The muon $g-2$ and $\alpha(M_Z^2)$: the hadronic part

Fred Jegerlehner, DESY Zeuthen/Humboldt University Berlin

The Evaluation of the Leading Hadronic Contribution
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Outline of Talk:

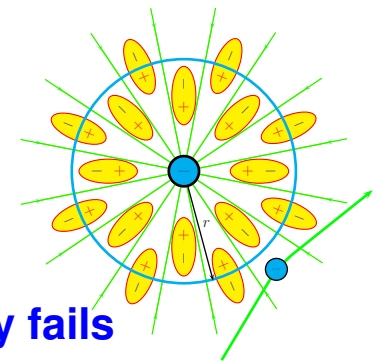
- Introduction
- $g-2$ Status: Theory vs experiment – do we see New Physics?
- Hadronic Vacuum Polarization (HVP) – Data & Status
- Evaluation of $\alpha(M_Z^2)$
- $\Delta\alpha^{\rm had}$ Adler function controlled: Euclidean split trick
- Issues in standard data based time-like approach
- A problem in DR for HVP and a first direct measurement of $\Pi'_\gamma(s)$
- Effective field theory: the Resonance Lagrangian Approach
- HVP from lattice QCD
- Alternative method: measure space-like $\alpha_{\rm QED,eff}(t) \to a_\mu^{\rm had}$
- Conclusion

1. Introduction

Non-perturbative hadronic effects in electroweak precision observables, main effect via

effective fine-structure "constant" $\alpha(E)$

(charge screening by vacuum polarization)



Of particular interest:

$$\alpha(M_Z) \text{ and } a_\mu \equiv (g-2)_\mu/2 \Leftrightarrow \alpha(m_\mu)$$

- **electroweak effects (leptons etc.) calculable in perturbation theory**
- **strong interaction effects (hadrons/quarks etc.) perturbation theory fails**

$\Longrightarrow$ **Dispersion integrals over** e^+e^-**–data**

encoded in $$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

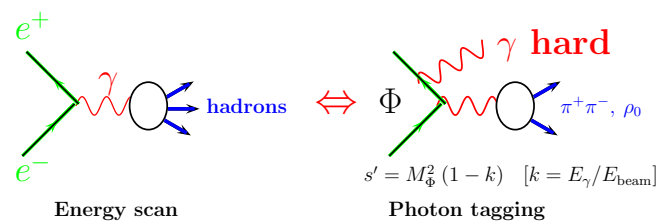
Errors of data $\Longrightarrow$ **theoretical uncertainties !!!**

The art of getting precise results from non-precision measurements !

The challenge for precision experiments on $\sigma(e^+e^- \to hadrons)$

σ_{hadronic} **via scan or radiative return:**

NSK, KLOE, BABAR, BESIII...



Muon $g-2$ to go

5 Numbers to establish the "g-2 Test"

(that is, 5 that have relevant uncertainties to keep watch on)

$$a_\mu(\text{New Physics}) \equiv a_\mu(\text{Expt}) - a_\mu(\text{SM})$$

Discussion today

$$a_\mu(\text{Expt}) = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p - \omega_a/\tilde{\omega}_p}$$

Expression in BNL PRD

Essentially experimental; limited at 120 ppb by μ_μ/μ_p

- $a_\mu(SM) = a_\mu(QED) + a_\mu(Weak) + a_\mu(HVP) + a_\mu(Had\ HO) + a_\mu(HLbL)$

✓ ✓ ✓

Discussion today

Goals:	Δa_μ(Expt)	~ 140 ppb
	Δa_μ(SM)	< 220 ppb

slide from D. Hertzog

2. Muon $g-2$ Status: Theory vs experiment

Given the CODATA/PDG recommended value of α the theory confronts experiment as follows: see Marc Knecht's Talk

Standard model theory and experiment comparison

Contribution	Value$\times 10^{10}$	Error$\times 10^{10}$	Reference
QED incl. 4-loops + 5-loops	11 658 471.886	0.003	Aoyama et al 12,Laporta 17
Hadronic LO vacuum polarization	689.46	3.25	
Hadronic light–by–light	10.34	2.88	
Hadronic HO vacuum polarization	-8.70	0.06	
Weak to 2-loops	15.36	0.11	Gnendiger et al 13
Theory	11 659 178.3	4.3	–
Experiment	11 659 209.1	6.3	BNL 04
The. - Exp. 4.0 standard deviations	-30.6	7.6	–

Standard model theory and experiment comparison [in units 10^{-10}]. What represents the 4 σ deviation: ❐ new physics? ❐ a statistical fluctuation? ❐ underestimating uncertainties (experimental, theoretical)?

❖do experiments measure what theoreticians calculate?

- a “New Physics” interpretation of the persisting 3 to 4 σ requires relatively strongly coupled states in the range below about 250 GeV.

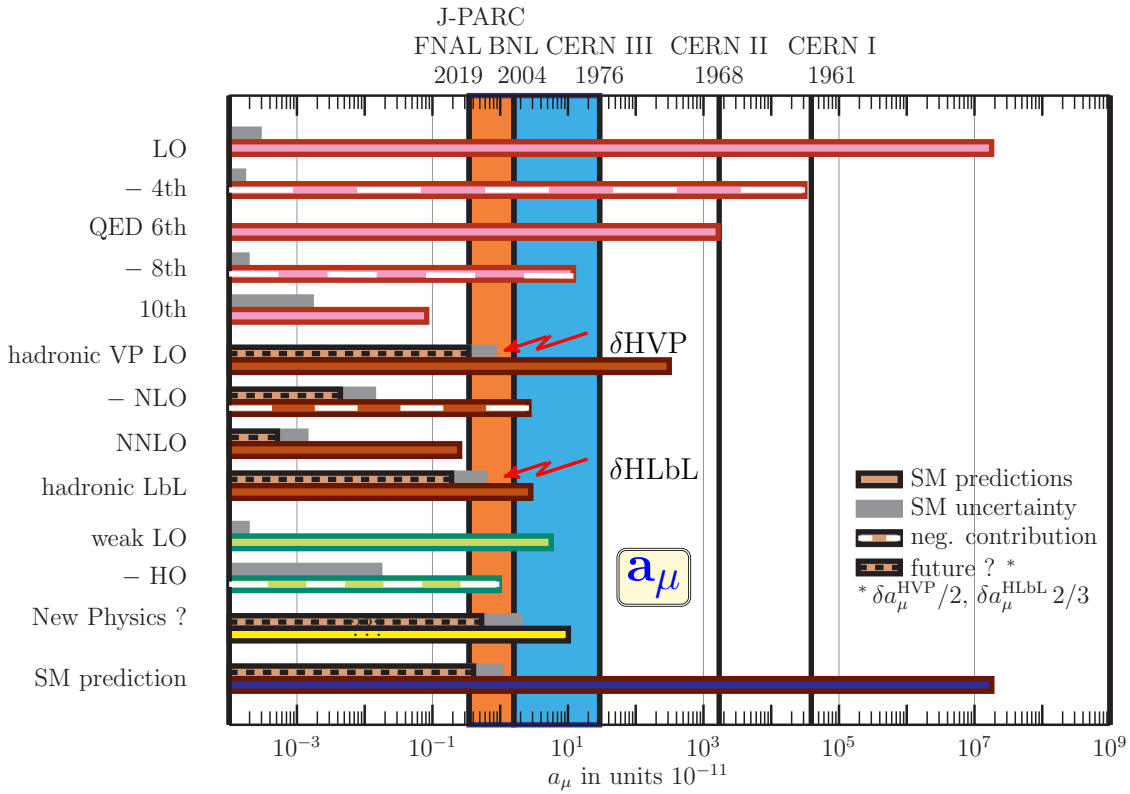
- Search bounds from LEP, Tevatron and specifically from the LHC already have ruled out a variety of Beyond the Standard Model (BSM) scenarios, so much hat standard motivations of SUSY/GUT extensions seem to fall in disgrace.

- There is no doubt that performing doable improvements on both the theory and the experimental side allows one to substantially sharpen (or diminish) the apparent gap between theory and experiment.

- ✳ Or is it unaccounted for real photon radiation effects?
  Do experiments measure what theory calculates?

- At the present/future level of precision a_μ depends on all physics incorporated in the SM: electromagnetic, weak, and strong interaction effects and beyond that all possible new physics we are hunting for.

here we are and hope to go:



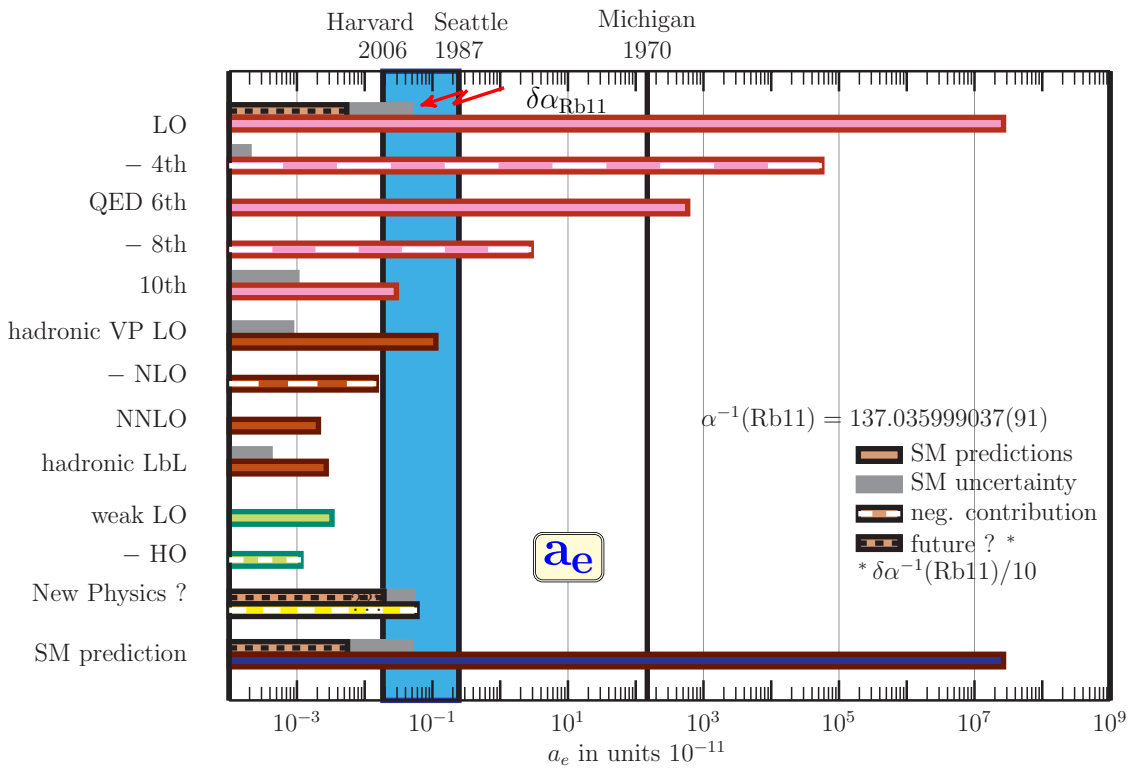
Past and future $g-2$ experiments testing various contributions.

New Physics $\stackrel{?}{=}$ deviation $(a_\mu^{\rm exp} - a_\mu^{\rm the})/a_\mu^{\rm exp}$.

Limiting theory precision: hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL)

*** digging deeper and deeper ***

same status for the electron:



Status and sensitivity of the a_e experiments testing various contributions. The error is dominated by the uncertainty of $\alpha(\mathrm{Rb11})$ from atomic interferometry. No "New Physics" $\stackrel{?}{=}$ deviation $(a_e^{\rm exp} - a_e^{\rm the})/a_\mu^{\rm exp}$. The blue band illustrates the improvement by the Harvard experiment. Note the very different sensitivities to non-QED contributions in comparison with a_μ.

*** still is and remains a QED test mainly ***

Experiment now:

$a_\mu^{\rm exp} = (11\,659\,209.1 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10}$ BNL updated

To come – :

New muon $g-2$ experiments at Fermilab and J-PARC: improve error by factor 4

$\Rightarrow$new muon $g-2$ experiment: $\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm the} = 8\,\sigma$ theory as today

Reduction of hadronic VP uncertainty by factor 2, same HLbL $\Rightarrow \quad \Delta a_\mu = 12\,\sigma$

That's what we hope to achieve!

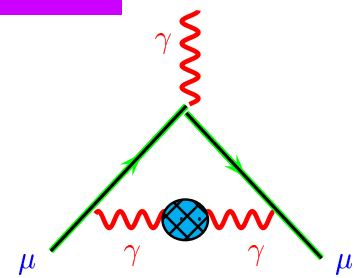
Key problem: limited accuracy of HVP! How to safely reduce and crosscheck?

- New lattice QCD now starts to have impact
- Ongoing improvement on R measurements
- New alternative methods: the workshops main theme here

❒ R-data Evaluation of $a_\mu^{\rm had}$

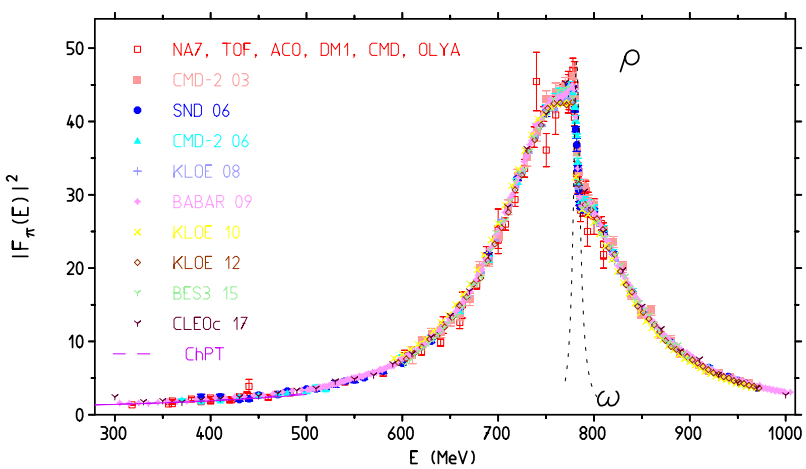
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via Dispersion Relation (DR):

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E_{\rm cut}^2} ds \, \frac{R_\gamma^{\rm data}(s)\, \hat{K}(s)}{s^2} + \int_{E_{\rm cut}^2}^{\infty} ds \, \frac{R_\gamma^{\rm pQCD}(s)\, \hat{K}(s)}{s^2} \right)$$



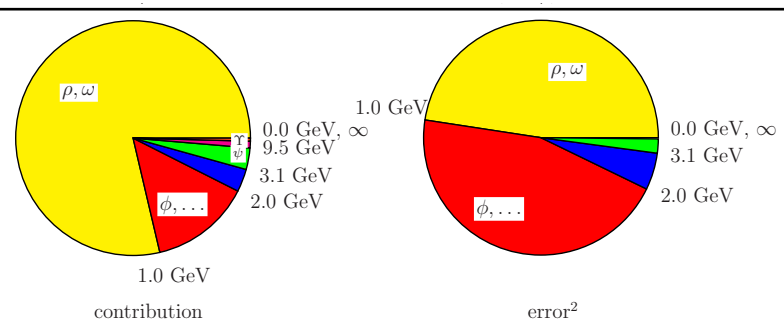
- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

Data: NSK, KLOE, BaBar, BES3, CLEOc



$$a_\mu^{\rm had(1)} = (686.99 \pm 4.21)[687.19 \pm 3.48]\ 10^{-10}$$

e^+e^-–data based [incl. τ]



Issues in standard data based time-like approach

❶ How to combine a pretty large number of data-sets to a truly reliable R-function: true uncertainty? How much just taken from pQCD? Choosing/selecting data-sets? Bare vs physical cross sections, how reliable is VP subtraction?

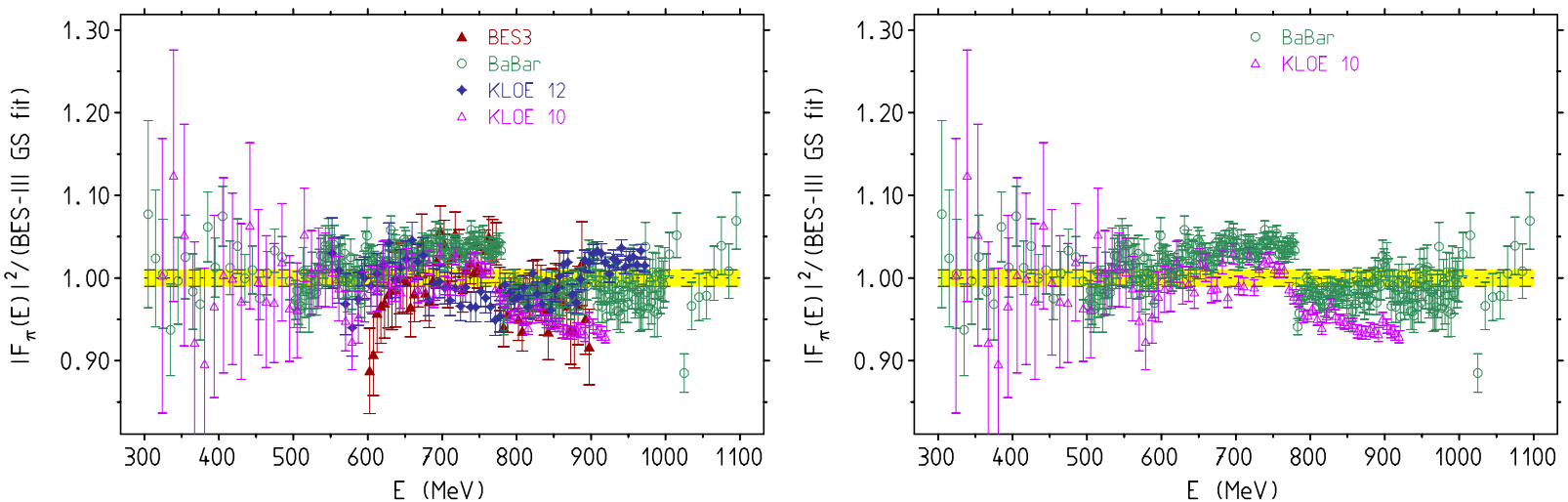
❷ Radiative corrections specifically for ISR method, sQED issues etc. The ISR method requires one order in α more precise RC calculation relative to SCAN method, at least full 2–loop Bhabha and/or $e^+e^- \to \mu^+\mu^-$ as well as ISR–FSR interference in $\pi^+\pi^-$ channel. What about RC to other more complicated channels (see e.g. F.J.&Karol Kołodziej 2017)? see Hernryk Czyż's Talk
What about disentangling 30 channels and recombining them in the 1 to 2 GeV region (quantum interference, missing parts, double counting issues)?

❸ What precisely do we need in the DR? The 1PI "blob", which is not a measurable quantity. Need undressing from QED effects, photon VP subtraction, FSR modeling, $\rho^0 - \gamma$ mixing? Do we do this at sufficient precision?

❹ Non-convergence of Dyson series for OZI suppressed narrow resonances

In addition:

❒ Data compatibility



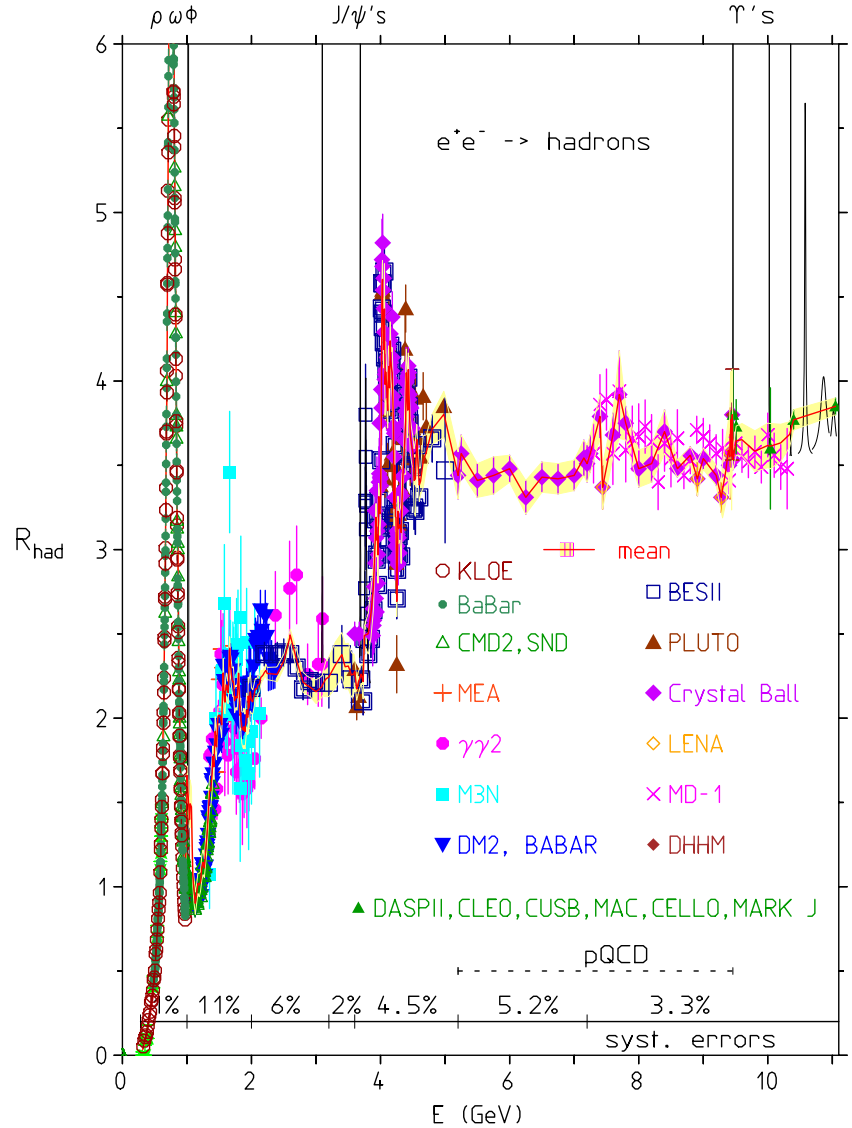
Recent BES-III vs BaBar and KLOE: dominant $\pi\pi$ channel still could be better

Recent results:

- $\pi^+\pi^-$ from BES-III, CMD-3 and CLEOc
- $\pi^+\pi^-\pi^0$ from Belle
- K^+K^- from CMD-3 and SND
- $\omega\pi^0 \to \pi^0\pi^0\gamma$ from SND
- $K_S K^\pm\pi^0\pi^\mp$, $K_S K^\pm\pi^\mp\eta$, $\pi^+\pi^-\pi^0\pi^0$, $K_S K_L\pi^0$, $K_S K_L\eta$, $K_S K_L\pi^0\pi^0$ from BaBar

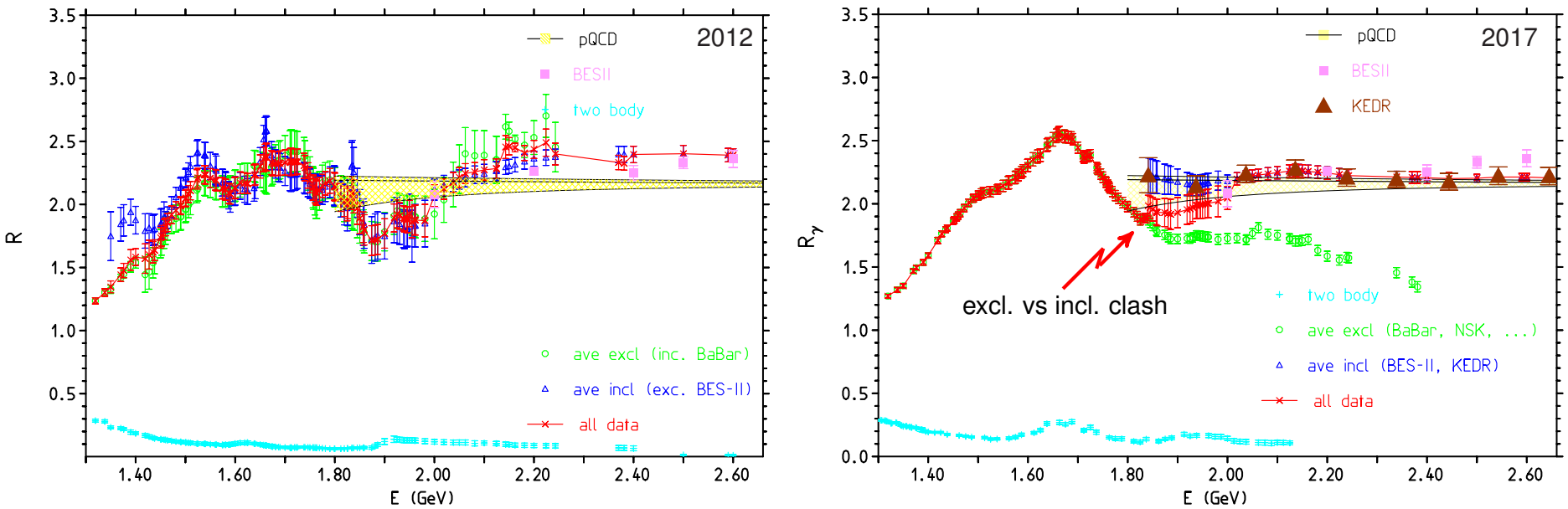
see Simon Eidelman's Talk

Energy range	$a_\mu^{\rm had}[\%](\text{error}) \times 10^{10}$	rel. err.	abs
ρ, ω ($E < 1$ GeV)	540.98 [78.6](2.80)	0.5 %	50
1 GeV $< E <$ 2 GeV	96.49 [14.0](2.54)	2.6 %	41
2 GeV $< E < \infty$ incl pQCD	51.09 [7.4](1.10)	2.2 %	7
total	688.65 [100.0](3.94)	0.6 %	100



❒ Still an issue in HVP

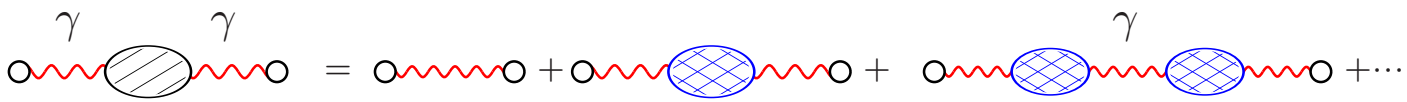
❒ region 1.2 to 2 GeV data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty



● illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

❐ A problem in DR for HVP?

Full photon propagator Dyson resummation of 1PI part (blue blob)



$$
\begin{aligned}
\mathrm{i}\, D'_\gamma(q^2) &\equiv \frac{-\mathrm{i}}{q^2} + \frac{-\mathrm{i}}{q^2}\,(-\mathrm{i}\Pi_\gamma)\,\frac{-\mathrm{i}}{q^2} + \frac{-\mathrm{i}}{q^2}\,(-\mathrm{i}\Pi_\gamma)\,\frac{-\mathrm{i}}{q^2}\,(-\mathrm{i}\Pi_\gamma)\,\frac{-\mathrm{i}}{q^2} + \cdots \\
&= \frac{-\mathrm{i}}{q^2}\left\{1 + \left(\frac{-\Pi_\gamma}{q^2}\right) + \left(\frac{-\Pi_\gamma}{q^2}\right)^2 + \cdots\right\} \\
&= \frac{-\mathrm{i}}{q^2}\left\{\frac{1}{1+\frac{\Pi_\gamma}{q^2}}\right\} = \frac{-\mathrm{i}}{q^2+\Pi_\gamma(q^2)} = \frac{-\mathrm{i}}{q^2}\,\frac{1}{1+\Pi'_\gamma(q^2)}\,.
\end{aligned}
$$

Including external e.m. coupling

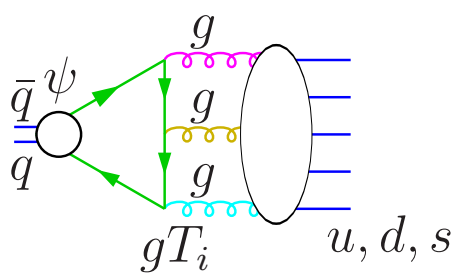
$$\mathrm{i}\, e^2\, D'_\gamma(q^2) = \frac{-\mathrm{i}}{q^2}\,\frac{e^2}{1+\Pi'_\gamma(q^2)}$$

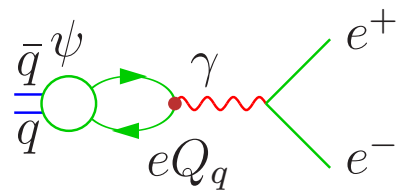
Effective charge

$$\frac{e^2}{1+\Pi'_\gamma(s)} = \frac{e^2}{1-\Delta\alpha(s)} = e^2(s)$$

Usually, $\Delta\alpha(s)$ is a correction i.e $\Delta\alpha(s) \ll 1$ and the Dyson series converges well.

Exceptions: narrow OZI suppressed resonances (below $q\bar{q}$-thresholds)





Γ_{ee} not much smaller than $\Gamma_{\rm QCD}$ (i.e strong decays): $J/\psi, \psi_2, \Upsilon_1, \Upsilon_2, \Upsilon_3$

Note: imaginary parts from narrow resonances, $\mathrm{Im}\,\Pi'(s)) = \frac{\alpha}{3} R(s) = \frac{3}{\alpha}\frac{\Gamma_{ee}}{\Gamma}$ at peak, are sharp spikes and are obtained correctly only by appropriately high resolution scans. For example,

$$\boxed{|1-\Pi'(s)|^2 - (\alpha/\alpha(s))^2 = (\mathrm{Im}\,\Pi'(s))^2}$$

at $\sqrt{s} = M_R$ is given by

ρ	1.23	$\times 10^{-3}$
ω	2.76	$\times 10^{-3}$
ϕ	1.56	$\times 10^{-2}$
J/ψ	594.81	
ψ_2	9.58	
ψ_3	2.66	$\times 10^{-4}$
Υ_1	104.26	
Υ_2	30.51	
Υ_3	55.58	

- What is measured in an experiment is the full propagator, corresponding to

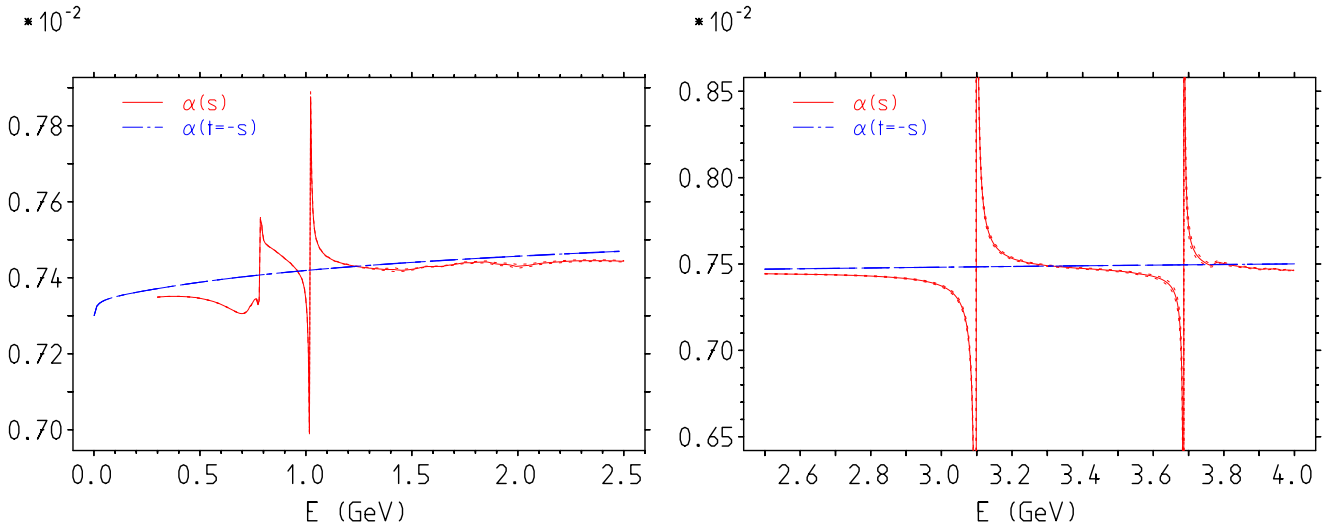
$$\boxed{\frac{1}{1-x}\ ; \quad x \ \text{ irreducible part}}$$

Object required in the DR:

$$R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}\ R^{\text{undressed}} = R^{\text{observed}} * |(1-x)|^2$$

VP subtraction is iterative procedure: does not converge!

The time-like vs space-like effective charge

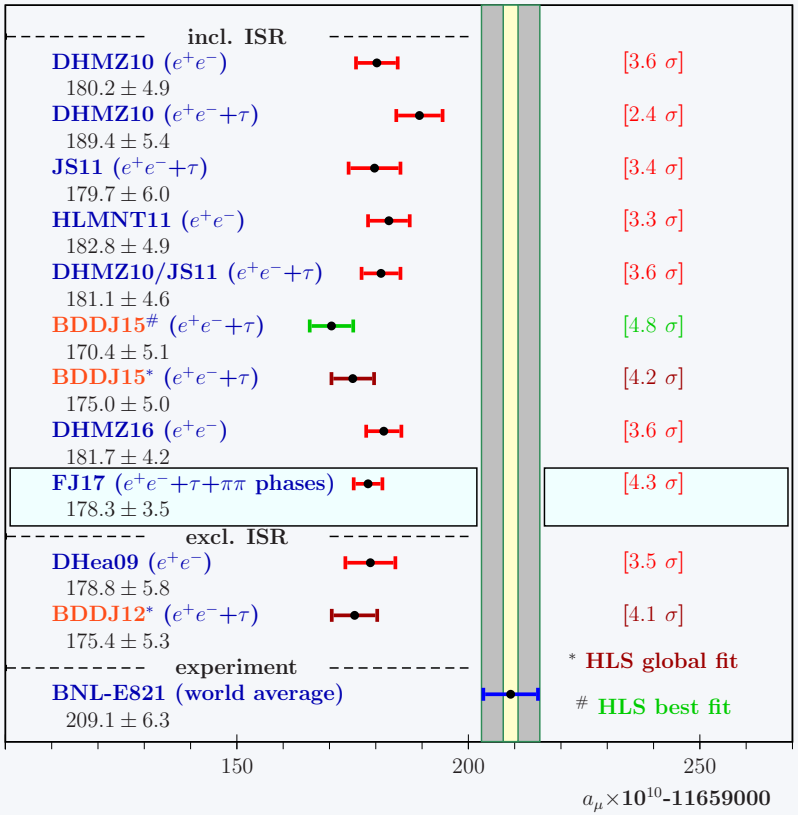


Note that the smooth space-like effective charge agrees rather well with the non-resonant "background" above the Φ (kind of duality)

No proof that this cannot produce non-negligible shifts!

Time-like VP-subtraction cannot be implemented locally near OZI suppressed resonances: $J/\psi, \psi'$ and $\Upsilon_1, \Upsilon_2, \Upsilon_3$

Present leading uncertainty: hard to improve by direct $R(s)$ measurements



Comparison with other Results. Note: results depend on which value is taken for HLbL. JS11 and BDDJ13 includes $116(39) \times 10^{-11}$ [JN], DHea09, DHMZ10, HLMNT11 and BDDJ12 use $105(26) \times 10^{-11}$ [PdRV].

HVP from lattice QCD

The need for ab initio calculation of $a_\mu^{\rm had}$ is well motivated:

– the problems to determine non-perturbative contributions to the muon $g-2$ from experimental data at sufficient precision persists and is not easy to improve,

– a model–independent extension of CHPT to the relevant energies ranges up to 2 GeV is missing while the new experiments E989 FNAL and E34 J-PARC

require an improvement of the hadronic uncertainties by a factor of four.

The hope is that LQCD can deliver estimates of accuracy

$$\delta a_\mu^{\rm HVP}/a_\mu^{\rm HVP} < 0.5\% \ , \quad \delta a_\mu^{\rm HLbL}/a_\mu^{\rm HLbL} \lesssim 10\%$$

in the coming years. see Marina Marinkovic's Talk

Primary object for HVP in LQCD: e.m. current correlator in configuration space

$$\langle J_\mu(\vec{x},t)\, J_\nu(\vec{0},0)\rangle\,, \quad J_\mu = \tfrac{2}{3}\bar{u}\gamma_\mu u - \tfrac{1}{3}\bar{d}\gamma_\mu d - \tfrac{1}{3}\bar{s}\gamma_\mu s + \cdots$$

In principle, a Fourier transform

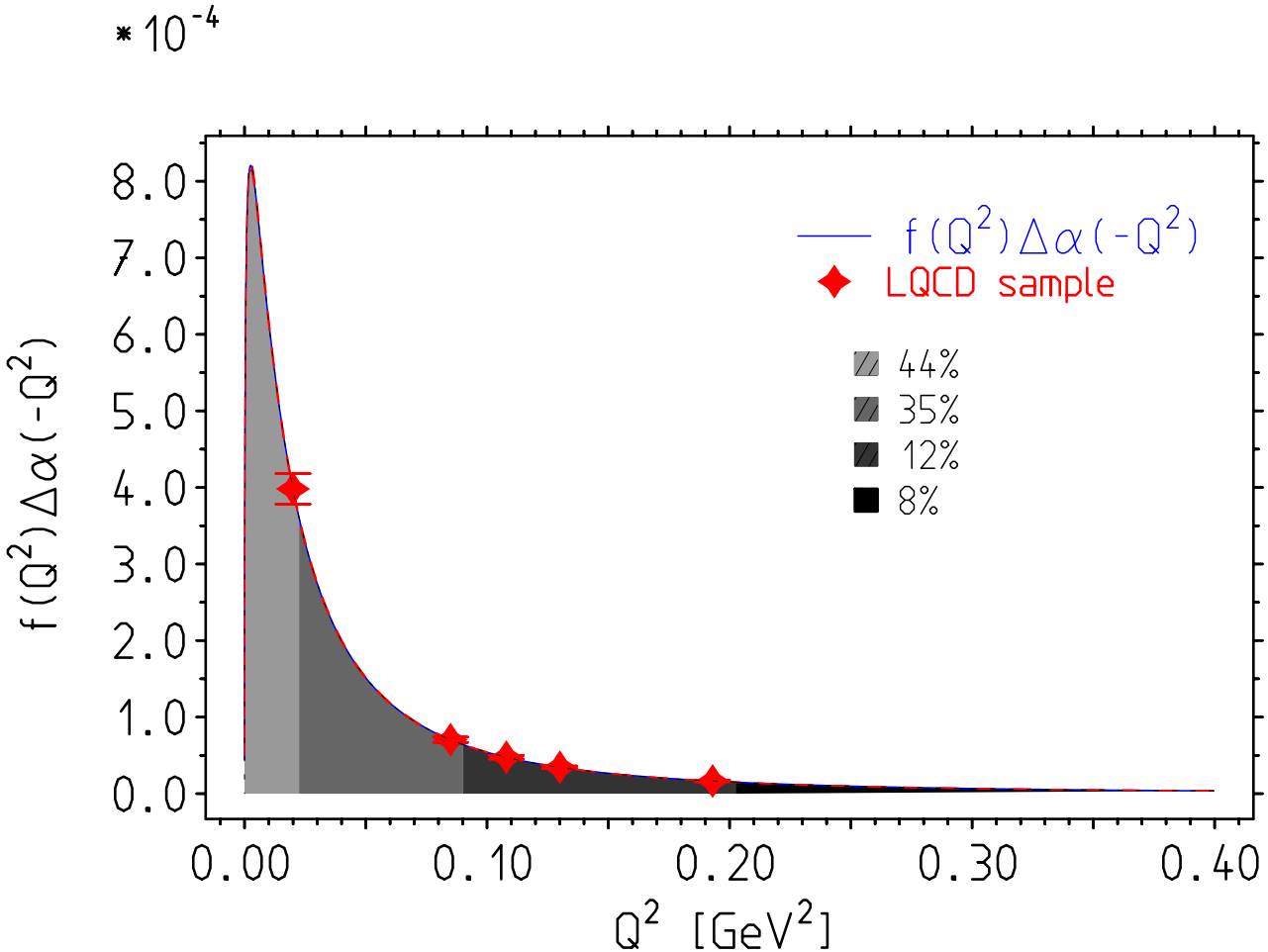
$$\Pi_{\mu\nu}(Q) = \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}\,Qx}\,\langle J_\mu(x)\, J_\nu(0)\rangle = \left(Q_\mu Q_\nu - \delta_{\mu\nu}\, Q^2\right)\,\Pi(Q^2)$$

yields the vacuum polarization function $\Pi(Q^2)$ needed to calculate

$$a_\mu^{\mathrm{HVP}} = 4\alpha^2 \int_0^\infty \mathrm{d}Q^2\, f(Q^2)\,\left\{\Pi(Q^2) - \Pi(0)\right\}$$

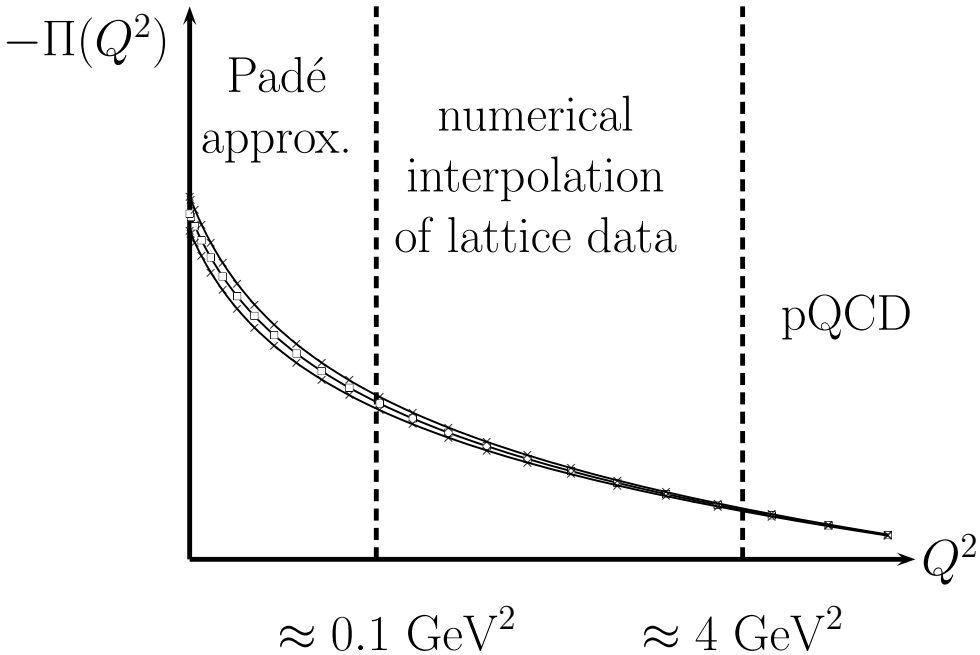
The integration kernel in this representation is

$$f(Q^2) = w(Q^2/m_\mu^2)/Q^2\ ; \quad w(r) = \frac{16}{r^2\left(1+\sqrt{1+4/r}\right)^4\,\sqrt{1+4/r}}\ .$$



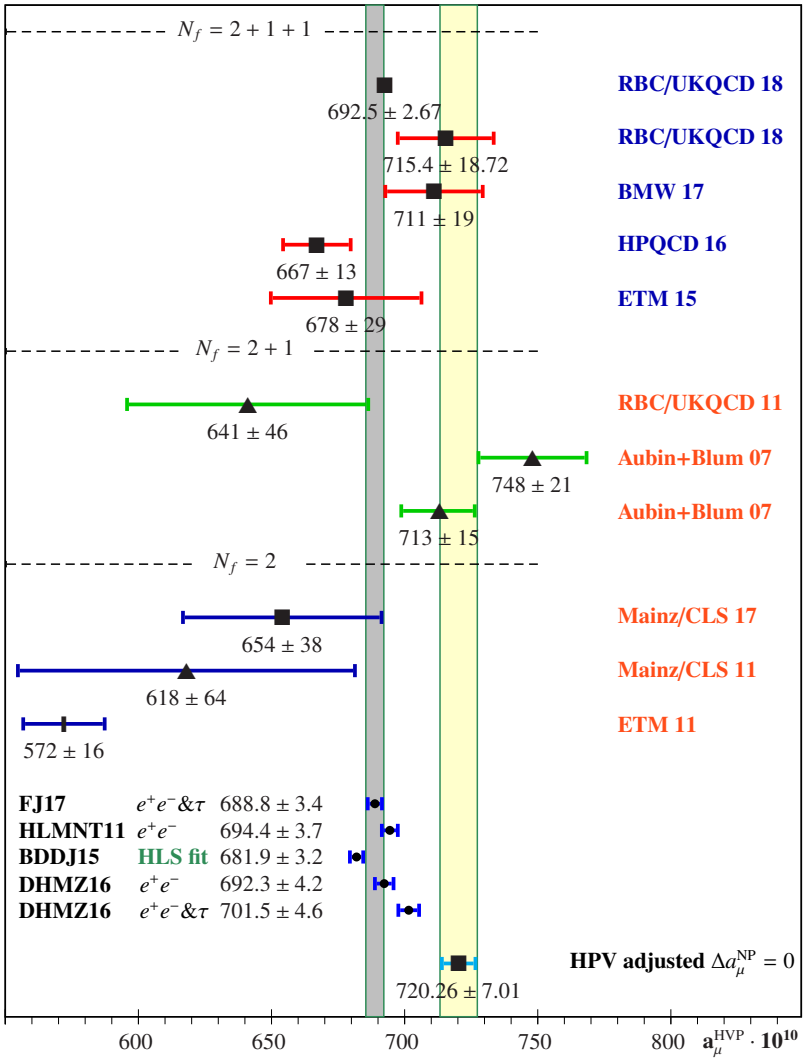
The integrand Q^2. Ranges between $Q_i = 0.00,\ 0.15,\ 0.30,\ 0.45$ and 1.0 GeV and their percent contribution to $a_\mu^{\rm had}$ and the "LQCD sample"

- LQCD lattice in finite box: momenta are quantized $Q_{\min} = 2\pi/L$ where L is the lattice box length. $Q_{\min} \to 0 \Leftrightarrow L \to \infty$ infinite volume limit
- $Q_{\min} = 2\pi/L$ with $m_\pi a L \gtrsim 4$ for $m_\pi \sim 200$ MeV, such that $Q_{\min} \sim 314$ MeV
- about 44% of the low x contribution to $a_\mu^{\rm had}$ is not covered by data yet



- lattice data: $Q^2 > (2\pi/L)^2$
- extrapolate to $Q^2 = 0$ via Padé's
- Note need $\Pi(0)$!
- required accuracy: needed LQCD data down to $Q^2_{\min} \approx 0.1$ GeV^2

New: RBC/UKQCD 18 use lattice between 0.1 and 4 GeV^2 and R–data for IR and UV tails $\Rightarrow$most precise evaluation so far: $a_\mu^{\rm HVP-LO} = (692.5 \pm 2.7) \times 10^{-10}$
[$a_\mu^{\rm lat} = (232.1 \pm 1.5) \times 10^{-10} \hat{=} 33.5\%$; $a_\mu^{\rm dat} = (460.4 \pm 2.2) \times 10^{-10}$]



Summary of recent LQCD results for the leading order $a_\mu^{\rm HVP}$, in units 10^{-10}. Labels: ■ marks u, d, s, c, ▲ u, d, s and ▮ u, d contributions. Individual flavor contributions from light (u, d) amount to about 90%, strange about 8% and charm about 2%. Budapest, Marseille, Wuppertal, Brookhaven, Zeuthen, Mainz, Edinburgh, ... The gray vertical band represents my evaluation. The wheat band represents the HVP required such that theory matches the experimental BNL result. The very precise RBC/UKQCD point is obtained by supplementing lattice results by R–data.

3. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

α , G_μ, M_Z **most precise input parameters** $\Rightarrow$ **precision predictions**

50% non-perturbative

$\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters for VB physics (Z,W) etc.**

$\frac{\delta\alpha}{\alpha}$	$\sim$	3.6	$\times$	10^{-9}	
$\frac{\delta G_\mu}{G_\mu}$	$\sim$	8.6	$\times$	10^{-6}	
$\frac{\delta M_Z}{M_Z}$	$\sim$	2.4	$\times$	10^{-5}	
$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)}$	$\sim$	$0.9 \div 1.6$	$\times$	10^{-4}	(present : lost 10^5 in precision!)
$\frac{\delta\alpha(M_Z)}{\alpha(M_Z)}$	$\sim$	5.3	$\times$	10^{-5}	(ILC requirement)

LEP/SLD: $\sin^2\Theta_{\rm eff} = (1 - g_{Vl}/g_{Al})/4 = 0.23148 \pm 0.00017$

$\delta\Delta\alpha(M_Z) = 0.00020 \quad \Rightarrow \quad \delta\sin^2\Theta_{\rm eff} = 0.00007$

affects Higgs mass bounds, precision tests and new physics searches!!!

For pQCD contributions very crucial: precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t \Rightarrow$ Lattice-QCD

❒ Relevance of $\alpha(M_Z^2)$

The Parameters of the Standard Model

– in four fermion and vector boson processes –

in addition QCD coupling α_s, y_t vs. M_t, λ_H vs. M_H etc.

unlike in QED and QCD in SM (SBGT)

parameter interdependence

➠

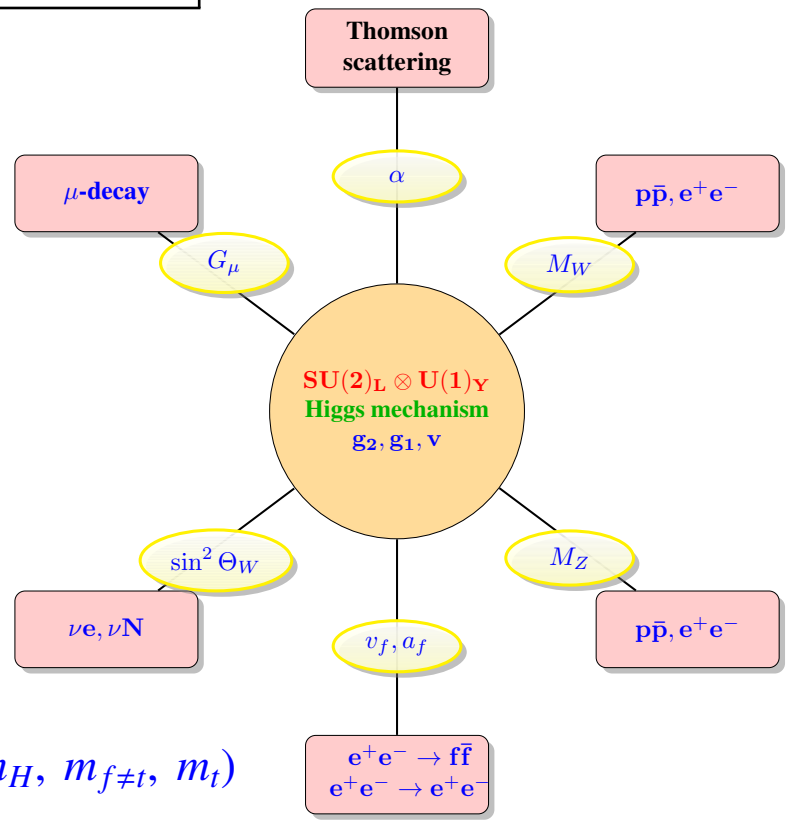
only 3 independent quantities

(besides fermion masses and mixing parameters)

α, G_μ, $M_Z \Rightarrow \alpha_{\rm eff}(M_Z^2) \Rightarrow$ large hadronic correction

$\Downarrow$

$$\sin^2\Theta_i\ \cos^2\Theta_i\ = \frac{\pi\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i}\ ;\quad \Delta r_i = \Delta r_i(\alpha,\ G_\mu,\ M_Z, m_H,\ m_{f\neq t},\ m_t)$$



parameter relationships between very precisely measurable quantities

precision tests, possible sign of new physics

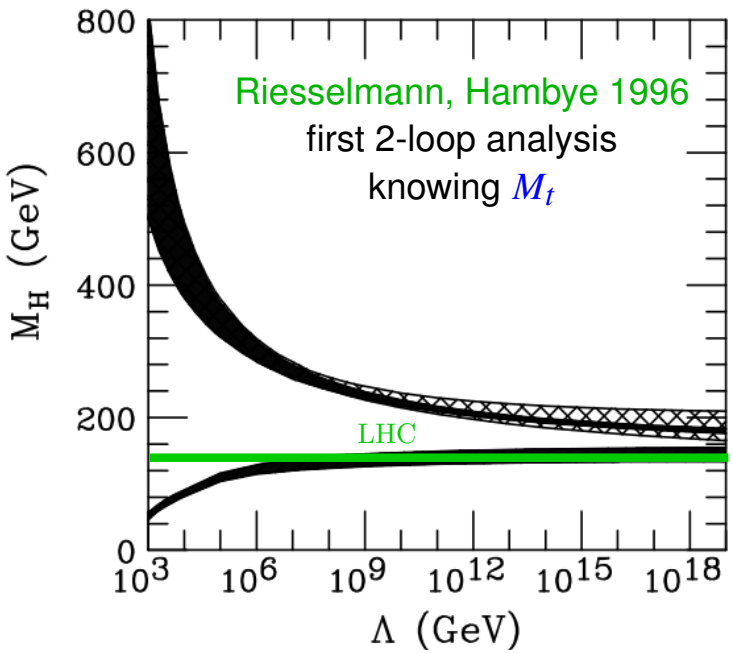
non-perturbative $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ is limiting precision predictions

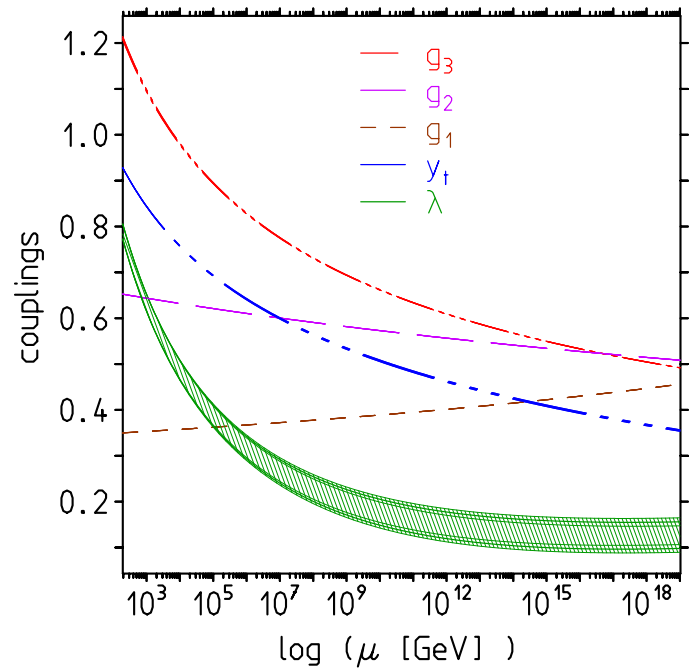
Note: 30 SD disagreement between SM prediction and experiment when subleading corrections are dropped!

SM extrapolation up to Planck scale?

After Higgs discovery: Higgs vacuum stability issue!

⇒Need very precise SM parameters: g', g, g_s, y_t, λ





The SM dimensionless couplings in the $\overline{\mathrm{MS}}$ scheme as a function of the renormalization scale for $M_H = 124 - 127$ GeV.

- perturbation expansion works up to the Planck scale!
  no Landau pole or other singularities, Higgs potential likely remains stable!
- ❒ $U(1)_Y$ screening (IR free), $SU(2)_L$, $SU(3)_c$ antiscreening (UV free): g_1, g_2, g_3

as expected (standard wisdom)

- ❒ Top Yukawa y_t and Higgs λ: screening if standalone (IR free, like QED)

as part of SM, transmutation from IR free to UV free

As SM couplings are as they are: QCD dominance in top Yukawa RG requires $g_3 > \frac{3}{4} y_t$, top Yukawa dominance in Higgs RG requires $\lambda < \frac{3(\sqrt{5}-1)}{2} y_t^2$ in the gaugeless ($g_1, g_2 = 0$) limit.

In the focus:

- ❒ does Higgs self-coupling stay positive $\lambda > 0$ up to $\Lambda_{\rm Pl}$?
- ❒ the key question/problem concerns the size of the top Yukawa coupling y_t
  decides about stability of our world! — [$\lambda = 0$ would be essential singularity!]

Will be decided by:
- more precise input parameters
- better established EW matching conditions

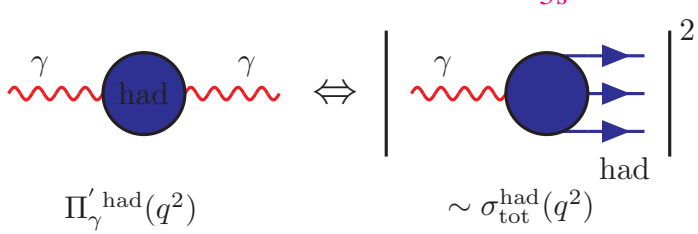
❒ R-data Evaluation of $\alpha(M_Z^2)$

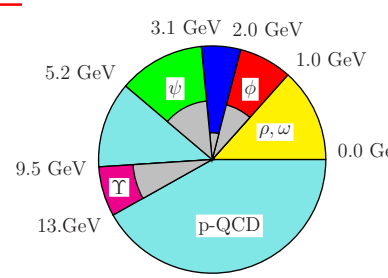
Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s) = -\left(\Pi'_\gamma(s) - \Pi'_\gamma(0)\right)$ **can be evaluated in terms of** $\sigma(e^+e^- \to \text{hadrons})$ **data via dispersion integral:**

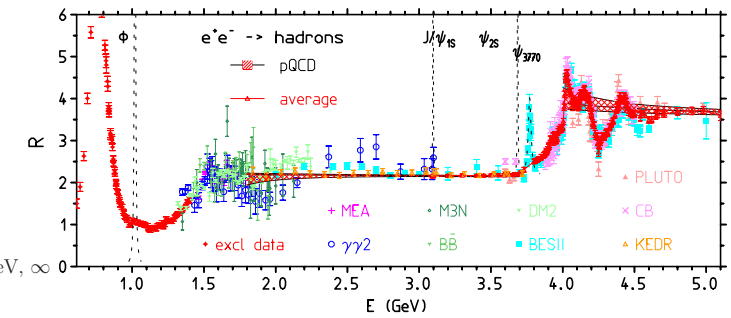
$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha\, s}{3\pi}\left(\oint\!\!\!\!\!\!\int_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \oint\!\!\!\!\!\!\int_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

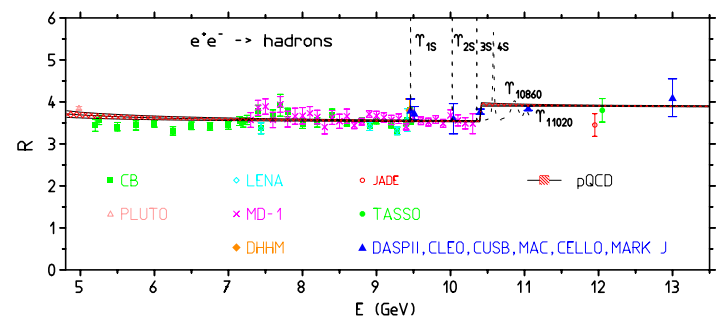
Compilation: FJ 15
Theory = pQCD: Gorishny et al. 91, Chetyrkin et al. 97...09

where $R_\gamma(s) \equiv \dfrac{\sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})}{\frac{4\pi\alpha^2}{3s}}$









hadronic vacuum polarization

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\ ; \quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)$$

Present situation: (after KLOE, BaBar and first BESIII results)

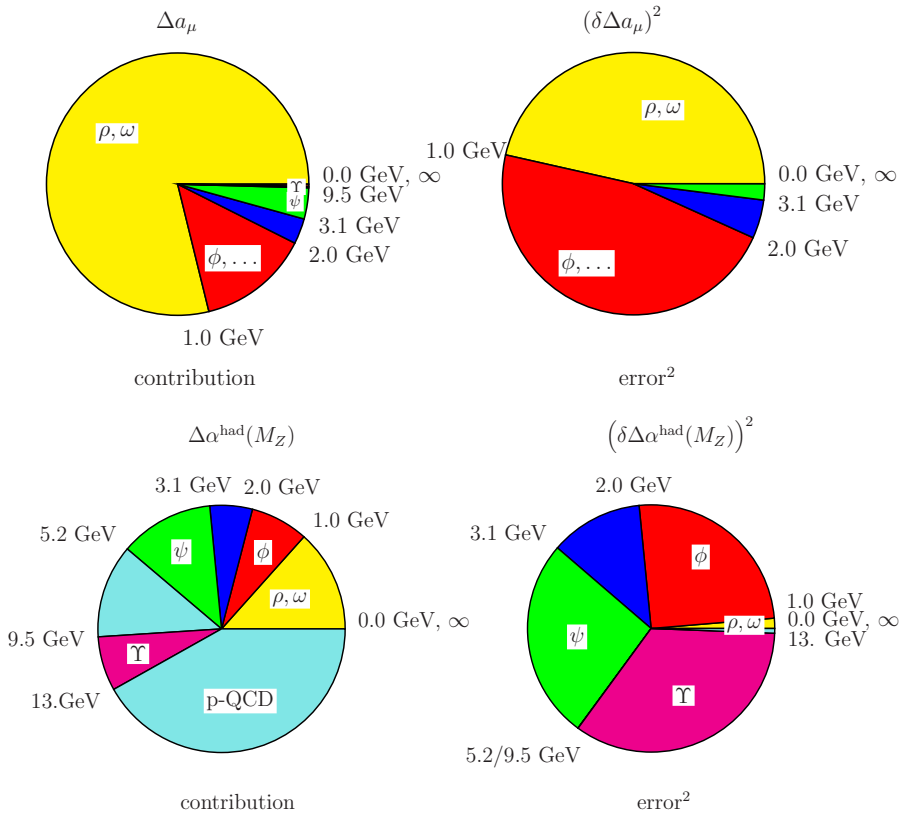
$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2)$	$=$	0.027738 ± 0.000158	
		0.027523 ± 0.000119	Adler
$\alpha^{-1}(M_Z^2)$	$=$	128.919 ± 0.022	
		128.958 ± 0.016	Adler

$\Delta\alpha_{\rm had}(M_Z^2)$ results from ranges:

for $M_Z = 91.1876$ GeV in units 10^{-4}. 2017 update in terms of e^+e^--data and pQCD. 43% data, 57% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 11.5 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had} \times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	33.91 (0.05) (0.18)[0.19]	0.6%	1.4%
ω	(0.42, 0.81)	3.10 (0.04) (0.08)[0.09]	3.0%	0.3%
ϕ	(1.00, 1.04)	4.76 (0.07) (0.11)[0.13]	2.7%	0.7%
J/ψ		12.38 (0.60) (0.67)[0.90]	7.2%	32.1%
Υ		1.30 (0.05) (0.07)[0.09]	6.9%	0.3%
had	(1.05, 2.00)	16.53 (0.06) (0.83)[0.83]	5.0%	27.4%
had	(2.00, 3.20)	15.34 (0.08) (0.61)[0.62]	4.0%	15.2%
had	(3.10, 3.60)	4.98 (0.03) (0.09)[0.10]	1.9%	0.4%
had	(5.20, 5.20)	16.84 (0.12) (0.21)[0.25]	0.0%	2.4%
pQCD	(5.20, 9.46)	33.84 (0.12) (0.25)[0.03]	0.1%	0.0%
had	(9.46,11.50)	11.12 (0.07) (0.69)[0.69]	6.2%	19.2%
pQCD	(11.50,∞)	123.29 (0.00) (0.05)[0.05]	0.0%	0.1%
data	(0.28,11.50)	120.25 (0.63) (1.45)[1.58]	1.0%	0.0%
total		277.38 (0.63) (1.45)[1.58]	0.6%	100.0%

Correlation between different contributions to $a_\mu^{\rm had}$ and $\Delta\alpha^{\rm had\,(5)}$



Contributions from e^+e^- data ranges and form pQCD to $a_\mu^{\rm had}$ and $\Delta\alpha^{\rm had\,(5)}$.

4. Reducing uncertainties via the Euclidean split trick: Adler function controlled pQCD

- experiment side: new more precise measurements of $R(s)$
- future direct measurements Patrick Janot, Luca Trentadue et al
- theory side: $\alpha_{\rm em}(M_Z^2)$ by the "Adler function controlled" approach

$$\alpha(M_Z^2) \quad = \quad \alpha^{\rm data}(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD} + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$$

where the space-like $-s_0$ is chosen such that pQCD is well under control for $-s < -s_0$. The monitor to control the applicability of pQCD is the Adler function

$$D(Q^2 = -s) = \frac{3\pi}{\alpha} s \frac{d}{{\rm d}s} \Delta\alpha_{\rm had}(s) = -(12\pi^2)\, s\, \frac{{\rm d}\Pi'_\gamma(s)}{{\rm d}s} = Q^2 \int_{4m_\pi^2}^{\infty} \frac{R(s)}{(s+Q^2)^2}$$

which also is determined by $R(s)$ and can be evaluated in terms of experimental e^+e^-–data. Perturbative QCD tail: $D(Q^2) \to N_c \sum_f Q_f^2\, (1 + O(\alpha_s))$ as $Q^2 \to \infty$.

S. Eidelman, F. J., A. Kataev, O. Veretin, Phys. Lett. B **454** (1999) 369

$\Delta\alpha^{\rm had}$ Adler function controlled

✓ **use old idea: Adler function: Monitor for comparing theory and data**

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

$$\Rightarrow \quad D(Q^2) = Q^2\left(\int_{4m_\pi^2}^{E^2_{\rm cut}} {\rm d}s\, \frac{R(s)^{\rm data}}{(s+Q^2)^2} + \int_{E^2_{\rm cut}}^{\infty} \frac{R^{\rm pQCD}(s)}{(s+Q^2)^2}\, {\rm d}s\right).$$

pQCD $\leftrightarrow R(s)$	**pQCD** $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	**smooth simple function in Euclidean region**

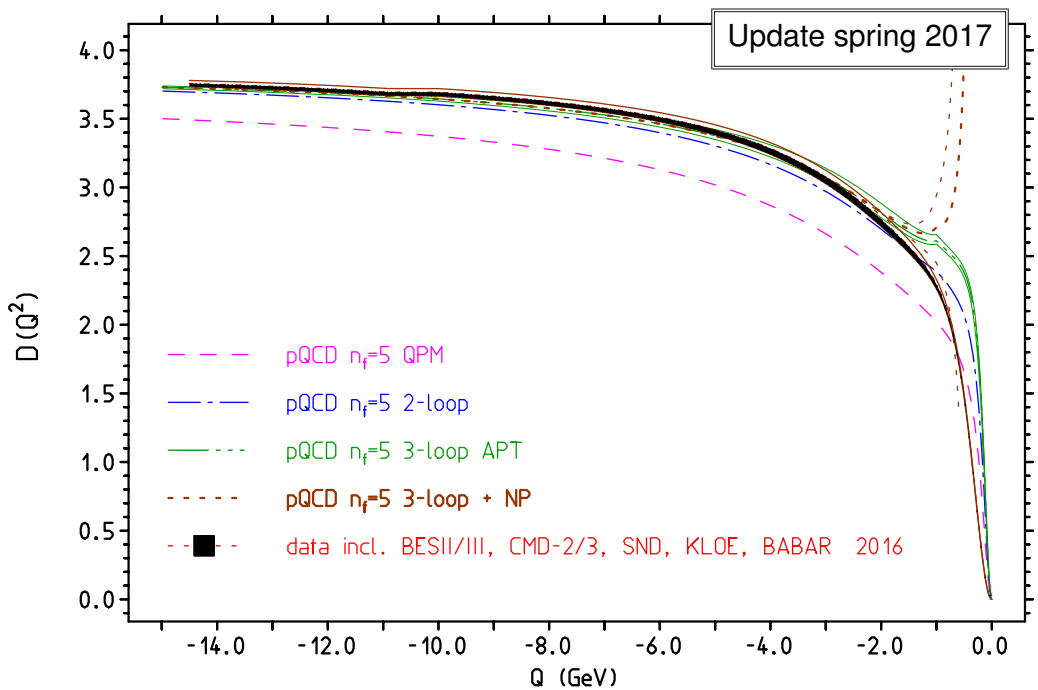
Conclusion:

❖**time-like approach: pQCD works well in "perturbative windows"**
3.00 - 3.73 GeV, 5.00 - 10.52 GeV and 11.50 - ∞ **Kühn,Steinhauser**

❖**space-like approach: pQCD works well for** $\sqrt{Q^2 = -q^2} > 2.0$ **GeV (see plot)**

"Experimental" Adler–function versus theory (pQCD + NP)

Error includes statistical + systematic here (in contrast to most R-plots showing statistical errors only)!



(Eidelman, F. J., Kataev, Veretin 98, FJ 08/17 updates)

theory based on results by Chetyrkin, Kühn et al.

⇒ **pQCD works well controlled to predict $D(Q^2)$ down to $s_0 = (2.0\,\textbf{GeV})^2$; use this to calculate**

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int {\rm d}Q'^2 \frac{D(Q'^2)}{Q'^2}$$

$$\boxed{\begin{aligned}\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) \;=\; & \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} \\ & + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}\end{aligned}}$$

and obtain, for $s_0 = (2.0\,\textbf{GeV})^2$: **(FJ 98/17)**

$$\begin{aligned}\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} &= 0.006409 \pm 0.000063 \\ \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) &= 0.027483 \pm 0.000118 \\ \Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.027523 \pm 0.000119\end{aligned}$$

❖**shift** $+0.000008$ **from the 5-loop contribution**

❖**error** ± 0.000100 **added in quadrature form perturbative part**

QCD parameters: ● $\alpha_s(M_Z) = 0.1189(20)$**,**

● $m_c(m_c) = 1.286(13)$ $[M_c = 1.666(17)]$ **GeV** , ● $m_b(m_c) = 4.164(25)$ $[M_b = 4.800(29)]$ **GeV**

based on a complete 3–loop massive QCD analysis Kühn et al 2007

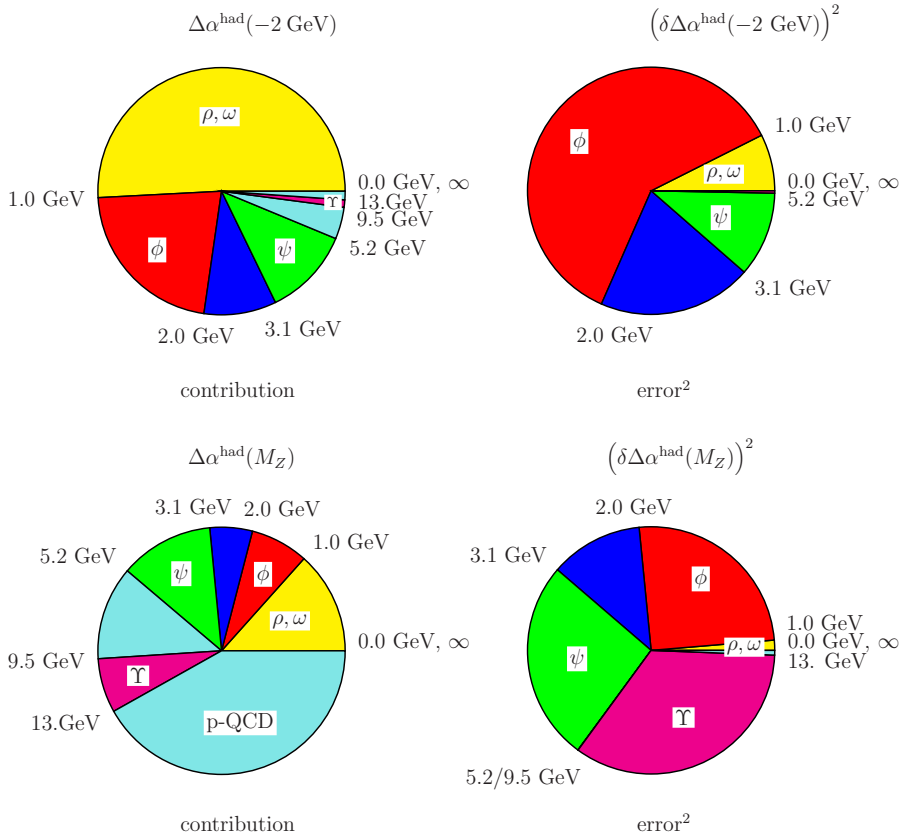
F. J., Nucl. Phys. Proc. Suppl. **181-182** (2008) 135

$\Delta\alpha_{\rm had}(-M_0^2)$ results from ranges:

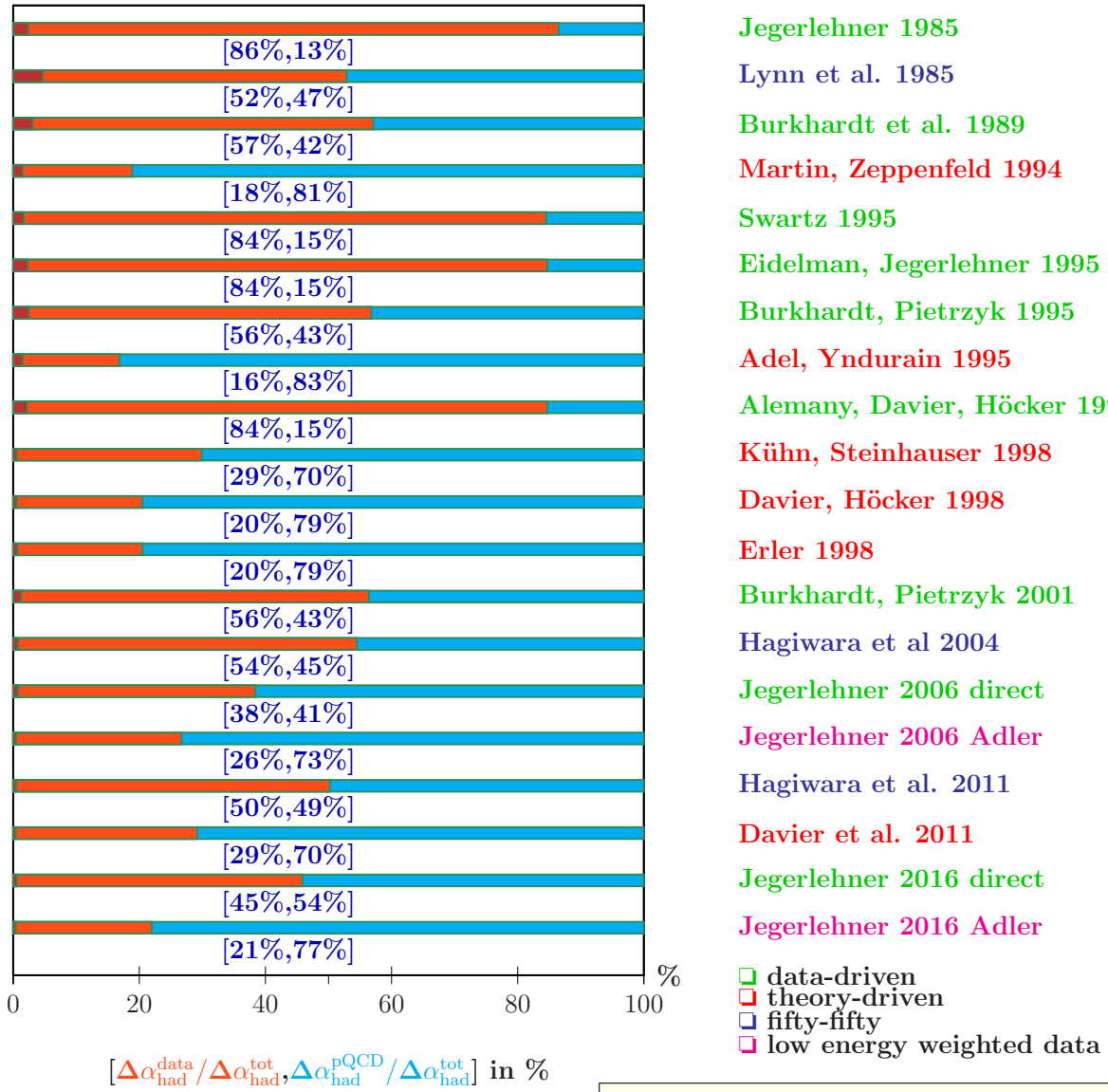
for $M_0 = 2$ GeV in units 10^{-4}. 2015 update in terms of e^+e^--data and pQCD. 94% data, 6% perturbative QCD. pQCD is used between 5.2 GeV and 9.5 GeV and above 11.5 GeV.

final state	range (GeV)	$\Delta\alpha^{(5)}_{\rm had}(-M_0^2)\times 10^4$ (stat) (syst) [tot]	rel	abs
ρ	(0.28, 1.05)	29.78 (0.04) (0.16)[0.16]	0.5%	6.6%
ω	(0.42, 0.81)	2.69 (0.03) (0.07)[0.08]	3.0%	1.6%
ϕ	(1.00, 1.04)	3.78 (0.05) (0.09)[0.10]	2.7%	2.6%
J/ψ		3.21 (0.15) (0.15)[0.21]	6.7%	11.4%
Υ		0.05 (0.00) (0.00)[0.00]	6.8%	0.0%
had	(1.05, 2.00)	10.36 (0.04) (0.49)[0.49]	4.8%	61.2%
had	(2.00, 3.20)	6.06 (0.03) (0.25)[0.25]	4.2%	16.1%
had	(3.10, 3.60)	1.31 (0.01) (0.02)[0.03]	1.9%	0.2%
had	(5.20, 5.20)	2.90 (0.02) (0.02)[0.03]	0.0%	0.2%
pQCD	(5.20, 9.46)	2.66 (0.02) (0.02)[0.00]	0.1%	0.0%
had	(9.46,11.50)	0.39 (0.00) (0.02)[0.02]	5.7%	0.1%
pQCD	(11.50,∞)	0.90 (0.00) (0.00)[0.00]	0.0%	0.0%
data	(0.28,11.50)	60.53 (0.18) (0.61)[0.63]	1.0%	0.0%
total		64.09 (0.18) (0.61)[0.63]	1.0%	100.0%

Of $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ 22% data, 78% pQCD!



Contributions from e^+e^- data ranges and form pQCD to $\Delta\alpha^{(5)}_{\rm had}(-M_0^2)$ vs. $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$.



How much pQCD?

Note: the Adler function monitored Euclidean data vs pQCD split approach is only moderately more pQCD-driven, than the time-like approach adopted by Davier et al. and others.

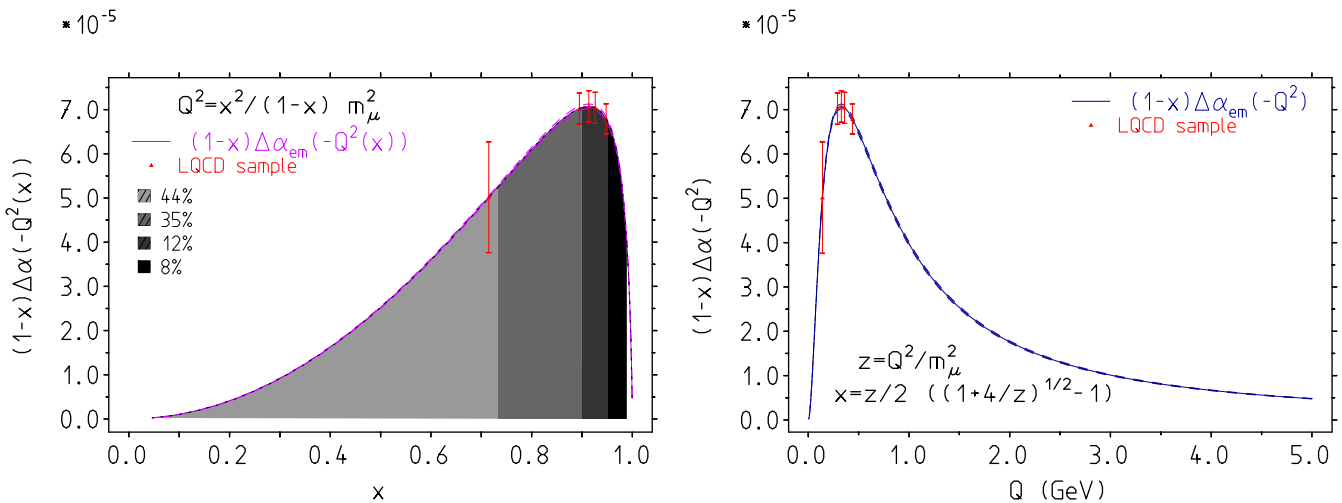
Alternative method: measure space-like $\alpha_{\rm QED,eff}(t)$

Newly proposed recently: [arXiv:1504.02228,1609.08987]
"A new approach to evaluate the leading hadronic corrections to the muon g-2"
Carloni Calame, Passera, Trentadue, Venanzoni 2015; Abbiendi et al. 2016

- space-like $\Delta\alpha_{\rm had}(-Q^2) = 1 - \frac{\alpha}{\alpha(-Q^2)} - \Delta\alpha^{\rm lep}(-Q^2)$ determines $a_\mu^{\rm had}$ via

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int_0^1 {\rm d}x\,(1-x)\,\Delta\alpha_{\rm had}\left(-Q^2(x)\right)$$

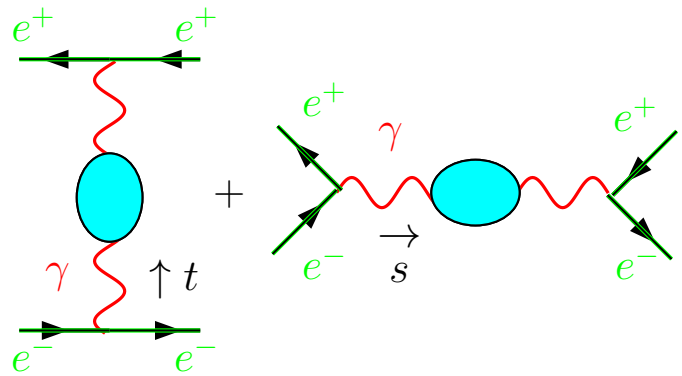
where $Q^2(x) \equiv \frac{x^2}{1-x}m_\mu^2$ is the space–like square momentum–transfer. Also in the Euclidean region the integrand is highly peaked, now around half of the ρ meson mass scale.



The integrand of $a_\mu^{\rm had}$ integral as functions of x and Q. Strongly peaked at about 330 MeV. Ranges between $Q_i = 0.00,\ 0.15,\ 0.30,\ 0.45$ and 1.0 GeV and their percent contribution to $a_\mu^{\rm had}$.

- measuring directly low energy running $\alpha_{\rm QED}(s)$ in space-like region via
- very different paradigm: no VP subtraction issue!
- no exclusive channel collection
- even 1% level measurement can provide important independent information

Bhabha scattering $\quad e^+(p_+)\ e^-(p_-) \to e^+(p'_+)\ e^-(p'_-)$



VP dressed tree level Bhabha scattering in QED

has two tree level diagrams the t– and the s–channel. With the positive c.m. energy square $s = (p^+ + p^-)^2$ and the negative momentum transfer square

$$t = (p_- - p'_-)^2 = -\frac{1}{2}\,(s - 4m_e^2)\,(1 - \cos\theta)\,,$$

θ the e^- scattering angle, there are two very different scales involved

The VP dressed lowest order cross–section is

$$\frac{d\sigma}{d\cos\Theta} = \frac{s}{48\pi} \sum_{ik} |A_{ik}|^2$$

where A_{ik} tree level helicity amplitudes,i, k =L,R left– and right–handed electrons.

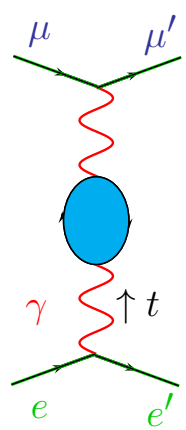
Dressed transition amplitudes: ($m_e \approx 0$)

$$\begin{aligned} |A_{\rm LL,RR}|^2 &= \frac{3}{8}(1+\cos\theta)^2 \left|\frac{e^2(s)}{s} + \frac{e^2(t)}{t}\right|^2 \\ |A_{\rm LR,RL}|^2 &= \frac{3}{8}(1-\cos\theta)^2 \left|\frac{e^2(s)}{s} + \frac{e^2(t)}{t}\right|^2 . \end{aligned}$$

Preferably one uses small angle Bhabha scattering (small $|t|$) as a normalizing process which is dominated by the t–channel $\sim 1/t$, however, detecting electrons and positrons along the beam axis often has its technical limitations.

Care also is needed concerning the ISR corrections because cuts for the Bhabha process ($e^+e^- \to e^+e^-$) typically are different from the ones applied to $e^+e^- \to$ hadrons. Usually, experiments have included corresponding uncertainties in their systematic errors, if they not have explicitly accounted for all appropriate radiative corrections.

μ^-e^- scattering $\qquad \mu^-(p_-)\ e^-(q_-) \to \mu^-(p'_-)\ e^-(q'_-)$



Get $a_\mu^{\rm had}$ from $\mu^-e^- \to \mu^-e^-$ process

G. Abbiendi et al. , arXiv:1609.08987 Massimo Passera's Talk

$$\frac{d\sigma^{\text{unpol.}}_{\mu^-e^-\to\mu^-e^-}}{dt} = 4\pi\,\alpha(t)^2\,\frac{1}{\lambda(s,m_e^2,m_\mu^2)}\left\{\frac{\left(s-m_\mu^2-m_e^2\right)^2}{t^2}+\frac{s}{t}+\frac{1}{2}\right\}$$

- The primary goal of [arXiv:1504.02228,1609.08987]: determining a_μ^{had} in an alternative way
- $\Pi'_\gamma(Q^2)-\Pi'_\gamma(0) = -\Delta\alpha_{\text{had}}(-Q^2) = \frac{\alpha}{\alpha(-Q^2)} + \Delta\alpha^{\text{lep}}(-Q^2) - 1$

  directly checks lattice QCD data
- My proposal here: determine very accurately

$$\boxed{\Delta\alpha_{\text{had}}\left(-Q^2\right) \text{ at } Q \approx 2.5 \text{ GeV}}$$

by this method (one single number!) as the non-perturbative part of $\Delta\alpha_{\text{had}}\left(M_Z^2\right)$ as in "Adler function" approach.

Conclusions

- Muon $g-2$ theory uncertainty remains the key issue.
- Are presently estimated (essentially agreed) evaluations in terms of R-data reliable?
- Lattice QCD estimates very close to be competitive, tending to larger central values?
- Novel hybrid method lattice + R-data optimized method looks very promising
  D. Bernecker, Harvey B. Meyer 2011, RBC/UKQCD Blum et al. 2018
- In any case on paper $e^-\mu^+ \to e^-\mu^+$ looks to be the ideal process to perform an unambiguous measurement of $\alpha(-Q^2)$, which determines the LO HVP to a_μ
- Radiative corrections much easier than for the time-like hadronic channels
- Key problem: how to control in a fixed target experiment the precision at the 1% level? What range is accessible at what precision?

Likely to be supplemented by lattice QCD results and/or using time-like data

- This experiment G. Abbiendi et al. is absolutely important also as it allows for for direct crosschecks with lattice QCD results and is has completely different systematics. Even a 5% crosscheck would be very helpful to scrutinize the HVP issue, and last bu not least whether the observed deviation is a real BSM effect.

Thanks you for your attention!