Comment on $H \rightarrow \gamma\gamma$ and the role of the decoupling theorem and the equivalence theorem

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Abstract

I am commenting on the recent paper [1] about a new calculation of the $H \rightarrow \gamma\gamma$ rate mediated by the W boson loop, and the lack of decoupling of the heavy states which mediate the decay. We remind the reader that the heavy Higgs limit is dominated by the contribution from the longitudinal W bosons, which in the limit $m_H \gg M_W$ are represented by the charged Higgs ghosts according to the equivalence theorem. The corresponding contribution is missing in [1].

1 Introduction

In a recent paper [1] previous one-loop calculations of the decay $H \rightarrow \gamma\gamma$ via the W boson loop in the electroweak Standard Model (SM) have been criticized to be incorrect. In [1] it is argued that in the limit $m_H \gg M_W$ the Higgs decay amplitude $A_{H\gamma\gamma}^{(W)}$ should stay bounded, while it is actually $\propto m_H^2$, a behavior which is claimed to be violating the decoupling theorem. In this short note we defend previous calculations and explain why previous results are correct. The main point is that the decoupling theorem [2] is a statement about the limit $M_W \gg m_H$ while the limit $m_H \gg M_W$ is ruled by the equivalence theorem [3]. It is well known that heavy Higgs and heavy top physics has little to do with the gauge sector (since the heavy Higgs, heavy top effective theory is there for vanishing gauge couplings $g, g' = 0$), but is determined entirely by the symmetry breaking sector, the Higgs and the Yukawa sector [4],

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} &= \partial_\mu \Phi^+ \partial^\mu \Phi + \bar{t}\gamma^\mu \partial_\mu t + \bar{b}\gamma^\mu \partial_\mu b + \mu^2 \, \Phi^+ \Phi + \lambda \left(\Phi^+ \Phi\right)^2 \\
&\quad - y_t \left(\bar{Q}_L \Phi^c t_R + \text{h.c.}\right)
\end{aligned}
\tag{1}
$$

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where covariant derivatives appear replaced by normal derivatives and the local $SU(2)_L \otimes U(1)_Y$ symmetry is replaced by a global $SU(2)_L$ symmetry with corresponding Ward-Takahashi identities. Φ is the $Y = 1$ Higgs doublet field

$$\Phi(x) = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix} \; ; \;\; \varphi^0 = \frac{H + v - \mathrm{i}\,\varphi}{\sqrt{2}} \; . \tag{2}$$

By $\Phi^c = \mathrm{i}\,\tau_2 \Phi^*$ we denote the Y charge conjugate $Y = -1$ Higgs doublet. Q_L is the left-handed (t, b) doublet. The b as a light field can be taken to be non-interacting. In our case of $H \to 2\gamma$ photons couple as usual to the charged particles in the heavy sector (charged Higgses and the top and bottom quarks). Furthermore, the limit $g, g' \to 0$ should be taken at fixed low energy constraint on $\sin^2 \Theta_W = 1 - \frac{M_W^2}{M_Z^2} = \frac{g'^2}{g^2 + g'^2}$. The adequate renormalization scheme takes G_μ and $\sin^2 \Theta_W$ as input parameters together with α for the QED part.

We first look at the decoupling limit: the **decoupling theorem** states that heavy virtual particles of mass M decouple like $O(E/M)$ as $M \to \infty$ where E is the fixed energy or light-mass scale of the "light" particle sector. In fact the Appelquist-Carrazone decoupling–theorem [2] holds in theories like QED and QCD only, where masses and couplings are independent and when some of the masses get large at fixed couplings. In the SM where masses are generated by the Higgs mechanism the decoupling theorem does not hold in general because of the well known mass coupling relations

$$\begin{aligned} M_W &= \frac{g\,v}{2} \quad , \quad M_Z = \frac{g\,v}{2\cos\Theta_W} \\ m_f &= \frac{y_f\,v}{\sqrt{2}} \quad , \quad m_H = \sqrt{2\lambda}\,v \; . \end{aligned} \tag{3}$$

In the SM masses can only get large either in the strong coupling regime, or by taking the Higgs vacuum expectation value $v \to \infty$, which then would violate the important low energy constraint

$$v = \left(\sqrt{2}\,G_\mu\right)^{-1/2} = 246.2186(16) \text{ GeV} \; , \tag{4}$$

and in addition would rescale the spectrum uniformly to large masses. In the SM a particle cannot be removed from the theory by just taking its mass to infinity. At fixed v it requires to take the strong coupling limit in which perturbative arguments fail to apply.

One drawback, relevant for the $H \to \gamma\gamma$, is that the HWW coupling is $2M_W^2/v$, and similarly, the $H\bar\psi_f\psi_f$ fermion couplings are m_f/v. As a consequence, masses appear as factors in the numerators of Feynman amplitudes in

addition to the masses in propagators [mass terms] which show up in the denominators. The mass factors in the numerators obviously lead to non decoupling effects for large masses.

In discussing various mass limits in the SM one should keep in mind that relations like the custodial symmetry constraint $\rho = \frac{M_Z^2}{M_W^2 \cos^2 \Theta_W} = 1$ or equivalently $\cos^2 \Theta_W = M_W^2/M_Z^2$ must be respected. In the gauge field sector we have three basic parameters, the gauge couplings g, g' and the Higgs vacuum expectation value v which usually (LEP parametrization) are mapped to α, G_μ and M_Z as the most precisely known input parameters. Then the W mass is given by

$$M_W^2/M_Z^2 = \frac{1}{2} \left(1 + \sqrt{1 - \frac{4 A_0^2}{M_Z^2} \frac{1}{1 - \Delta r}} \right) = \cos^2 \Theta_W \tag{5}$$

considered to be kept fixed. Here $A_0 = \left(\frac{\pi \alpha}{\sqrt{2} G_\mu} \right)^{1/2} = 37.2802(3)$ GeV and Δr represents known radiative corrections. In any case we must respect $M_W < M_Z$, or more precisely the relation (5). We just wanted to point out that non-decoupling effects in the SM are experimentally well established, and one cannot conclude a SM calculation to be incorrect because of lack of decoupling. Of course the lack of decoupling naively looks unnatural, but in fact constraints from the ρ–parameter on heavy states are quite intriguing and extensions of the SM in most cases end up in a fine tuning problem [5], because decoupling of new heavy states, in theories where masses are generated by spontaneous symmetry breaking, is more the exception than the rule. Therefore, experimental low energy constraints actually are much more severe than often anticipated.

The **equivalence theorem** has to do with the massive gauge bosons in the limits where gauge boson masses are expected to become irrelevant. This concerns high energies $E \gg M_W, M_Z$ as well as the effective regime when gauge boson masses are small relative to other masses, like $m_t, m_H \gg M_W, M_Z$. Specifically for the heavy top effects this has been worked out and discussed in [4] and [6].

The root of the equivalence theorem is the following: in the unbroken phase of the SM the gauge bosons are massless and have two transverse degrees of freedom. When the SM undergoes spontaneous symmetry breaking (Higgs mechanism) the gauge bosons become massive and get a third physical degree of freedom, the longitudinal one, by "eating up the Higgs ghosts", which become unphysical. The number of physical degrees of freedom remains conserved. The broken symmetry is recovered as an asymptotic symmetry when

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energies get large relative to the gauge boson masses. Equivalently, in the limit $M_W, M_Z \ll E$ the gauge bosons become transversal again, however, the longitudinal modes transmute back into physical scalar Higgs degrees of freedom. Three of the four scalar Higgses had turned into unphysical Higgs ghosts during symmetry breaking. At high energies the symmetry is restored.

Formally, one may use 't Hooft's gauge fixing conditions (linear covariant gauge)

$$
\begin{array}{llll}
W_\mu^\pm & : & C^\pm = -\partial_\mu W^{\mu\pm} \pm i\xi_W M_W \varphi^\pm & = 0 \\
Z_\mu & : & C_Z = -\partial_\mu Z^\mu - \xi_Z M_Z \varphi & = 0 \\
A_\mu & : & C_A = -\partial_\mu A^\mu & = 0
\end{array}
\tag{6}
$$

which imply a correspondence like $\partial_\mu W^{\mu\pm} \propto \varphi^\pm$. While the transversal modes couple with gauge coupling $g\, HW^+W^-$ the ghosts couple with the Higgs self coupling $\lambda\, H\varphi^+\varphi^-$ and when $m_H \gg M_W$, meaning $\lambda \gg g$, the longitudinal modes dominate. This is the limit referred to in Ref. [1] as decoupling limit, mistakenly the dominating longitudinal mode has been lost in the calculation.

The relevant Ward-Takahashi identities derive from the standard model Slavnov-Taylor identities as follows. We use the notation of Ref. [7]. In the 't Hooft gauge we denote by ξ the gauge parameter, a, ζ and $\eta^\pm$ are the photon-associated, the neutral and charged Faddeev–Popov ghost fields, respectively. The W boson propagator satisfies

$$
\begin{aligned}
& < T\partial_\mu W^{\mu+}(x)W_\nu^-(y) > +\xi M_W < T\varphi^+(x)W_\nu^-(y) > \\
= \quad & -\xi < T\bar\eta^+(x)\partial_\nu \eta^-(y) > +i\xi\,[e < T\bar\eta^+(x)\,(W_\nu^-\,a - A_\nu\,\eta^-)(y) > \\
& -g\,\cos\Theta_W\, < T\bar\eta^+(x)(W_\nu^-\zeta - Z_\nu\eta^-)(y) >]
\end{aligned}
$$

and

$$
\begin{aligned}
& < T\partial_\mu W^{\mu+}(x)\partial_\nu W^{\nu-}(y) > + \xi M_W < T\partial_\mu W^{\mu+}(x)\varphi^-(y) > \\
& + \xi M_W < T\varphi^+(x)\partial_\nu W^{\nu-}(y) > + \xi^2 M_W^2 < T\varphi^+(x)\varphi^-(y) >= -i\xi\delta(x-y)
\end{aligned}
$$

for the longitudinal parts of the gauge field propagators. Using the usual tensor decomposition for the self-energy functions (inverse propagators) in Fourier space

$$
\begin{array}{llll}
< TW^{\mu+}(x)W^{\nu-}(y) > & \rightarrow & i\,(g^{\mu\nu}A_1 + q^\mu q^\nu A_2) & \equiv & -i\,(g^{\mu\nu}\Pi_W(q^2) + \cdots) \\
< TW^{\mu+}(x)\varphi^-(y) > & \rightarrow & M_W p^\mu B_1 & & \\
< T\varphi^+(x)\varphi^-(y) > & \rightarrow & i\,C_1 & \equiv & i\,\Pi_\varphi(q^2) \\
< T\bar\zeta^+(x)\zeta^-(y) > + \cdots & \rightarrow & -i\,M_W^2 D_1 & &
\end{array}
$$

the above identities read:

$$
\begin{aligned}
\xi\,\left(A_1 + q^2 A_2 + \ B_1\right) + D_1 &= 0 \\
q^2\,\left(A_1 + q^2 A_2 + 2B_1\right) + C_1 &= 0 \;.
\end{aligned}
$$

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By D_1 we have denoted the full Faddeev–Popov ghost contribution which includes the three terms on the r.h.s. of the first of the above Slavnov-Taylor identities .

In the limit $g \to 0$ the Faddeev-Popov ghosts do not contribute and therefore $D_1 \simeq 0$ such that $A_1 + q^2 A_2 + B_1 \simeq 0$. Since the self–energy amplitude A_2 does not exhibit a pole at $q^2 = 0$ we have $q^2 A_2 \to 0$. Thus we obtain the relevant Ward-Takahashi identity $A_1 \simeq \frac{C_1}{q^2}$ which we may write in the form

$$\frac{\Pi_W(q^2)}{M_W^2} \simeq -\frac{\Pi_{\varphi^\pm}(q^2)}{q^2} = -\Pi'_{\varphi^\pm}(q^2) \tag{7}$$

and which expresses the physical transversal part of the W self–energy in terms of the self–energy of the charged scalar Higgs ghosts. For the HW^+W^- vertex the Slavnov-Taylor identities look the same as for the W^+W^- propagator with a Higgs field in addition under the time-ordering prescription: like

$$\langle T \partial_\mu W^{\mu+} W^{\nu-} H \rangle - \mathrm{i}\,\xi\, M_W \langle T \varphi^+ W^{\nu-} H \rangle = -\xi \langle T \bar{\eta}^+ \partial_\mu \eta^- H \rangle + \cdots \tag{8}$$

modulo the gauge variation of the Higgs field, which vanishes when the Higgs is taken on-shell. Again in the limit of vanishing gauge couplings the r.h.s is vanishing, meaning that the longitudinal component of the W is replaced by its charged Higgs ghost.

For illustration of a similar mechanism we remind about another example of non-decoupling and the play of the equivalence theorem: the heavy top contribution to the W self–energy. In the limit $m_t \gg M_W, M_Z$ the gauge coupling $g\, W^{\mu+}(\bar{t} \gamma_\mu \Pi_- b)$ turns into $y_t\, \varphi^+ (\bar{t} \Pi_- b) - y_b\, \varphi^+(\bar{t} \Pi_+ b)$ ($\Pi_\pm = (1 \pm \gamma_5)/2$ the chiral projectors). Most prominent example of an electroweak non-decoupling heavy top effect is the well known low energy effective neutral to charged current coupling ratio $\rho = G_{\mathrm{NC}}/G_{\mathrm{CC}}$. It gets renormalized as $\rho = 1 + \Delta\rho$ where $\Delta\rho = \frac{\Pi_Z(0)}{M_Z^2} - \frac{\Pi_W(0)}{M_W^2} \simeq \Pi'_{\varphi^\pm}(0) - \Pi'_\varphi(0) = \frac{\sqrt{2} G_\mu N_c}{16\pi^2} |m_t^2 - m_b^2|$ at one loop. This leading top effect is Veltman's "flag pole" [8] and allowed to "measure" the top mass indirectly at LEP [9], prior to the direct top discovery at the Tevatron [10]. Similarly, $B - \bar{B}$ oscillations discovered by Argus at DESY [11] were possible because the effect is enhanced by a non-decoupling heavy top contribution (see e.g. [12, 13]). Non-decoupling effects in extensions of the SM have been discussed in Ref. [5]. In the context of Higgs production and decay non-decoupling phenomena of heavy fermions where investigated long time ago in [14] (see also [15, 16]). Last but not least, the present indirect Higgs mass bound from LEP is due to a non-decoupling effect. When we try to remove the virtual Higgs

from the SM by increasing its mass the SM would turn into a non-renormalizable theory as we know. In fact m_H^2 effects in the limit $m_H \gg M_W, M_Z$ at one loop are screened by the custodial symmetry of the minimal Higgs system and only a logarithmic Higgs mass dependence persists [17] in this case.

In summary: in the limit under consideration physical S-matrix elements are dominated by the longitudinal vector boson degrees of freedom and according to the *equivalence theorem*, with $m_H\,[m_t]$ as a high energy scale, one is allowed to replace (up to a phase and up to $O(M/m_H)\,[O(M/m_t)]$ corrections) a longitudinally polarized vector boson by its corresponding unphysical scalar. An equivalent relationship is obtained in the limit of vanishing gauge couplings, $g', g \to 0$, from the Ward-Takahashi identities which derives from the remaining global symmetry [4].

## 2 $H \to 2\gamma$ at one-loop

For $H \to \gamma\gamma$ the correct SM results are well known. The W-loop amplitude is (see e.g. [18, 19, 20, 21])

$$
\begin{aligned}
A_{H\gamma\gamma}^{(W)} \quad &= \quad C_W \left[3\, C_0(M_W, M_W, M_W; 0, 0, m_H^2)\,(2\,M_W^2 - m_H^2) - 3 - \frac{1}{2}\,x_W \right] \\
&\stackrel{x_W \to 0}{\sim} \quad C_W \left[\frac{7}{4}\,x_W + \frac{11}{120}\,x_W^2 + \frac{19}{1680}\,x_W^3 + \cdots \right] \\
&\stackrel{z_W \to 0}{\sim} \quad 3 C_W \left\{ \left[\boxed{\frac{1}{6z_W}} + \left(\pi^2 - \ln^2 z_W + 2 \right)/2 \right. \right. \\
&\qquad\qquad \left. - \left(\pi^2 - \ln^2 z_W + 2\,\ln z_W \right)\, z_W + \left(\ln z_W - 2 \right) z_W^2 + \cdots \right] \\
&\qquad\qquad \left. + \mathrm{i}\,\pi\, \left[-\ln z_W + 2\,\left(\ln z_W - 1 \right)\, z_W + z_W^2 + \cdots \right] \right\} \qquad (9)
\end{aligned}
$$

with $C_W = \frac{\alpha}{2\pi}\,M_W^2$ and $C_0(M, M, M; 0, 0, s) = \frac{2}{s}\left(\arctan \frac{1}{\sqrt{4M^2/s - 1}} \right)^2$ for $s \le 4M^2$. Furthermore, we denoted $x_W = m_H^2/M_W^2$ and $z_W = 1/x_W$. For $s > 4M^2$ we have $C_0 = -\frac{1}{2s}\left(\ln \frac{1 - \sqrt{1-y}}{1 + \sqrt{1-y}} + \mathrm{i}\pi \right)^2$ with $y = 4M^2/s$. For fixed HWW coupling C_W is fixed and the amplitude exhibits decoupling i.e. it is $O(x_W)$ as $x_W \to 0$. Taking into account the growth of the coupling, however, the complete amplitude for $m_W \gg m_H$ tends to a constant and lacks decoupling in the naive sense. The other limit $z_W \to 0$, i.e. $m_H \gg M_W$, is exhibiting the singular term in the box. This term is the one which has been questioned in [1]. The equivalence theorem requires this term to be there without question.

We have calculated these amplitudes in the 't Hooft gauge with an arbitrary gauge parameter ξ as well as in the unitary gauge using dimensional regularization. The off-shell Slavnov-Taylor identities have been checked as well to be satisfied. In the 't Hooft gauge with free gauge parameter ξ there are 13 diagrams contributing. In the unitary gauge there are 2[*]:
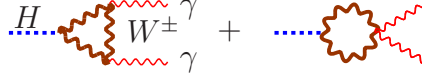
Both calculations give identical results[†].

Using the equivalence theorem, we may calculate the leading contribution for $m_H \gg M_W$ by calculating the much simpler diagrams
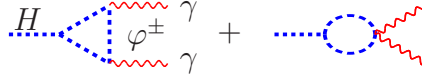
exhibiting loops of the charged Higgs ghosts only. The result in the 't Hooft gauge with arbitrary gauge parameter ξ reads

$$
\begin{aligned}
A_{H\gamma\gamma}^{(\varphi)} &= C_W \left[-\frac{1}{2} x_W + C_0(M_\varphi, M_\varphi, M_\varphi; 0, 0, m_H^2) \, M_\varphi^2 \, x_W \right] \\
&\overset{M_W \to 0}{\sim} 3 C_W \left\{ \boxed{\frac{1}{6 z_W}} + O(M_W^2/m_H^2) \right\} .
\end{aligned}
\tag{10}
$$

In the full SM in the 't Hooft gauge $M_\varphi^2 = \xi M_W^2$ is gauge dependent, however, in the limit where the equivalence theorem applies we have $M_\varphi/m_H \to 0$ where the second term vanishes. The remaining physical (gauge invariant) leading term agrees precisely with the leading term of the full SM calculation. The subleading term is gauge dependent and hence unphysical. We conclude that physics uniquely fixes that questioned leading term, and it also implies that Slavnov-Taylor (ST) identities are obviously not respected in the calculation of Ref. [1].

For comparison, the corresponding result for a heavy top loop is given by

$$
A_{H\gamma\gamma}^{(f)} = 2 Q_f^2 C_f \left[1 + C_0(M_f, M_f, M_f; 0, 0, m_H^2) \left(\frac{1}{2} m_H^2 - 2 M_f^2 \right) \right]
$$

---

[*]In a renormalizable gauge each W is represented by a W or φ line which yields $2^3 + 2^2 = 12$ diagrams plus the Faddeev-Popov ghost loop.

[†]In the unitary gauge some technicalities with so called Lee-Yang terms [22] must be taken into consideration also if dimensional regularization is applied.
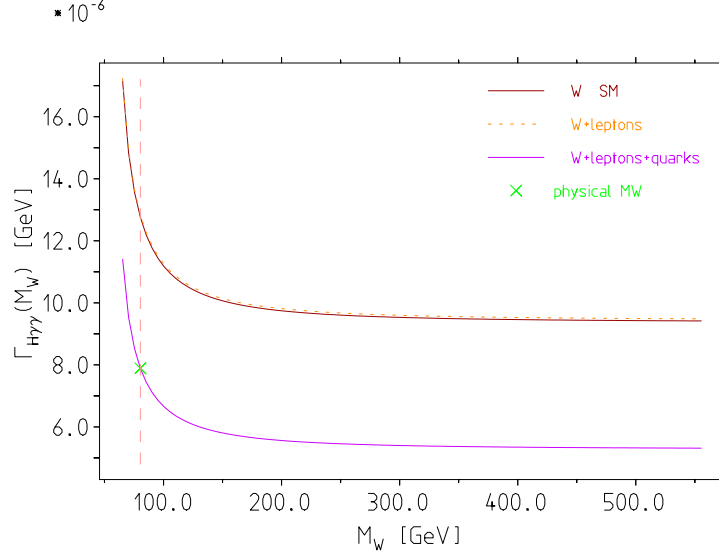
Figure 1: The $H \to \gamma\gamma$ width as a function of the W mass at one loop order ($m_H = 120$ GeV, $m_t = 171.3$ GeV).

$$
\begin{aligned}
\overset{x_f \to 0}{\sim} \quad & 2\, Q_f^2\, C_f \left[\frac{1}{6}\, x_f + \frac{7}{720}\, x_f^2 + \frac{1}{1008}\, x_f^3 + \cdots \right] \\
\overset{z_f \to 0}{\sim} \quad & 2\, Q_f^2\, C_f \Bigg\{ \left[1 + \frac{1}{4} \left(\pi^2 - \ln^2 z_f \right) \right. \\
& \left. \qquad - \left(\pi^2 - \ln^2 z_f + \ln z_f \right)\, z_f + \left(\frac{5}{2}\, \ln z_f - 1 \right)\, z_f^2 + \cdots \right] \\
& \quad + \mathrm{i}\, \pi \left[-\frac{1}{2}\, \ln z_f + \left(2\, \ln z_f - 1 \right)\, z_f + \frac{5}{2}\, z_f^2 + \cdots \right] \Bigg\}
\end{aligned}
\tag{11}
$$

with $C_f = \frac{\alpha}{2\pi}\, M_f^2$, $x_f = m_H^2/M_f^2$ and $z_f = 1/x_f$.

The $H \to \gamma\gamma$ width is given by

$$
\Gamma_{H\gamma\gamma} = \frac{\sqrt{2}\, G_\mu}{4\pi\, m_H}\, |\mathcal{A}_{H\gamma\gamma}|^2
\tag{12}
$$

where $\mathcal{A} = A_{H\gamma\gamma}^{(W)} + \sum_f A_{H\gamma\gamma}^{(f)}$. Figure 1 shows how the $H \to \gamma\gamma$ partial width as a function of M_W for $M_W \to \infty$ tends to a constant. Figure 2 compares the W mediated Higgs width in the heavy Higgs limit, with and without the proper leading term. In Fig. 3 we finally compare the full SM prediction with the W mediated result, all in one-loop approximation. By GWW we denoted the result from Ref. [1].
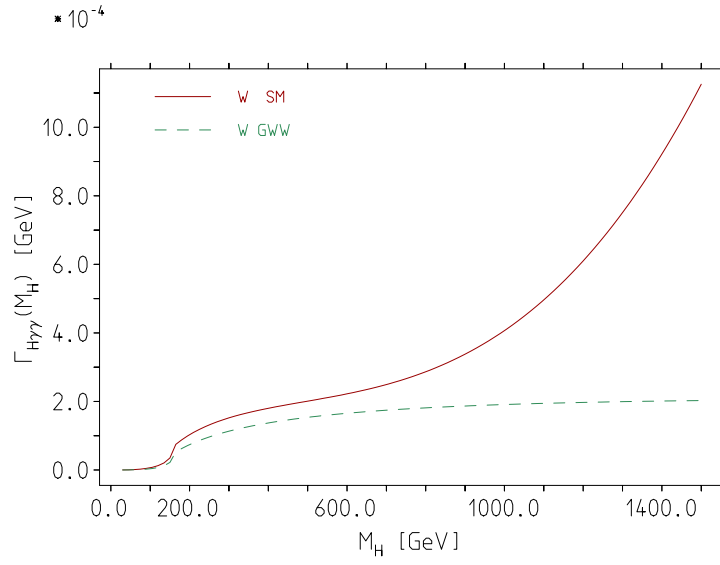
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Figure 2: The partial Higgs width $\Gamma(H \to \gamma\gamma)$ as a function of the Higgs mass m_H. The full line is the standard SM result, the dashed one the one calculated in Ref. [1].
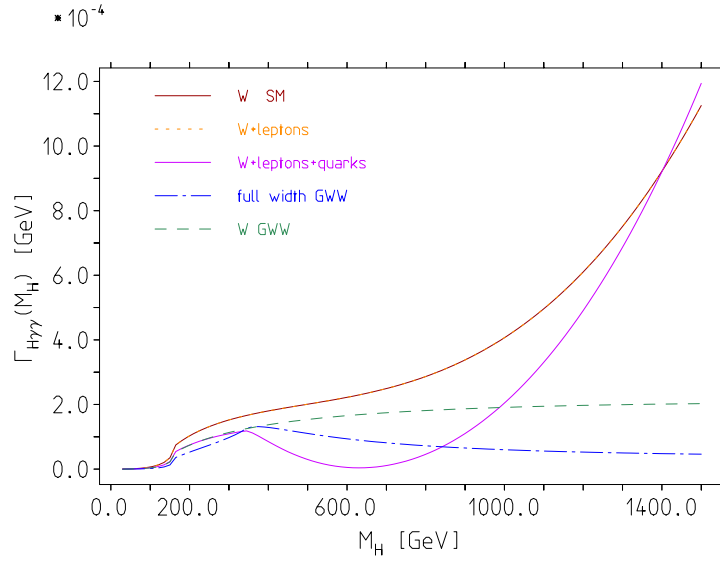

Figure 3: Comparison of the $\Gamma(H \to \gamma\gamma)$ standard SM predictions (W loop only, + leptons, + quarks) with the alternative prediction from Ref. [1]

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The decay $H \to 2\gamma$ has been investigated in great details including higher order effects. The two-loop QCD corrections to the quark loops have been presented in [23, 24, 25, 26]. The limit $m_H >> M_W$ has been investigated at the two loop level in [27]. The complete two-loop corrections are also known [28, 29]. Not surprisingly, they get large in the heavy Higgs range where strong coupling problems show up. For a comprehensive overview of the Standard Model Higgs profile see [30] and references therein.

Comments like the ones presented in this note have been presented in Refs. [31, 32, 33] in response of [1]. In our view, the result in question may be interpreted as follows: in the regime $m_H \gg M_W$ only the contribution from the massless transversal W's have been taken into account, while the contribution from the [in this limit] physical charged massless Higgses has not been taken into account.

We conclude that the symmetry properties of the SM uniquely fix the correct answer for $H \to 2\gamma$ rate. Dimensional regularization (DREG) remains the most adequate technical tool to preserve the gauge symmetry properties in calculations exhibiting ultraviolet divergent integrals at some stage of a calculation. This even is so if no overall renormalization is required like in the $H \to 2\gamma$ process. Whatever regularization prescription is utilized at the end one has to make sure that the Slavnov-Taylor and Ward-Takahashi identities are respected. Limitations of dimensional regularization are well known in connection with chiral structures: the anticommuting γ_5 problem and the related Adler-Bell-Jackiw triangle anomaly (see e.g. [34] and references therein). Similarly, for supersymmetric structures dimensional reduction (DRED) is a more adequate [35] regularization procedure. These technicalities however do not play a role in the SM $H \to 2\gamma$ decay.

Because of the importance of the result in view of the Higgs search at the LHC I find it appropriate to publicize these comments in spite of the fact that essentially only known results are reviewed.

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