Lecture 2:

the SM as a low energy effective theory (LEESM scenario)

Outline of Lecture 2:

③ **Low energy effective QFT of a cutoff system**

- ❏ **The emergence of multipole forces**
- ❏ **The emergence of non-Abelian gauge structures**
- ❏ **Conspiracies tuning for criticality**

A summary and outlook of Lecture 1

Connection between classical statistical mechanics and QFT

The Theory of Critical Phenomena tells us that for tuned values of external parameters like temperature, external magnetic field etc. when the correlation length $\xi \to \infty$ (corresponding to the continuum limit) a renormalizable Euclidean quantum field theory describes critical points, critical behavior, universality,...
Prototypes: Ising, Heisenberg, Landau-Ginzburg, Dzyaloshinskii-Moria, etc
Main issue: new view of elementary particle interaction, where most of its regularity and "simplicity" may be attributed to the universality of long range structures of statistical mechanics systems near a critical point.

Since Ken Wilson 1971 we know:

- **long range asymptote of statistical mechanics system near critical point =**

renormalizable Euclidean continuum quantum field theory (quantization for free)

- **the generating functional of the statistical mechanics correlation functions in long range effective form = generating functional of an Euclidean field theory (path integral quantization based on classical fields)**

- **renormalizability including the required symmetries to preserve it emerge naturally, non-renormalizable effective couplings are irrelevant i.e. power suppressed at long distances**

- **Wilson type RG analysis confirms the Landau criterion that statistical mechanics systems all behave Gaussian (free fields) in $d > 4$ dimensions for all systems with a stable ground state. Long range modes are uncoupled in $d > 4$ (no need for compactification)**

behind this: leading long range dynamics normalized by kinetic term $p^2\,\tilde{\phi}^2$

- **for the borderline case $d = 4$ interaction sets in as $d \gtrsim 4$. This happens to coincide with dimension of physical space-time. And the perturbative nature of the interaction of the long range modes finds its natural physical justification.**

- **by universality: long range physics does not depend on details of the short distance structure, except for properties (dimension, number and type of long range effective degrees of freedom) which affect the universality class. A universality class exhibits a whole plethora of different systems, like Landau-Ginsburg superconductor, ferromagnetic, liquid-gas systems in condensed matter physics. Cutoff at long distances only sets the scale (renormalization scale)**

- **interactions only restricted by such general principles as positivity, existence of the infinite volume limit and to some extent translation and rotational invariance (homogeneity and isotropy)**

- **the Osterwalder-Schrader conditions are satisfied by Long Range Tail (LRT) correlation functions and the 4-dim Euclidean effective theory is mathematically equivalent to a $(3,1)$ local QFT in Minkowski space (analyticity permits a Wick rotation)**

- **consider elementary particle interactions as long range effective interactions (Van der Waals like) of an appropriate statistical mechanics system which exhibits a fundamental cutoff.**

- **we expect the Planck length $\ell_0 \sim \times 10^{-33}$ cm, the characteristic fundamental length intrinsic to gravity (Newton constant), to define the required cutoff $M_{\rm Planck} \sim \times 10^{19}$ GeV. The Planck medium (condensed matter type system), which we may call "ether" is a system unifying gravity with all other interactions, which are effective interactions emerging as collective phenomena at longer distances.**

③ Low energy effective QFT of a cutoff system

Example: Higgs self-interacting system:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm int} = \frac{1}{2}\,\partial^\mu\phi(x)\,(1+\Box/\Lambda^2)\,\partial_\mu\phi(x) - \frac{1}{2}\,m_0^2\,\phi(x)^2 - \frac{\lambda_0}{4!}\,\phi^4(x)\ .$$

- **regularization here as a Pais-Uhlenbeck higher-derivative kinetic term, equivalent to a Pauli-Villars cutoff**
- **vertex functions of N scalar fields** $\Gamma^{(N)}_{\Lambda,b}(p;m_0,\lambda_0) = \langle\tilde\phi(p_1)\cdots\tilde\phi(p_{N-1})\phi(0)\rangle^{\rm prop}$
- **bare functions are related to the renormalized ones by reparametrizing parameters and fields**

$$\Gamma^{(N)}_{\Lambda\,r}(p;m,\lambda) = Z^{N/2}(\Lambda/m,\lambda)\,\Gamma^{(N)}_{\Lambda\,b}(p;\Delta m_0(\Lambda,m,\lambda),\lambda_0(\Lambda/m,\lambda))\ .$$

- **satisfy a RG equation for the response to a change of the cutoff Λ for fixed renormalized parameters** $\Lambda\frac{\partial}{\partial\Lambda}\,\Gamma(N)_{\Lambda\,b}\Big|_{m,\lambda}$**, which by applying**

the chain rule of differentiation reads

$$\left(\Lambda\frac{\partial}{\partial\Lambda}+\beta_0\frac{\partial}{\partial\lambda}-N\,\gamma_0+\delta_0\,\Delta m_0^2\,\frac{\partial}{\partial\Delta m_0^2}\right)\Gamma^{(N)}_{\Lambda\,b}(p;m_0,\lambda_0)=Z^{-N/2}\,\Lambda\frac{\partial}{\partial\Lambda}\,\Gamma^{(N)}_{\Lambda\,r}(p;m,\lambda)\ .$$

m_{0c}^2 **is the "critical value" of the bare mass for which the renormalized mass is zero i.e.** $m^2\propto\Delta m_0^2=m_0^2-m_{0c}^2$ **is related to renormalized mass. Since the renormalized vertex functions have a regular limit as** $\Lambda\to\infty$**, to all orders in perturbation theory the inhomogeneous part behaves as**

$$Z^{N/2}\,\Lambda\frac{\partial}{\partial\Lambda}\,\Gamma^{(N)}_{\Lambda\,r}(p;m,\lambda)=O(\Lambda^{-2}(\ln\Lambda)^l)\ ,$$

i.e., the inhomogeneous part, representing a cutoff insertion, falls off faster than the l.h.s. by two powers in the cutoff for large cutoffs.

In addition, all the RG equation coefficients exist as non-trivial functions in

the limit of infinite cutoff:

$$\lim_{\Lambda\to\infty} \alpha_0(\Lambda/m,\lambda) = \alpha(\lambda)\,,\ \alpha = \beta,\gamma,\delta$$

- **proper vertex-functions have a large cutoff Λ-expansion**

$$\Gamma^{(N)}_{\Lambda\, b}(p;\Delta m_0,\lambda_0) = \sum_{j,l\geq 0} \Lambda^{-2j}(\ln\Lambda)^l\, f^{(N)}_{jl}(p\ \Delta m_0,\lambda_0)$$

and for large Λ we obtain the <u>preasymptote</u> of $\Gamma^{(N)}_{\Lambda\, b}$

$$\Gamma^{(N)}_{\Lambda\,\mathrm{as}}(p;\Delta m_0,\lambda_0) = \sum_{l\geq 0} (\ln\Lambda)^l\, f^{(N)}_{0l}(p\ \Delta m_0,\lambda_0)$$

such that

$$\left|\Gamma^{(N)}_{\Lambda\, b}(p;\Delta m_0,\lambda_0) - \Gamma^{(N)}_{\Lambda\,\mathrm{as}}(p;\Delta m_0,\lambda_0)\right| = O(\Lambda^{-2}(\ln\Lambda^{l_x}))$$

where l_x is bounded to all orders in the perturbation expansion. The key point is that the still cutoff dependent preasymptote satisfies a homogeneous RG equation, a special property of the long range tail of the bare theory:

$$\left(\Lambda\frac{\partial}{\partial\Lambda}+\beta_{\rm as}(\Lambda/\Delta m_0,\lambda_0)\frac{\partial}{\partial\lambda_0}-N\,\gamma_{\rm as}(\Lambda/\Delta m_0,\lambda_0)\right.$$
$$\left.+\delta_{\rm as}(\Lambda/\Delta m_0,\lambda_0)\,\Delta m_0^2\,\frac{\partial}{\partial\Delta m_0^2}\right)\Gamma^{(N)}_{\Lambda\,{\rm as}}(p;\Delta m_0,\lambda_0)=0\ .$$

Note on mass-dependent renormalization:

The response of the on-shell renormalized theory to a change in the mass, in the limit $\Lambda \to \infty$, is given by the Callan-Symanzik equation (Callan 1970, Symanzik 1970)

$$\left(m\frac{\partial}{\partial m} + \beta(\lambda)\frac{\partial}{\partial \lambda} - N\,\gamma(\lambda)\right)\Gamma_r^{(N)}(p;m,\lambda) = -m^2\,\left(2-\delta(\lambda)\right)\,\Delta_0\,\Gamma_r^{(N)}(p;m,\lambda)\,,$$

where Δ_0 is the integrated mass operator insertion. For large momenta the r.h.s. is suppressed $O(m^2\,\ln(m)^l)$ by the small mass-square $m^2 \ll p^2$ up to logarithms, such that for large momenta asymptotically

$$\left(m\frac{\partial}{\partial m} + \beta(\lambda)\frac{\partial}{\partial \lambda} - N\,\gamma(\lambda)\right)\Gamma_{r\,as}^{(N)}(p;m,\lambda) = 0\,.$$

The mass asymptotically only plays the role of a renormalization scale, $\Gamma_{r\,as}^{(N)}(p;m,\lambda)$ are vertex functions of an effectively massless theory. Up to appropriate finite reparametrization and a rescaling $m = \kappa\mu$ the homogeneous CS equations are nothing but the standard RG equation in the $\overline{\mathrm{MS}}$ scheme.

- the homogeneity tells us that Λ has lost it function as a cutoff and takes the role of a renormalization scale, i.e., the homogeneous RG represents the response of a rescaling of the system: a change in Λ is compensated by a finite renormalization of the fields, the couplings and the masses

- by a finite renormalization we may reparametrize (renormalization conditions) the preasymptote. By finite rescaling $\Lambda = \kappa\mu$ the usual RG in the renormalization scale μ of a non-trivial continuum QFT are recovered.

- this allows for a precise interrelation to $\overline{\mathrm{MS}}$ renormalized quantities.

- what we observe as the SM is a physical reparametrization (renormalization) of the preasymptotic bare theory

- one of the impacts of the very high Planck scale is that the local renormalizable QFT structure of the SM is presumably valid up to 10^{17} GeV at the 0.1% accuracy level. This also justifies the application of the SM RG up to high scales.

- **infinite tower of dim > 4 irrelevant operators not seen at low energy (simplicity of LEET)**

- **problems are the dim < 4 relevant operators , in particular the mass terms,**

  require "tuning to criticality" .

Our Hierarchy Problem!

In the symmetric phase of the SM, where there is only one mass (the others are forbidden by the known chiral and gauge symmetries), the one in front of the Higgs doublet field, the fine tuning has the form

$$m_0^2 = m^2 + \delta m^2 \; ; \quad \delta m^2 = \frac{\Lambda^2}{16\pi^2} C$$

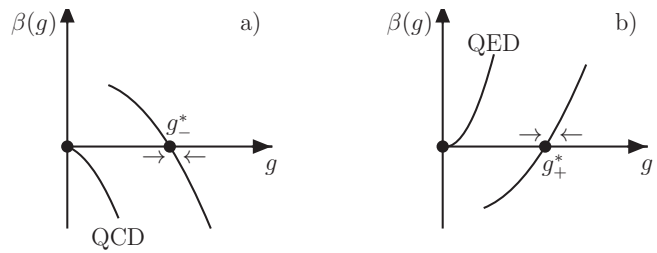
with a coefficient typically $C = O(1)$. To keep the renormalized mass at some small value, which can be seen at low energy, m_0^2 has to be adjusted to compensate the huge number δm^2 such that about 35 digits must be adjusted in order to get the observed value around the electroweak scale.

In the following we consider the SM as a strictly renormalizable theory, regularized as usual by dimensional regularization in $D = 4 - \varepsilon$ space-time dimensions, such that the $\overline{\mathrm{MS}}$ parametrization and the corresponding RG can be used in the well known form.

- **Key observation: elementary particle interactions have rather weak coupling such that perturbation theory works in general, except QCD below 2 GeV**

- **IR fixed points in $d > 4$ couplings go to zero power-like with the cut off**

- **in $d = 4$ boarder case $\lambda = 0$ is IR stable fixed point and coupling gets weak logarithmically in the cut off**



RG fixed points are zeros of the β–function: a) UV fixed points, b) IR fixed points

The precise RG fixed point structure of the SM will be discussed later

The low energy expansion at a glance

	dimension	operator	scaling behavior
	.	∞–many	
	.	irrelevant	
$\uparrow$	.	operators	
no			
data	$d = 6$	$(\Box\phi)^2, (\bar\psi\psi)^2, \cdots$	$(E/\Lambda_{\rm Pl})^2$
|	$d = 5$	$\bar\psi\sigma^{\mu\nu}F_{\mu\nu}\psi, \cdots$	$(E/\Lambda_{\rm Pl})$
|	$d = 4$	$(\partial\phi)^2, \phi^4, (F_{\mu\nu})^2, \cdots$	$\ln(E/\Lambda_{\rm Pl})$
experimental	$d = 3$	$\phi^3, \bar\psi\psi$	$(\Lambda_{\rm Pl}/E)$
data	$d = 2$	$\phi^2, (A_\mu)^2$	$(\Lambda_{\rm Pl}/E)^2$
$\downarrow$	$d = 1$	ϕ	$(\Lambda_{\rm Pl}/E)^3$

Up to date and for a long time to come there is and will be no direct experimental information on $O(E/\Lambda_{\rm Pl})$ effects (but bounds on absence of such terms).

Note: if we compare the two opposing ways to approach the Planck scale

TOE Paradigm:

$$\text{M–THEORY} \sim \text{STRINGS} \leftarrow \text{SUGRA} \leftarrow \text{SUSY} \leftarrow \text{SM},$$

versus

Emergence Paradigm:

$$\text{ETHER} \sim \text{Planck medium} \rightarrow \text{low energy effective QFT} \rightarrow \text{SM}.$$

in both cases the low energy expansion must predict the SM as a renormalizable effective theory, i.e. the tail below $\sim$ 1 TeV should be identical.

However, the STRINGy path necessarily must have as a “pre SM tail” the much more complex minimal super symmetric extended SM (MSSM) as a renormalizable LEET.

While the emergent SM is the result of not seeing the infinite tower of irrelevant operators and we naturally expect symmetries to be explicitly broken as the irrelevant operators come into play, the “String relict SM” looks to get more symmetric by including additional terms in the low energy expansion, in particular also by including at the end all irrelevant operators. The extended symmetry in the TOE approach acts as a collective fine tuning of the entire operator tower.

The necessary unnatural fine tuning which must take place to increase symmetries when going to higher energies can be achieved only by imposing the symmetries at the Planck scale by hand.

Naturally emergent structures at low energies

Above we outlined that known empirical facts about structural properties of elementary particle theory find a natural explanation in low energy effective theories. Usually,

- ❑ **quantum mechanics,**
- ❑ **special relativity,**
- ❑ **four dimensionality,**
- ❑ **renormalizability**

are independent inputs. Detailed investigations confirm that all these properties may be understood as consequences of the existence of an "stochastic cutoff system" in the appropriate universality class.

At long distances we further expect to observe

- **multipole fields together with their interactions**

- **non-Abelian gauge symmetries, broken as the SM $SU(2)_L$ or unbroken like the SM $SU(3)_c$**

Important remark:

- **renormalizability usually has been imposed as a "technical" requirement to get a predicting theory, and by the success of renormalizable QFTs in phenomenological applications. Only operators of dimension $d \leq 4$ observed.**

- **in our setup renormalizability is a predictable structure emerging automatically in a low energy expansion!**

Note: the Higgs was a prediction resulting from the requirement of a minimal renormalizable extension of QED+ low energy effective Fermi theory (weak charged current interaction) ➜ Yang-Mills interaction, neutral currents, Fermions in families and the Higgs and its interaction

- **the Higgs boson was invented 1964 by Higgs and others [Brout-Englert, Guralnik-Hagen-Kibble, Kibble] to formulate a renormalizable theory of weak interactions [allows us to make finite unambiguous predictions, formally only operators of mass dimension $d \leq 4$ allowed] responsible e.g. for β–decay of the neutron [$n \to p + e^- + \nu_e$]**

Englert and Higgs NP 2013: for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider.

Did the Nobel Committee (and CERN physicists) understand what the Higgs was invented for? Indeed most colleagues seem not to be aware that what is important for achieving renormalizability is not only mass generation but the Higgs interaction (the Higgs as an exchange particle mediating specific interactions) and the resulting mass-coupling relations.

❑ The emergence of multipole forces

Assume there exists a potential $\Phi \sim -\frac{1}{r^{d-2}}$ for ($r \to \infty$) sitting at $M_{\rm Planck}$

1. For weaker decay thermodynamic limit not existent.

2. For stronger decay non–leading term.

If we perform a multipole expansion, we get

$$
\begin{aligned}
\partial_i \Phi &= (d-2)\,\frac{x_i}{r^d} \\
\partial_j \partial_i \Phi &= (d-2)\left\{\frac{\delta_{ij}}{r^d} - d\frac{x_i x_j}{r^{d+2}}\right\} \\
\partial_k \partial_j \partial_i \Phi &= (d-2)\,d\,\left\{(d+2)\,\frac{x_i x_j x_k}{r^{d+4}} - \frac{\delta_{ij}x_k + \delta_{ik}x_j + \delta_{jk}x_i}{r^{d+2}}\right\} \\
\partial_l \partial_k \partial_j \partial_i \Phi &= (2-d)\,d\,\left\{(d+2)(d+4)\,\frac{x_i x_j x_k x_l}{r^{d+6}} - (d+2)\,\frac{\delta_{ij}x_k x_l + \text{perm}}{r^{d+4}} + \frac{\delta_{ij}\delta_{kl} + \text{perm}}{r^{d+2}}\right\} \\
&\vdots
\end{aligned}
$$

In the stochastic cutoff system naturally scalar-, vector-, tensor- etc fluctuation variables

$$q, A_i, Q_{ij} \cdots$$

exist, interacting with the "potential". The corresponding interaction Hamiltonians read:

$$\begin{aligned}
H_{qq} &= q_1 q_2 \Phi \\
H_{qA} &= -q_1 A_{2i} \partial_i \Phi \\
H_{qQ} &= q_1 Q_{2ij} \partial_j \partial_i \Phi \\
H_{AA} &= A_{1i} A_{2j} \partial_j \partial_i \Phi \\
H_{AQ} &= -A_{1k} Q_{2ij} \partial_k \partial_j \partial_i \Phi \\
H_{QQ} &= -Q_{1ij} Q_{1kl} \partial_l \partial_k \partial_j \partial_i \Phi \\
&\vdots
\end{aligned}$$

"Charge neutrality" for large distances requires $q = 0$. Remaining lowest bilinear "emergences" thus are H_{AA} and H_{QQ}, which we may work out in momentum space

Dipole–dipole interaction:

$$\mathbf{H = -\sum K_{x-y,ik} A^i_x A^k_y}$$

$$\tilde{K}_{ik}(q) = m^2 \left(d\,\frac{q_i q_k}{q^2} + \delta_{ik} \right) + c \left(-d q_i q_k + q^2 \delta_{ik} \right) + O(q^4)$$

Quadrupole–quadrupole interaction:

$$\mathbf{H = -\sum K_{x-y,ijkl} Q_{xij} Q_{ykl}}$$

$$\tilde{K}_{ijkl}(q) = a_0\,\frac{q_i q_j q_k q_l}{q^2} + b_0 \left(\delta_{ij} q_k q_l + \cdots \right) + c_0 q^2 \left(\delta_{ij}\delta_{kl} + \cdots \right) + O(q^4)$$

$$c_0 = \int_0^\infty dr\, r^{2/2+2} J_{d/2+3}(r)\ ; \quad b_0 = (d+2)\,c_0\ , \quad a_0 = (d+4)\,b_0$$

Propagators including n.n. exchange interactions:

"Vector boson":

$$\tilde{G}_{q,ij} = \left(\tilde{K}_q\right)^{-1}_{ij} = \left(\delta_{ij} - \frac{g + bq^2}{g + bq^2 + m^2 + q^2}\,\frac{q_i q_j}{q^2}\right)\frac{1}{m^2 + q^2}$$

"Graviton":

$$\tilde{G}_{q,ijkl} = \left(\tilde{K}_q\right)^{-1}_{ijkl} = \sum_{i=0}^{4} b_i X_i \;;\quad \tilde{K}_{q,ijkl} = \sum_{i=0}^{4} a_i X_i$$

X_0	X_1	X_2	X_3	X_4
$\delta_{ij}\delta_{kl}$	$\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}$	$\delta_{ij}\,\frac{q_k q_l}{q^2} + \delta_{kl}\,\frac{q_i q_j}{q^2}$	$\delta_{ik}\,\frac{q_j q_l}{q^2} + \text{perm}$	$\frac{q_i q_j q_k q_l}{q^4}$

with

$$b_1 = \frac{1}{2a_1} \;;\quad b_3 = -2b_1\,\frac{a_3}{2(a_1 + a_3)}$$

Note: by renormalization of the boson fields and the couplings one may always arrange the q^2 term in the bilinear part to be Euclidean invariant (Liu–Stanley theorem). A similar statement should hold for fermions.

The RG analysis $\mathbf{K}'(q', s) = \alpha_x^2 s^{-d}\,\mathbf{K}(q'/s)$ for the vector boson reveals two possible fixed points
(i) $m^* = 0, g^* = 0$
(ii) $m^* = 0, g^* = \infty$
At the bipolar fixed point (ii)

$$\mathbf{G}_{ik} = \frac{1}{q^2}\left\{\delta_{ik} - \frac{q_i q_k}{q^2}\right\}$$

a massless spin 1 propagator in a non-local gauge. If dipolar interactions are absent, we are at the isotropic fixed point (i) and $\mathbf{G}_{ik}$ has the form of a massive vector-boson propagator in the 't Hooft (R_ξ) gauge with h as the

gauge parameter.

$$\mathbf{G}_{ik} = \frac{1}{q^2}\left\{\delta_{ik} - \frac{h}{m^2 + (1-h)\,q^2}\,q_i q_k\right\}$$

for detail and the case of the graviton see F.J. HPA 1978.

❑ W–Pair Production and Gauge Field Dynamics

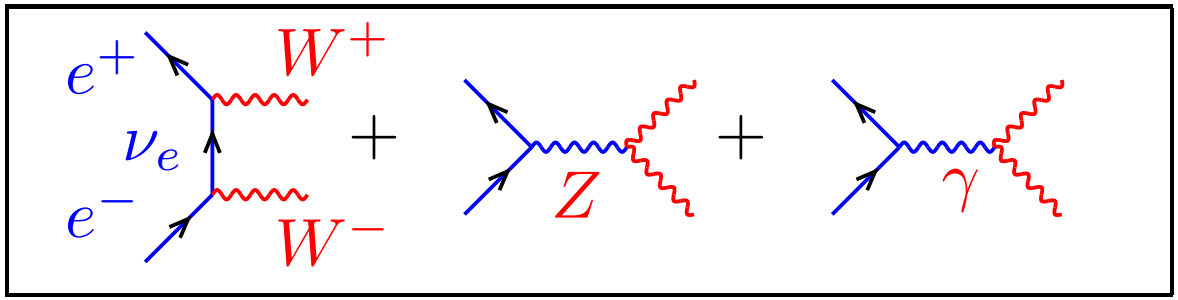
Spin 1 bosons in QFT are special

We consider in more detail the idealized process

$$e^+(p_+,\bar{\sigma}) + e^-(p_-,\sigma) \to W^+(q_+,\bar{\lambda}) + W^-(q_-,\lambda)$$

in the approximation of stable W's. There are three diagrams contributing to the process at the tree level. The first diagram only exhibits known interaction vertices, while the last two are new to the extent that no *direct* experimental test has

been possible so far.



SM born diagrams $e^+e^- \to W^+W^-$

The differential cross section for definite helicities is given by

$$\frac{d\sigma(\pm;\lambda,\bar{\lambda})}{d(\cos\theta)} = \frac{\beta_W}{32\pi s} \mid M_0(\pm;\lambda,\bar{\lambda}) \mid^2$$

where $s = (2E_b)^2$, E_b **is the beam energy, and** $\cos\theta$ **is the angle between the three-momentum vectors of the** e^- **and the** W^- **in the c.m. system.** $\beta_W = \sqrt{1 - 4M_W^2/s}$ **is the** W **velocity and** $\gamma^2 = s/(4M_W^2)$ **the Lorentz factor. The**

helicity matrix elements have the form

$$M_0(-;\lambda,\bar\lambda) = S^{(-)}_{1(0)}\mathbf{M}_{S_1}(-;\lambda,\bar\lambda) + T^{(-)}_{1(0)}\mathbf{M}_{T_1}(-;\lambda,\bar\lambda)$$
$$M_0(+;\lambda,\bar\lambda) = S^{(+)}_{1(0)}\mathbf{M}_{S_1}(+;\lambda,\bar\lambda) \quad .$$

The gauge cancellation may be observed directly by writing the invariant amplitudes in the form:

$$\begin{aligned}
S^{(-)}_{1(0)} &= -\frac{e^2}{s}(1^\gamma_V - \frac{1^Z_V}{1-M_Z^2/s}) - \frac{g^2}{2s}\frac{1^Z_A}{1-M_Z^2/s} \\
S^{(+)}_{1(0)} &= -\frac{e^2}{s}(1^\gamma_V - \frac{1^Z_V}{1-M_Z^2/s}) \\
T^{(-)}_{1(0)} &= \frac{g^2}{s}\frac{1}{\frac{1+\beta_W^2}{2} - \beta_W\cos\theta}
\end{aligned}$$

where the labels of $1^\gamma_V \equiv 1$ are just indicating the origin of the term, V and A

denoting, respectively, vector and axial–vector current, and γ and Z denoting the s-channel exchange particle. While the gauge cancellation for the vector happens between the γ and Z directly in the invariant amplitudes the axial part of the Z exchange contribution only cancels in the sum ME0 against the $T^{(-)}_{1(0)}$ amplitude. Here it is crucial that the ratio

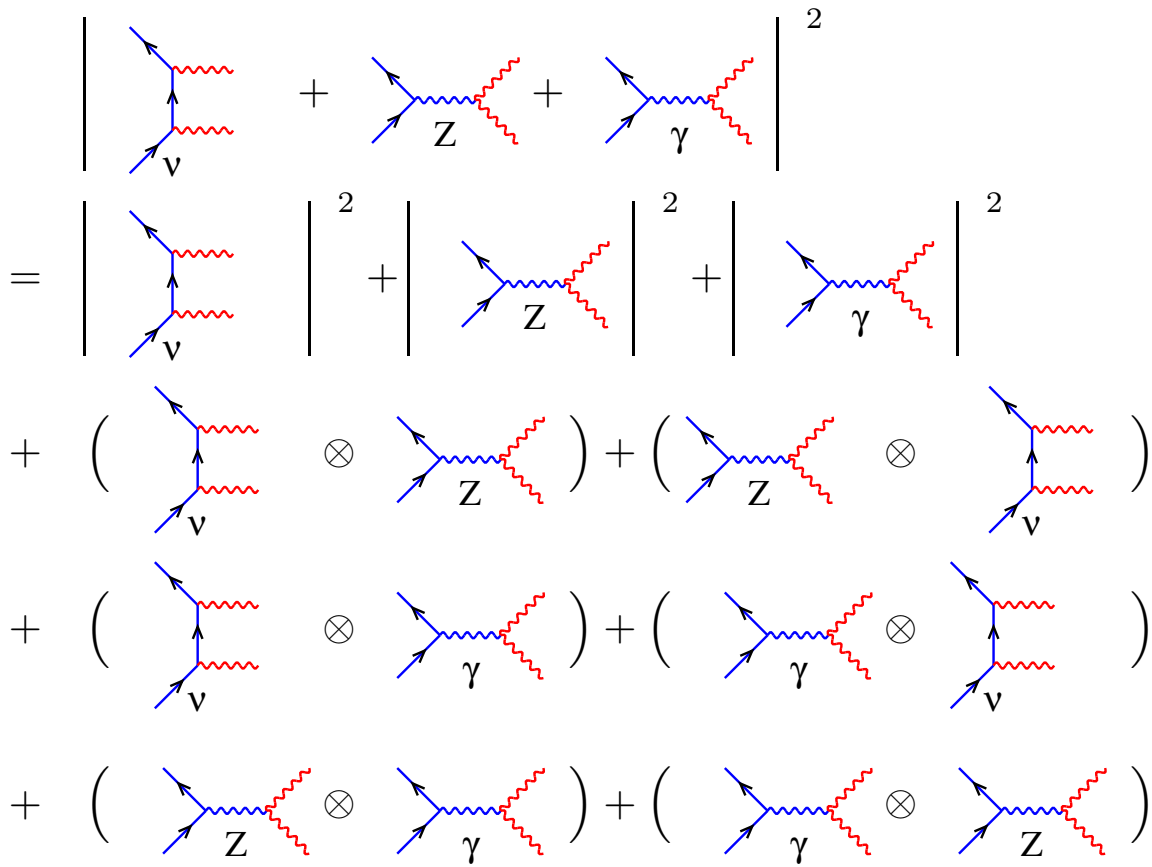
$$\frac{\mathbf{M}_{T_1}(h_e;\lambda,\bar{\lambda})}{\mathbf{M}_{S_1}(h_e;\lambda,\bar{\lambda})} = \frac{1-\cos\theta}{2} \quad \text{for} \;\; \gamma \to \infty \; (\beta_W \to 1)$$

is equal for all final states with $|\Delta\lambda| < 2$. This allows to fix the axial Z coupling relative to the established t-channel ν exchange contribution.

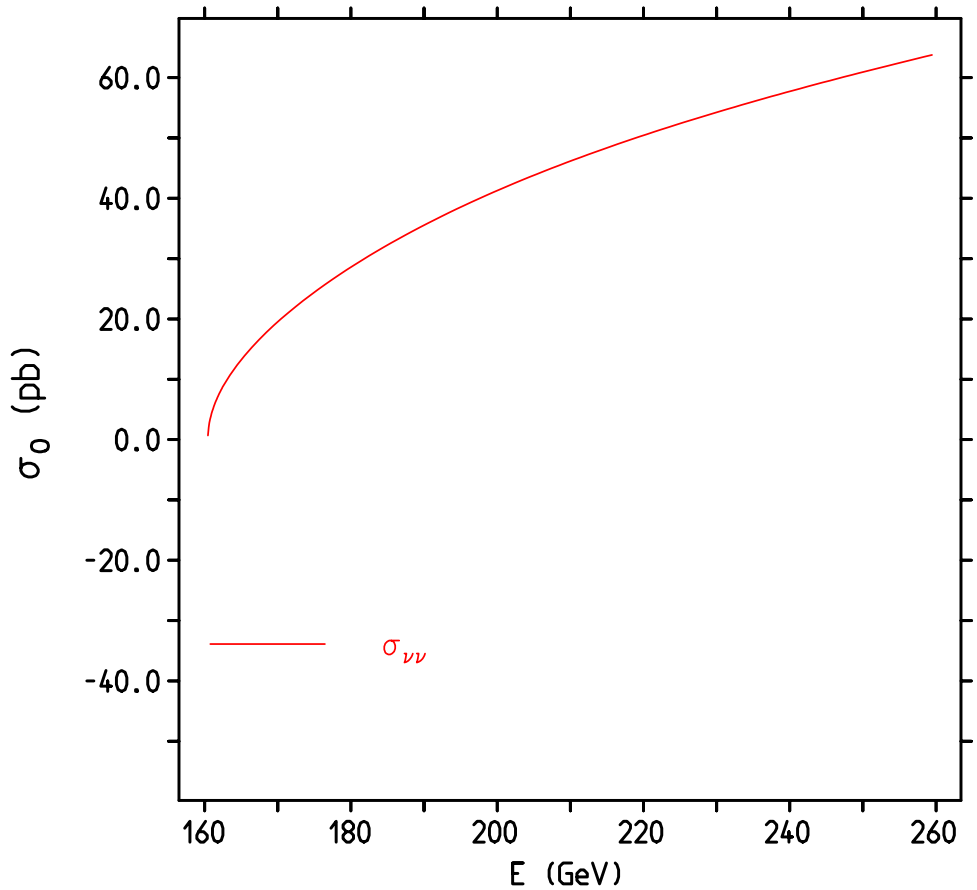
The gauge cancellation dictated by the gauge invariance of the theory implies that

- **individual Feynman diagrams have no physical meaning**
- **the observable total is the result of a strongly destructive interference.**

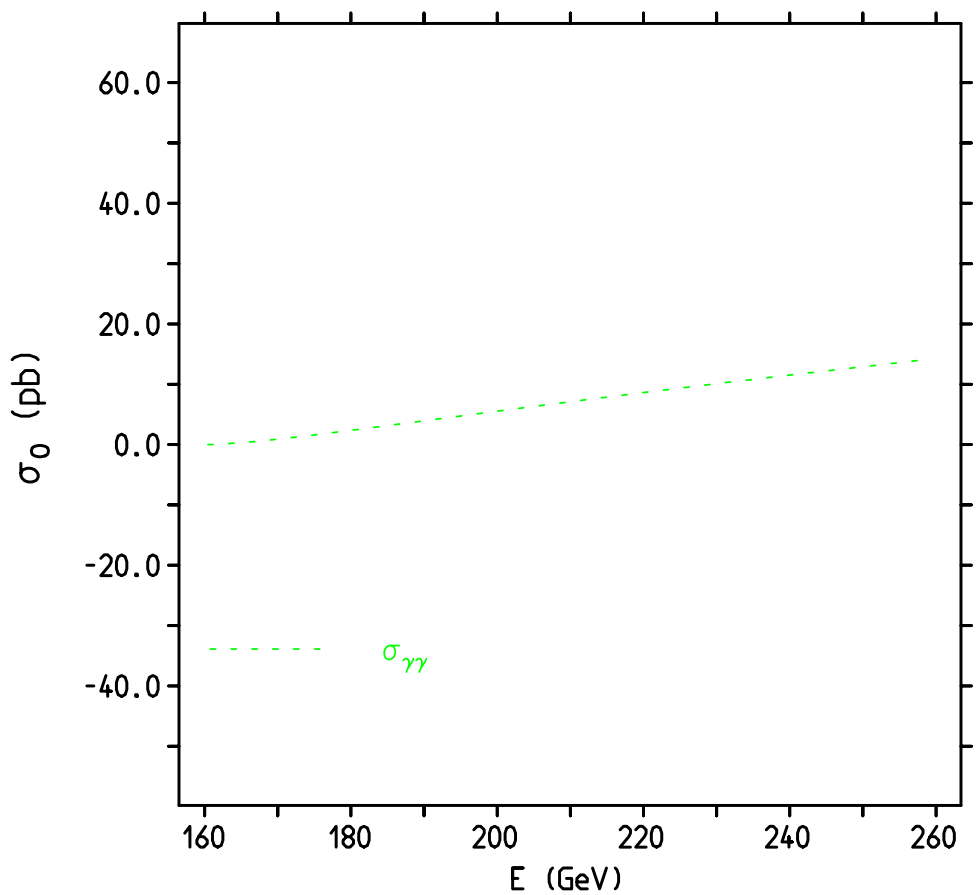
This is a new situation.



$|\, M_0(\pm;\lambda,\bar{\lambda})\,|^2$ **the different cross-section contributions**



Gauge cancellation in $e^+e^- \to W^+W^-$.



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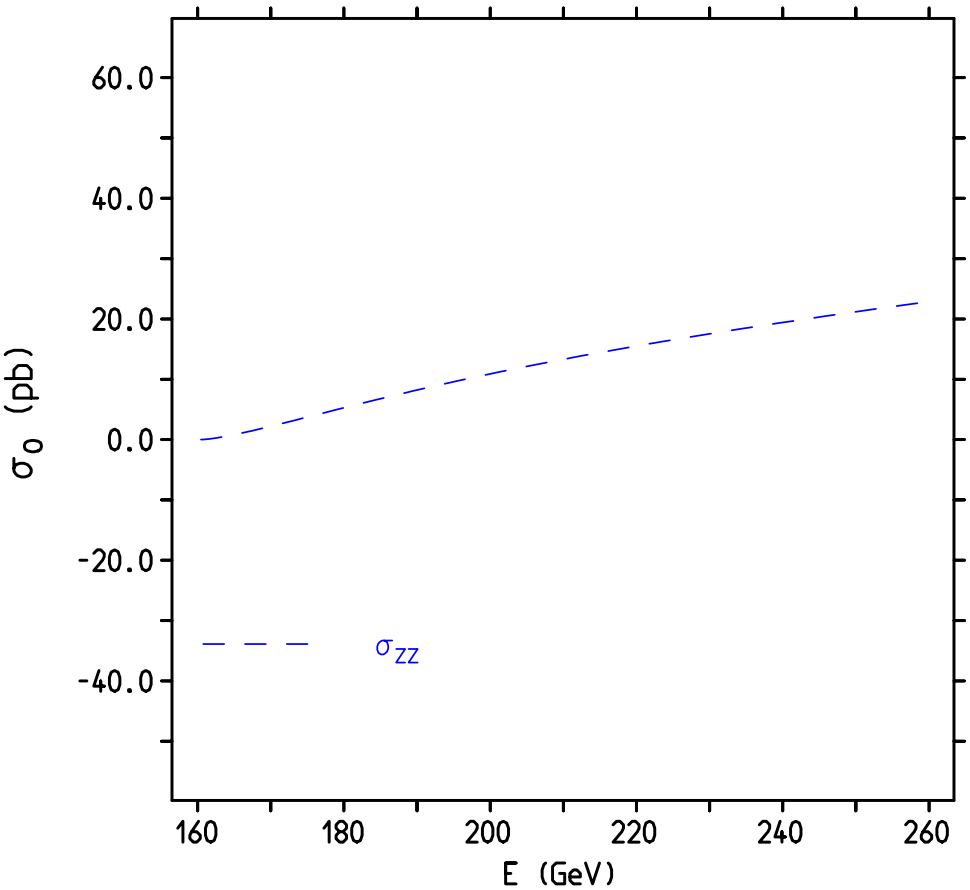

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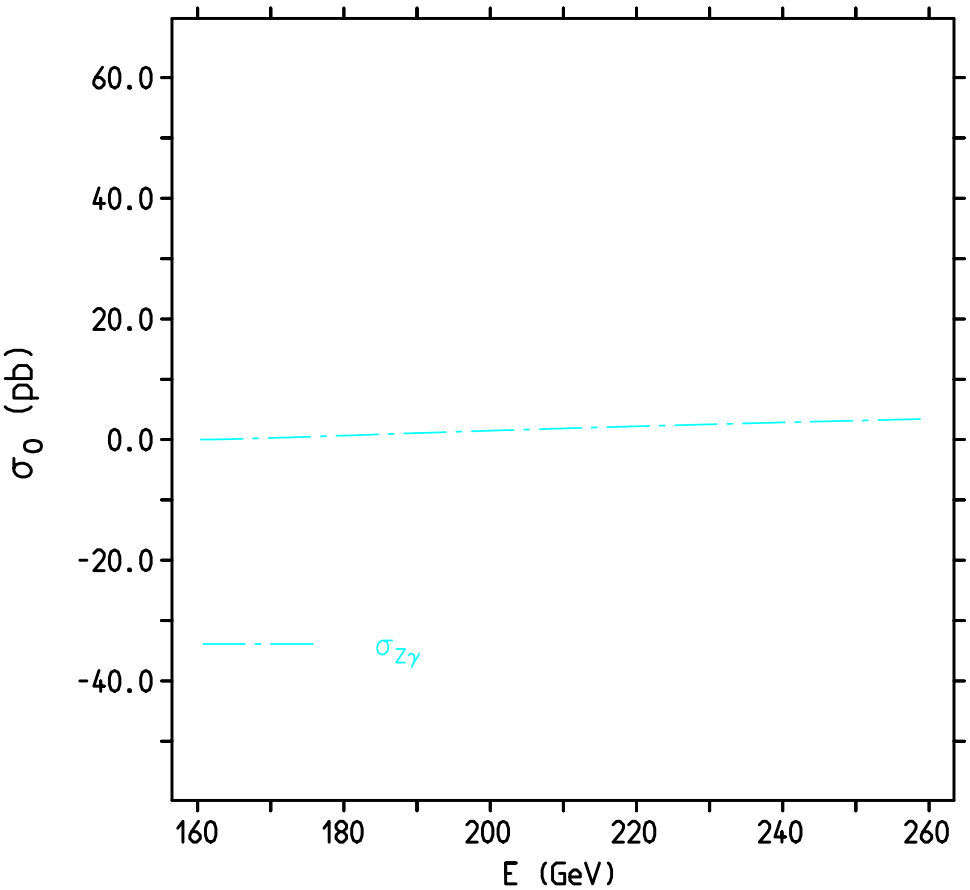

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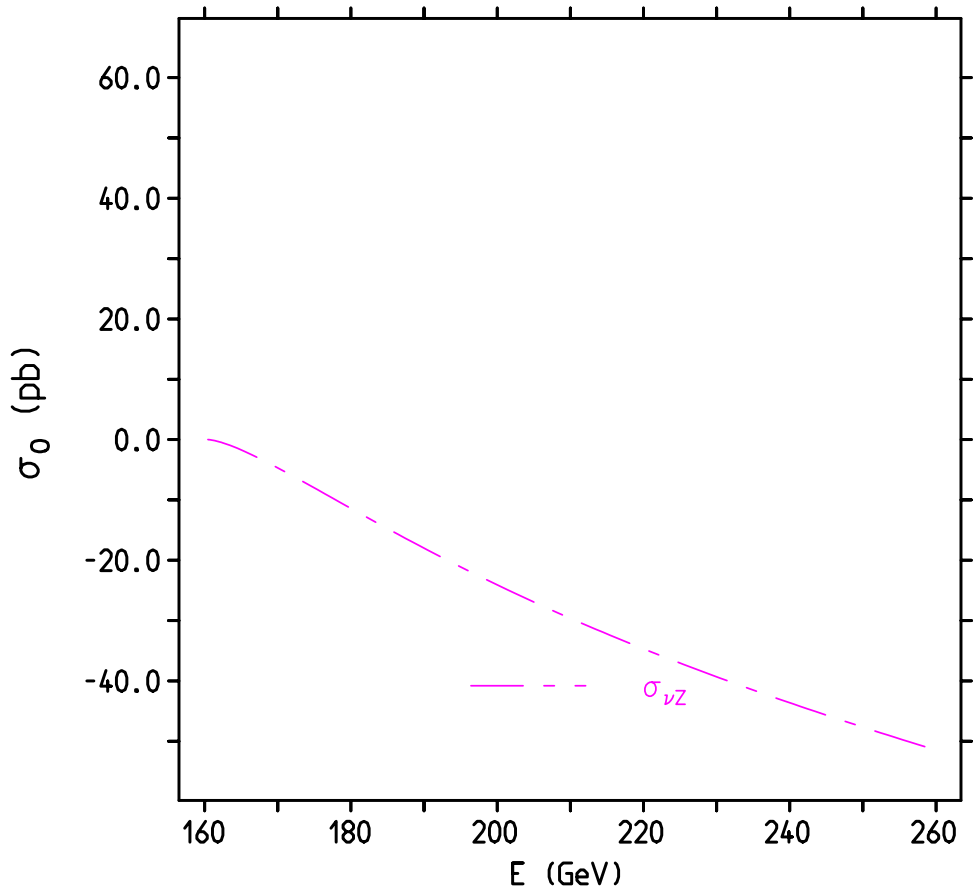

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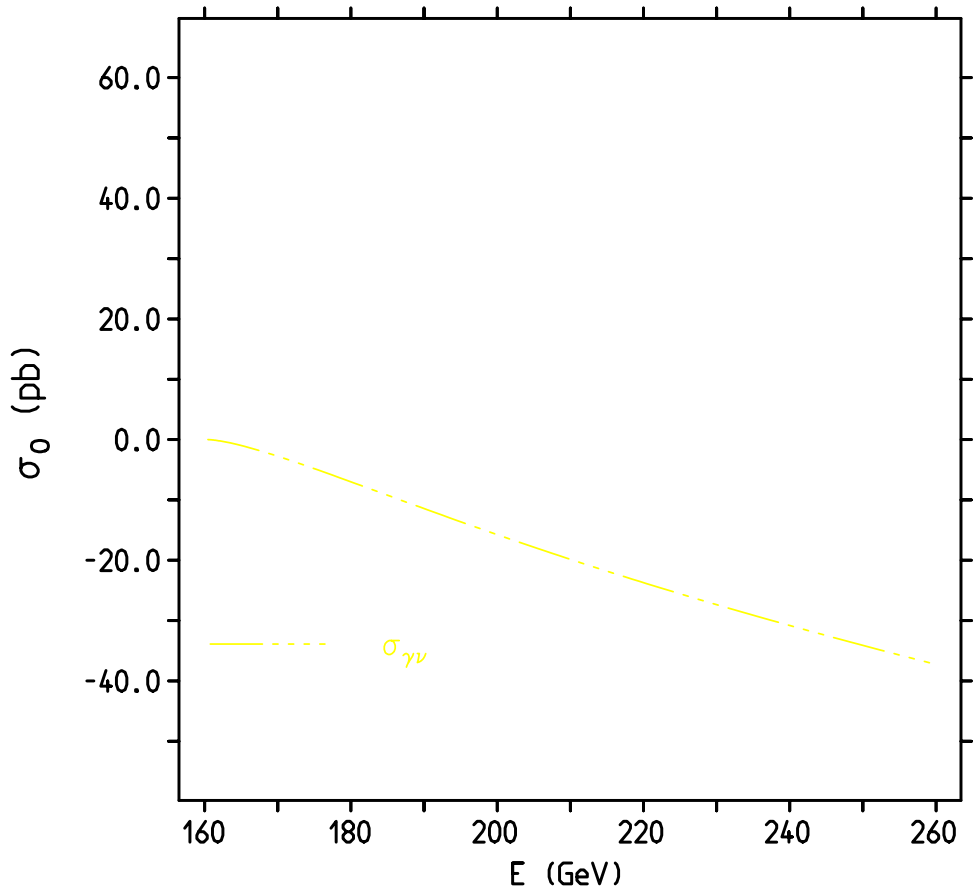

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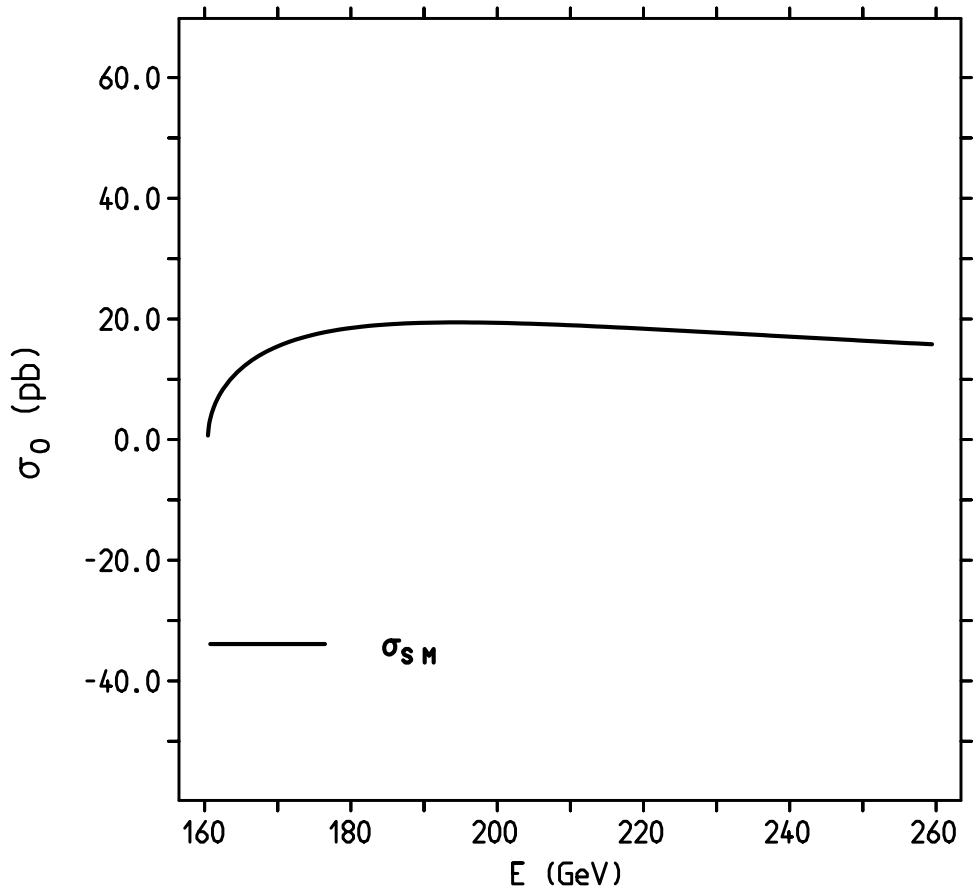
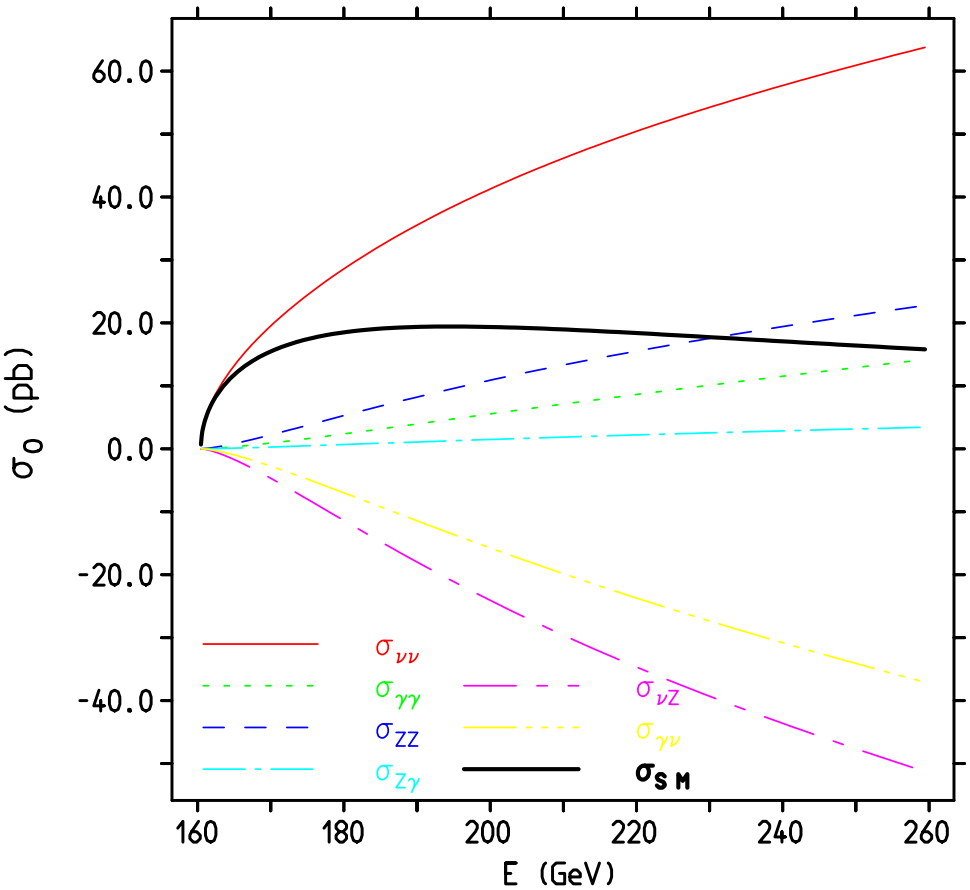


Gauge cancellation in $e^+e^- \to W^+W^-$.



Gauge cancellation in $e^+e^- \to W^+W^-$.

Up to the present, the phenomenology of electroweak processes (from Fermi (1934) until LEP II will be running) was the history of fixing the structure of the $SU(2)_L \otimes U(1)_Y$ fermion current couplings, determined in "low energy" four fermion processes like μ–decay, neutrino scattering and e^+e^-–annihilation, either at low momentum transfer or near some resonance (e.g., the Z at LEP I). Couplings (form–factors) measured in such circumstances are dominated by single one–particle exchange processes and consequently provide simple interpretations for the measurements. In W–pair production in e^+e^-–annihilation the situation is much more complex. It takes place in the continuum of states and one only observes interferences, so that form–factors have *no direct observable meaning*. This not only makes the interpretation of the results unclear, it makes precise predictions more difficult. Consequently, if deviations from the SM would be found the physics origin could remain obscure for quite some time.

❑ The emergence of non-Abelian gauge structures

A simplified model:

Consider 3 particle species only, scalars ϕ_a, fermions ψ_α, vector bosons $W_{i\mu}$, with covariant propagators. Because of low energy expansion we need consider only terms which are not manifestly irrelevant

$$
\begin{aligned}
\mathcal{L}_1 &= \bar\psi_\alpha \left\{ L^i_{\alpha\beta} P_- + R^i_{\alpha\beta} P_+ \right\} \gamma^\mu \psi_\beta W_{\mu i} \\
\mathcal{L}_2 &= \tfrac{1}{2} D_{ijk} W^k_\mu \left(W^j_\alpha \partial_\mu W^{\alpha i} - W^i_\alpha \partial_\mu W^{\alpha j} \right) \\
\mathcal{L}_3 &= \bar\psi_\alpha \left\{ C^{+b}_{\alpha\beta} P_+ + C^{-b}_{\alpha\beta} P_- \right\} \psi_\beta \phi^b \\
&\quad + \tfrac{1}{2} K^b_{ij} W^i_\mu W^j_\mu \phi_b \\
&\quad + \tfrac{1}{2} T^i_{ba} W^i_\mu \left(\phi_a \partial_\mu \phi_b - \phi_b \partial_\mu \phi_a \right) \\
&\quad + \tfrac{1}{4} M^{ij}_{ab} W_{\mu i} W_{\mu j} \phi_a \phi_b
\end{aligned}
$$

Since the couplings are generic this effective theory is not renormalizable, in any case a cutoff is needed. It is a "pre-effective" theory, which has not yet build in the typical gauge cancellations, necessary to render the theory

renormalizable. It is sufficient to calculate tree level amplitudes and to expand them for low momenta. Terms with "bad" high energy behavior can be made to be absent by constraining the generic couplings appropriately.

Predictions by Cornwall, Levin & Tiktopoulos 1974 also Veltman, Bell, Lewellyn Smith and others

- **Non-Abelian gauge structure automatic! Conspiracy in small multiplets doublets, triplets $\rightarrow SU(2), SU(3), \cdots$ natural [E_6 and other GUT's unnatural!]**
- **At least one Higgs required to render low energy tail renormalizable**
- **Einstein causality may be lost once $E/\Lambda_{\rm Pl}, \quad (E/\Lambda_{\rm Pl})^2, \cdots$ terms come into play: say at 1% level, i.e., above $E \simeq 10^{16}$ GeV**

The relationship between bare and renormalized parameters must be

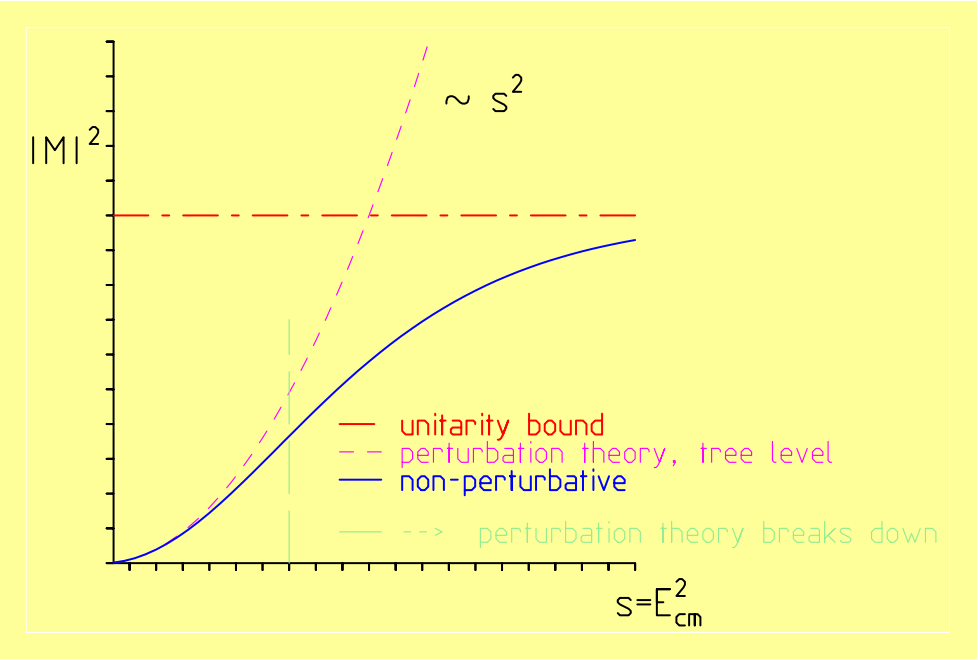
physical (stability of the Hamiltonian, in particular of the scalar potential etc.)

Note: the emergence of non-Abelian gauge structures often is misleadingly attributed to perturbative unitarity bounds, in fact it is a consequence of the low energy expansion in a large cutoff!

Some more details on this: a cross section is given by

$$\frac{d\sigma}{d\cos\theta} = \frac{\beta}{32\pi s}|M|^2$$

where $|M|^2$ is proportional to a transition probability. It thus must be a bounded function of the energy for the sake of unitarity. Our starting Lagrangian specified above is non-renormalizable.



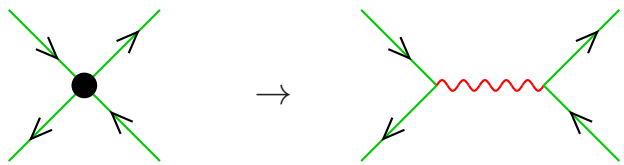
Unitarity and perturbative predictions of non-renormalizable theories.

It is well known that in a non–renormalizable theory one gets into trouble with perturbative unitarity. For example, in the Fermi theory, where

 $\simeq G_\mu s$ **implies** $|M|^2 \propto s^2$ **,**

one is led to a breakdown of perturbation theory at the Fermi scale $v = (\sqrt{2}G_\mu)^{-1/2} \simeq$ **250 GeV. But, surprisingly, this did not mean that we had to go to non-perturbative methods. It meant that we had to change the theory.**

Since this problem does not show up in a renormalizable theory it was natural to try to extend the low-energy theory to render it renormalizable. This path led to the construction of the "minimal renormalizable extension" of the phenomenological four fermion interactions and it directly led to the electroweak SM. The first step was introducing the massive intermediate weak gauge bosons



which renders the four fermion sector "renormalizable" at tree and up to the one–loop level (Veltman 1968). More generally, one requires $|M|^2$ to grow with energy at most as E^{4-N} for $E \to \infty$, where N is the number of external particles, so that perturbative unitarity is retained at tree level (necessary condition for renormalizability). The starting point is the general form of the fermion current. Let the spinor $\Psi_\alpha = (e, \nu_e, \mu, \nu_\mu, \ldots)$ include the known

fermion fields. The current has the form

$$J^{\mu a} = \bar{\Psi}_\alpha \gamma^\mu \{L^a_{\alpha\beta} P_- + R^a_{\alpha\beta} P_+\} \Psi_\beta$$

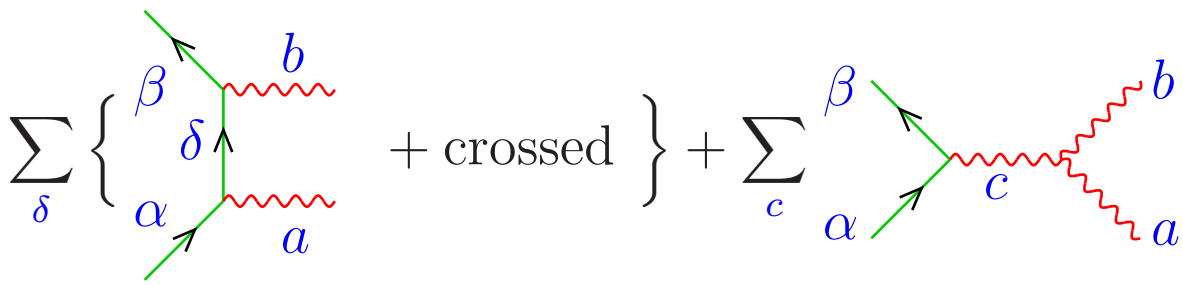
where $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$ **are the right and left handed projections, respectively. The intermediate vector boson Lagrangian then reads**

$$\mathcal{L}_1 = J^{\mu a} W_{\mu a}$$

which leads to s^2**–terms in the pair production of intermediate vector bosons in fermion–antifermion annihilation. This can be cured by adding new interactions, the triple gauge vertices (TGV's):**

$$\mathcal{L}_2 = D_{abc} W^c_\mu W^b_\nu \partial^\mu W^{\nu a} \quad .$$

The condition of compensation of the s^2**-terms in**



leads to the algebraic relationships

$$\begin{aligned}\left[L^a, L^b\right] &= iD_{abc}L^c \\ \left[R^a, R^b\right] &= iD_{abc}R^c\end{aligned}$$

telling us that there is symmetry needed, the coupling structure of a Lie–group $G_L \otimes G_R$. Next, inspection of $WW \to WW$ scattering again exhibits s^2-terms. They can be canceled by new quartic interactions

$$\mathcal{L}_3 = \frac{1}{4}E_{abcd}W^a_\mu W^{\mu b}W^c_\nu W^{\nu d} \quad .$$

The absence of s^2-terms implies that D_{abc} is antisymmetric and satisfies the Jacobi identity and

$$2E_{abcd} = D_{ade}D_{cbe} + D_{ace}D_{dbe} \quad .$$

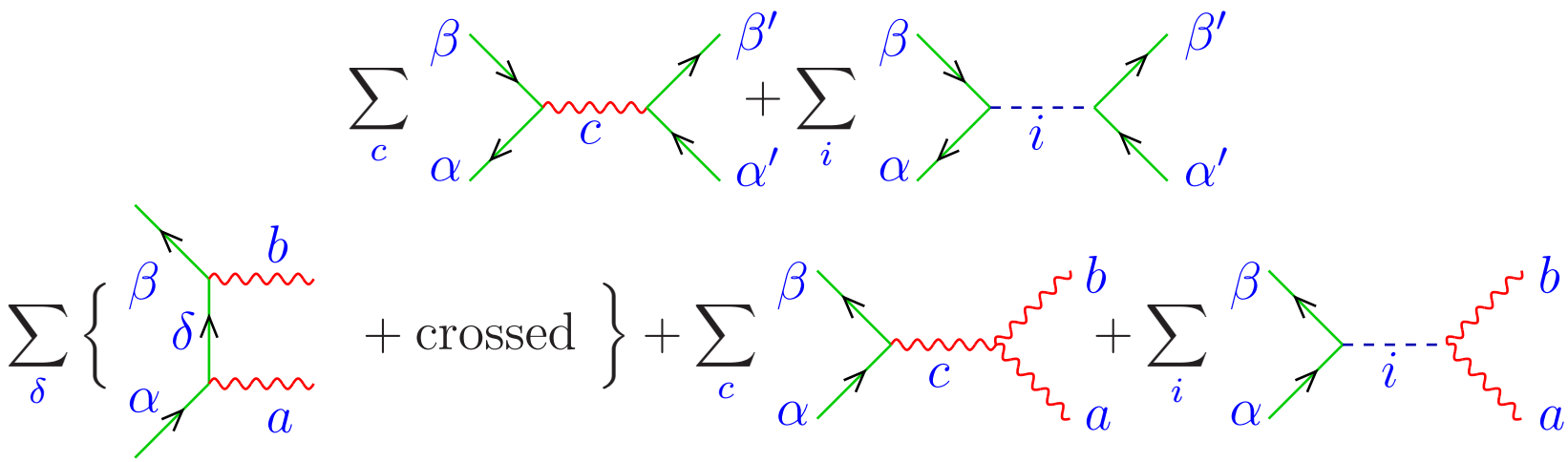
Thus one finds that, provided

- **all couplings have dimension ≤ 0 (otherwise they are not renormalizable anyway), and**
- **s^2–terms are absent,**

we obtain a theory with *Yang–Mills couplings* in the fermion and gauge boson sector. This works for all channels at the tree level.

There are still unacceptable terms growing linearly with s. Since spin 1/2 and spin 1 fields have been taken into consideration in a general way there are

no further compensations possible without adding new types of fields. Adding particles of spin higher than 1 in any case would led to a non-renormalizable theory. Hence, we are left with only one possibility: to introduce spin 0 particles, *the Higgs boson*. The condition of compensation of the s-terms in



fixes the couplings of the Higgs to the gauge bosons as well as the Yukawa couplings. The Higgs self-coupling remains a free parameter.

In fact the condition of absence of linear s-terms requires the "Goldstone solution with locally gauged fields," which means the *Higgs mechanism* (Cornwall, Levin, Tiktopoulos 1973).

Here we consider the SM to be a LEET and we do not have to impose the absence of s - power enhanced terms, because powers of s only come into play in conjunction with a $\Lambda_{\rm Pl}^2$ suppression. And since $\Lambda_{\rm Pl}$ is so large the $s/\Lambda_{\rm Pl}^2$ terms are not seen. What is seen is what conspires as a non-Abelian gauge symmetry structure or more generally, as in the SM, as 'SBGT'.

This way to look at "spontaneously broken gauge theories" naturally leads us to the effective field theory scenario. It is reasonable to assume that the SM is a low-energy asymptotic (effective) theory of some unknown underlying theory which could show up at some larger scale Λ. Because the SM works so well this new scale is expected to be substantially higher than the present "high energy" scale, which is set by the Higgs vacuum expectation value $v \simeq 246$ GeV. The particle content of the SM represents the

light particles of the underlying theory at the scale Λ. The Higgs could be an exception because of the "naturalness" problem. The Higgs mass is not protected (required to vanish) by the "nearby" unbroken SU(2)$\times$U(1) symmetry (i. e. the limit $v \to 0$), which requires all other fields to be massless. The low-energy effective theory obtained by an asymptotic expansion in $\sqrt{s}/\Lambda \leq v/\Lambda$ exhibits corrections to the SM described by operators of increasing dimensions which are suppressed by increasing powers of Λ. Anomalous couplings naturally emerge here as next-to-leading order perturbations, which scale away at low energies but become more and more important as we go to higher energies.

Important comment: we do not claim that we are able to derive the SM just by a low energy expansion. The claim is that certain structures like renormalizable QFT, Yang-Mills coupling structure are "automatic" in a low energy expansion. The light particle content in the long range structure has to be known in order to specify the universality class. However, the emergence of the SM is by far not 'ad hoc', as it realizes the simplest possibilities of the given richness.

Note that the family structure of the SM is also a consequence of the low energy scenario: terms which do not match the family patters have an anomaly which spoils renormalizability and hence is suppressed by the cutoff

Quadratic divergences in the SM: Higgs mass renormalization

$$\delta m_H^2 = \frac{1}{16\pi^2 v^2} \quad \Big\{ A_0(m_H)\, 3m_H^2 + A_0(M_Z)\,(m_H^2 + 6M_Z^2) + A_0(M_W)\,(2m_H^2 + 12M_W^2) + \sum_{f_s} A_0(m_f)\,(-8m_f^2) + \cdots \Big\}$$

$$A_0(m) = \Lambda^2 \left(\frac{m^2}{\mu^2}\right)^{(d/2-1)} (4\pi)^{-d/2}\,\Gamma(1-d/2)$$

pole at $d = 2$

$$\delta m_H^2 \sim 6\,(\Lambda^2/v^2)\left(m_H^2 + M_Z^2 + 2M_W^2 - \frac{4}{3}N_{cf}m_f^2\right)$$

Conformal conspiracy: $\boxed{\delta m_H^2 \sim 0}$

$$\Rightarrow \qquad m_H = \left(4(m_t^2 + m_b^2) - M_Z^2 - 2M_W^2\right)^{1/2} \sim 318 \ \ \text{GeV}$$

Veltman 1980

conspiracy between states allows for a light Higgs
renormalization and threshold effects not included

Naturalness:
Small masses are natural only if setting them to zero increases the symmetry of the system (G. 't Hooft 1979)

This will be revisited below.

❑ Conspiracies tuning for criticality

Light spectrum (IR relevant terms) only possible by "conspiracy" i.e. modes conspire to form "almost multiplets" of some symmetry which protects the masses from large renormalizations:

- **Light fermions require approximate chiral symmetry**
- **Light vector bosons require approximate local gauge symmetry**
- **Light scalars require approximate super symmetry [standard dogma to be reconsidered later]**
- **On classical level: in the symmetric phase the Higgs doublet mass term is the only mass term, setting $m^2 = 0$ enhances the symmetry by rendering it conformally symmetric (dilatation symmetry implies conformal symmetry) (on quantum level conformal symmetry is broken by radiative corrections, non-trivial renormalization group = dilatation Ward-identity)**

Summary: the SM in the Spirit of Emergence

Natural framework: think in effective theories, asymptotic understanding of tail of the "substrate" living at the Planck scale.

Plenty of open questions which have not been investigated in this direction

what we learned about the emergence of long range structures

➠ **space(-time) dimension 4 plays special role, Euclidean $O(4)$ rotational invariant renormalizable QFT ➜ relativistic invariance, if we accept the possible Wick rotation (possible because of intrinsic properties of any QFT) from Euclidean to Minkowski space as something just happening in nature. Likely, $(d-1,1)$ space-time at Planck scale, but is easier to argue with $O(4)$ invariance.**

➠ **emergence of conspiring spin 1 modes get imprinted non-Abelian gauge structure in smallest possible multiplets: doublets, triplets besides singlets (15-plets etc. very unlikely).**

➠ **of course we cannot derive the SM in its specific form without specifying the low energy particle content, which is specific for the universality class of the SM.**

Non-perturbative approach to QFT

➠ **Non-perturbative definition of QFT – only known way is to introduce a cutoff a and a box L and simulate the system having in the focus the continuum limit $a \to 0$ and infinite volume limit (thermodynamic limit) $L \to \infty$: i.e.**

$$a \ll x \ll L$$

➠ **In fact lattice QCD or any lattice version of a QFT are making use of such automatic recovery of the proper symmetries in the continuum limit: like relativistic invariance, chiral invariance etc.**

Should we try to work out the theory of the "ether" now?

Remind Galileo Galilei's (1564-1642) scientific revolution, the root of physics

and science

Experiment was, is and remains the key to progress in natural sciences

The construction of a theory residing at $M_{\rm Planck}$ ($n \times M_{\rm Planck}$ with $(n = 1, 2, \cdots, \infty)$ in case of string theory) is beyond any direct testability and hence at best is Meta Physics.

Of course we have to figure out which experiments are able to answer more or less fundamental questions, i.e, the right speculations are the meat for progress in physics

Nevertheless, as we shall see, it is possible to learn a lot about the Planck medium by trying to extrapolate back to the Planck scale from what we see at human accessible scales. That's what these lectures are about. To learn how the cutoff enters the "Planck medium" in any case is beyond what can be addressed as "real physics".

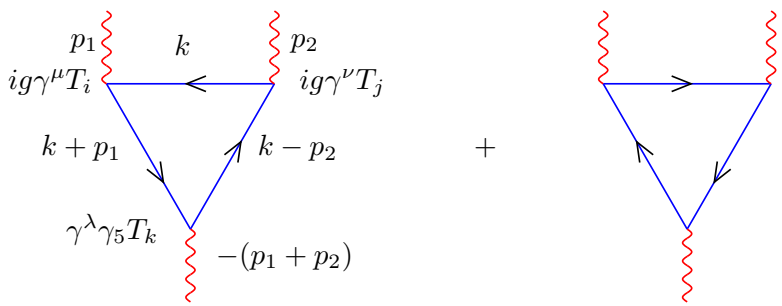
Remark concerning our emergence symmetry pattern:

$$\text{ETHER} \to \text{QFT} \overset{???}{\simeq} \text{SM (symmetric)} \underset{symmetry\ breaking}{\overset{spontaneous}{\longrightarrow}} \text{SM (broken phase).}$$

mentioned earlier: a typical misinterpretation which is the origin for the belief that symmetries increase with increasing energy is the spontaneous SB in the low energy region, which concerns the relevant operators only. Indeed at first the symmetry increases as the relevant operators get less and less relevant when going to higher energies. In the framework of renormalizable theories that's it, the symmetry gets restored. However, in a low energy effective scenario there exists an infinite tower of irrelevant terms, which get more and more relevant with increasing energy, and their appearance is not expected to enhance the symmetry unless there is some fine tuning, e.g. a symmetry imposed at a higher scale. It is rather natural to expect that the symmetry gets lost.

Axial Vector Anomaly and Anomaly Cancellation

Axial vector currents lead to the axial anomaly, which is associated with the triangle fermion loop diagram:



Triangle diagram exhibiting the axial anomaly

More generally, anomalies show up in diagrams which exhibit an odd number of axial vector current vertices and which are UV divergent (and hence need regularization at intermediate steps). One can show that all anomalies are related to the triangle anomaly, which we briefly discuss now.

The amplitude for the triangle graph is given by the integral

$$\tilde{T}^{\mu\nu\lambda}_{ijk}(p_1,p_2) = -iTr(T_jT_iT_k)\cdot \frac{g^2}{(2\pi)^4}\int d^4k Tr\left(\frac{1}{\not k - \not p_2 + i\epsilon}\gamma^\nu\frac{1}{\not k + i\epsilon}\gamma^\mu\frac{1}{\not k + \not p_1 + i\epsilon}\gamma^\lambda\gamma_5\right).$$

Adding the diagram we obtain by interchanging the two vector vertices we get an amplitude which is ***bose symmetric***

$$T^{\mu\nu\lambda}_{ijk}(p_1,p_2) = \tilde{T}^{\mu\nu\lambda}_{ijk}(p_1,p_2) + \tilde{T}^{\nu\mu\lambda}_{jik}(p_2,p_1)$$

and for which we *impose* **vector current conservation** **(condition on possible renormalization counter term)**

$$p_{1\mu}T^{\mu\nu\lambda}_{ijk}(p_1,p_2) = p_{2\nu}T^{\mu\nu\lambda}_{ijk}(p_1,p_2) = 0\ .$$

The divergence of the axial vector current is non-vanishing and uniquely determined by the mass independent anomaly

$$-(p_1+p_2)_\lambda T^{\mu\nu\lambda}_{ijk}(p_1,p_2) = i\frac{g}{16\pi^2}D_{ijk}4\varepsilon^{\mu\nu\rho\sigma}p_{1\rho}p_{2\sigma} \neq 0$$

(Adler, Bell and Jackiw 1969). We have introduced the abbreviation $D_{ijk} \equiv Tr(\{T_i,T_j\}T_k)$ for the representation dependent coefficient of the anomaly. The result can be obtained as a matrix element of the anomalous divergence equation

$$\partial_\lambda j^\lambda_{5k}(x) = \frac{g^2}{16\pi^2}D_{ijk}\tilde{G}^{\mu\nu}_i(x)G_{j\mu\nu}(x)$$

where $G_{i\mu\nu}$ is the (Abelian or non-Abelian) field strength tensor and $\tilde{G}^{\mu\nu}_i = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}G_{i\rho\sigma}$ its dual tensor. This is a very surprising result because the canonical Ward-Takahashi identities reading

$$\partial_\mu \left(\bar{\psi}_1 \gamma^\mu \psi_2\right)(x) = i(m_1 - m_2)\left(\bar{\psi}_1 \psi_2\right)(x)$$
$$\partial_\mu \left(\bar{\psi}_1 \gamma^\mu \gamma_5 \psi_2\right)(x) = i(m_1 + m_2)\left(\bar{\psi}_1 \gamma_5 \psi_2\right)(x)$$

do not exhibit such a term and for massless fields both currents are conserved.

- **anomaly unaltered by higher order effects ➜ anomaly is exact (non-perturbative).**
- **crucial point: the anomaly spoils renormalizability of a theory!**
- **only anomaly free theories are renormalizable**

The appearance of anomalies in a gauge field theory is strongly related to

the fermion representations. Which representations are anomaly free?

- **Real representations ($R \sim R^*$) are anomaly free, since $D_{ijk} = 0$ for all real representations.**
  The groups which have only real representations are: $SO(2\ell+1)$ for $(\ell > 1)$, $Sp(2\ell)$, G_2, F_4, E_7, E_8. In addition $D_{ijk} = 0$ also holds for $SO(2\ell)$ for $(\ell > 1)$ with one exception: $SO(6) \simeq SU(4)$.

- **Since for any representation R one has $D_{ijk}(R) = D_{ijk}(R_0) \cdot K(R)$ where R_0 denotes the fundamental representation and $K(R)$ is a representation dependent invariant, all representations are anomaly free if $D_{ijk}(R_0) = 0$. In particular, this is the case for $SU(2)$, for which $(R_0 \sim R_0^*)$, and for E_6.**

- **The groups $SU(n)$, $(n \geq 3)$ have complex representations $(R \nsim R^*)$ and $D_{ijk}(R_0) \neq 0$. These groups are *not anomaly save* !**
  If we write

$$j_i^\mu = \bar{\psi}_L \gamma^\mu T_{Li} \psi_L + \bar{\psi}_R \gamma^\mu T_{Ri} \psi_R$$

at the γ_5-vertex and use

$$\gamma_5 \frac{1 \pm \gamma_5}{2} = \pm \frac{1 \pm \gamma_5}{2}$$

we obtain

$$D_{ijk} \equiv Tr(\{T_{Li}, T_{Lj}\}T_{Lk}) - Tr(\{T_{Ri}, T_{Rj}\}T_{Rk})$$

which tells us that left-handed and right-handed fields give independent contributions to the anomaly. Of particular interest for us is the color group $SU(3)_c$ and the quark representations. The quarks are in the fundamental representation 3, the antiquarks in 3^*. Under charge conjugation we have

$$\psi_L \xrightarrow{C} \psi_L^c = i\gamma^2 \psi_R^* \, .$$

Therefore it follows that ψ_L and ψ_R are in the same representation and

hence $D_{ijk} \equiv 0$. Evidently, renormalizability of QCD requires parity conservation and thus the absence of axial current couplings.

- **in SM: anomalies are obtained from Abelian axial current couplings. Here we have to worry about the $U(1)_Y$. Per doublet $\Psi = (\psi_1, \psi_2)$, using $Q = T_3 + Y/2$, $Q_1 - Q_2 = 1$ and $Q_{Ri} = Q_{Li}$, we get**

$$D = \sum_i (Y_{Li}^3 - Y_{Ri}^3) = -12Q_1 + 6$$

which yields $D_{lepton} = 6$ and $D_{quark} = -6N_c\,(2Q_1^q - 1) = -6$.

As a consequence we find that the $U(1)_Y$ subgroup of

the SM is renormalizable
if and only if
there is the lepton-quark family structure!

This lepton-quark duality is one of the most surprising properties of the SM. Nature seems to take very serious the mathematical consistency of the theory. In a LEESM scenario the family structure shows up because terms disturbing this structure are suppressed as $E/\Lambda_{\rm Pl}$ and hence not observable!

How natural is the minimal SM?

We finally try to derive the SM by starting from some general assumptions as suggested by M. Veltman.
Let us make the following assumptions:
1) local field theory
2) interactions follow from a local gauge principle
3) renormalizability
4) masses derive from the minimal Higgs system
5) ν_R is absent or if it exists it does not carry hypercharge.
We admit that the last assumption looks quit ad hoc, but nevertheless we make it. From the above assumptions the following picture develops:

- **For the gauge interactions the simplest non-trivial possibility is that the fundamental massless matter fields group into doublets and triplets which are the fundamental representations of $SU(2)$ and $SU(3)$.**

- **Since fields are massless all fields can be chosen left-handed. Left-handed**

particles and left-handed antiparticles at this stage are uncorrelated.

- **We must have *pairing* for particles that are going to be massive, since a mass term (we ignore the possibility to have Majorana fields here) has the form $\bar{\psi}\psi = \bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L$. Notice that for massive particles, only, we know which left-handed antiparticle belongs to which left-handed particle to form a Dirac field.**

- **For $SU(3)_c$ triplets we *must* have pairing in order to avoid axial anomalies. $SU(3)$ is the simplest group having complex representations. This allows to put particles in 3 and antiparticles in the inequivalent 3^*. As a consequence a rich color singlet structure ($\equiv$ hadron spectrum) results. Furthermore, confinement requires $SU(3)_c$ to be unbroken !**

- **$SU(2)_L$ is anomaly free and hence there is no anomaly condition associated with this group. To generate mass we have to break $SU(2)_L$ by a Higgs mechanism. The simplest and natural possibility is to chose one Higgs**

field in the fundamental representation of $SU(2)_L$. There is no hypercharge for the moment. The Higgs field may be written in the form

$$\Phi_b = \tilde{\Phi}\chi_b \; ; \; \chi_b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

in terms of a 2×2 matrix field

$$\tilde{\Phi} = \frac{1}{\sqrt{2}}(H_s + i\tau_i\phi_i) \; .$$

The covariant derivative being given by

$$D_\mu \Phi_b = (\partial_\mu - i\frac{g}{2}\tau_a W_{\mu a})\Phi_b \; ,$$

the Higgs system exhibits an extra global $SU(2)_R$-symmetry $\chi_b \rightarrow V^+\chi_b$.

One easily checks that the transformation

$$\tilde{\Phi} \to U(x)\tilde{\Phi}V^+$$

with $U(x) \in SU(2)_{L,local}, V \in SU(2)_{R,global}$ **leaves the Higgs Lagrangian invariant. This implies that the fields** (W^+, W_3, W^-) **form an isospin triplet with** $M_Z = M_{W^\pm}$**.**
Now consider the fermions (still no hypercharge). Since L_f **and** Φ_b **are doublets** R_f ***must*** **be a singlet ! otherwise we would not be able to write down an invariant and renormalizable fermion-Higgs coupling. Therefore** $SU(2)_L$ **must be parity violating of V-A-type! The Yukawa term has the general form**

$$\mathcal{L}_{\text{Yukawa}} = -\bar{L}_f\tilde{\Phi}\begin{pmatrix} g_1 g_2 \\ g_3 g_4 \end{pmatrix} R_f + h.c.$$

with 4 complex couplings g_i **and** R_f **a "doublet" having to right-handed singlets as entries. Although we have not used hypercharge to restrict**

these couplings the existence of a global $SU(2)_R$-symmetry of the Higgs system allows to transform the Yukawa couplings

$$\tilde{\Phi}(\cdot)R_f \to \tilde{\Phi}V^+(\cdot)WR_f$$

to standard form, $V^+(\cdot)W$= real diagonal. Since $V \in SU(2)_R$ has 3 parameters and W is an arbitrary unitary matrix with 4 parameters we end up with one free parameter such that the system exhibits a global $U(1)$ invariance. This is not surprising since in the unitary gauge we always can end up only with $\mathcal{L}_{\text{Yukawa}}$ in the simple standard form.

- The global $U(1)$ which is a consequence of the minimal Higgs mechanism may be *interpreted* as a global $U(1)_Y$. We are free to assign to Φ_b $Y = 1$, which means nothing else than that we measure Y in units of the Φ_b-hypercharge. Then

$$\Phi_t = \tilde{\Phi}\chi_t \; ; \; \chi_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

has $Y = -1$, and we may write $\tilde{\Phi} = (\Phi_b, \Phi_t)$. Since we have the global $U(1)_Y$ for free, we may *assume* this symmetry to be *local*. The covariant derivative for $\tilde{\Phi}$ now reads

$$D_\mu \tilde{\Phi} = \partial_\mu \tilde{\Phi} + i\frac{g'}{2} B_\mu \tilde{\Phi} \tau_3 - i\frac{g}{2} \tau_a W_{\mu a} \tilde{\Phi}$$

and we find back the usual Higgs Lagrangian. The 3 real fields ϕ_a $a = 1, 2, 3$ can be gauged away and only 3 out of 4 gauge fields can acquire a mass. Hence there must exist one massless field, the photon! Evidently we obtain the relations $g' = g \tan \Theta_W$ and $\rho = M_W^2/(M_Z^2 \cos^2 \Theta_W) = 1$! instead of $M_Z = M_{W^\pm}$ when $g' = 0$.
Now, what can we say about the hypercharge of the fermions?:
A left-handed doublet transforms like

$$L \to e^{i\frac{g'}{2} Y_L} L$$

where Y_L is arbitrary. By inspection of $\mathcal{L}_{\text{Yukawa}}$ we find for the hypercharges

of the singlets: ψ_{1R} must have $Y_{1R} = Y_L + 1$ and ψ_{2R} must have $Y_{2R} = Y_L - 1$. One consequence is that $U(1)_Y$ *must* violate parity. The astonishing thing is that the fermion current which couples to the photon preserves parity. By inspection we find

$$\begin{aligned} D_\mu L_f &= (\partial_\mu - i\frac{g'}{2}Y_L B - \mu - i\frac{g}{2}\tau_3 W_{\mu 3} - \cdots)L_f \\ D_\mu R_f &= (\partial_\mu - i\frac{g'}{2}Y_L B - \mu - i\frac{g}{2}\tau_3 B_\mu - \cdots)R_f \end{aligned}$$

and the couplings of L_f and R_f to A_μ read

$$\begin{aligned} L_f &: \quad -i(g\sin\Theta_W\,\frac{\tau_3}{2} + g'\cos\Theta_W\frac{Y_L}{2})A_\mu \\ R_f &: \quad -i(g'\cos\Theta_W\,\frac{\tau_3}{2} + g'\cos\Theta_W\frac{Y_L}{2})A_\mu\,. \end{aligned}$$

Because we have $g'\cos\Theta_W = g\sin\Theta_W = e$ we find the Gell-Mann-Nishijima

(GMN) relation

$$Q = T_3 + \frac{Y}{2}$$

as a consequence of a minimal Higgs structure! What we find is, that, whatever the hypercharge of L_f is L_f and R_f must couple identically to photons. Thus QED must be parity conserving! Furthermore the charges of the upper (1) and lower (2) components of the doublets satisfy

$$Q_{Li} = Q_{Ri}\ ,\ Q_1 - Q_2 = 1\ and\ Q_1 + Q_2 = Y_L\ .$$

So far we have no charge quantization. Here we need a last assumption.

- **If ν_R does not exist we have to set $Y_{\nu R} = 0$ and consequently we must have $Y_{\nu L} = -1 = Y_{\ell L} = 0$ and $Q_\nu = 0$, $Q_\ell = -1$. For the $U(1)_Y$ anomaly cancellation we need lepton-quark duality and the charges of the quarks must have**

their known values if they appear in three colors. One thus must have the usual charge quantization.

We finally summarize the consequences of the assumptions stated above:

- **Breaking $SU(2)_L$ by a minimal Higgs automatically leads to a global $U(1)_Y$, which can be gauged**
- **parity violation of $SU(2)_L$**
- $\rho = M_W^2/(M_Z^2 \cos^2 \Theta_W) = 1$
- **existence of the photon**
- **parity conservation of QED**
- **validity of the Gell-Mann-Nishijima relation**

- **family structure**
- **charge quantization**

We do not know of course why right-handed neutrinos do not exist or not couple in the real world and it remains a mystery why there exist family replica.

In our context assumptions 1) - 4) are direct consequences of the LEET scenario.

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