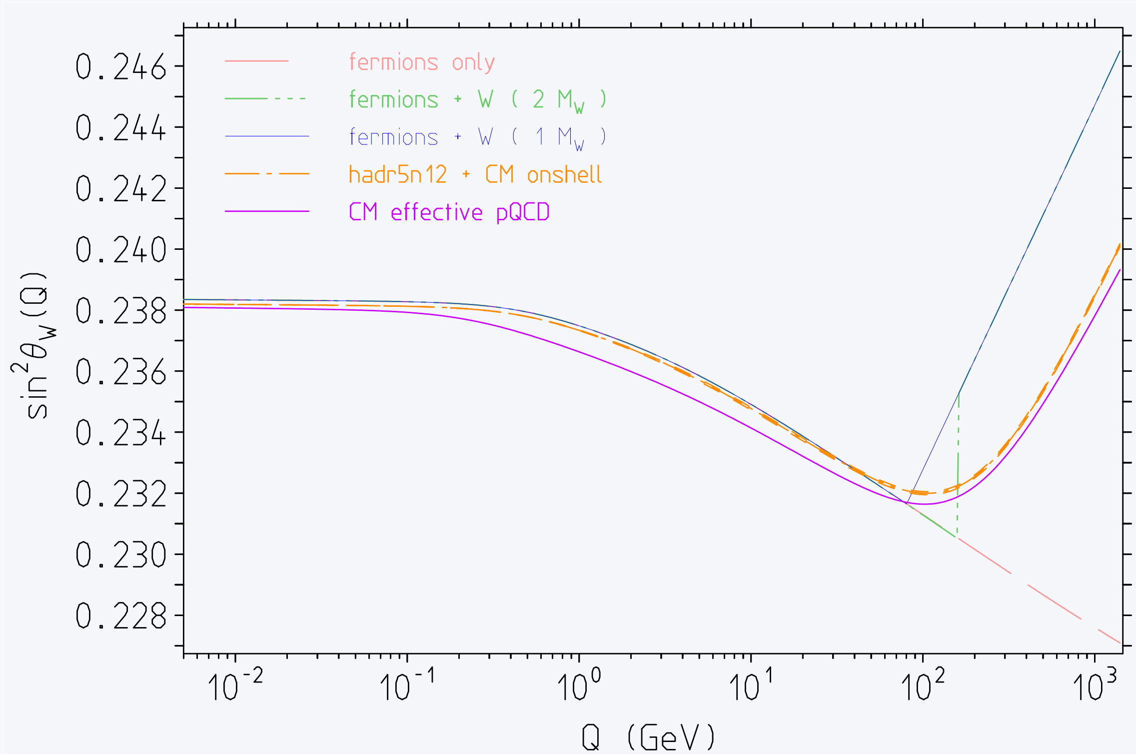


the weak charge is defined by

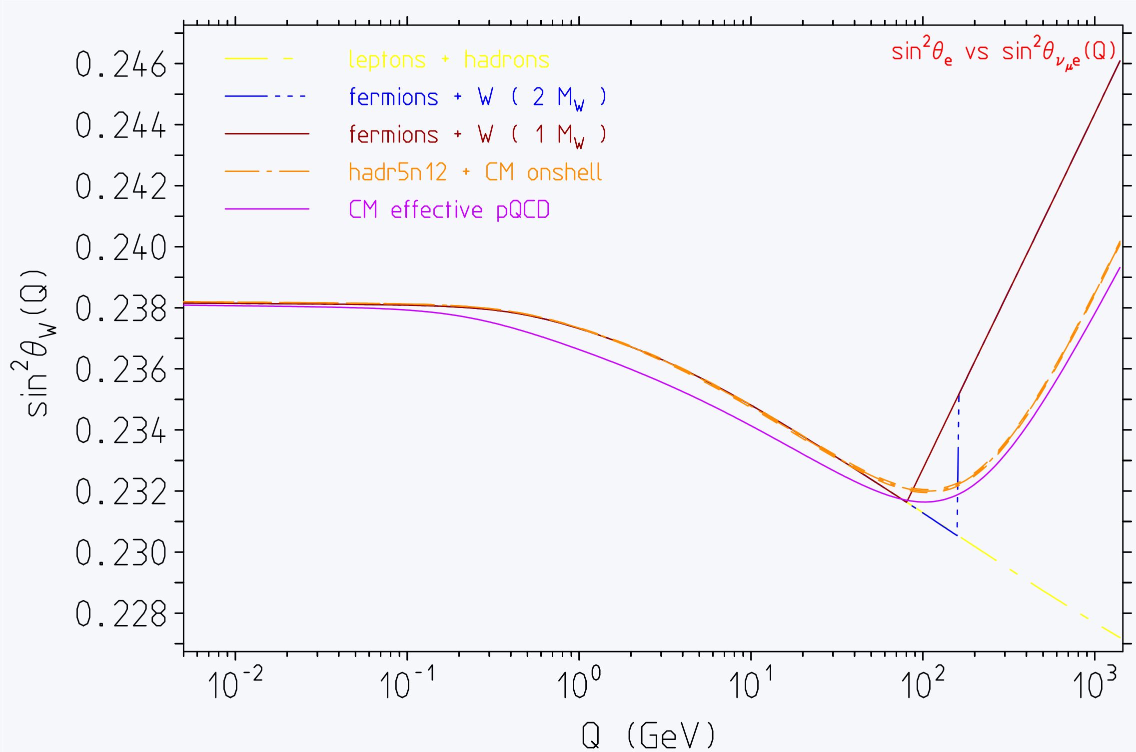
$$Q_w = -4a_e \ \{v_u\,(2Z+N) + v_d\,(Z+2N)\} \simeq Z\,(1-4\ \sin^2\Theta_{\rm eff})\ -N$$

is particularly sensitive to Z' in GUT scenarios.
Møller scattering:





Scheme dependence in $\sin^2\Theta_{\rm eff}$ predictions. Compared: alphaQED/alpha2SM no W, W in $\overline{\rm MS}$ $2M_W$ as threshold, same M_W as threshold, on-shell Møller scattering and the same using pQCD with appropriate effective quark masses.



On-shell $\nu_\mu e^-$ scattering corrections included. Compared: `alphaQED/alpha2SM` no W, W in $\overline{\mathrm{MS}}$ $2M_W$ as threshold, same M_W as threshold, on-shell Møller scattering and the same using pQCD with appropriate effective quark masses.

How to calculate $\Delta\alpha^{(5)}_{2\,\mathrm{had}}$

All evaluations based on end of 2011 update of e^+e^-–annihilation data:

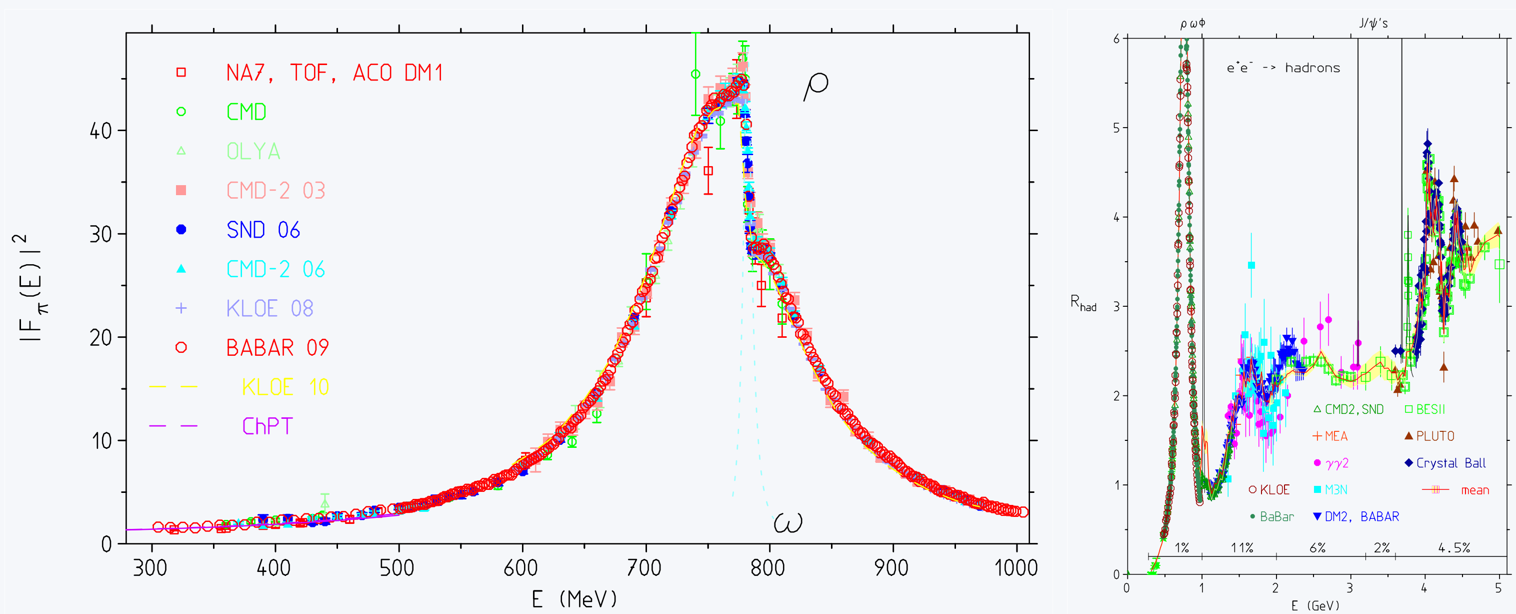


Figure 1: A recent compilation of the available *R*-data

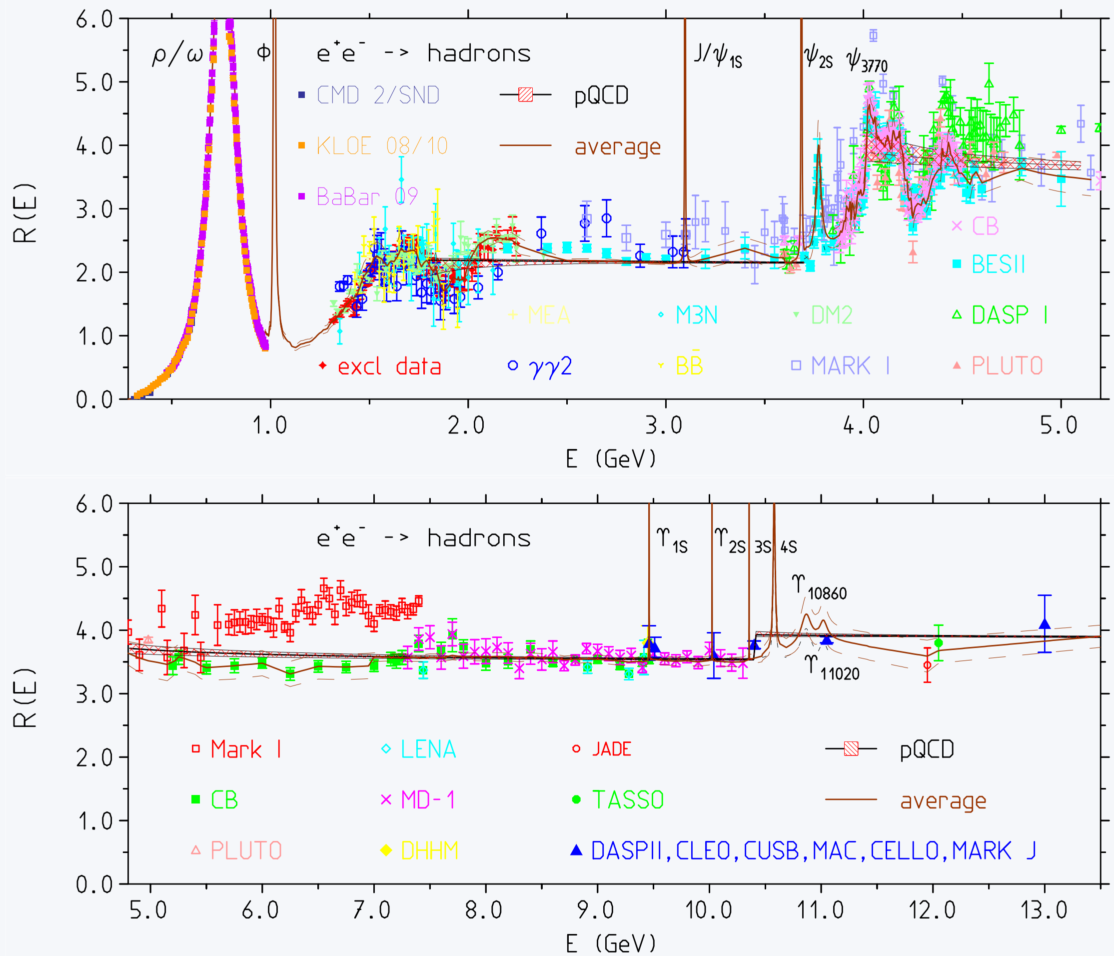


Figure 2: R-data versus pQCD

New evaluation of $\alpha_2(s)$ relies on available

exclusive channels data below 2.1 GeV.

Above that energy many missing channels. Most important BaBar exclusive data.

SM radiative corrections are dominated by gauge boson self-energy effects! A major part are non-perturbative hadronic correction in $\gamma\gamma$, γZ and ZZ self energies, as

$$
\begin{aligned}
\Pi^{\gamma\gamma} &= e^2\,\hat{\Pi}^{\gamma\gamma}\;; \\
\Pi^{Z\gamma} &= \frac{eg}{c_\Theta}\,\hat{\Pi}_V^{3\gamma} - \frac{e^2\,s_\Theta}{c_\Theta}\,\hat{\Pi}_V^{\gamma\gamma}\;; \\
\Pi^{ZZ} &= \frac{g^2}{c_\Theta^2}\,\hat{\Pi}_{V-A}^{33} - 2\,\frac{e^2}{c_\Theta^2}\,\hat{\Pi}_V^{3\gamma} + \frac{e^2\,s_\Theta^2}{c_\Theta^2}\,\hat{\Pi}_V^{\gamma\gamma}\;; \\
\Pi^{WW} &= g^2\,\hat{\Pi}_{V-A}^{+-}
\end{aligned}
$$

Leading non-perturbative effects, with $\hat{\Pi}(s) = \hat{\Pi}(0) + s\hat{\pi}(s)$,

$$\Delta\alpha = -e^2 \left[\mathrm{Re}\,\hat{\pi}^{\gamma\gamma}(s) - \hat{\pi}^{\gamma\gamma}(0)\right] ,$$

$$\Delta\alpha_2 = -\frac{e^2}{s_\Theta^2} \left[\mathrm{Re}\,\hat{\pi}^{3\gamma}(s) - \hat{\pi}^{3\gamma}(0)\right] .$$

The latter exhibit the

leading hadronic non-perturbative parts,

i.e. the ones involving the photon field via mixing.

- $\langle 33 \rangle$ is most closely related to the charged channel $\langle +- \rangle$ correlator
- $\langle +- \rangle$ is accessible directly via τ–decay spectra accessible for energies below 1.8 GeV. The decay spectra have been measured only for a few channels so far.
- $V - A$ correlators $VV + AA$ are decomposeable into $2VV$ plus $(AA - VV)$ where the latter is expected to be much smaller: $|AA - VV|/2VV \ll 1$ at larger energies.

On hadronic currents and correlators

Electromagnetic current:

$$j^\mu_{\mathrm{em}} = \frac{2}{3}\,\bar{u}\gamma^\mu u - \frac{1}{3}\,\bar{d}\gamma^\mu d - \frac{1}{3}\,\bar{s}\gamma^\mu s + \cdots$$

Weak isovector current:

$$j^\mu_3 = \frac{1}{2}\,\bar{u}\gamma^\mu u - \frac{1}{2}\,\bar{d}\gamma^\mu d - \frac{1}{2}\,\bar{s}\gamma^\mu s + \cdots$$

Correlators in SU(3) limit: $m_u = m_d = m_s$; $\langle uu \rangle \simeq \langle dd \rangle \simeq \langle ss \rangle$ etc.

$$
\begin{aligned}
\langle \gamma\gamma \rangle &\sim \frac{6}{9}\langle uu \rangle - \frac{6}{9}\langle ud \rangle = \frac{2}{3}\,(\langle uu \rangle - \langle ud \rangle) \\
\langle \gamma 3 \rangle &\sim \frac{4}{6}\langle uu \rangle - \frac{4}{6}\langle ud \rangle = \frac{2}{3}\,(\langle uu \rangle - \langle ud \rangle) \\
\langle 33 \rangle &\sim \frac{3}{4}\langle uu \rangle - \frac{2}{4}\langle ud \rangle = \frac{3}{4}\,(\langle uu \rangle - \langle ud \rangle) + \frac{1}{4}\,\langle ud \rangle
\end{aligned}
$$

In this case

$$
\langle \gamma 3 \rangle_{uds} = \langle \gamma\gamma \rangle_{uds}\ ;\ \ \langle 33 \rangle_{uds} \simeq \frac{9}{8}\,\langle \gamma\gamma \rangle_{uds} + O(\frac{\langle ud \rangle}{\langle uu \rangle})\ .
$$

Correlators in SU(2) limit: $m_u = m_d$; $\langle uu\rangle \simeq \langle dd\rangle$ etc.

$$\begin{aligned}
\langle\gamma\gamma\rangle &\sim \frac{5}{9}\langle uu\rangle - \frac{4}{9}\langle ud\rangle + \frac{1}{9}\langle ss\rangle - \frac{2}{9}\langle us\rangle \sim \frac{5}{9}\langle uu\rangle + \frac{1}{9}\langle ss\rangle + O(\frac{\langle ud\rangle}{\langle uu\rangle}, \frac{\langle us\rangle}{\langle ss\rangle}) \\
\langle\gamma 3\rangle &\sim \frac{1}{2}\langle uu\rangle - \frac{1}{2}\langle ud\rangle + \frac{1}{6}\langle ss\rangle - \frac{1}{6}\langle us\rangle \sim \frac{1}{2}\langle uu\rangle + \frac{1}{6}\langle ss\rangle + O(\frac{\langle ud\rangle}{\langle uu\rangle}, \frac{\langle us\rangle}{\langle ss\rangle}) \\
\langle 33\rangle &\sim \frac{1}{2}\langle uu\rangle - \frac{1}{2}\langle ud\rangle + \frac{1}{4}\langle ss\rangle \sim \frac{1}{2}\langle uu\rangle + \frac{1}{4}\langle ss\rangle + O(\frac{\langle ud\rangle}{\langle uu\rangle})
\end{aligned}$$

As indicated, here there are no simple relations between (in the symmetry limit) known combinations. The only way is to assume that the off-diagonal elements are sub-dominant $|\langle ud\rangle| \ll |\langle uu\rangle| = |\langle dd\rangle|$ as well as $|\langle us\rangle| = |\langle ds\rangle| \ll |\langle ss\rangle|$ i.e.

we are assuming OZI-rule violation small in this particular observable $\Delta\alpha^{(5)}_{2\,\mathrm{had}}$.

This yields the re-weightings:

$$\begin{aligned}
\langle uu \rangle &\simeq \frac{9}{5}\langle \gamma\gamma \rangle_{u,d}\ ;\ \ \langle ss \rangle \simeq 9\langle \gamma\gamma \rangle_s \\
\langle \gamma 3 \rangle_{ud} &\simeq \frac{9}{10}\langle \gamma\gamma \rangle_{ud}\ ;\ \ \langle \gamma 3 \rangle_s \simeq \frac{9}{6}\langle \gamma\gamma \rangle_s \\
\langle 33 \rangle_{ud} &\simeq \frac{9}{10}\langle \gamma\gamma \rangle_{ud}\ ;\ \ \langle 33 \rangle_s \simeq \frac{9}{4}\langle \gamma\gamma \rangle_s
\end{aligned}$$

SM gauge boson self-energy contributions are expressed in terms of $J_3^\mu = \frac{1}{2}\, j_3^\mu$ such that $\hat{\pi}^{3\gamma}(s) - \hat{\pi}^{3\gamma}(0) \ \Leftrightarrow \frac{1}{2}\langle 3\gamma \rangle$ and $\hat{\pi}^{3\gamma}(s) - \hat{\pi}^{3\gamma}(0) \ \Leftrightarrow \frac{1}{4}\langle 33 \rangle$.

Flavor separation by hand:

we skip all final states involving photons like: $\pi^0\gamma$, $\eta\gamma$ channels, including η' (data not yet available?)

- as $ud,\ I=0$ we include states with odd number of pions
- as $ud,\ I=1$ we include states with even number of pions
- as $\bar{s}s$ we count all states with Kaons

States ηX with X some other hadrons are collected separately, and then split into $q=u,d$ and s components by appropriate mixing.

Flavor separation is possible only in regions where exclusive channel cross sections are available. We perform this in the region 0.61 GeV to 2.1 GeV. Above this energy only inclusive $R(s)$ measurements are available.

Above about 1.5 GeV there is an off-set of -2.2% (mainly due to SU(3) breaking by Kaon mass effects).

The following tabular illustrates where the main difference comes from, here evaluated at $s = M_Z^2[\,500\ \mathrm{GeV}]$ (values in units $\sin^2\Theta_{\mathrm{eff}}\,\Delta g \times 10^4$)

	old SU(3) scheme	new SU(2) scheme
below c threshold	$\Delta g_{uds} = 35.183\,[35.171]$	$\Delta g_{usd} = 31.679\,[31.667]$
above c threshold	$\Delta g_{\mathrm{rem}} = 95.784[144.735]$	$\Delta g_{\mathrm{rem}} = 95.809[144.761]$

As expected the surprisingly large shift comes from the low energy region. The shift persists at higher energies since, with t_c the charm threshold,

$$\Delta g(q^2) = -\frac{g^2\,q^2}{12\pi^2}\int_{s_0}^{t_c}\frac{\mathrm{d}s}{s}\,\frac{R^{3\gamma}(s)}{s-q^2-\mathrm{i}\varepsilon} \overset{q^2 \gg t_c}{\sim} \frac{\alpha_2}{3\pi}\int_{s_0}^{t_c}\frac{\mathrm{d}s}{s}\,R^{3\gamma}(s)$$

independent of q^2. It means that at large q^2 our effective coupling cannot be expected to agree with the perturbative result. This remains true without applying a cut-off of course.

In a way the “observed” shift in the high energy tail is a violation of global quark-hadron duality. The latter is expected to be exact only in the large N_c limit $N_c \to \infty$ anyway.

Correlators $\langle\gamma\gamma\rangle$, $\langle 3\gamma\rangle$ and $\langle 33\rangle$ can be simulated from first principles in

lattice QCD.

Thus in coming years we expect progress in true checks of the “flavor separation” in corresponding approximations made.

Active groups: H. Wittig, Uni Mainz, H. Meyer, Uni Mainz, K. Jansen, NIC Zeuthen

Standard Model $SU(2)_L$ coupling $\alpha_2 = g^2/4\pi$

Calculation implemented in package

`alphaQED`,

- $\Delta\alpha$ real and complex: `alphaQEDreal.f`, `alphaQEDcomplex.f`
- $\Delta\alpha_2$ real and complex: `alpha2SMreal.f`, `alpha2SMcomplex.f`
- tables of hadronic shifts `der,errder,deg,errdeg` in file `hadr5n12.f` [updated/extended],

In order to change the reference coupling one has to rescale the factor

$$1/\sin^2\Theta_e$$

This is automatically done by changing the input line

```
st2=0.23153d0 !  Reference value for weak mixing parameter
```

in the main program. The tables for the hadronic shift `deg,errdeg` contained in the file `hadr5n12.f` are given for $\alpha/(\mathrm{st2} = 0.23153)$ as an overall factor. Contributions are rescaled to the st2 value specified in the main program, or in the calling routine

```
call hadr5n12(e,st2,der,errder,deg,errdeg)
```

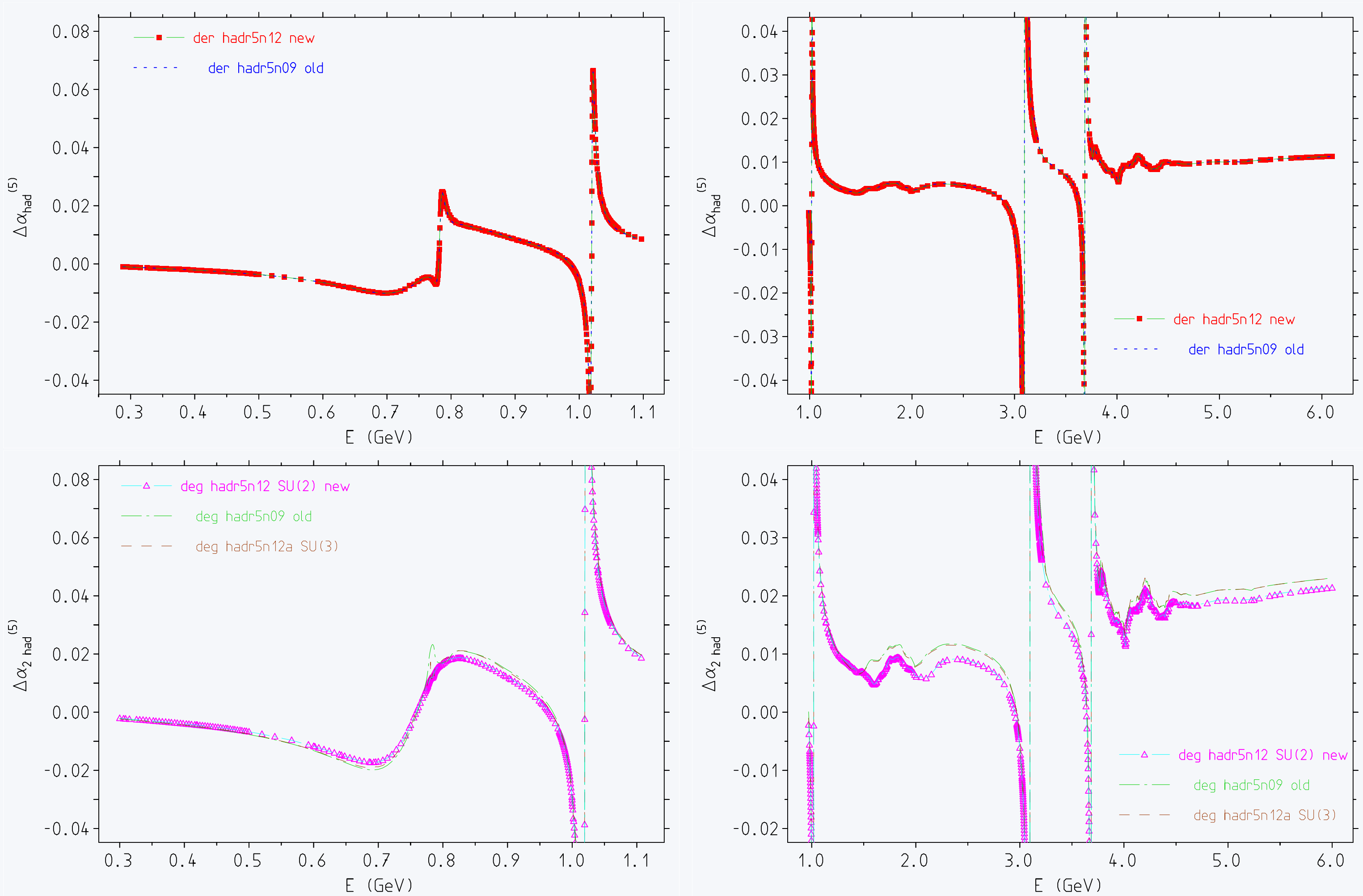


Figure 3: ρ, ω, ϕ and J/ψ regions: 2012 vs 2009 routines

Complex vs. real α VP correction

In our analysis we have subtracted vacuum polarization (VP) effects by replacing the real running $\alpha(s)$ by α, i.e. $R(s)$ is corrected by $(\alpha/\alpha(s))^2 = |1 - \mathrm{Re}\,\Pi'(s)|^2$ ($\Pi'(0)$ subtracted). More precisely, one actually has to subtract $|1 - \Pi'(s)|^2 = \alpha/|\alpha_c(s)|)^2$ where $\alpha_c(s)$ is the complex generalization of its real counterpart. This is what the Novosibirsk CMD-2 Collaboration has been using in recent analyzes. The corresponding code has been made public recently and is available from Fedor Ignatov's Web page ✲≫. In the following figure we plot the correction

$$1 - |1 - \Pi'(s)|^2/(\alpha/\alpha(s))^2$$

as a function of energy. Typically, corrections are below the one per mill level, except at resonances where corrections are the larger the smaller the widths:

Note: imaginary parts from narrow resonances, $\mathrm{Im}\,\Pi'(s)) = \frac{\alpha}{3} R(s) = \frac{3}{\alpha} \frac{\Gamma_{ee}}{\Gamma}$ at peak, are sharp spikes and are obtained correctly only by appropriately high resolution scans. For example,

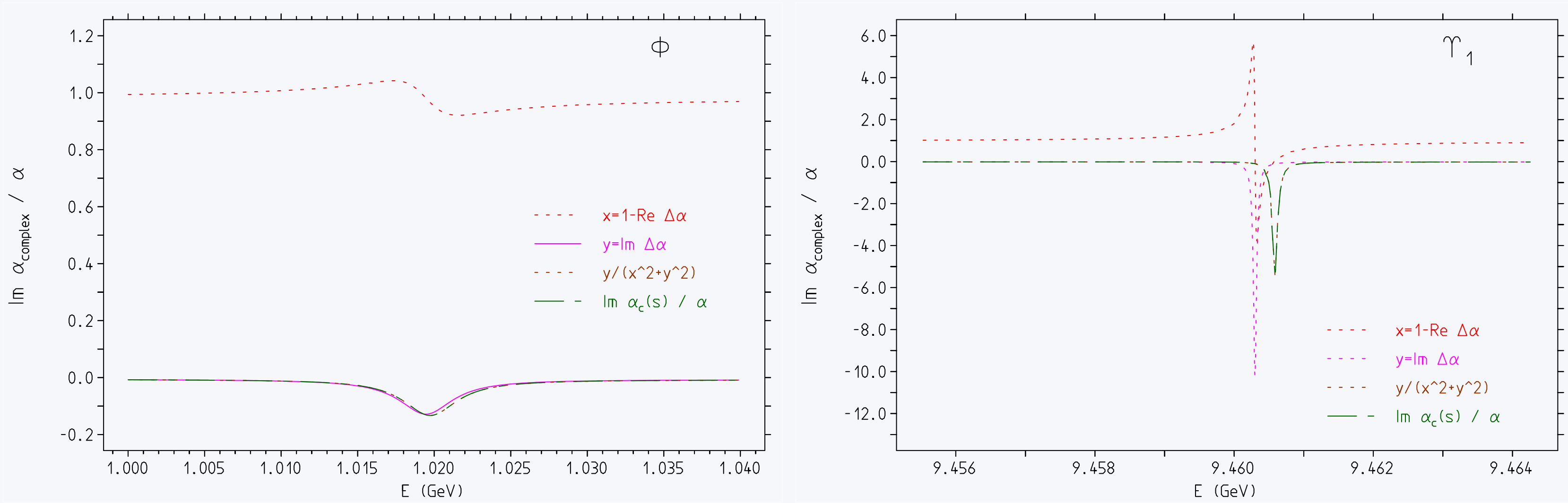
$$|1-\Pi'(s)|^2-(\alpha/\alpha(s))^2=(\mathrm{Im}\,\Pi'(s))^2$$

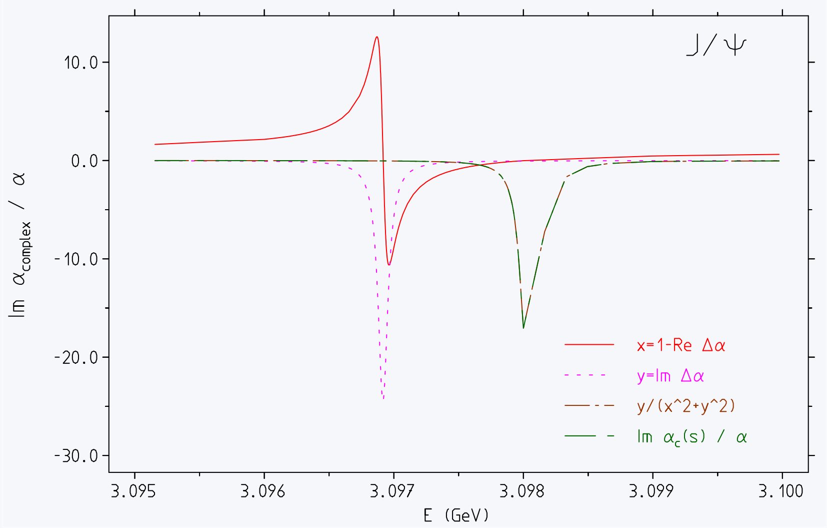
at $\sqrt{s}=M_R$ is given by

ρ	1.23×10^{-3}	J/ψ	594.81	Υ_1	104.26
ω	2.76×10^{-3}	ψ_2	9.58	Υ_2	30.51
ϕ	1.56×10^{-2}	ψ_3	2.66×10^{-4}	Υ_3	55.58

Except for the ρ and ψ_{4-6} narrow resonances are taken into account as Breit-Wigner resonances, starting with physical parameters as listed by the PDG. They thus have to be undressed (VP subtraction) by renormalizing it with $(\alpha/\alpha(s))^2$. We actually apply the complex running coupling, because the real version has Landau poles at the resonances $J/\psi, \psi_2$ and $\Upsilon_1, \Upsilon_2, \Upsilon_2$.

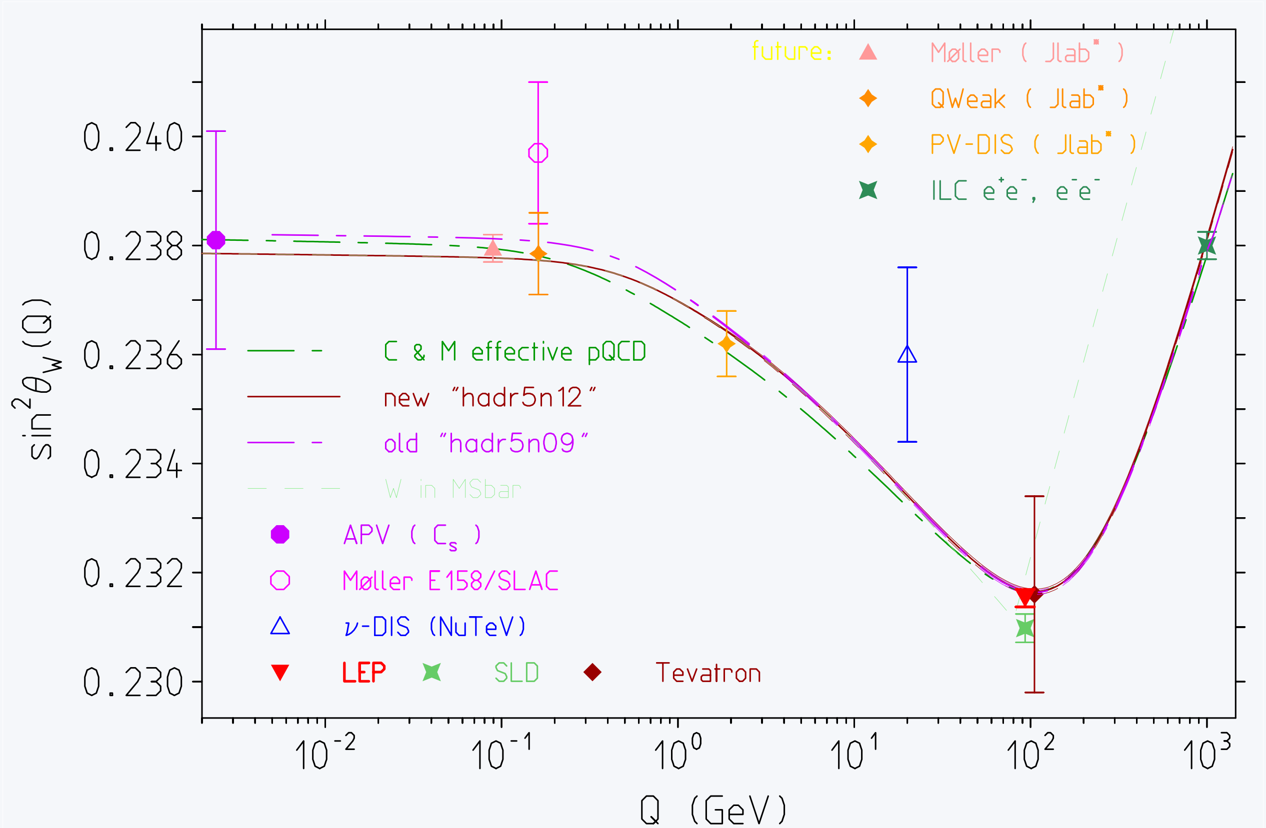
The tables in hadr5n12.f have been obtained by integrating the VP subtracted data collected in Rdat_all.f and processed via Rdat_fun.f without the narrow resonances (options iresonances=0 and IRESON=0). The resonance contributions are integrated separately with appropriate renormalization (undressing = VP subtraction). If resonances are switched on in Rdat_fun.f and Rdat_fit.f renormalization has to be applied in the Breit-Wigner function FUNCTION BW(S,M,G,P) attached in Rdat_fun.f. Complex renormalization effects (provided by FUNCTION BWRENO(s,fracerr) are dramatic for the very narrow resonances in particular for J/ψ and Υ_1.


φ
Im α_complex / α
E (GeV)
x=1-Re Δα
y=Im Δα
y/(x^2+y^2)
Im α_c(s) / α
ϒ 1
Im α_complex / α
E (GeV)
x=1-Re Δα
y=Im Δα
y/(x^2+y^2)
Im α_c(s) / α



For very narrow resonances the Breit-Wigner "bump" appears shifted dramatically (about 6 $\times\Gamma_{J/\psi}$ for the J/ψ) in the imaginary part of $\alpha_c(s)$. The real version of $\alpha(s)$ would have a Landau pole. For the ϕ corrections are in the "perturbative regime", i.e. $|\mathrm{Re}\ \Delta\alpha|, |\mathrm{Im}\ \Delta\alpha| \ll 1$.

Final result for $\sin^2\Theta_{\rm eff}$ compared with data



All predictions are adjusted to $\sin^2\Theta_{\rm eff}(M_Z)$ from LEP1

Legend:

- Curves show various predictions:
  - ❶ Czarnecki & Marciano using pQCD with effective quark masses together with bosonic ($W, Z-\gamma$) corrections (self-energy,vertex+box) determined by on-shell polarized Møller scattering,
  - ❷ the same using hadronic VP effects based on new SU(2) flavor splitting (`hadr5n12.f`),
  - ❸ the same using hadronic VP effects based on old SU(3) flavor splitting (`hadr5n09.f`),
  - ❹ as "❷ " but W contribution in $\overline{\mathrm{MS}}$ scheme, switching on the W at M_W (not $2M_W$)

- Data points as selected by Erler & Langacker in their PDG Review (see Fig. reproduced above)

- Expected "data points" from measurements planned at JLab illustrating future precisions possible. Mainz plans to measure the weak charge of the proton $Q_W(p)$.

- Expected "data points" from measurements possible a future linear collider.

Outlook

- Precision measurements of the “running” of $\sin^2\Theta_{\mathrm{eff}}$ able to provide sensitive tests of new physics scenarios

- Hadronic effects not obtainable from data only, e.g. assumptions on OZI rule violation. Above 2 GeV pQCD fairly save for non-leading flavors. Part below 2 GeV will be obtained in not far future from lattice QCD. (Mainz & Zeuthen)

- Crosscheck $\langle 33\rangle$ vs. $\langle +-\rangle$ from τ–decay may provide direct test of quality of flavor separation schemes.

- More exclusive channel results expected: Belle, CMD-3, SND, BES III, ... will help to improve

- At ILC 1000 in particular: polarized Møller scattering provides promising test.

- Refined SM running couplings key input for GUT unification tests.

Thanks!

Thanks a lot to the Organizers for the kind invitation to the interesting workshop and to TARI for support.