Sigma Hadronic and Precision Tests of the SM

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EURODAΦNE Collaboration Meeting

April 12-17, 1999

Laboratori Nazionali di Frascati - INFN

Frascati (Rome) - Italy

❶ Introduction

The total cross section

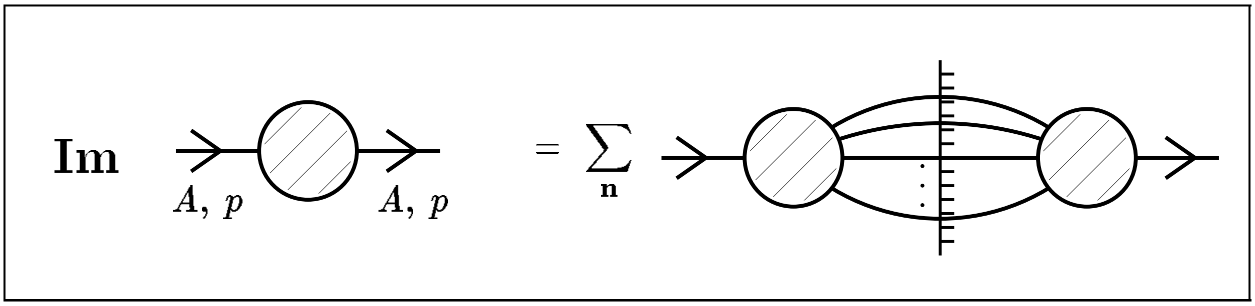
$$\sigma(e^+e^- \to hadrons)$$

for hadroproduction in e^+e^-– annihilation directly determines the hadronic contribution to the photon vacuum polarization function $\Pi'_\gamma(q^2)$ via a dispersion integral as follows:

❏ ***Causality*** **implies** ***analyticity*** **(subtracted dispersion relation)**

$$\Pi'_\gamma(q^2) - \Pi'_\gamma(0) = \frac{q^2}{\pi} \int\limits_0^\infty ds \frac{\mathrm{Im}\Pi'_\gamma(s)}{s\,(s - q^2 - i\varepsilon)}$$

❏ ***Unitarity*** **implies the** ***optical theorem:*** **For the photon propagator reads**



$$\mathrm{Im}\Pi'_\gamma(s) = \frac{1}{12\pi} R(s)$$

where

$$R(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to hadrons)}{\frac{4\pi\alpha^2(s)}{3s}}$$

What is the precise definition of R(s)?

Theoretically:

$$R_{\rm had} \equiv \frac{\sigma_{\rm had}(s)}{\sigma_{\mu\mu}(s)} = \frac{\sigma^0_{\rm had}(s)}{\sigma^0_{\mu\mu}(s)}$$

- **"undressed" from electroweak effects**

Experimentally:

$$R_{\rm had} = \frac{N_{\rm had}\;(1+\delta_{RC})}{N_{\rm norm}\;\epsilon} \cdot \frac{\sigma_{\rm norm}(s)}{\sigma^0_{\mu\mu}(s)}$$

- δ_{RC}

  all radiative corrections to hadronic yield

- ϵ

  efficiency-acceptance product for hadronic events

- $\sigma_{\rm norm}$

  physical cross section for the normalizing events (including all radiative events) integrated over the acceptance used for the luminosity measurement

need detailed understanding of detector (efficiency, acceptance) and theory (unfolding of photon radiaton etc.)

(Davier, Höcker 97)

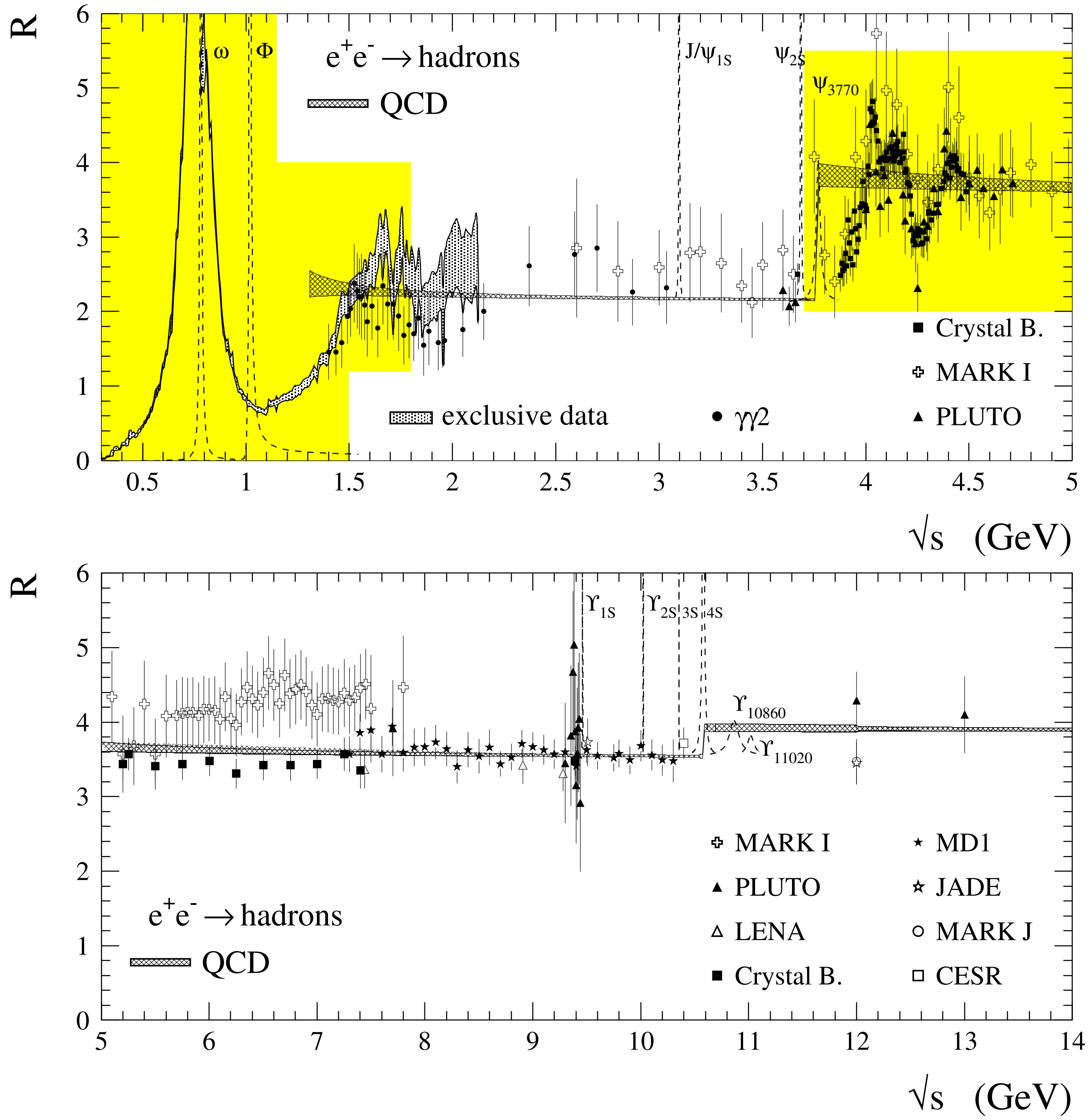
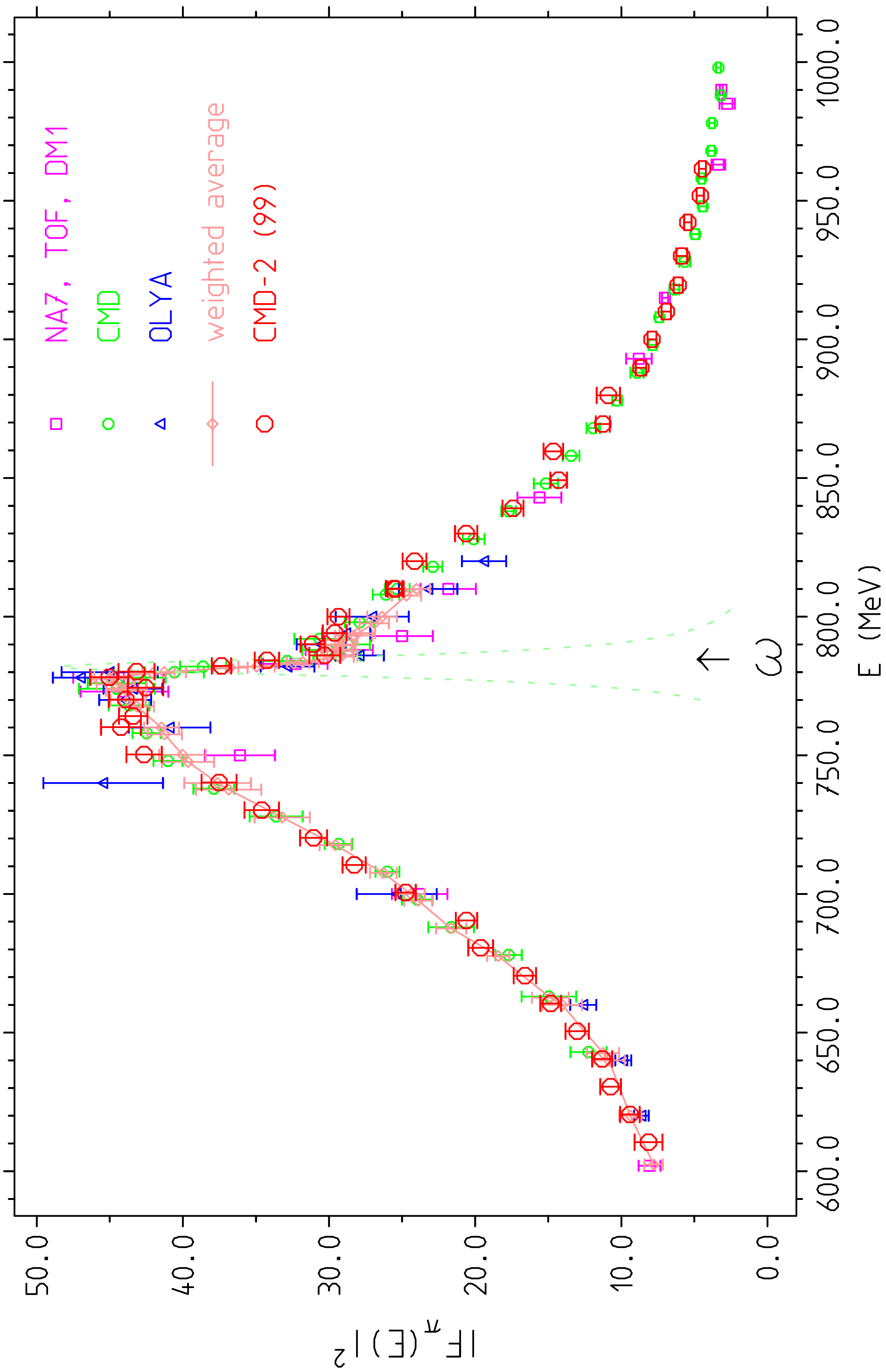
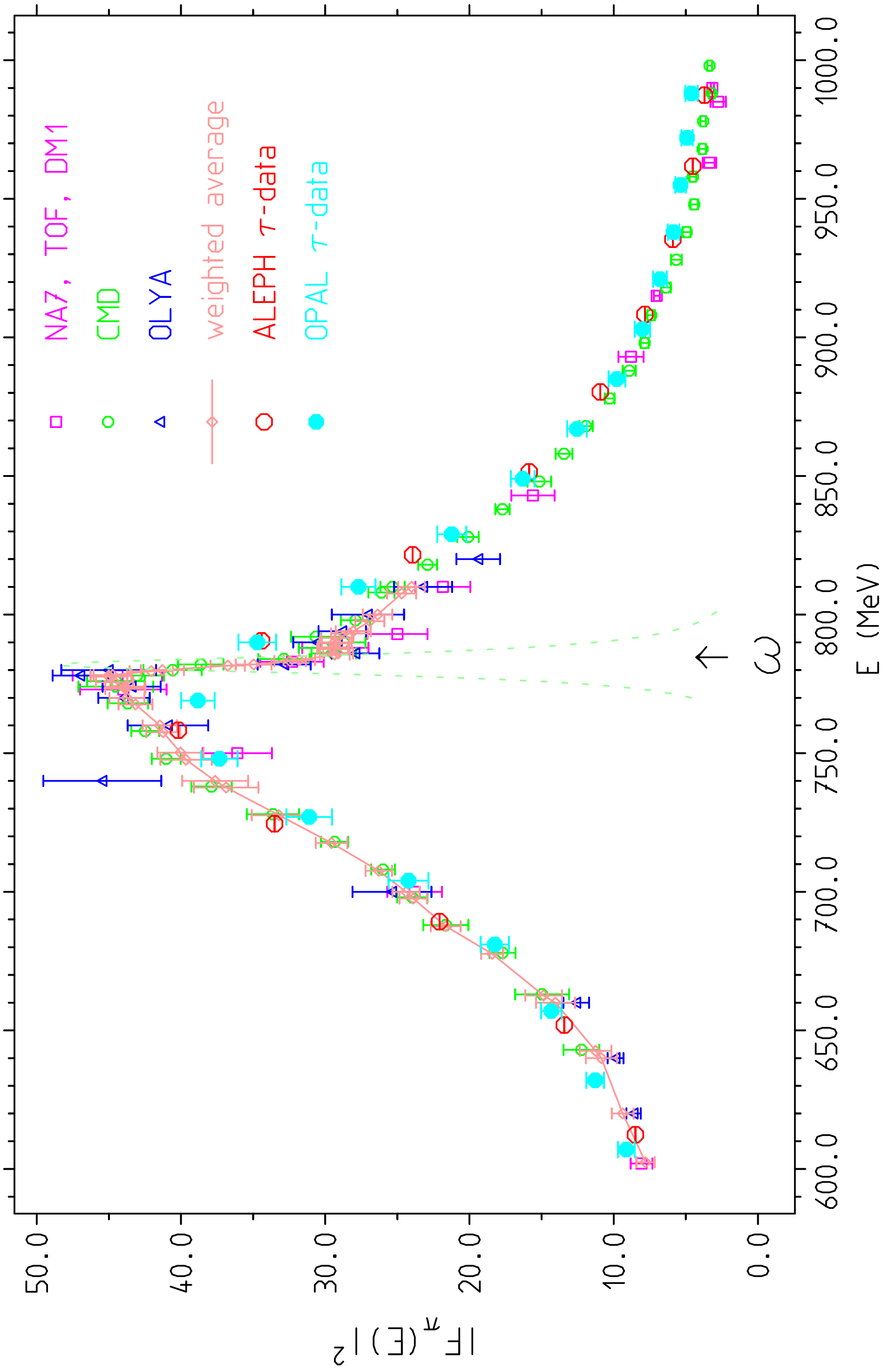
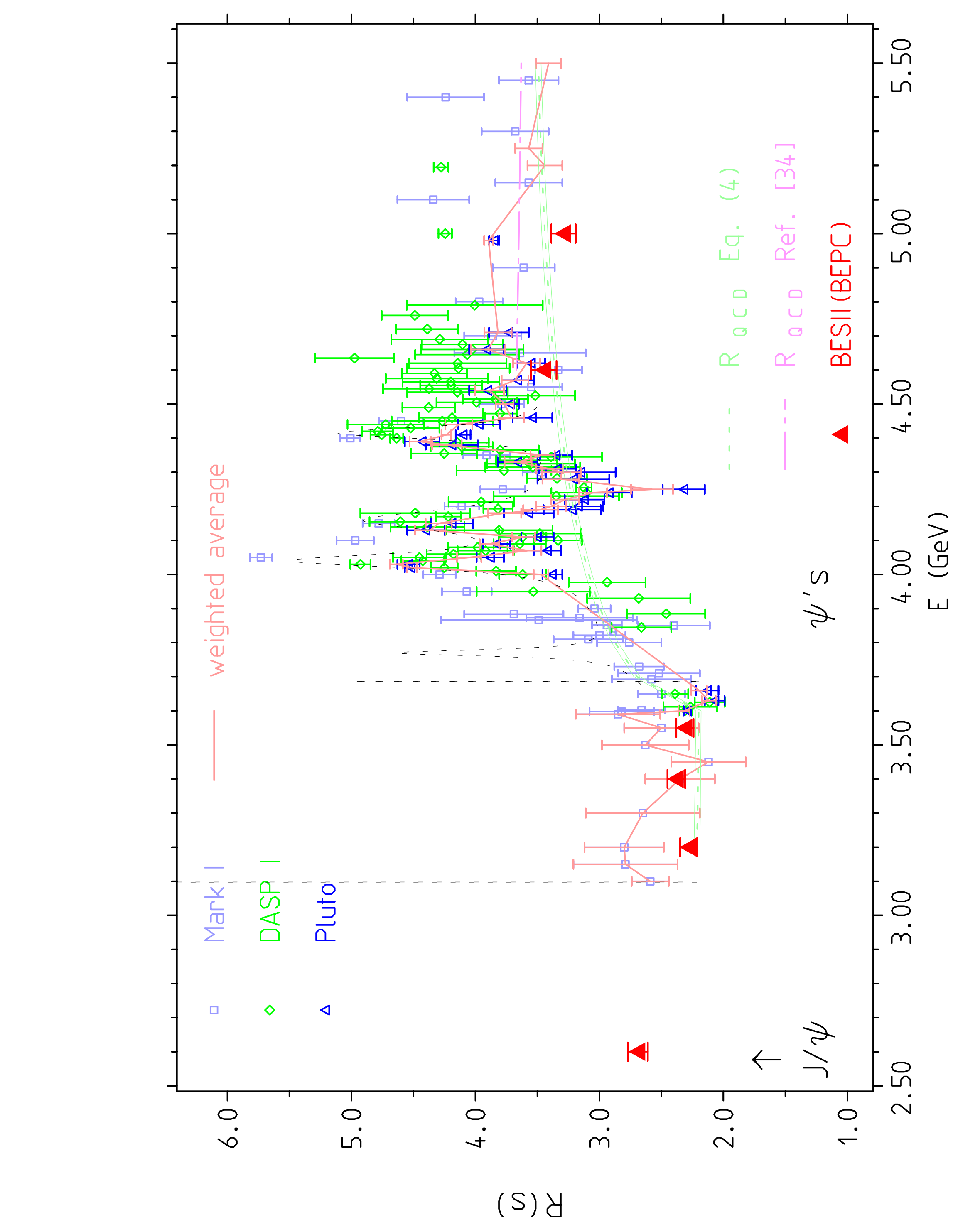


Figure 1: *Inclusive hadronic cross section ratio in e^+e^- annihilation versus the c.m. energy $\sqrt{s}$. Additionally shown is the QCD prediction of the continuum contribution as explained in the text. The shaded areas depict regions were experimental data are used for the evaluation of $\Delta\alpha_{\rm had}(M_Z^2)$ and $a_\mu^{\rm had}$ in addition to the measured narrow resonance parameters. The exclusive e^+e^- cross section measurements at low c.m. energies are taken from DM1,DM2,M2N,M3N,OLYA,CMD,ND and τ data from ALEPH (see Ref. [1] for detailed information).*

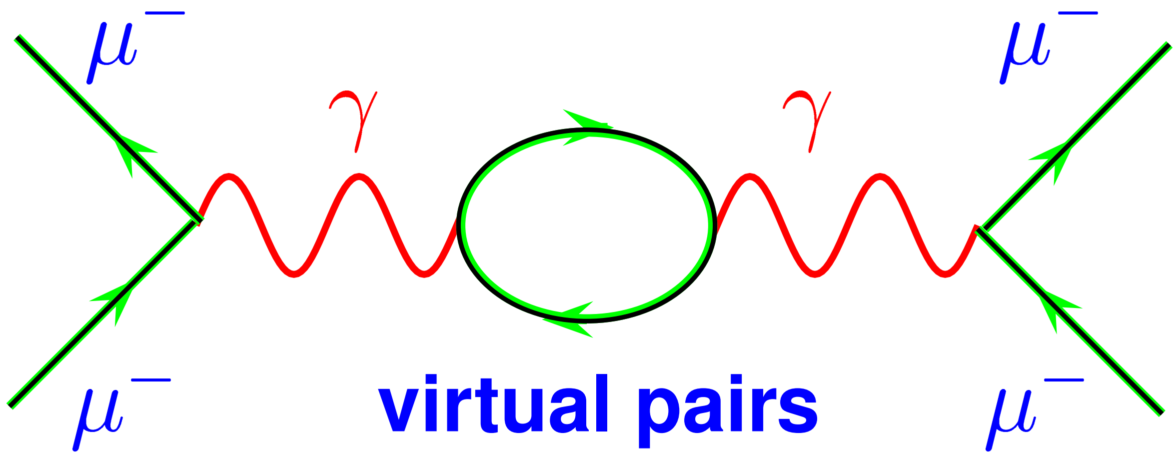

NA7, TOF, DM1
CMD
OLYA
weighted average
CMD-2 (99)
ω
E (MeV)
|F_π(E)|^2
600.0
650.0
700.0
750.0
800.0
850.0
900.0
950.0
1000.0
0.0
10.0
20.0
30.0
40.0
50.0



systematic deviations not yet understood!


R(s)
E (GeV)
Mark I
DASP I
Pluto
weighted average
R QCD Eq. (4)
R QCD Ref. [34]
BESII(BEPC)
J/ψ
ψ's

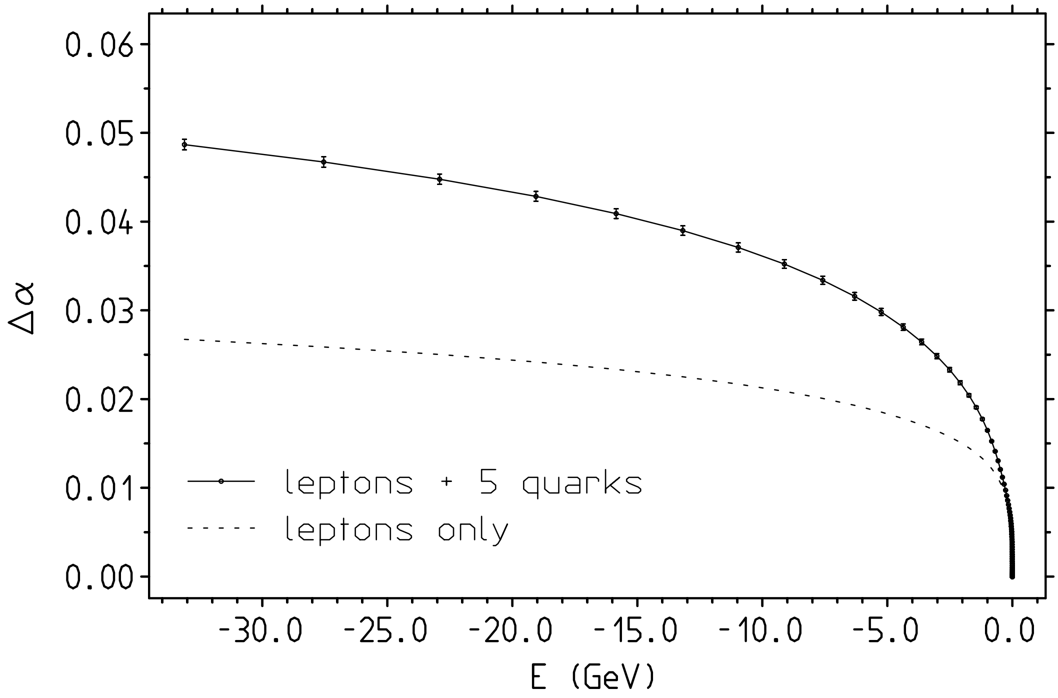
❷ Vacuum polarization and charge screening



$$\gamma^* \to e^+e^-, \mu^+\mu^-, \tau^+\tau^-, \underbrace{u\bar{u}, d\bar{d}, \cdots}_{\text{hadrons}} \to \gamma^*$$

Effective finestructure "constant" $\alpha(E)$

appears in many places in physics! **large EWRC !**



Formal definition:

Effective QED coupling constant at scale $\sqrt{s}$:

$$\alpha(s) = \frac{\alpha}{1 - \Delta\alpha(s)}$$

with

$$\Delta\alpha(s) = -4\pi\alpha \mathrm{Re}\left[\Pi'_\gamma(s) - \Pi'_\gamma(0)\right]$$

where $\Pi'_\gamma(s)$ is the photon vacuum polarization function

$$i\int d^4x\, e^{iq\cdot x}\langle 0|\mathrm{T} j^\mu_{em}(x) j^\nu_{em}(0)|0\rangle = -(q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi'_\gamma(q^2)$$

and $j^\mu_{em}(x)$ is the electromagnetic current.

Contributions:

- **Leptons**

Calculable in perturbation theory: Free lepton loop affected by small electromagnetic corrections.

$$\begin{aligned}
\Delta\alpha_{\mathrm{leptons}}(s) &= \\
&= \sum_{\ell=e,\mu,\tau} \frac{\alpha}{3\pi}\left[-\frac{8}{3} + \beta_\ell^2 - \frac{1}{2}\beta_\ell(3-\beta_\ell^2)\ln\left(\frac{1-\beta_\ell}{1+\beta_\ell}\right)\right] \\
&= \sum_{\ell=e,\mu,\tau} \frac{\alpha}{3\pi}\left[\ln\left(s/m_\ell^2\right) - \frac{5}{3} + O\left(m_\ell^2/s\right)\right] \\
&= 0.03142 \text{ for } s = M_Z^2
\end{aligned}$$

where $\beta_\ell = \sqrt{1-4m_\ell^2/s}$.

❑ Quarks

The contribution of the five light quarks (u, d, s, c, b) is not calculable in perturbation theory: Free quark loop strongly modified by strong interactions at low energy. Way out: unitarity and the analyticity of $\Pi'_\gamma(s)$ allows us to write

(Cabibbo, Gatto 1961)

$$\Delta\alpha^{(5)}_{\mathrm{had}}(s) = -\frac{\alpha s}{3\pi}\left(\mathrm{P}\!\!\int_{4m_\pi^2}^{E^2_{\mathrm{cut}}} ds' \frac{R_\gamma^{\mathrm{data}}(s')}{s'(s'-s)} + \mathrm{P}\!\!\int_{E^2_{\mathrm{cut}}}^{\infty} ds' \frac{R_\gamma^{\mathrm{pQCD}}(s')}{s'(s'-s)} \right)$$

where

$$R_\gamma(s) \equiv \frac{\sigma(e^+e^- \to \gamma^* \to \mathrm{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} = 12\pi \mathrm{Im}\Pi'_\gamma(s)$$

has been measured in e^+e^-–annihilation experiments up to about $E_{\mathrm{cut}} \sim 40$ GeV. By virtue of the asymptotic freedom of QCD the high energy tail can be safely calculated using pQCD (Gorishny, Kataev, Larin 91).

$E_{\mathrm{cut}} \gtrsim 12. GeV$

$$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2) = 0.0280 \pm 0.0007 \quad \mathbf{E} \quad (EJ95)$$

$$\rightarrow 0.02763 \pm 0.00016 \quad \mathbf{T} \quad (DH98)$$

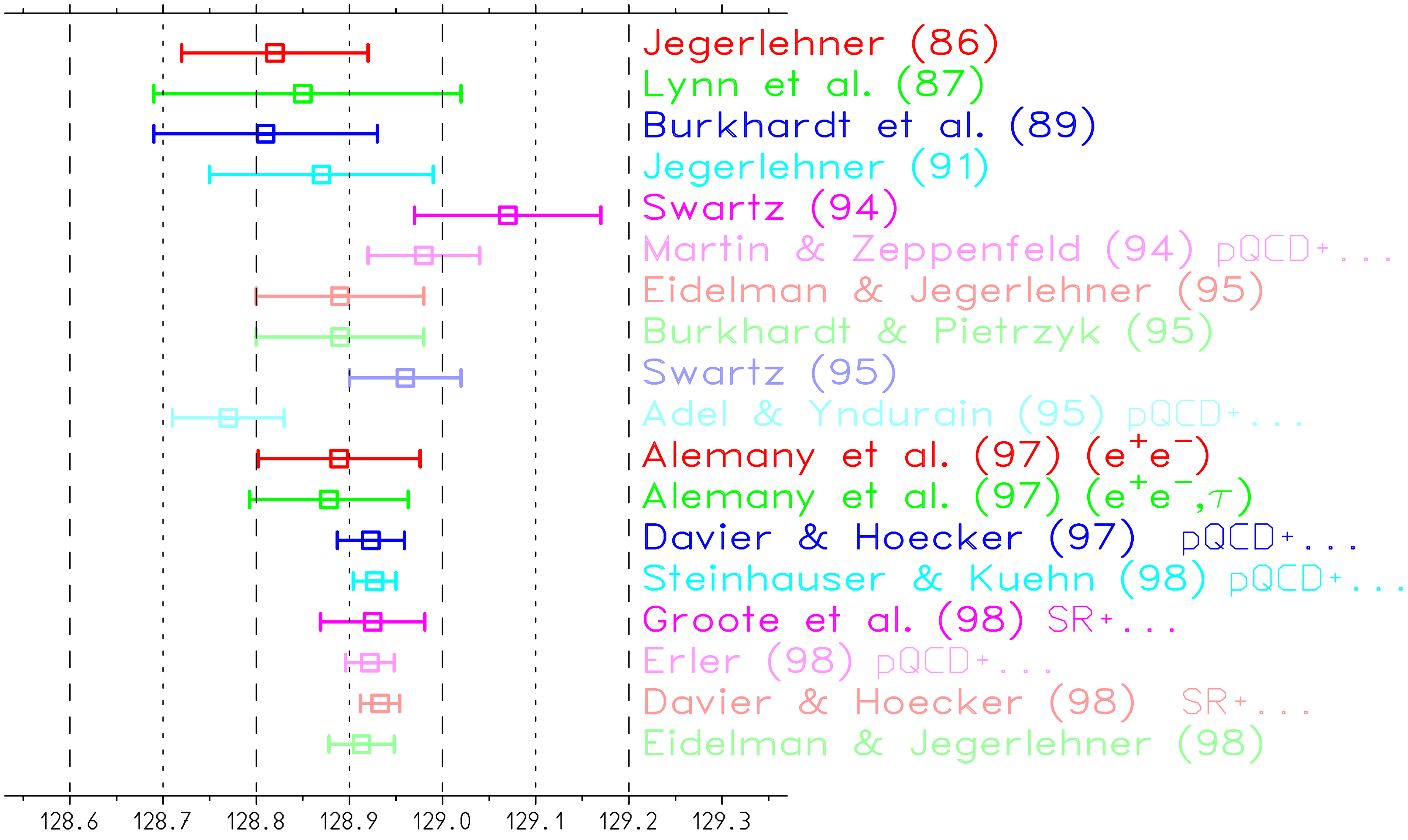
$E_{\mathrm{cut}} \sim 1.7 GeV$

$$\alpha^{-1}(M_Z) = 128.89 \pm 0.09 \quad \mathbf{E} \quad (EJ95)$$

$$\rightarrow 128.933 \pm 0.021 \quad \mathbf{T} \quad (DH98)$$

Evaluations of $\alpha(M_Z)$ by different authors

All values have been rescaled to $M_Z = 91.1887$ GeV. The small top quark and the W contributions are not included.



$\Delta\alpha$ in electroweak precision physics

Calculate $\sin^2\Theta_i$ from α, G_μ and M_Z :

$$\boxed{\sqrt{2}\, G_\mu\, M_Z^2\, \sin^2\Theta_i\, \cos^2\Theta_i = \pi\alpha\, \frac{1}{1-\Delta r_i}}$$

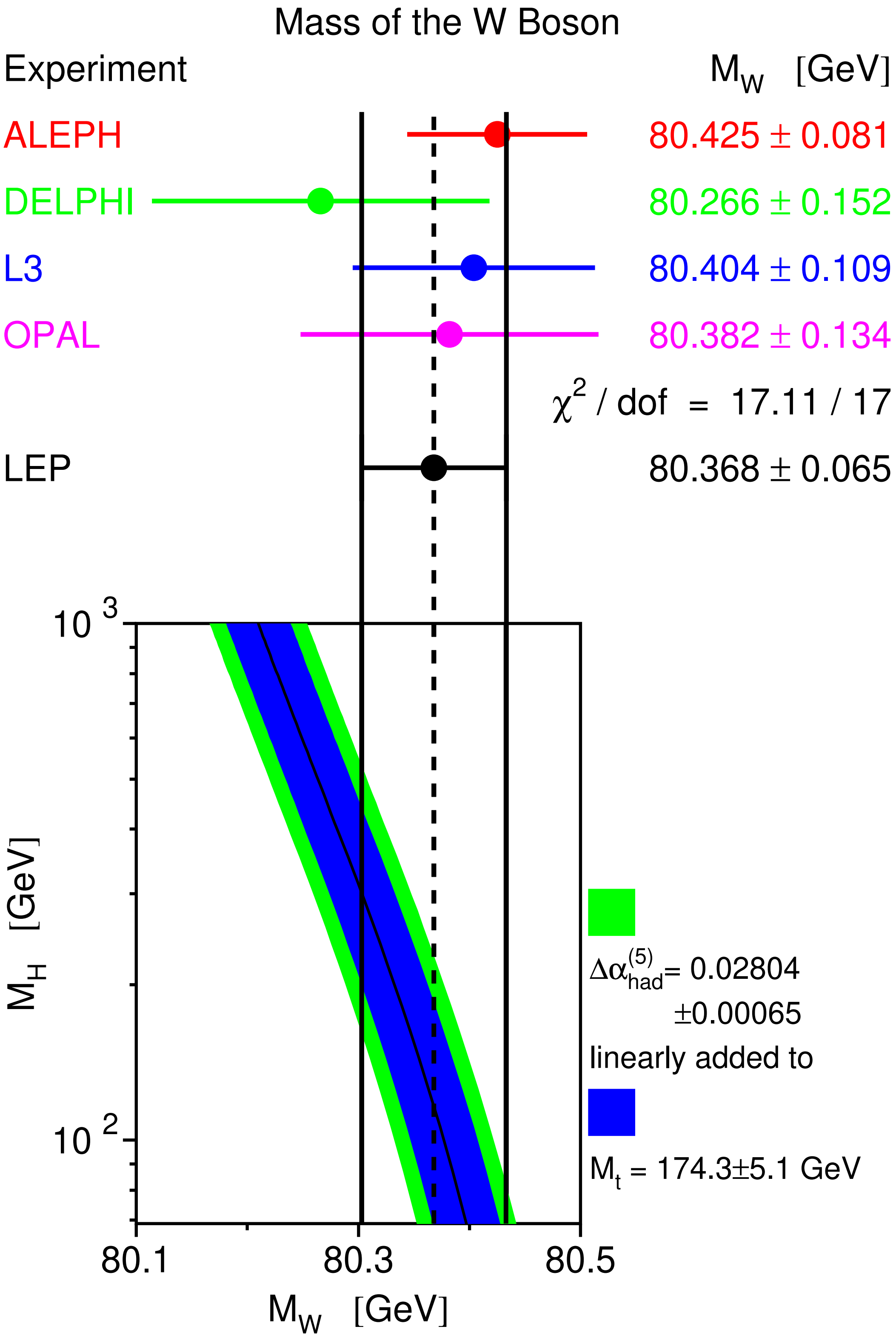
Various definitions of $\sin^2\Theta_i$, which coincide at tree level: From weak gauge boson masses, from electroweak gauge couplings and from the neutral current coupling of the charged fermions:

$$\begin{aligned} \sin^2\Theta_W &= 1 - \frac{M_W^2}{M_Z^2} \\ \sin^2\Theta_g &= e^2/g^2 \\ \sin^2\Theta_f &= \frac{1}{4|Q_f|}\left(1 - \frac{v_f}{a_f}\right) \;, \quad f \neq \nu \end{aligned}$$

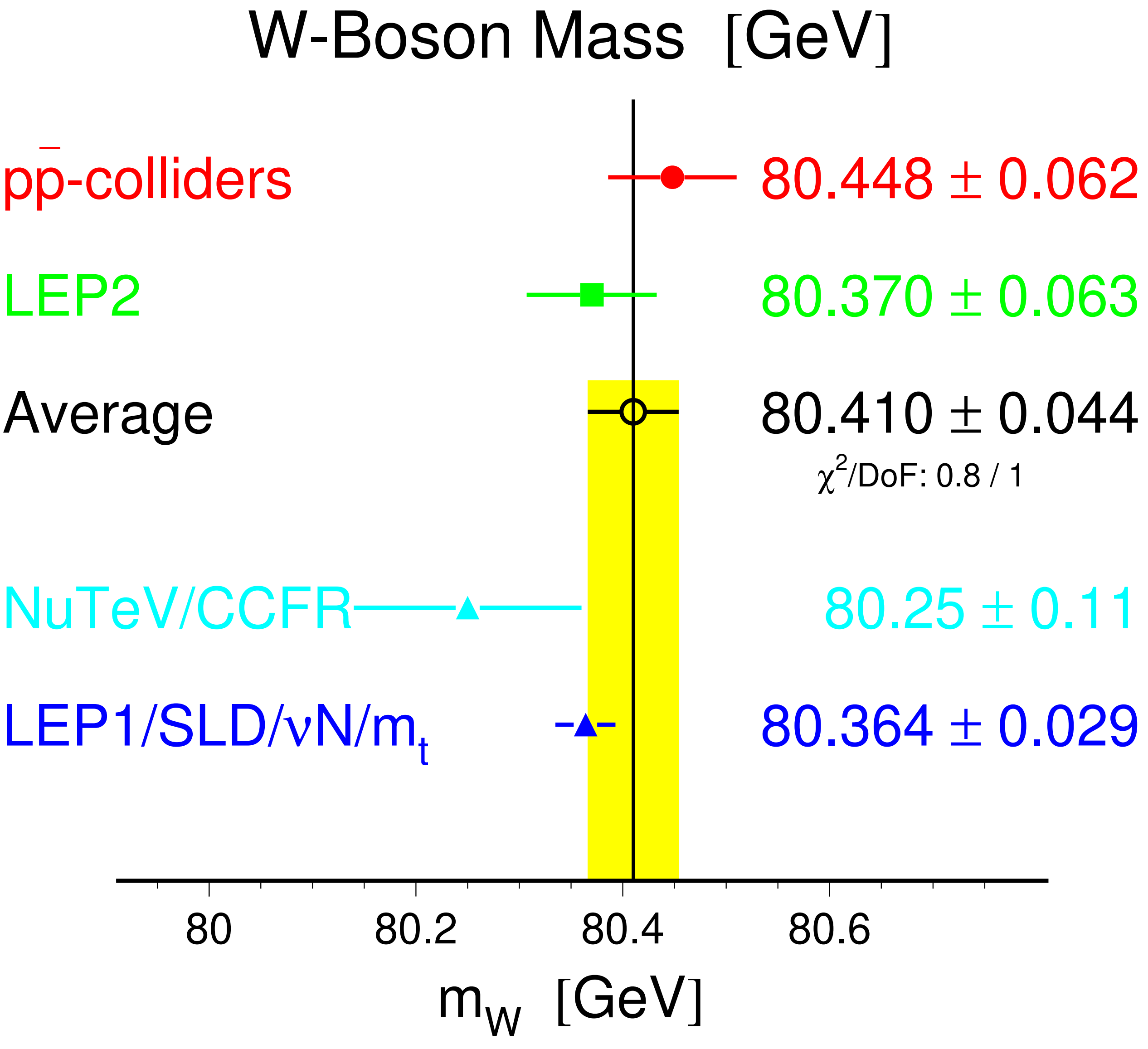
$$\begin{aligned} \Delta r_i &= \Delta r_i(\alpha,\, G_\mu,\, M_Z, m_H, m_{f\neq t},\, m_t) \\ \Delta r_i &= \underset{\substack{\uparrow \\ \text{universal}}}{\Delta\alpha} - f_i(\sin^2\Theta_i)\, \Delta\rho + \Delta r_{i\,\mathrm{rem}} \end{aligned}$$

$\rightarrow$ **predictions for** M_W, A_{LR}, A_{FB}^f, Γ_f, $\cdots$

$$\delta\Delta\alpha \rightarrow \delta M_W \;, \quad \delta \sin^2\Theta_i \;, \; \cdots$$



(LEP Electroweak Working Group: J. Alcaraz et al. 99)



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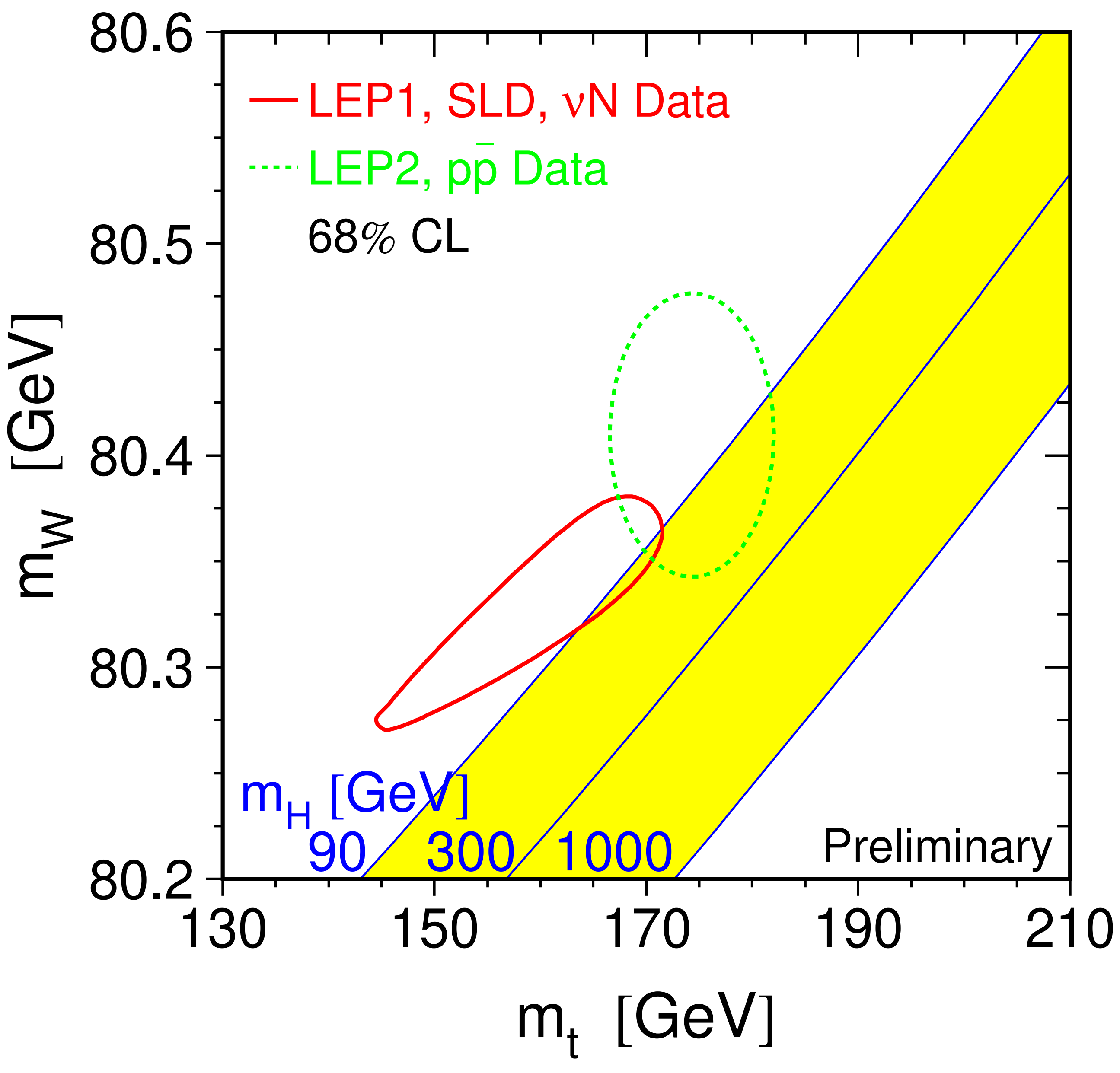
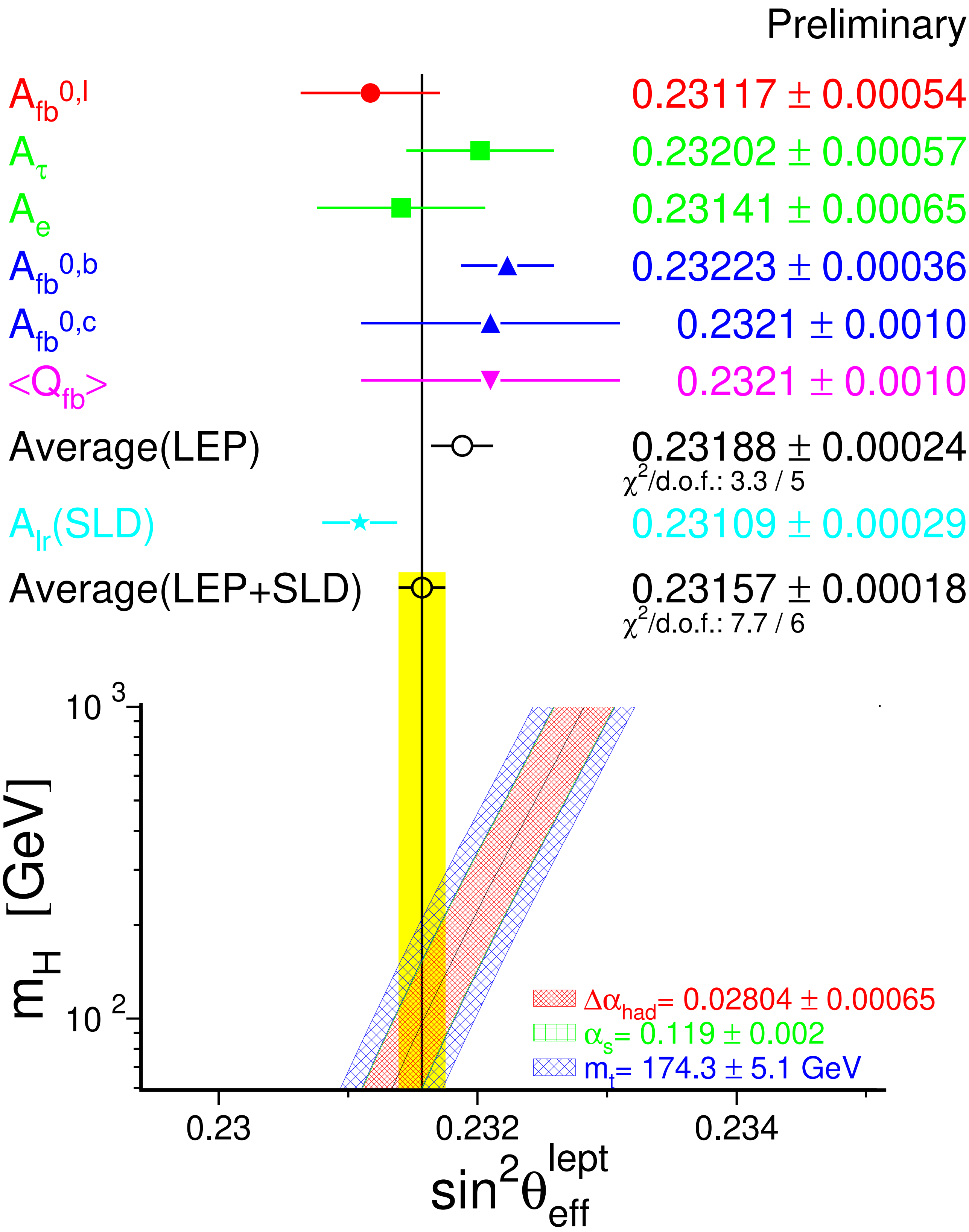


(LEP Electroweak Working Group: J. Alcaraz et al. 99)



(LEP Electroweak Working Group: J. Alcaraz et al. 99)

Sensitivity of indirect Higgs mass bounds for $m_H \sim 100$ GeV:

$$\frac{\delta m_H}{m_H} \sim 1300 \, \frac{\delta M_W}{M_W}$$

$$\frac{\delta m_H}{m_H} \sim 440 \, \frac{\delta \sin^2\theta_{\rm eff}^{\rm lep}}{\sin^2\theta_{\rm eff}^{\rm lep}}$$

EW–fits Moriond 99:

$$M_W = 80.410 \pm 0.044 \, \text{GeV}$$

$$\sin^{2\,\rm lep}_{\rm eff} = 0.23157 \pm 0.0018$$

$$m_t = 174.3 \pm 5.1 \, \text{GeV}$$

$\Rightarrow \quad \Rightarrow \quad \Rightarrow$

$$\delta m_H = (\cdots)^{+127}_{-\ 74}\,, \quad m_H < 366 \, \text{GeV} \quad \text{from} \quad M_W$$

$$\delta m_H = (102)^{+\ 89}_{-\ 47}\,, \quad m_H < 276 \, \text{GeV} \quad \text{from} \quad \sin^2\theta_{\rm eff}^{\rm lep}$$

All data:

$$m_H = 71^{+\ 75}_{-\ 42}\,, \quad m_H < 220 \, \text{GeV} \ \text{at} \ 95\% \ \text{CL}$$

direct $\quad m_H > 95$ GeV at 95% CL

Future:

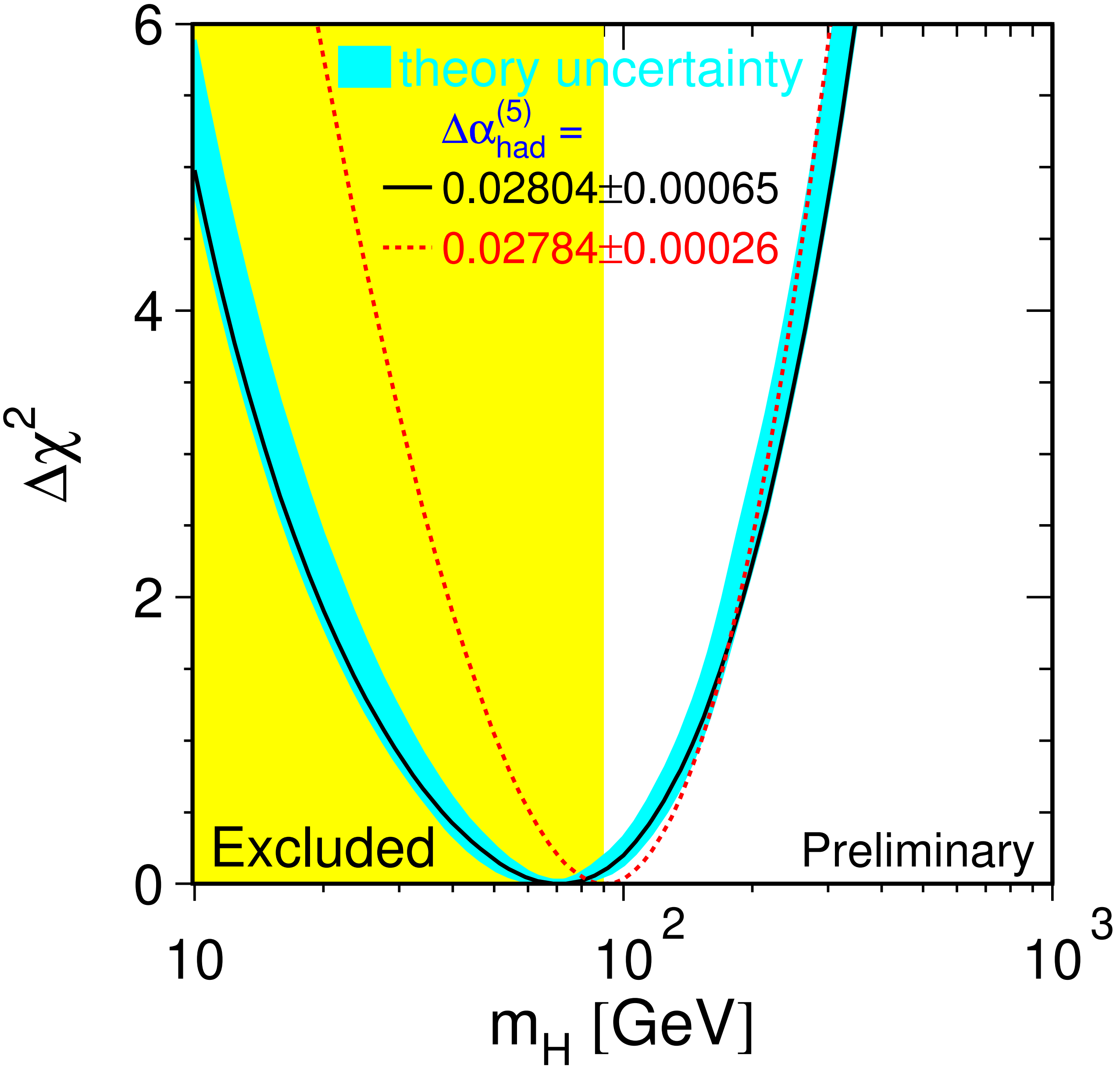
$$\delta m_t \sim 2 \, \text{GeV}\,, \quad \delta\Delta\alpha \sim ???$$

$$\frac{\delta M_W}{M_W} \sim \frac{1}{2}\frac{\sin^2\theta}{\cos^2\theta-\sin^2\theta}\,\delta\Delta\alpha \sim 0.23\;\delta\Delta\alpha$$

$\delta\Delta\alpha$	δM_W **(MeV)**	
0.00065	**12.0**	**[EJ95]**
0.00016	**3.0**	**[DH98]**
0.00025	**4.6**	**[FJ98]**
0.00007	**1.3**	$\delta\sigma \lesssim 1\%$ **up to** J/ψ
0.00005	**0.9**	$\delta\sigma \lesssim 1\%$ **up to** Υ
WA	**44.0**	**Moriond99**

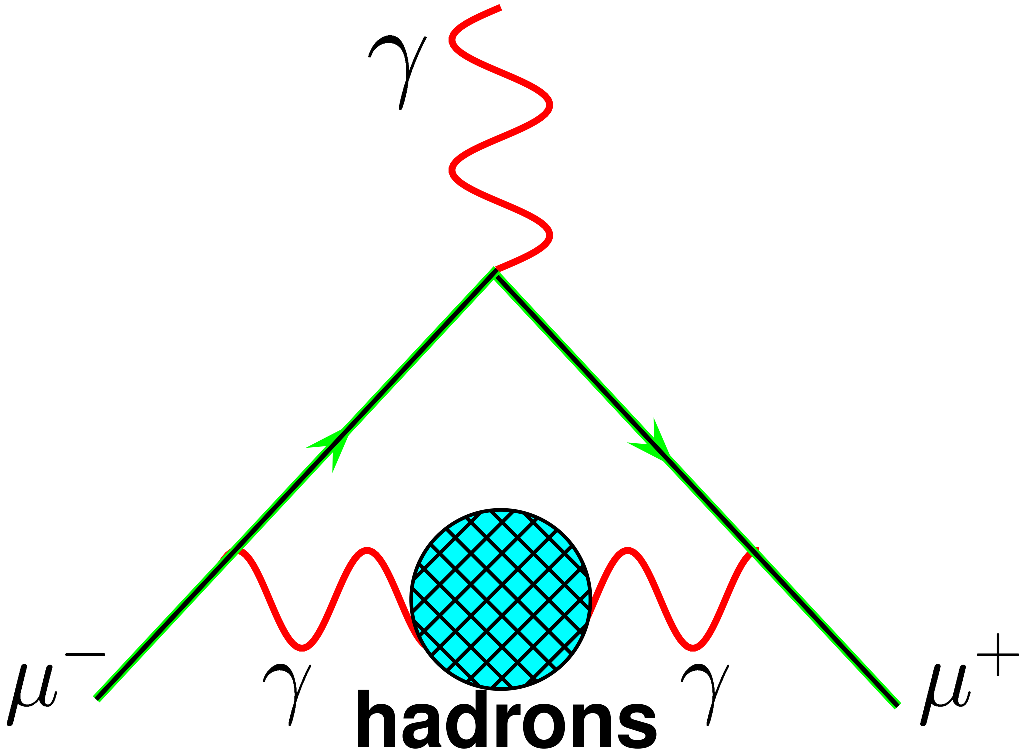
$$\frac{\delta\sin^2\theta}{\sin^2\theta} \sim \frac{\cos^2\theta}{\cos^2\theta-\sin^2\theta}\,\delta\Delta\alpha \sim 1.54\;\delta\Delta\alpha$$

$\delta\Delta\alpha$	$\delta\sin^2\theta$	
0.00065	**0.000232**	**[EJ95]**
0.00016	**0.000057**	**[DH98]**
0.00025	**0.000089**	**[FJ98]**
0.00007	**0.000025**	$\delta\sigma \lesssim 1\%$ **up to** J/ψ
0.00005	**0.000018**	$\delta\sigma \lesssim 1\%$ **up to** Υ
WA	**0.000180**	**LEP+SLD**



(LEP Electroweak Working Group: J. Alcaraz et al. 99)

❸ Vacuum polarization contribution to $g-2$



$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E^2_{\rm cut}} ds\, \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int_{E^2_{\rm cut}}^{\infty} ds\, \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right)$$

(Bouchiat, Michel 1961, Durand III 1962, Gourdin, de Rafael 1969)

- The kernel $\hat{K}(s)$ is a smooth bounded function growing from 0.63... at threshold to 1 at ∞.
- Note the extra $1/s$–enhancement of contributions from low energies in a_μ as compared to $\Delta\alpha$.

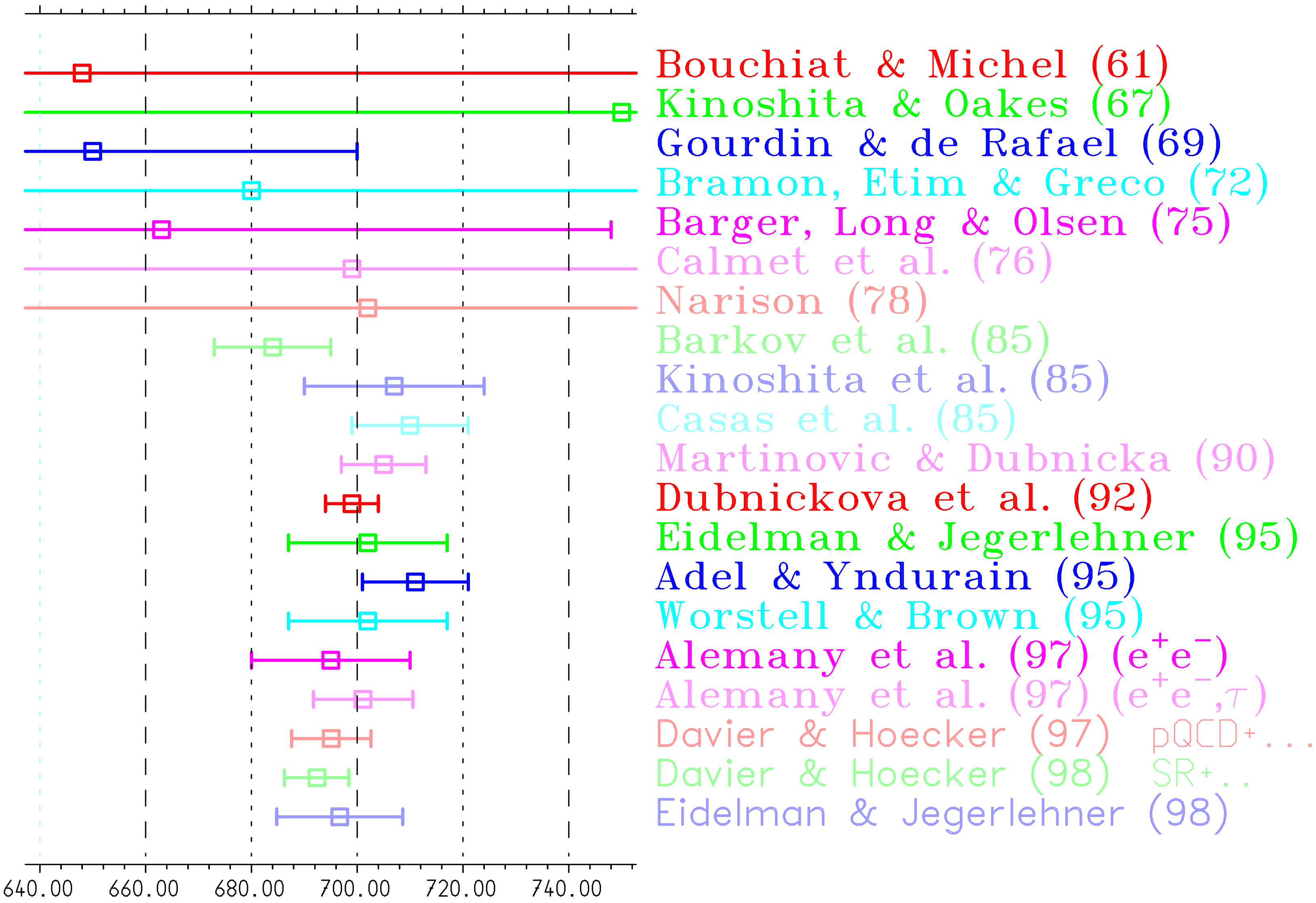
$$a_\mu^{\rm had} = (696.7 \pm 11.9) \times 10^{-10} \qquad \textbf{(EJ 98)}$$

For this observable the use of τ spectral–functions in addition to e^+e^- cross–sections leads to a dramatic reduction of the uncertainty. Extended use of pQCD and application of QCD sum rule techniques leads to further improvements

$$\begin{aligned}
a_\mu^{\rm had} &= (701.1 \pm 9.4) \times 10^{-10} && \textbf{(ADH 98)} \\
a_\mu^{\rm had} &= (695.1 \pm 7.5) \times 10^{-10} && \textbf{(Davier, Höcker 98a)} \\
a_\mu^{\rm had} &= (692.4 \pm 6.2) \times 10^{-10} && \textbf{(Davier, Höcker 98b)}
\end{aligned}$$

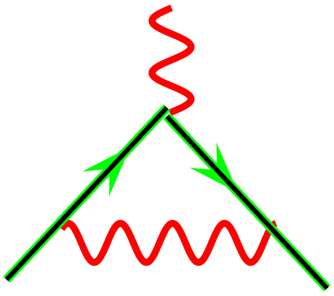
Evaluations of $a_\mu^{\rm hadrons} \times 10^{10}$ by different authors

Only the leading hadronic vacuum polarization diagram is included.



a_μ status:

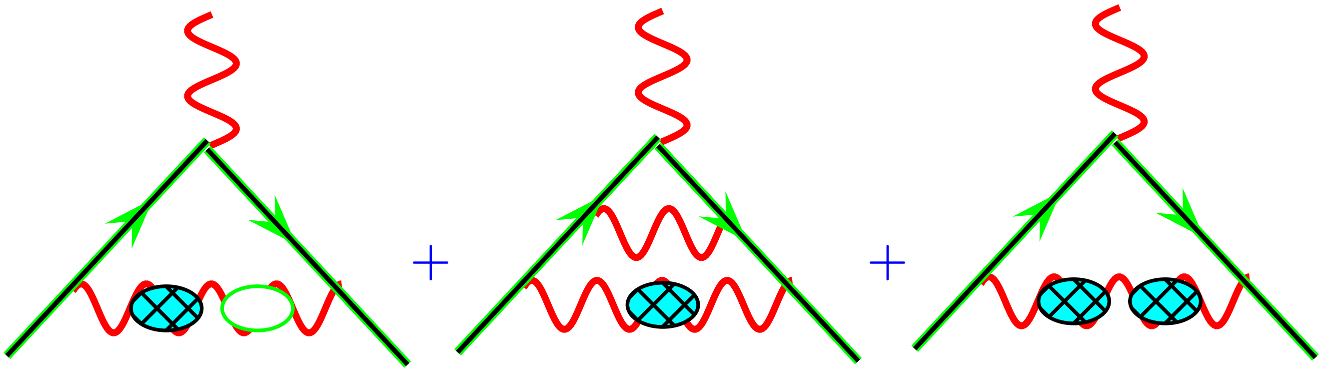
- QED



$$a_\mu^{\mathrm{QED}(1)} = (116140981 \pm 6) \times 10^{-11}$$
$$\vdots$$
$$a_\mu^{\mathrm{QED}(5)} = (116584706 \pm 3) \times 10^{-11}$$

(Schwinger 48, · · ·, Kinoshita 93, · · ·, Laporta, Remiddi 96)

- Higher order hadronic contributions

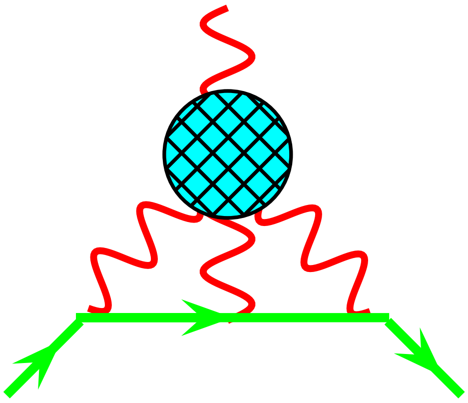


$$a_\mu^{\mathrm{had}(2)} = \frac{1}{4\pi^3} \int\limits_{4m_\pi^2}^{\infty} ds\, K^{(2)}(s)\, \sigma_{\mathrm{had}}(s)$$

$$a_\mu^{\mathrm{had}(2)} = -(100 \pm 6) \times 10^{-11}$$

(Krause 96)

- Hadronic light–by–light scattering contribution

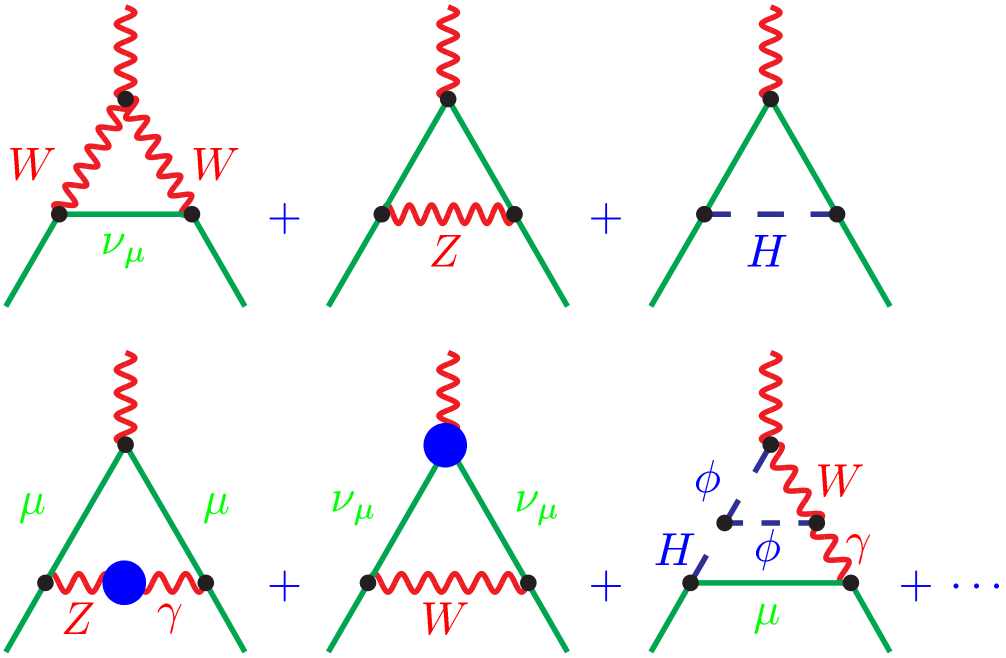


QCD inspired models used: ENJL and HGS models supplemented by VMD

$$a_\mu^{\mathrm{light-by-light}} = \left\{ \begin{array}{lcr} -(92 \pm 32) & & BPP96 \\ -(79 \pm 15) & \times 10^{-11} & HK98 \\ -(85 \pm 25) & & CM98 \end{array} \right.$$

model ambiguity? (Bijnens, Pallante, Prades 96, Hayakawa, Kinoshita 98)

- Higher order electroweak contributions



$$a_\mu^{\mathrm{weak}(2)} = -(44 \pm 4) \times 10^{-11}$$

(Czarnecki, Krause, Marciano 98)

a_μ summary:

	$a_\mu \times 10^{11}$		
CERN experiment (79)	116592400	$\pm$	850
BNL E821 exp. (97)	116592500	$\pm$	1520
BNL E821 goal $\to$	$\cdots$	$\pm$	40
Theory (98)	116591595	$\pm$	67
QED	116584706	$\pm$	3
1–loop EW	195		
2–loop EW	− 44	$\pm$	4
hadronic (1)	7025	$\pm$	102
$\to$	6925	$\pm$	62[a]
hadronic (2)	− 100	$\pm$	6
light–by–light	− 92	$\pm$	32
$\to$	− 79	$\pm$	15[b]

[a] Davier, Höcker 98

[b] Hayakawa, Kinoshita 98

❹ Comments on recent progress

❶ Additional data: τ–data $+$ CVC

ALEPH–Coll., (OPAL–Coll.,), Alemany, Davier, Höcker 96

$$\tau^- \to X^- \nu_\tau \quad \leftrightarrow \quad e^+ e \to X^0$$

where X^- and X^0 are hadronic states related by isospin rotation. The e^+e^- cross–section is then given by

$$\sigma^{I=1}_{e^+e^- \to X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-} \quad , \quad \sqrt{s} \leq M_\tau$$

in terms of the τ spectral function v_1.

- mainly improves the knowledge of the $\pi^+\pi^-$ channel (ρ–resonance contribution)
- which is dominating in $a_\mu^{\rm had}$ (72%)

$$I = 1 \sim 75\% \,; \; I = 0 \sim 25\%$$

τ–data cannot replace e^+e^-–data

$$\begin{array}{ccccc} \delta a_\mu & : & 15.6 \times 10^{-10} & \to & 10.2 \times 10^{-10} \\ \delta \Delta\alpha & : & 0.00067 & \to & 0.00065 \end{array}$$

- usual problem of matching and averaging data
- requires splitting of the e^+e^- data into $I = 0$ and $I = 1$ parts
- introduces additional systematic error, non trivial to estimate

How accurate is CVC? (Eidelman, Ivanchenko 96)

compare τ–channels with corresponding e^+e^- channels

$$\tau^- \to X^- \nu_\tau \; , \quad X^- = \pi^- \pi^0, \pi^- 3\pi^0, (3\pi)^- \pi^0, \cdots$$

Branching Ratios of $\tau^- \to X^- \nu_\tau$ in percent.

Hadronic State X	World Average 1996	CVC Predictions
$\pi^-\pi^0$	25.24 ± 0.16	24.74 ± 0.79
$\pi^-3\pi^0$	1.14 ± 0.14	1.07 ± 0.10
$(3\pi)^-\pi^0$	4.25 ± 0.09	4.36 ± 0.55
Total	30.63 ± 0.23	30.17 ± 1.00

CVC prediction good to -1.5% $\to$ 1.5% uncertainty should be taken into account while calculating $R^{I=1}(s)$ from the τ spectra.

❷ Progress in pQCD calculations

Assume pQCD to be valid down to M_τ

$$\sigma_{\rm tot}(e^+e^- \to \text{hadrons}) \sim \sigma_{\rm tot}(e^+e^- \to \text{quarks and gluons})$$

The assumption seems to be supported by

- **the apparent applicability of pQCD to τ physics. In fact the running of $\alpha_s(M_\tau) \to \alpha_s(M_Z)$ from the τ mass up to LEP energies agrees well with the LEP value. The estimated uncertainty may be debated, however.**
- **non–perturbative (NP) effects if parametrized as prescribed by the operator product expansion (OPE) of the electromagnetic current correlator are seemingly small.**

For a more conservative approach see below (scheme dependence, mass effects, Euclidean approach)

❸ Non perturbative effects?

Parametrize NP effects at sufficiently large energies and away from resonances as prescribed by the OPE. Non–vanishing gluon and light quark condensates (Shifman, Vainshtein, Zakharov 79) imply the leading power corrections

$$
\begin{aligned}
\Pi_\gamma^{\prime\,\mathrm{NP}}(Q^2) &= \frac{4\pi\alpha}{3} \sum_{q=u,d,s} Q_q^2 N_{cq} \cdot \left[\frac{1}{12}\left(1-\frac{11}{18}a\right)\frac{<\frac{\alpha_s}{\pi}GG>}{Q^4} \right. \\
&+ \quad 2\left(1+\frac{a}{3}+\left(\frac{11}{2}-\frac{3}{4}l_{q\mu}\right)a^2\right)\frac{<m_q\bar{q}q>}{Q^4} \\
&\left. + \left(\frac{4}{27}a+\left(\frac{4}{3}\zeta_3-\frac{257}{486}-\frac{1}{3}l_{q\mu}\right)a^2\right)\sum_{q'=u,d,s}\frac{<m_{q'}\bar{q}'q'>}{Q^4}\right] \\
&+ \quad \cdots
\end{aligned}
$$

where $a \equiv \alpha_s(\mu^2)/\pi$ and $l_{q\mu} \equiv \ln(Q^2/\mu^2)$. $<\frac{\alpha_s}{\pi}GG>$ and $<m_q\bar{q}q>$ are the scale-invariantly defined condensates. Sum rule estimates of the condensates yield typically (large uncertainties)

$$
\begin{aligned}
&<\frac{\alpha_s}{\pi}GG>\sim \quad (0.389\ \mathbf{GeV})^4 \\
&<m_q\bar{q}q>\sim -(0.098\ \mathbf{GeV})^4 \quad \text{for} \quad q=u,d \\
&<m_q\bar{q}q>\sim -(0.218\ \mathbf{GeV})^4 \quad \text{for} \quad q=s
\end{aligned}
$$

Note that the above expansion is just a parametrization of the high energy tail of NP effects associated with the existence of non–vanishing condensates. There are other kind of NP phenomena like bound states, resonances, instantons and what else. The dilemma with is that it works only for Q^2 large enough and it has been successfully applied in heavy quark physics. It fails do describe NP physics at lower Q^2, once it starts to be numerically relevant pQCD starts to fail because of the growth of the strong coupling constant.

❹ Global duality and sum rules

Assuming global duality and using sum rules (SR):

Starting point is the OPE:

$$< h|J_\mu(x)\, J_\nu(0)|h' > \simeq \sum_i C_{\mu\nu\, i}(x)\, H_i$$

$H_i =< h|\mathcal{O}_i(0)|h' >$ **hadronic matrix element**

$C_{\mu\nu\, i}(x)$ **Wilson coefficient**

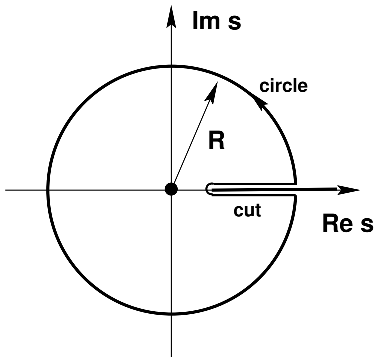
global duality assumption asserts that all non–perturbative physics resides in the H_i's and $C_{\mu\nu\, i}(x)$ is given by pQCD

(no precise tests yet, quality of convergence?)

The following schemes have no justification:

- $\sigma_{\rm had} = \sigma_{\rm pQCD} + \sigma_{\rm resonances}$
- **local duality: i.e., duality applied to energy intervals (resonance regions)**

❺ Pitfalls with dispersion relations?



Cauchy's theorem: Let $\Delta\Pi'(s)$ be analytic, then

$$\int_{\rm cut} \frac{ds}{s} \Delta\Pi'(s) = - \int_{\rm circle} \frac{ds}{s} \Delta\Pi'(s)$$

Optical theorem: relates $\Delta\Pi'(s)$ on cut to $R(s)$

Can we get non-perturbative physics from pQCD?

Naive argument:

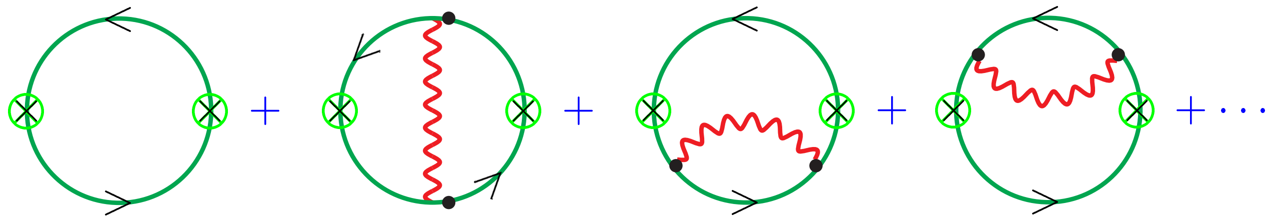
- **for large enough R pQCD may be applied on circle**
- **consequently the non–perturbative integral along the cut seems to be determined by the perturbative integral of the circle**

The naive argument is misleading:

- **uncertainty is $\delta = 2\pi R\varepsilon$, ε small error by truncation of the perturbative series, however, δ is not under control due to R large**
- **order by order analyticity tells us that from pQCD we precisely reproduce the perturbative answer for the cut–integral, nothing more!**
- **inclusion of the OPE (power corrections) does not really improve for similar resons**

❺ Progress in utilizing pQCD in a safe way

❶ pQCD calculations of vacuum polarization amplitudes



up to 4–loops massless — Groshny, Kataev, Larin 91

up to 3–loops massive — Chetyrkin, Kühn et al. 97

up to 2–loops massive BF–MOM RG — F. J., Tarasov 98

❷ use old idea: testing non–perturbative effects with help of the Adler function

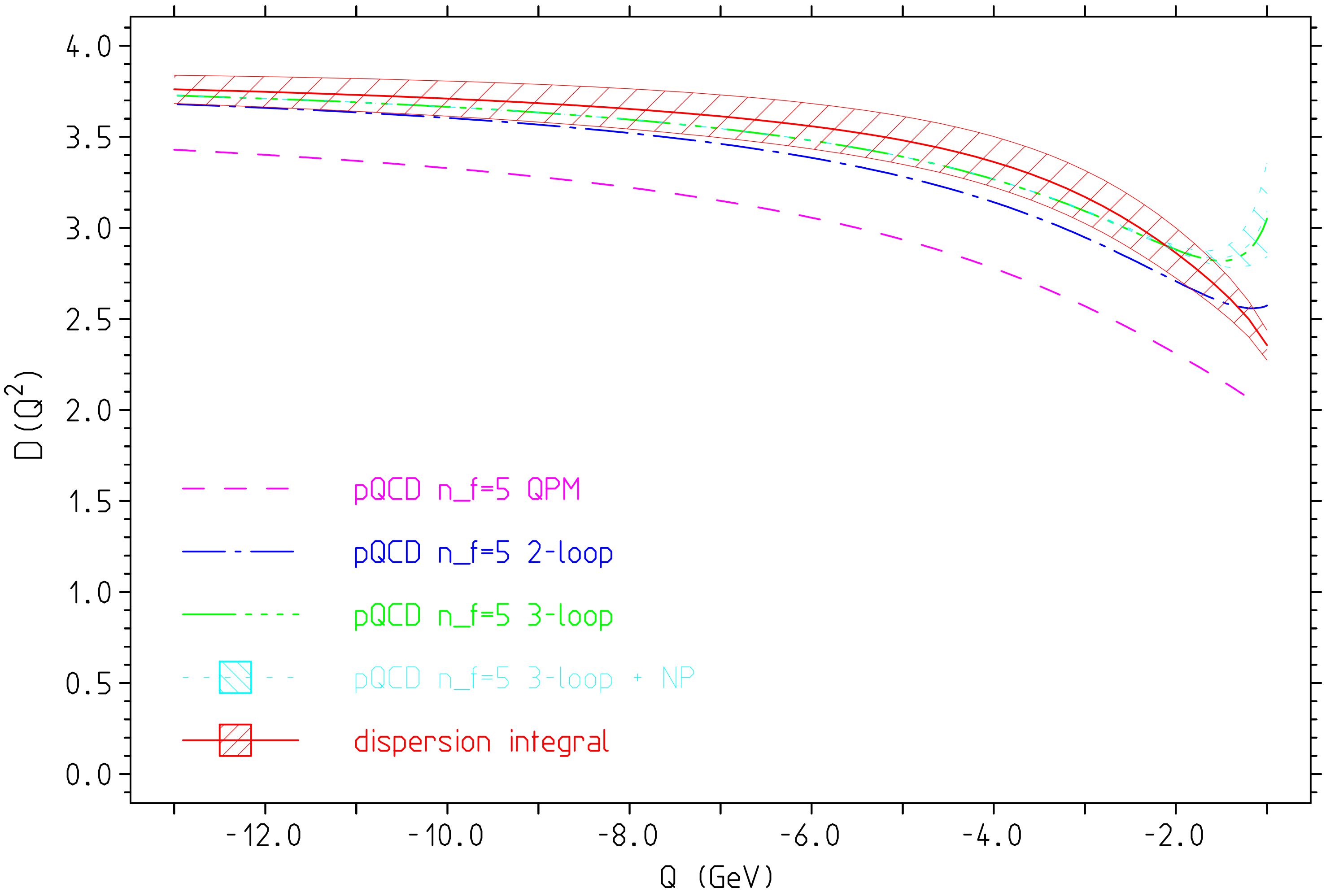
Eidelman, F. J., Kataev, Veretin 98

$$D(-s) \doteq \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\, s\, \frac{d\Pi'_\gamma(s)}{ds}$$

$$\Rightarrow \qquad D(Q^2) = Q^2 \int\limits_{4m_\pi^2}^{\infty} ds\, \frac{R(s)}{(s+Q^2)^2}$$

pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

“Experimental” Adler–function versus theory (pQCD + NP)



(Eidelman, F.J., Kataev, Veretin 98)

$\Rightarrow$ pQCD works well to predict $D(Q^2)$ down to $s_0 = (2.5\,\text{GeV})^2$ (not down to m_τ ! however); use this to calculate

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int dQ'^2\,\frac{D(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0 = (2.5\,\text{GeV})^2$: **(FJ 98)**

$$\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} = 0.007541 \pm 0.000254$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.027737 \pm 0.000254$$

$$\alpha^{-1}(-M_Z^2) = 128.917 \pm 0.035$$

The virtues of this analysis are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.5 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!**

Uncertainties for $\Delta\alpha^{(5)}_{\mathrm{had}}(-s_0)^{\mathrm{data}}$ in comparison to $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)^{\mathrm{data}}$

final state	energy range (GeV)	$\Delta\alpha^{(5)}_{\mathrm{had}}(-s_0)$ (stat) (syst)	$\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ (stat) (syst)
χPT	(0.28, 0.32)	0.03 (0.00) (0.00)	0.03 (0.00) (0.00)
ρ	(0.28, 0.81)	23.49 (0.27) (0.56)	25.33 (0.29) (0.60)
ω	(0.42, 0.81)	2.70 (0.03) (0.07)	2.96 (0.04) (0.08)
ϕ	(1.00, 1.04)	4.41 (0.06) (0.10)	5.14 (0.07) (0.12)
J/ψ		4.21 (0.20) (0.20)	11.93 (0.58) (0.64)
Υ		0.07 (0.00) (0.00)	1.27 (0.05) (0.07)
hadrons	(0.81, 1.40)	11.96 (0.13) (0.67)	13.99 (0.15) (0.80)
hadrons	(1.40, 3.10)	16.25 (0.16) (2.23)	28.23 (0.33) (4.11)
hadrons	(3.10, 3.60)	2.15 (0.11) (0.41)	5.98 (0.31) (1.15)
hadrons	(3.60, 9.46)	8.17 (0.04) (0.52)	50.50 (0.24) (3.33)
hadrons	(9.46,12.00)	0.72 (0.01) (0.06)	13.85 (0.17) (1.22)
perturb	(12.0,∞)	1.28 (0.00) (0.00)	120.92 (0.00) (0.34)
data	(0.28,12.00)	74.13 (0.42) (2.50)	159.18 (0.86) (5.67)
total		75.41 (0.42) (2.50)	280.10 (0.86) (5.68)

Remaining problems are the following:

a) contributions to the Adler function up to three–loops all have the same sign and are substantial. Four– and higher–orders could still add up to non-negligible contribution. An error for missing higher order terms is not included.

b) the link between space–like and time-like region is the difference

$$\Delta = \Delta\alpha^{(5)}_{\rm had}(M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.000045 \pm 0.000002$$

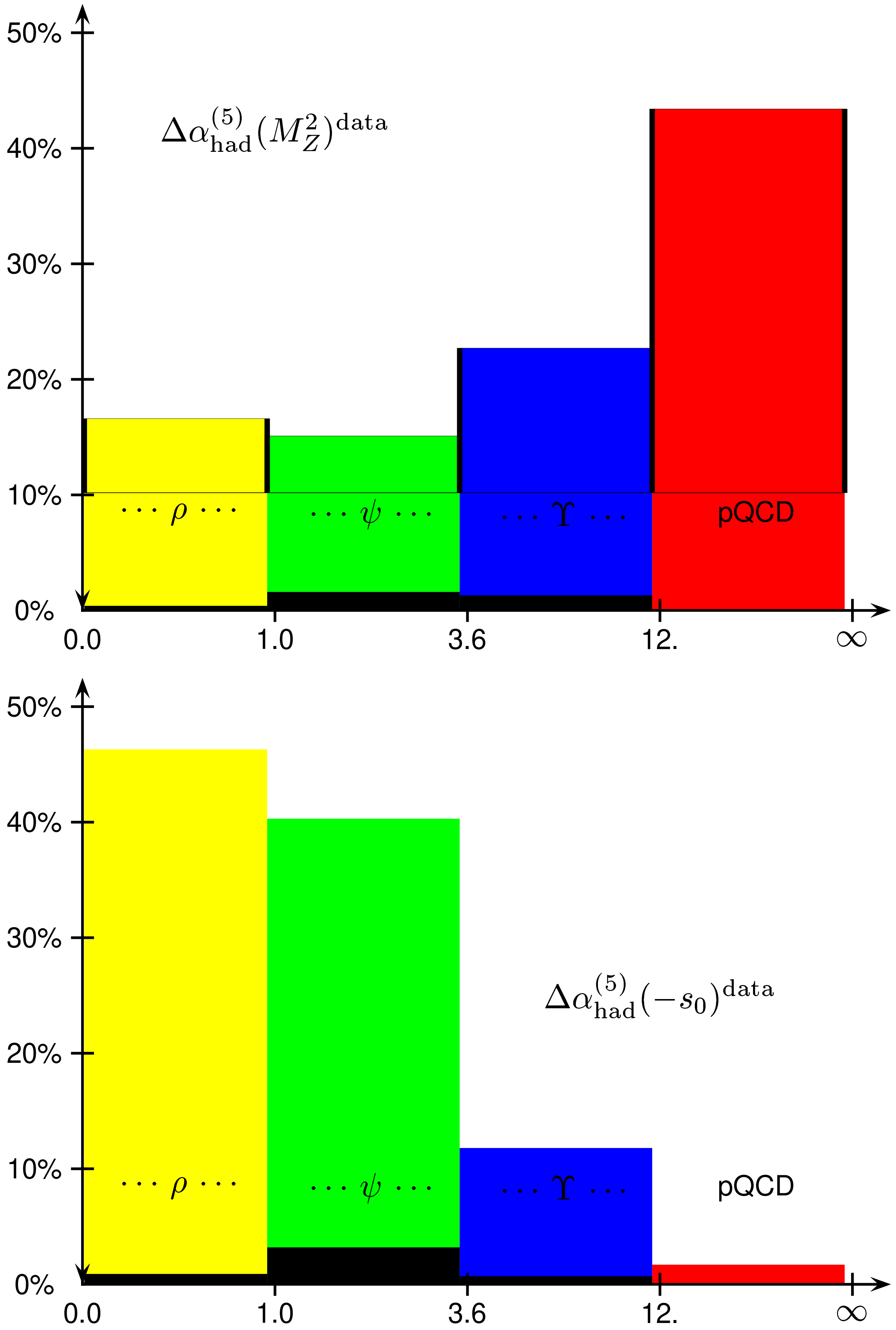
which can be calculated in pQCD. It accounts for the $i\pi$–terms from the logs $\ln(-q^2/\mu^2) = \ln(|q^2/\mu^2|) + i\pi$: Including this term yields

$$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) = 0.027782 \pm 0.000254$$

$$\alpha^{-1}(M_Z^2) = 128.913 \pm 0.035$$

One may ask the question whether terms like Δ should be resummed at all, i.e., included in the running coupling. Usually such terms tend to cancel against constant rational terms which are not included in the renormalization group (RG) evolution. Note that the Dyson resummation in general is not a systematic resummation of leading, subleading etc. terms as the RG resummation is.

Distribution of hadronic contributions to $\Delta\alpha$



❻ Precise measurement of $\sigma(e^+e^- \to hadrons)$ at DAΦNE

Aim: Reduce hadronic uncertainty in $(g-2)_\mu, \alpha(E), \cdots$

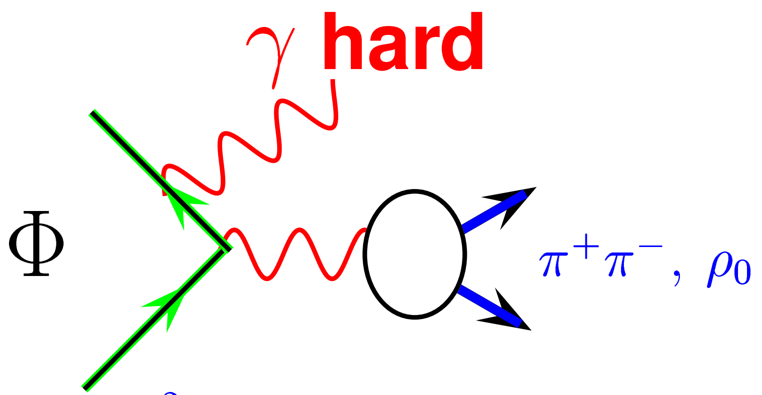
crucial for future of precision physics

Possible at ϕ–factory DAΦNE:

$\sigma(e^+e^- \to hadrons)$ **at** $\sqrt{s} \lesssim 2$ **GeV**

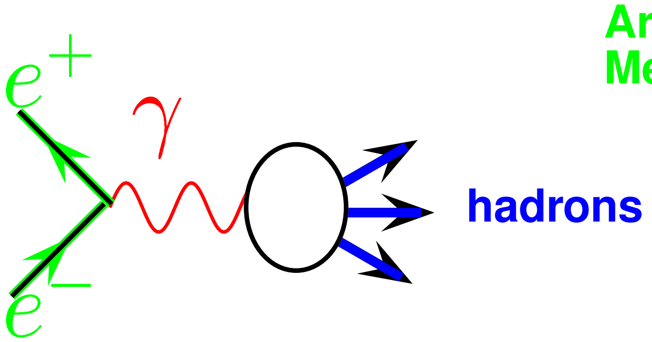
❶ $F_\pi(s)$ $\sqrt{s} \lesssim 1$ GeV

❷ $R(s)$ 1 GeV $\lesssim \sqrt{s} \lesssim 2$ GeV



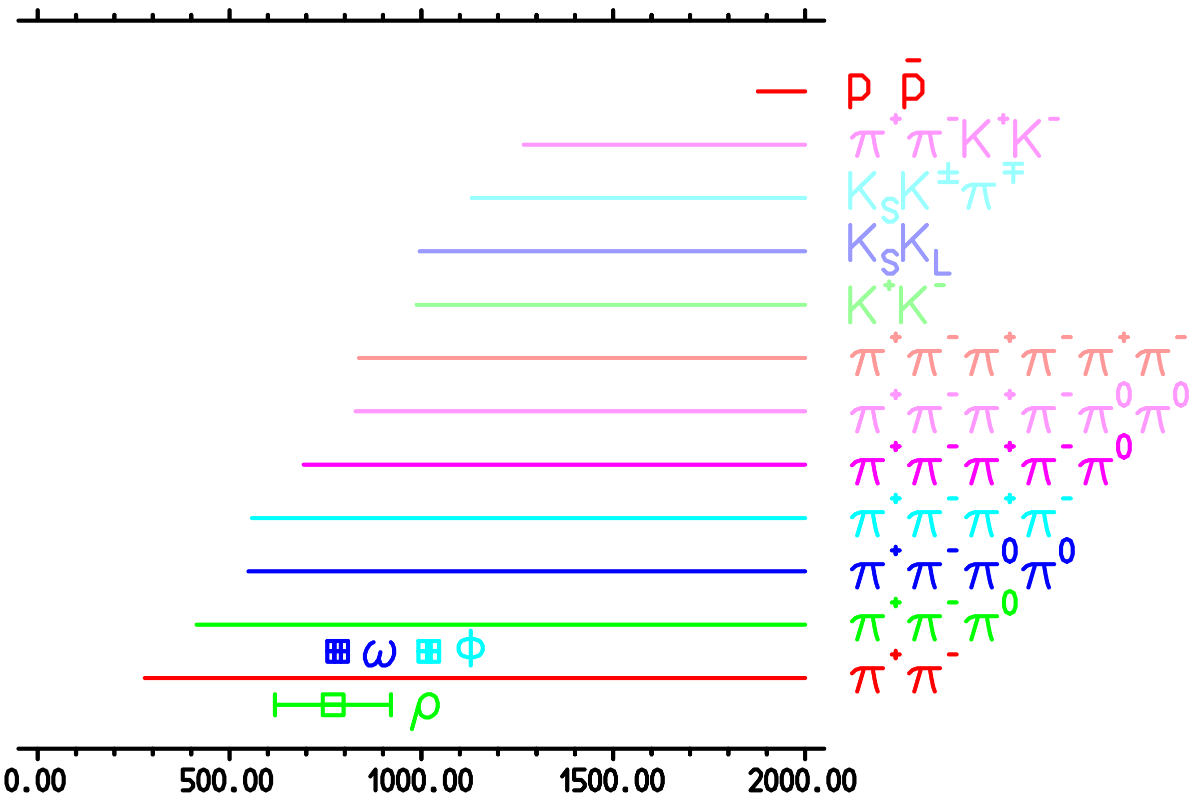
$s = M_\Phi^2$ $\quad s' = s\,(1-k) \quad [k = E_\gamma/E_{\mathrm{beam}}]$

Arbuzov et al. 97
Melnikov et al. 98



requires non trivial upgrade of DAΦNE such that energy scan is posssible $\to F_\pi(s), R(s)$ direct $\sqrt{s} \lesssim 2$ GeV.

Multiparticle channels in e^+e^-–annihilation at low energy



F_π, R : **inclusive quantities ! How to measure?**

competing two body channels

$$e^+e^-, \; \mu^+\mu^-, \; \pi^+\pi^-, \; K^+K^-, \; \cdots$$

separation requires particle identification

$\rightarrow$ **exclusive measurements**

requires detailed understanding of individual channels.

Contributions to $\tilde{a}_\mu^{\rm had} = a_\mu^{\rm had} \times 10^{10}$ from exclusive channels.

channel	$\tilde{a}_\mu^{\rm had}$	acc.
$\rho,\omega \to \pi^+\pi^-$	506	0.3%
$\omega \to 3\pi$	47	$\sim$ 1%
ϕ	40	$\downarrow$
$\pi^+\pi^-\pi^0\pi^0$	24	$\cdot$
$\pi^+\pi^-\pi^+\pi^-$	14	$\cdot$
$\pi^+\pi^-\pi^+\pi^-\pi^0\pi^0$	5	10%
3π	4	$\downarrow$
K^+K^-	4	$\cdot$
K_SK_L	1	$\cdot$
$\pi^+\pi^-\pi^+\pi^-\pi^0$	1.8	$\cdot$
$\pi^+\pi^-\pi^+\pi^-\pi^+\pi^-$	0.5	$\cdot$
$p\bar{p}$	0.2	$\cdot$
$2\ \mathrm{GeV} \leq E \leq M_{J/\psi}$	22	
$M_{J/\psi} \leq E \leq M_\Upsilon$	20	
$M_\Upsilon < E$	$\lesssim 5$	

Theoretical work with the aim to calculate radiative corrections at the level of precision as indicated in the Table is in progress. $\to \delta a_\mu^{\rm had} \lesssim 26 \times 10^{-11}$ **from** $\sqrt{s} \lesssim 2$ **GeV.**

❼ Summary and Conclusion

$a_\mu^{\rm had}$: uncertainty from $\pi^+\pi^-, \rho^0$ dominates

Uncertainties of $a_\mu^{\rm had}$ in units 10^{-11}.

δa_μ	input
~ 156	e^+e^- data (EJ95, ADH98)
~ 60	DaΦne + additional τ data
~ 62	theory-driven (DH98b)
~ 62	"Adler based" (EJV99)
~ 102	e^+e^-, τ data (ADH98)
~ 40	BNL a_μ–experiment (CM98)

Precision measurement of a_μ provides most sensitive test of magnetic helicity flip transition

$$\bar{\psi}_L\, \sigma_{\mu\nu}\, F^{\mu\nu}\, \psi_R \qquad (\,\mathrm{dim}\; 5\; \mathrm{operator}\,)$$

such a term must be absent for any fermion in any renormalizable theory at tree level

$\rightarrow$ finite <u>prediction</u>

e.g. non–Yang Mills $WW\gamma$ coupling would spoil this prediction!

$\boxed{\Delta\alpha^{(5)}_{\rm had}(M_Z^2)}$ **:**

Traditional approach:

here pQCD at $E > 12$ GeV and for 5GeV $< E < 9.5$GeV uncertainty distributed everywhere below Υ

$\delta\Delta\alpha \sim 0.00065$ **timelike (present data EJ compilation)**

$\delta\Delta\alpha \sim 0.00057$ **spacelike (present data EJ compilation)**

$\delta\Delta\alpha \sim 0.00036$ **(data with 1% up do 3.6 GeV)**

$\delta\Delta\alpha \sim 0.00008$ **(data with 1% up do Υ)**

$\delta\Delta\alpha \sim 0.00016$ **("theoy-driven") (KS98, DH98b)**

A real breakthrough in improving on the hadronic uncertainties would require dedicated efforts in high precision measurements of $R(s)$ in a wide energy range up to about 10 GeV.

Alternative:

conservative approach of combining data and pQCD

Euclidean approach:

uncertainty distributed essentially below J/ψ

$\delta\Delta\alpha \sim 0.00025$ **spacelike (present data EJ compilation)**

$\delta\Delta\alpha \sim 0.000070$ **(data with 1% up do 3.6 GeV)**

$\delta\Delta\alpha \sim 0.000047$ **(data with 1% up do Υ)**

Major improvement can be done by DaΦne, BEPC, τ–charm like facilities.

$\Rightarrow$ **Is good news for future of precision physics !**

Typical consequences: see e.g. D'Agostini, Degrassi 99

$$\Delta\alpha = 0.02804(65) \text{ [EJ95]}$$

$$\Downarrow$$

$$M_W = 80.375 \pm 0.027 \text{ GeV, } M_H = 130 \pm 80 \text{ GeV}$$

$$\Delta\alpha = 0.02763(16) \text{ [DH98]}$$

$$\Downarrow$$

$$M_W = 80.366 \pm 0.025 \text{ GeV, } M_H = 150 \pm 70 \text{ GeV}$$

Alternative strategies:

- **try to determine $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ by a direct measurement (would be model dependent, one looses an important prediction)**
- **simplest: find the Higgs and determine m_H precisely; this would allow us to determine $\Delta\alpha^{(5)}_{\mathrm{had}}(M_Z^2)$ from the precise value of $\sin^2\Theta^{\ell}_{\mathrm{eff}}$ measured at LEP/SLD.**
- **For what concerns theory, which is essentially pQCD, we need better estimates of uncertainties (scheme dependence, higher orders etc. which are a problem at M_τ), extended studies in MOM schemes are needed.**

Concluding Remarks:

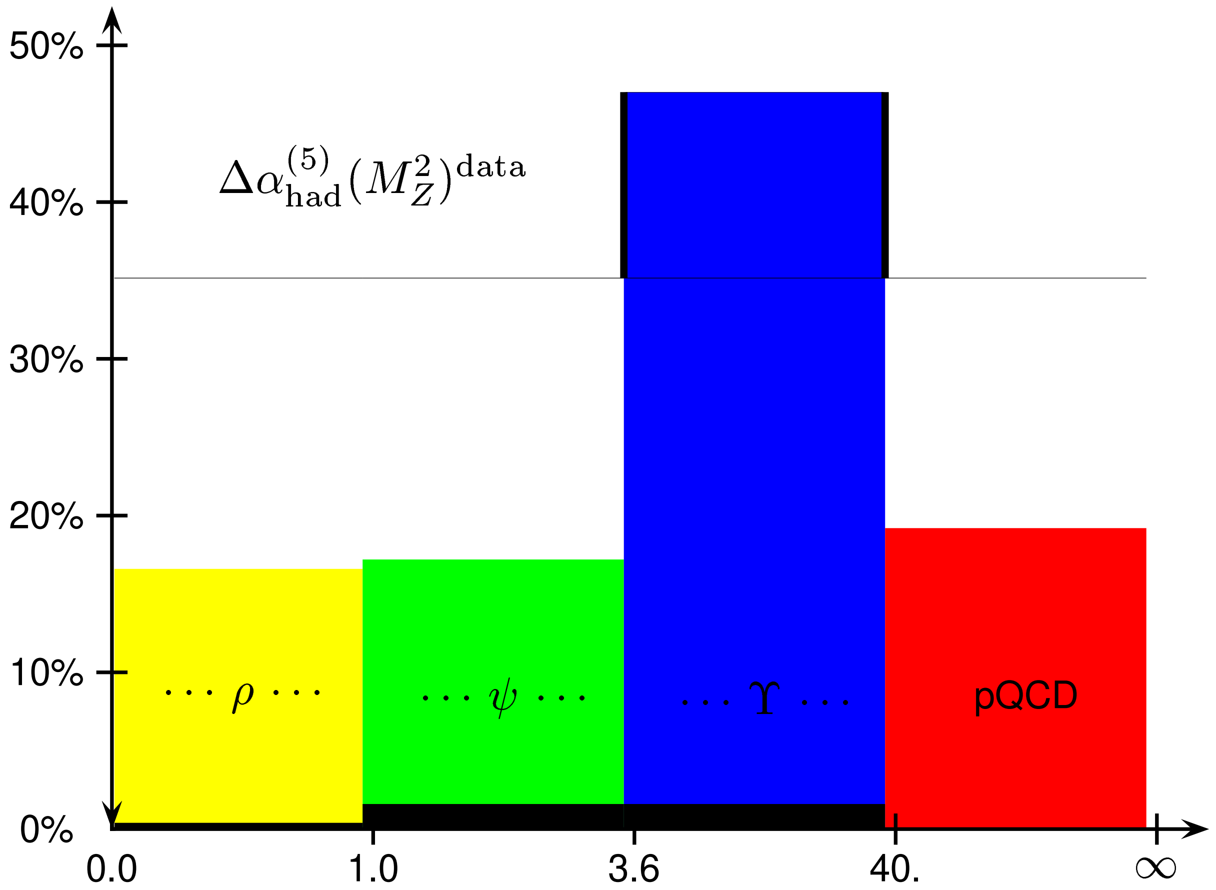
Experimental efforts to measure very precisely the total cross section $\sigma(e^+e^- \to hadrons)$ at low energies are mandatory for the future of electroweak precision physics. Taking into account recent theoretical progress, these "low energy" measurements are not only important for testing a_μ but as well for the effective fine structure constant $\alpha(M_Z)$. A real beakthrough would be possible by measuring $\sigma(e^+e^- \to hadrons)$ at 1% accuracy below the τ–threshold. Do it soon!

- **VEPP-2M Novosibirsk (CMD-2, SND): can still improve to 1.5% up to 1.4 GeV (upgrade?)**
- **DAΦNE Frascati (KLOE): upgrade for direct measurement at better than 1% level needed (2004?)**
- **BEPC Beijing (BES): can improve a lot in the important J/Ψ region (upgrade planned)**
- **future "τ–charm facility" tunable between 2 GeV and 3.6 GeV would settle the remaining problems.**

Distribution of hadronic contributions to $\Delta\alpha$

Data based approach (E):

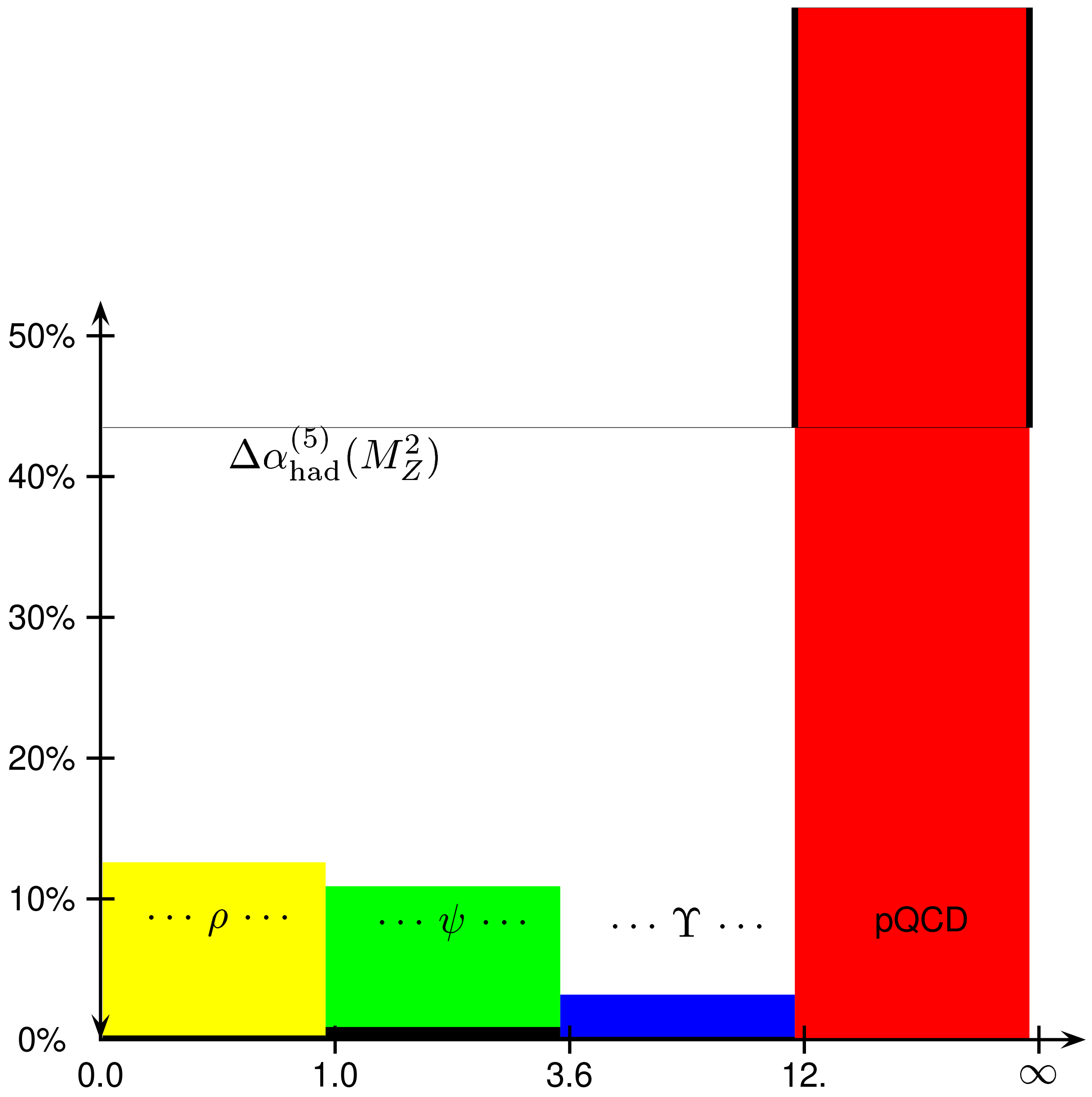
Based on available data up to 40 GeV $(EJ95)$



Distribution of hadronic contributions to $\Delta\alpha$

Euclidean improved approach (T):

Based on available data up to 12 GeV $(FJ98)$



Note: Our Euclidean approach (based on the Adler function $D(Q^2)$) allows us to rely on pQCD in a safe way, this is in contrast to the so called "theory driven" approaches, which try to predict $R(s)$ directly.