The closer you look the more there is to see

The muon g-2 in progress

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New updated and expanded edition
Dedicated to the memory of Maria Kawczyk
Outline of Talk:

- Introduction
- Hadronic Vacuum Polarization (HVP) – Data & Status
- Hadronic Light-by-Light (HLbL): Setup and Problems
- Theory vs experiment: do we see New Physics?
- What could fill the gap? BSM Physics?
- Prospects
**Introduction**

Particle with spin $\vec{s} \Rightarrow$ magnetic moment $\vec{\mu}$ (internal current circulating)

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c} \vec{s} ; \quad g_\mu = 2 \left( 1 + a_\mu \right)$$

**Dirac:** $g_\mu = 2 \quad , \quad a_\mu = \frac{\alpha}{2\pi} + \cdots$ muon anomaly

**Electromagnetic Lepton Vertex**

$$= (-ie) \bar{u}(p') \left[ \gamma^\mu F_1(q^2) + i \frac{\sigma^\mu\nu}{2m_\mu} q^\nu F_2(q^2) \right] u(p)$$

$$F_1(0) = 1 \quad ; \quad F_2(0) = a_\mu$$

The simplest object you can think of in the static limit

$a_\mu$ responsible for the Larmor (spin) precession $\Rightarrow$ need polarized muons orbiting

Shoot protons on target producing pions which decay by $P$ violating weak process

$$\pi^+ \rightarrow \mu^+ \nu_\mu \quad ; \quad \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$
Larmor precession \( \vec{\omega} \) of beam of spin particles in a homogeneous magnetic field \( \vec{B} \)

\[
\omega_a = a_\mu \frac{eB}{mc}
\]

actual precession \( \times 2 \) \( \sim 12' \) /circle

Magic Energy: \( \vec{\omega} \) is directly proportional to \( \vec{B} \) at magic energy \( \sim 3.1 \) GeV

\[
\vec{\omega}_a = \frac{e}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{B} \times \vec{E} \right]_{E \sim 3.1 \text{ GeV}} \\
\approx \frac{e}{m} \left[ a_\mu \vec{B} \right]
\]

CERN, BNL g-2 experiments
Stern, Gerlach 22: \( g_e = 2 \); Kusch, Foley 48: \( g_e = 2 \ (1.00119 \pm 0.00005) \)
Crucial: 3.1 GeV muons life-time in lab frame $\gamma \tau_\mu$ 29 times longer!

$$a_\mu^{\text{exp}} = (11\,659\,209.1 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10} \text{ BNL updated}$$

To come – two complementary experiments: magic $\gamma$ improved and $\vec{E} = 0$ novel

New muon $g - 2$ experiments at Fermilab $(a_\mu - \frac{1}{\gamma^2 - 1}) = 0$ and J-PARC $\vec{E} = 0$: improve error by factor 4

- ultra relativistic (CERN, BNL, Fermilab)) vs. ultra cold (J-PARC) muons

$\Rightarrow$ very different systematics

$\Rightarrow$ $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{the}} = 6.1 \sigma$ if theory as today

Reduction of hadronic uncertainty by factor 2 $\Rightarrow$ $\Delta a_\mu = 10.7 \sigma$
That's what we hope to achieve!

A Case That Promises New Physics

\( a_\mu \) measured via a ratio of frequencies (measurement of \( a_\mu \) and \( B \))

\[
B = \frac{\hbar \omega_p}{2 \mu_p}, \quad \omega_a = \frac{ea_\mu}{m_\mu c} B, \quad \mu = (1 + a_\mu) \frac{e\hbar}{2m_\mu c} \iff \mu = \left(1 + a_\mu\right) \frac{\hbar}{2} \frac{\omega_a}{a_\mu B} = \left(\frac{1}{a_\mu} + 1\right) \frac{\omega_a}{\omega_p} \mu_p \iff \frac{a_\mu}{\lambda - \bar{R}}
\]

in terms of 3 frequency measurables

- \( \omega_p = (e/m_\mu) \langle B \rangle \) free proton NMR frequency
- \( \bar{R} = \omega_a/\omega_p \) Larmor precession from E-821
- \( \lambda = \omega_L/\omega_p = \mu/\mu_p \) from hyperfine splitting of muonium

value used by E-821 3.18334539(10)
new value 3.183345137(85) Mohr et al RMP 80 (2008) 633
For the improvements of the muon $g - 2$ experiments see Lusiani’s Talk
Hadronic stuff: the limitation to theory

General problem in electroweak precision physics:
contributions from hadrons (quark loops) at low energy scales

Leptons
\[ e, \mu, \tau \]

\[ \alpha : \text{weak coupling} \]

\[ \text{pQED} \checkmark \]

Quarks
\[ u, d, s, \cdots \]

\[ \alpha_s : \text{strong coupling} \]

\[ \text{pQCD} \times \]

(a) Hadronic vacuum polarization \( O(\alpha^2), O(\alpha^3) \)
(b) Hadronic light-by-light scattering \( O(\alpha^3) \)
(c) Hadronic effects in 2-loop EWRC \( O(\alpha G F m^2_{\mu}) \)

Light quark loops
\[ \downarrow \]
Hadronic “blobs”
Evaluation of non-perturbative effects:
- data in conjunction with Dispersion Relations (DR),
- low energy effective modeling, RLA, HLS, ENJL
- lattice QCD

(a) HVP via dispersion integral over $e^+e^- \rightarrow \text{hadrons}$-data
   \text{(1 independent amplitude to be determined by one specific data set)},
   HLS global fit, lattice QCD

(b) HLbL via Resonance Lagrangian Approach (RLA) (CHPT extended by VDM
   in accord with chiral structure of QCD), $\gamma\gamma \rightarrow \text{hadrons}$-data dispersive
   approach \text{(28/19 independent amplitudes to be determined by as many
   independent data sets)}, lattice QCD Blum et al, Wittig et al, ...

(c) quark and lepton triangle diagrams: VVV = 0 by Furry $\Rightarrow$ only VVA
   \text{(of $f\bar{f}Z$-vertex) contributes $\Rightarrow$ABJ anomaly is perturbative
   and non-perturbative simultaneously i.e. leading effects calculable
   (anomaly cancellation) de Rafael, Knecht, Perrottet, Melnikov, Vainshtein}
Hadronic Vacuum Polarization (HVP) – Data & Status

Leading non-perturbative hadronic contributions \( a_{\mu}^{\text{had}} \) can be obtained in terms of \( R_{\gamma}(s) \equiv \sigma^{(0)}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})/\frac{4\pi\alpha^2}{3s} \) data via Dispersion Relation (DR):

\[
a_{\mu}^{\text{had}} = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \left( \int_{4m^2_\pi}^{E^2_{\text{cut}}} ds \frac{R_{\gamma}^{\text{data}}(s) \hat{K}(s)}{s^2} + \int_{E^2_{\text{cut}}}^{\infty} ds \frac{R_{\gamma}^{\text{PQCD}}(s) \hat{K}(s)}{s^2} \right)
\]

- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: \( \sim 75\% \) come from region \( 4m^2_\pi < m^2_\pi\pi < M^2_\Phi \)

Data: NSK, KLOE, BaBar, BES3, CLEOc

\[ a_{\mu}^{\text{had}(1)} = (686.99 \pm 4.21)[687.19 \pm 3.48] \times 10^{-10} \]

\( e^+e^- \)–data based [incl. \( \tau \)]
Experimental input for HVP: BESII, CMD, SND KEDR-scans, KLOE, BaBar, BESIII, CLEOc-ISNR

\[ s = M_{\phi}^2; \quad s' = s (1 - k), \quad k = E_{\gamma}/E_{\text{beam}} \]

a) Initial state radiation (ISR)
fixed energy meson factories
“radiative return”

b) Standard energy scan
tunable storage rings
Comparison of ISR $\pi\pi$ data: ratio $|F_\pi(E)|^2$ in units of a GS fit from BES-III. Left panel: all sets. Right panel: BaBar vs. KLOE10, which exhibits the largest relative deviations. Most precise ISR measurements in conflict

- New KLOE combined $0.316 < M_{\pi\pi} < 0.975$ GeV
  $a_\mu(\pi^+\pi^-) = (489.8 \pm 1.7 \pm 4.8) \times 10^{-10}$. arXiv:1711.03085

- New CLEO-c ISR $\pi\pi$ data (CESR storage ring collider) $0.3 < M_{\pi\pi} < 1.00$ GeV
  $a_\mu(\pi^+\pi^-) = (500.4 \pm 3.6 \pm 7.5) \times 10^{-10}$. arXiv:1712.04530
Improvements using $\tau$ decay spectra and $\pi\pi$ scattering phase shifts

$|F_\pi(E)|^2$ in units of $e^+e^- I = 1$ (CMD-2 GS fit): a) $\tau$ data uncorrected for $\rho - \gamma$ mixing Szafron, F.J. 11, Benayoun et al 11, and b) after correcting for mixing.

Right: $\pi\pi$ $[+5\% -10\%]$ scattering phase-shift data $\delta_1(s)$ constraining $|F_\pi(E)|^2$.

Theory: Leutwyler 02, Colangelo 03, Caprini 16

Data: Hyams 73, Grayer 74, Protopopescu 73

to constrain $|F_\pi|^2$ below 0.63 GeV, one obtains

$$ F_\pi(s)=|F_\pi(s)| e^{i\delta(s)} = P(s) \exp \left\{ \frac{s}{\pi} \int_{4m_\pi^2}^\infty ds' \frac{\delta(s')}{s'(s'-s)} \right\} $$

$$ a_{\mu}^{\text{had}(1)} = (689.46 \pm 3.25) \times 10^{-10}, $$

best estimate combining $e^+e^-$, $\tau$–decay and $\pi\pi$ scattering phase shift data.
Higher energies:

Experimental results for $R_{\gamma}^{\text{had}}(s)$ in the range $1 \text{ GeV} < E = \sqrt{s} < 13 \text{ GeV}$, obtained at the $e^+e^-$ storage rings. The perturbative quark–antiquark pair–production cross section is also displayed (pQCD). Parameters: $\alpha_s(M_Z) = 0.1185 \pm 0.0006$, $m_c = 1.275 \pm 0.025 \text{ GeV}$, $m_b = 4.18 \pm 0.03 \text{ GeV}$ and $\mu \in (\frac{\sqrt{s}}{2}, 2 \sqrt{s})$.

<table>
<thead>
<tr>
<th>Energy range</th>
<th>$d\mu^{\text{had}}<a href="%5Ctext%7Berror%7D">%</a> \times 10^{10}$</th>
<th>rel. err.</th>
<th>abs. err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho, \omega$ ($E &lt; 1$ GeV)</td>
<td>540.98 [78.6][2.80]</td>
<td>0.5 %</td>
<td>50.7 %</td>
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<tr>
<td>1 GeV &lt; $E &lt; 2$ GeV</td>
<td>96.49 [14.0][2.54]</td>
<td>2.6 %</td>
<td>41.5 %</td>
</tr>
<tr>
<td>2 GeV &lt; $E &lt; \infty$ incl pQCD</td>
<td>51.09 [7.4][1.10]</td>
<td>2.2 %</td>
<td>7.8 %</td>
</tr>
<tr>
<td>total</td>
<td>668.65 [100.0][3.94]</td>
<td>0.6 %</td>
<td>100.0 %</td>
</tr>
</tbody>
</table>
Still an issue in HVP

- region 1.2 to 2 GeV data; test-ground exclusive vs inclusive $R$ measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty

- illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR
Cross sections (nb)

Eidelman et al 2011
NLO and NNLO HVP effects

Hadronic higher order contributions: involving LO, NLO and NNLO vacuum polarization, and light-by-light scattering insertions.

Barbieri, Remiddi 74, Krause 97, Kurz et al 14, my 2017 update below

- Uncertainties of the subleading NLO and NNLO and EW below 1 $\sigma$ even after future experimental precision will have been reached
HVP for the muon anomaly

Present status for the hadronic and weak contributions summarized

\[
\begin{align*}
a_{\mu}^{\text{had}(1)} &= (689.46 \pm 3.25)[688.77 \pm 3.38][688.07 \pm 1.14] \times 10^{-10} \\
\text{(LO)} \\
a_{\mu}^{\text{had}(2)} &= (-9.93 \pm 0.067) \times 10^{-10} \\
\text{(NLO)} \\
a_{\mu}^{\text{had}(3)} &= (1.224 \pm 0.010) \times 10^{-10} \\
\text{(NNLO)} \\
a_{\mu}^{\text{had,LbL}} &= (10.34 \pm 2.88) \times 10^{-10} \\
\text{(HLbL)} \\
a_{\mu}^{\text{weak}} &= (15.36 \pm 0.11[m_H, m_t] \pm 0.023[\text{had}]) \times 10^{-10} \\
\text{(LO+NLO)}
\end{align*}
\]

For details I refer to my new book 2017 and references therein (see also Zhang 15, Hagiwara et al 17). The QED prediction of \( a_\mu \) is given by (see Aoyama et al 12, Laporta 17, Steinhauser et al 17)

\[
a_\mu^{\text{QED}} = \frac{\alpha}{2\pi} + 0.765\ 857\ 423(16) \left(\frac{\alpha}{\pi}\right)^2 \\
+ 24.050\ 590\ 82(28) \left(\frac{\alpha}{\pi}\right)^3 + 130.8734(60) \left(\frac{\alpha}{\pi}\right)^4 + 750.798(891) \left(\frac{\alpha}{\pi}\right)^5. 
\]
Laporta’s quasi exact universal 4–loop result

Sample diagrams of the 25 gauge-invariant subsets of the 891 universal $O(\alpha^4)$ contributions to $a_\ell$. 
A contribution to \( A^{(8)} \) of the 25 gauge-invariant sets amounts to know \( a_{e}^{\text{QED}} \) and \( a_{\mu}^{\text{QED universal}} \) to \( O(\alpha^4) \) exactly confirming Aoyama, Kinoshita & Nio numerical results.

<table>
<thead>
<tr>
<th></th>
<th>( a_{e}^{\text{QED}} )</th>
<th>( a_{\mu}^{\text{QED universal}} )</th>
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<td>1.97107561683581894364569965537264406980</td>
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<td>2</td>
<td>-0.142487379799872157235945291684857370994</td>
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<tr>
<td>25</td>
<td>-0.117949868787420797062780493486346339829</td>
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</table>
Present leading uncertainty:  hard to improve by direct $R(s)$ measurements

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Value</th>
<th>Error</th>
<th>Significance</th>
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<tbody>
<tr>
<td>DHMZ10 ($e^+e^-$)</td>
<td>180.2</td>
<td>±4.9</td>
<td>[3.6 σ]</td>
</tr>
<tr>
<td>DHMZ10 ($e^+e^-+\tau$)</td>
<td>189.4</td>
<td>±5.4</td>
<td>[2.4 σ]</td>
</tr>
<tr>
<td>JS11 ($e^+e^-+\tau$)</td>
<td>179.7</td>
<td>±6.9</td>
<td>[3.4 σ]</td>
</tr>
<tr>
<td>HLMNT11 ($e^+$)</td>
<td>182.8</td>
<td>±4.9</td>
<td>[3.3 σ]</td>
</tr>
<tr>
<td>DHMZ10/JS11 ($e^+e^-+\tau$)</td>
<td>181.1</td>
<td>±4.6</td>
<td>[3.6 σ]</td>
</tr>
<tr>
<td>BDDJ15$^*$ ($e^+e^-+\tau$)</td>
<td>170.4</td>
<td>±5.1</td>
<td>[4.8 σ]</td>
</tr>
<tr>
<td>BDDJ15$^*$ ($e^+e^-+\tau$)</td>
<td>175.0</td>
<td>±5.0</td>
<td>[4.2 σ]</td>
</tr>
<tr>
<td>DHMZ16 ($e^+e^-$)</td>
<td>181.7</td>
<td>±4.2</td>
<td>[3.6 σ]</td>
</tr>
<tr>
<td>FJ17 ($e^+e^-+\tau+\pi\pi$ phases)</td>
<td>178.3</td>
<td>±3.5</td>
<td>[4.3 σ]</td>
</tr>
<tr>
<td>DHea09 ($e^+e^-$)</td>
<td>178.8</td>
<td>±5.8</td>
<td>[3.5 σ]</td>
</tr>
<tr>
<td>BDDJ12$^*$ ($e^+e^-+\tau$)</td>
<td>175.4</td>
<td>±5.3</td>
<td>[4.1 σ]</td>
</tr>
</tbody>
</table>

Comparison with other Results. Note: results depend on which value is taken for HLbL. JS11 and BDDJ13 includes $116(39) \times 10^{-11}$ [JN], DHea09, DHMZ10, HLMNT11 and BDDJ12 use $105(26) \times 10^{-11}$ [PdRV].
On the way to be competitive! From first principles approach

Recent LQCD results for the leading order $a_{\mu}^{\text{HVP}}$, in units $10^{-10}$. Individual flavor contributions from light ($u, d$) amount to about 90%, strange about 8% and charm about 2%. Plot from Budapest-Marseille-Wuppertal 17 Brookhaven, Zeuthen, Mainz, Edinburgh, ...

Most recent: arXiv:1711.04980 [hep-lat] $a_{\mu}^{\text{LO HVP}} = 711.0(7.5)(17.3) \times 10^{-10}$
Hadronic Light-by-Light (HLbL): Setup and Problems

Hadrons in $\langle 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0\rangle$

Key object full rank-four hadronic vacuum polarization tensor

$$\Pi_{\mu\nu\lambda\rho}(q_1, q_2, q_3) = \int d^4x_1\, d^4x_2\, d^4x_3\, e^{i(q_1x_1+q_2x_2+q_3x_3)}$$

$$\times\langle 0|T\{j_\mu(x_1)j_\nu(x_2)j_\lambda(x_3)j_\rho(0)\}|0\rangle.$$

- non-perturbative physics
- covariant decomposition involves 138 Lorentz structures (43 gauge invariant)
- 28 can contribute to $g - 2$; by permutation symmetry 19 independent
- fortunately, dominated by the pseudoscalar exchanges $\pi^0, \eta, \eta', ...$ described by the effective Wess-Zumino Lagrangian
general, pQCD useful to evaluate the short distance (S.D.) tail

dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar pions as well as the vector mesons (ρ, ⋯) which play a dominant role (vector meson dominance mechanism); HLS, ENJL, general RLA, large $N_c$ inspired ansätze, and others

Need appropriate low energy effective theory ⇒ amount to calculate the following type diagrams

LD contribution requires low energy effective hadronic models: simplest case $π^0 γγ$ vertex
Data show almost background free spikes of the PS mesons!

Basic problem: \((s, s_1, s_2)\)–domain of \(\mathcal{F}_{\pi^0\gamma^*\gamma^*}(s, s_1, s_2)\); here \((0, s_1, s_2)\)–plane

- Data + Dispersion Relation, OPE,
- QCD factorization,
- Brodsky-Lepage approach
- Models constrained by data
Constraint I: $\Gamma(\pi^0 \gamma \gamma) \leftrightarrow$ effective WZW-Lagrangian

- The constant $e^2 F_{\pi^0 \gamma \gamma}(m_{\pi}^2, 0, 0) = \frac{e^2 N_c}{12\pi^2 f_{\pi}} = \frac{\alpha}{\pi f_{\pi}} \approx 0.025 \text{ GeV}^{-1}$ well determined by $\pi^0 \to \gamma \gamma$ decay rate (from Wess-Zumino Lagrangian); experimental improvement needed!

- Information on $F_{\pi^0 \gamma^* \gamma}(m_{\pi}^2, -Q^2, 0)$ from $e^+ e^- \to e^+ e^- \pi^0$ experiments

CELLO and CLEO measurement of the $\pi^0$ form factor $F_{\pi^0 \gamma^* \gamma}(m_{\pi}^2, -Q^2, 0)$ at high space–like $Q^2$. Outdated by BaBar? Belle conforms with theory expectations!
Constraint II: VMD mechanism ↔ Brodsky-Lepage behavior

\[ F_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0) \simeq \frac{1}{4\pi^2 f_\pi} \frac{1}{1 + (Q^2 / 8\pi^2 f_\pi^2)} \sim \frac{2f_\pi}{Q^2} \]

then cannot miss to get reasonable result!

The leading pseudoscalar meson exchange matrix element is

\[ M_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2; q_1^2, q_2^2), \]

which can be evaluated in LQCD. Gérardin, Meyer, Nyffeler arXiv:1607.08174. for the first time \( F_{\pi^0\gamma^*\gamma}(m_\pi^2; -Q^2, -Q^2) \) measured on the lattice

Discriminates all simple VMD model ansätze!
The large--\( N_c \) QCD (OPE constrained) LMD+V ansatz

\[
\mathcal{F}^{\text{LMD+V}}_{\pi^0, \gamma^* \gamma^*} (p^2_{\pi}, q^2_1, q^2_2) = \frac{F_\pi}{3} \frac{\mathcal{P}(q^2_1, q^2_2, p^2_\pi)}{Q(q^2_1, q^2_2)},
\]

\[
\mathcal{P}(q^2_1, q^2_2, p^2_\pi) = h_0 q^2_1 q^2_2 (q^2_1 + q^2_2 + p^2_\pi) + h_1 (q^2_1 + q^2_2)^2 + h_2 q^2_1 q^2_2
\]
\[
+ h_3 (q^2_1 + q^2_2) p^2_\pi + h_4 p^4_\pi + h_5 (q^2_1 + q^2_2) + h_6 p^2_\pi + h_7,
\]

\[
Q(q^2_1, q^2_2) = (M^2_{V_1} - q^2_1) (M^2_{V_2} - q^2_2) (M^2_{V_1} - q^2_2) (M^2_{V_2} - q^2_2),
\]

\( p^2_\pi = m^2_{\pi} \), well constrained now, i.e. parameters \( h_i \) (\( i = 0, \cdots, 7 \)) rather well under control (QCD asymptotics, experimental and lattice data)

QCD: \( h_0 = -1 \), \( h_1 = 0 \) + constraints by data ; \( h_3, h_4, h_6 \) absent in chiral limit remain \( h_2, h_5 \) and \( h_7 \) as essential parameters

VMD masses: \( M^2_{V_1}, M^2_{V_2} \) identified with \( \rho, \rho' \) masses
the need of analytic continuation

Measured is $F_{\pi^0\gamma^*\gamma}(m_{\pi}^2, -Q^2, 0)$ at high space–like $Q^2$, needed at external vertex is $F_{\pi^0\gamma^*\gamma}(-Q^2, -Q^2, 0)$ or $F_{\pi^0\gamma^*\gamma}(q^2, q^2, 0)$ if integral to be evaluated in Minkowski space.

Can we check such questions experimentally or in lattice QCD?
### $\pi^0$-exchange

<table>
<thead>
<tr>
<th>Model</th>
<th>$a_\mu^{\pi^0} \cdot 10^{10}$</th>
<th>Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>EJLN/BPP</td>
<td>5.9(0.9)</td>
<td>Bijnens, Pallante, Prades 1995</td>
</tr>
<tr>
<td>Non-local quark model</td>
<td>6.72</td>
<td>Dorokhov, Broniowski 2008</td>
</tr>
<tr>
<td>Dyson-Schwinger Eq. Approach</td>
<td>5.75</td>
<td>Goecke, Fisher, Williams 2011</td>
</tr>
<tr>
<td>LMD+V/KN</td>
<td>5.8 – 6.3</td>
<td>Knecht, Nyffeler 2002</td>
</tr>
<tr>
<td>MV: LMD+V+OPE[WZW]</td>
<td>6.3(1.0)</td>
<td>Melnikov, Vainshtein 1997</td>
</tr>
<tr>
<td>Form-factor inspired by AdS/QCD</td>
<td>6.54</td>
<td>Cappiello, Cata, d’Ambrosio 2011</td>
</tr>
<tr>
<td>Chiral quark model</td>
<td>6.8</td>
<td>Greynat, de Rafael 2012</td>
</tr>
<tr>
<td>Magnetic susceptibility constraint</td>
<td>7.2</td>
<td>Nyffeler 2009</td>
</tr>
</tbody>
</table>

My estimation: Leading LbL contribution from PS mesons:

$$a_\mu[\pi^0, \eta, \eta'] \sim (93.91 = [63.14 + 14.87 + 15.90] \pm 12.40) \times 10^{-11}$$

Still controversial: □ VMD at external vertex ?
□ Pion-pole approximation ?
\[ a_\mu = F_2(0) ; \quad F_2(0) = \frac{1}{2\pi i} \int \frac{dq^2}{q^2} \text{Abs} F_2(q^2) \]

Both time-like 
\[ e^+e^- \to P\gamma \]

And space-like 
\[ \gamma^*\gamma^* \to P \]

Data needed as input

Dispersive approach to \[ \gamma^*\gamma^* \to \gamma^*\gamma^* \]: Colangelo, Hoferichter, Procura, Stoffer

Very ambitious long term project, requires all kind of data not yet available
Example: **pion–loop contribution** may be evaluated in terms of $\sigma(\gamma\gamma \rightarrow \pi\pi)$ evaluated for the first time 2017 by the **Bern group**

**Di-pion production in $\gamma\gamma$ fusion.** At low energy we have direct $\pi^+\pi^-$ production and by strong rescattering $\pi^+\pi^- \rightarrow \pi^0\pi^0$, however with very much suppressed rate. With increasing energy, above about 1 GeV, the strong $q\bar{q}$ resonance $f_2(1270)$ appears produced equally at expected isospin ratio $\sigma(\pi^0\pi^0)/\sigma(\pi^+\pi^-) = \frac{1}{2}$. This demonstrates convincingly that we may safely work with point-like pions below 1 GeV
Summary of selected estimates for the different contributions to $a_\mu^{\text{HLbL}} \times 10^{11}$. For comparison, the first line shows some results when no form factors are used (undressed). The only complete calculations are HKS and BPP, but only the latter has been updated recently. Note: LMD+V [KN,JN] and (LMD+V,WZW) [MV] are applying the large–$N_c$ concept plus OPE to different objects. (LMD+V,WZW) [MV] respect QCD asymptotics on the LbL scattering level which requires the external vertex to be the WZW point vertex, the quark loop contribution is then included already. LMD+V [KN,JN] apply the LMD+V transition form factor also at the external vertex, QCD asymptotics then requires an extra S.D. quark loop contribution.

<table>
<thead>
<tr>
<th>Model</th>
<th>Model</th>
<th>$\pi^0, \eta, \eta'$</th>
<th>axial-mesons</th>
<th>scalars</th>
<th>$\pi, K$-loops</th>
<th>quark-loop</th>
<th>Total</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>QM</td>
<td></td>
<td>$+\infty$</td>
<td></td>
<td></td>
<td>$-45$</td>
<td>$60$</td>
<td>$-$</td>
<td>no FF</td>
</tr>
<tr>
<td>HLS [HKS]</td>
<td></td>
<td>$82.7(6.4)$</td>
<td>$1.7(1.7)$</td>
<td></td>
<td>$-4.5(8.1)$</td>
<td>$9.7(11.1)$</td>
<td>$89.6(15.4)$</td>
<td>HKS95</td>
</tr>
<tr>
<td>ENJL [BPP]</td>
<td></td>
<td>$85(13)$</td>
<td>$2.5(1.0)$</td>
<td>$-6.8(2.0)$</td>
<td>$-19(13)$</td>
<td>$21(3)$</td>
<td>$83(32)$</td>
<td>BijnensLBL</td>
</tr>
<tr>
<td>LMD+V [KN]</td>
<td></td>
<td>$83(12)$</td>
<td></td>
<td>$-6.8(2.0)$</td>
<td>$-19(13)$</td>
<td></td>
<td>$80(40)$</td>
<td>KN01</td>
</tr>
<tr>
<td>(LMD+V,WZW) [MV]</td>
<td></td>
<td>$114(10)$</td>
<td>$22(5)$</td>
<td></td>
<td>$0(10)$</td>
<td></td>
<td>$0$</td>
<td>136(25)</td>
</tr>
<tr>
<td>LENJL</td>
<td></td>
<td>$95.5(17.0)$</td>
<td></td>
<td></td>
<td>$12.3(2.4)$</td>
<td></td>
<td>$107.7(16.8)$</td>
<td>Bartos01</td>
</tr>
<tr>
<td>ENJL update</td>
<td></td>
<td>$-$</td>
<td></td>
<td></td>
<td>$-11(40)$</td>
<td></td>
<td>$-110(40)$</td>
<td>BP07</td>
</tr>
<tr>
<td>PdRV consensus</td>
<td></td>
<td>$114(13)$</td>
<td>$15(10)$</td>
<td>$-6.8(2.0)$</td>
<td>$-19(13)$</td>
<td>$2.3$ [c-quark]</td>
<td>105(26)</td>
<td>PdeRV09</td>
</tr>
<tr>
<td>LMD+V [JN]</td>
<td></td>
<td>$99(16)$</td>
<td>$22(5)$</td>
<td>$-6.8(2.0)$</td>
<td>$-19(13)$</td>
<td></td>
<td>$116(40)$</td>
<td>Nyffeler09</td>
</tr>
<tr>
<td>RLA</td>
<td></td>
<td>$104.7(5.4)$</td>
<td></td>
<td>$-6.8(2.0)$</td>
<td>$-19(13)$</td>
<td></td>
<td>$-188(4)$</td>
<td>Roig14</td>
</tr>
<tr>
<td>RLA</td>
<td></td>
<td>$65.8(5.4)$ [$$\pi^0$$]</td>
<td></td>
<td></td>
<td>$11(9)$</td>
<td></td>
<td>$168(13)$</td>
<td>Dorokhov15</td>
</tr>
<tr>
<td>N$\chi$QM</td>
<td></td>
<td>$58.5(8.7)$</td>
<td>$3.4(4.8)$</td>
<td></td>
<td>$-107(2)$</td>
<td></td>
<td>$188(4)$</td>
<td>SDE2</td>
</tr>
<tr>
<td>pQCD</td>
<td></td>
<td>$81(2)$</td>
<td></td>
<td>$-6.8(2.0)$</td>
<td></td>
<td></td>
<td>$188(4)$</td>
<td>Boughezal11</td>
</tr>
<tr>
<td>DSE</td>
<td></td>
<td>$65.8(5.4)$ [$$\pi^0$$]</td>
<td></td>
<td></td>
<td>$11(9)$</td>
<td></td>
<td>$168(13)$</td>
<td>Dorokhov15</td>
</tr>
<tr>
<td>CQM</td>
<td></td>
<td>$68(3)$ [$$\pi^0$$]</td>
<td></td>
<td></td>
<td>$-107(2)$</td>
<td></td>
<td>$188(4)$</td>
<td>SDE2</td>
</tr>
<tr>
<td>RLA</td>
<td></td>
<td>$-20(5)$</td>
<td></td>
<td>$-20(5)$</td>
<td>$11(9)$</td>
<td></td>
<td>$168(13)$</td>
<td>Dorokhov15</td>
</tr>
<tr>
<td>ENJL+a$_1$</td>
<td></td>
<td>$-20(5)$</td>
<td></td>
<td>$-20(5)$</td>
<td>$11(9)$</td>
<td></td>
<td>$168(13)$</td>
<td>Dorokhov15</td>
</tr>
<tr>
<td>LMD [PV]</td>
<td></td>
<td>$-20(5)$</td>
<td></td>
<td>$-20(5)$</td>
<td>$11(9)$</td>
<td></td>
<td>$168(13)$</td>
<td>Dorokhov15</td>
</tr>
<tr>
<td>LMD+V</td>
<td></td>
<td>$95.5(12.4)$</td>
<td>$7.6(2.0)$</td>
<td>$-6.0(1.2)$</td>
<td>$-20(5)$</td>
<td>$22.3(4)$</td>
<td>$103.4(28.8)$</td>
<td>my est.</td>
</tr>
<tr>
<td>DRA [CHPS]</td>
<td></td>
<td>$-24(1)$</td>
<td></td>
<td>$-24(1)$</td>
<td>$-24(1)$</td>
<td></td>
<td>$-24(1)$</td>
<td>Colangelo17</td>
</tr>
</tbody>
</table>

F. Jegerlehner  
– Cracow EPIPHANY–  
January 12, 2018  
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### Results from various HLbL calculations. The plot also illustrates the history of HLbL calculations. Some of the estimates do not attempt to provide any model uncertainties, which are generally not easy to estimate reliably. The narrow band illustrates the expected uncertainty from the next generation experiments.

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Value</th>
<th>Uncertainty</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>HKS (HLS)</td>
<td>8.96</td>
<td>±1.54</td>
<td>[4.6 σ]</td>
</tr>
<tr>
<td>BPP (ENJL)</td>
<td>8.3</td>
<td>±3.2</td>
<td>[4.4 σ]</td>
</tr>
<tr>
<td>KN (LMD+V)</td>
<td>8.0</td>
<td>±4.0</td>
<td>[4.3 σ]</td>
</tr>
<tr>
<td>MV (LMD+V+new SD)</td>
<td>13.6</td>
<td>±2.5</td>
<td>[3.9 σ]</td>
</tr>
<tr>
<td>Bartos et al LENJL</td>
<td>10.77</td>
<td>±1.68</td>
<td>[4.4 σ]</td>
</tr>
<tr>
<td>BP ENJL update</td>
<td>11.0</td>
<td>±2.6</td>
<td>[3.9 σ]</td>
</tr>
<tr>
<td>PdRV consensus</td>
<td>10.5</td>
<td>±2.6</td>
<td>[4.3 σ]</td>
</tr>
<tr>
<td>JN LMD+V offshell</td>
<td>11.6</td>
<td>±4.0</td>
<td>[3.8 σ]</td>
</tr>
<tr>
<td>Dorokhov et al NχQM</td>
<td>16.8</td>
<td>±1.3</td>
<td>[3.6 σ]</td>
</tr>
<tr>
<td>Goecke et al DSE</td>
<td>18.8</td>
<td>±0.4</td>
<td>[3.4 σ]</td>
</tr>
<tr>
<td>Melnikov et al pQCD</td>
<td>13.3</td>
<td>±1.5</td>
<td>[4.1 σ]</td>
</tr>
<tr>
<td>de Rafael et al CQM</td>
<td>15.0</td>
<td>±0.3</td>
<td>[3.9 σ]</td>
</tr>
<tr>
<td>my update</td>
<td>10.34</td>
<td>±2.88</td>
<td>[4.2 σ]</td>
</tr>
<tr>
<td>BNL-E821 (world average)</td>
<td>209.1</td>
<td>±6.3</td>
<td></td>
</tr>
</tbody>
</table>

F. Jegerlehner – Cracow EPIPHANY – January 12, 2018
## Theory vs experiment: do we see New Physics?

Given the CODATA/PDG recommended value of $\alpha$ the theory confronts experiment as follows:

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value $\times 10^{10}$</th>
<th>Error $\times 10^{10}$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED incl. 4-loops + 5-loops</td>
<td>11 658 471.886</td>
<td>0.003</td>
<td>Aoyama et al 12, Laporta 17</td>
</tr>
<tr>
<td>Hadronic LO vacuum polarization</td>
<td>689.46</td>
<td>3.25</td>
<td></td>
</tr>
<tr>
<td>Hadronic light–by–light</td>
<td>10.34</td>
<td>2.88</td>
<td></td>
</tr>
<tr>
<td>Hadronic HO vacuum polarization</td>
<td>-8.70</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Weak to 2-loops</td>
<td>15.36</td>
<td>0.11</td>
<td>Gnendiger et al 13</td>
</tr>
<tr>
<td>Theory</td>
<td>11 659 178.3</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>Experiment</td>
<td>11 659 209.1</td>
<td>6.3</td>
<td>BNL 04</td>
</tr>
<tr>
<td>The. - Exp. 4.0 standard deviations</td>
<td>-30.6</td>
<td>7.6</td>
<td></td>
</tr>
</tbody>
</table>

Standard model theory and experiment comparison [in units $10^{-10}$]. What represents the 4 $\sigma$ deviation: □ new physics? □ a statistical fluctuation? □ underestimating uncertainties (experimental, theoretical)?
icted uncertainties (experimental, theoretical)?

enerima
New evaluations included:

<table>
<thead>
<tr>
<th>New contribution</th>
<th>Reference</th>
<th>$a_\mu \cdot 10^{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NNLO HVP</td>
<td>Kurz et al 2014</td>
<td>12.4 ± 0.1</td>
</tr>
<tr>
<td>NLO HLBL</td>
<td>Colangelo et al 2014</td>
<td>3 ± 2</td>
</tr>
<tr>
<td>New axial exchange HLBL</td>
<td>Pauk, Vanderhaeghen, F.J. 2014</td>
<td>7.55 ± 2.71</td>
</tr>
<tr>
<td>Old axial exchange HLBL</td>
<td>Melnikov, Vainshtein 2004</td>
<td>22 ± 5</td>
</tr>
<tr>
<td>Tensor exchange HLBL</td>
<td>Pauk, Vanderhaeghen 2014</td>
<td>1.1 ± 0.1</td>
</tr>
<tr>
<td>Total change</td>
<td></td>
<td>+2.1 ± 3.4 [← 5]</td>
</tr>
</tbody>
</table>
What could fill the gap? BSM Physics?

Most natural New Physics contributions: (examples)

neutral boson exchange: a) scalar or pseudoscalar and c) vector or axialvector, flavor changing or not, new charged bosons: b) scalars or pseudoscalars, d) vector or axialvector

Left: $m_\mu = M \ll M_0$ 
Right: $m_\mu \ll M_0 = M$, 
Couplings $f/(2\pi) = 0.1$
In general:

\[ \Delta a^\mu_{NP} = \alpha^{NP} \frac{m^2_\mu}{M^2_{NP}} \]

NP searches (LEP, Tevatron, LHC): typically \( M_{NP} \gg M_W \), then \( \Delta a^\mu_{\text{exp-the}} = \Delta a^\mu_{NP} \) requires \( \alpha^{NP} \sim 1 \) spoiling perturbative arguments. Exception: 2HDM, SUSY \( \tan \beta \) enhanced coupling! Note: NP sensitivity enhanced for muon by \( \sim 40,000 \) relative to electron, while \( a_e \) is only 2250 times more precise than \( a_\mu \Rightarrow \sim 19 \) in sensitivity! Note: no deviation \( a^\text{exp}_e - a^\text{the}_e \) within uncertainties.

Problem: LEP, Tevatron and LHC direct bounds on masses of possible new states

[typically \( M_X > 800 \text{ GeV} \)]

Need enhanced couplings! as in SUSY extensions of SM
SUSY and all that

\[
a_{\mu}^{\text{SUSY}} \approx \frac{\text{sign}(\mu M_2) \alpha(M_Z)}{8\pi \sin^2 \Theta_W} \frac{\left(5 + \tan^2 \Theta_W \right)}{6} \frac{m_{\mu}^2}{M_{\text{SUSY}}^2} \tan \beta \left(1 - \frac{4\alpha}{\pi} \ln \frac{M_{\text{SUSY}}}{m_{\mu}} \right)
\]

Before LHC: just would fit great the MSSM

Physics beyond the SM: leading SUSY contributions to \( g - 2 \) in a supersymmetric extension of the SM. The Higgs mass is constrained, here \( \tan \beta = 5 \) and \( m_A = 60 \text{ GeV} \).
Higgs found at 125 GeV (CERN “observed”) we must have $m_{1/2} > 800$ GeV or higher! More specifically: **heavy stop!**

- If universal sfermion masses: all sfermion masses go up!

- $M_{\text{SUSY}}$ lightest SUSY particle; SUSY requires two Higgs doublets

- $\tan \beta = \frac{v_1}{v_2}$, $v_i = \langle H_i \rangle$ ; $i = 1, 2$ ; $\tan \beta \sim m_t/m_b \sim 40 \ [4 − 40]$

To be precise: $a_\mu$ depends on masses of sneutrino, chargino, smuon and neutralino, only direct constraints on them are unambiguous!

On these states no direct bounds from LHC!

Unless MSSM parameter space is deformed to suit the purpose, SUSY likely is exclude to be able to explain $\Delta a_\mu^{\text{NP}}$!

Almost all other known BSM scenarios are disfavored even more to be able to account for the $a_\mu^{\text{exp}} - a_\mu^{\text{the}}$ deviation!
The LHC changed it all! e.g. CMSSM scenario

Right: the pre–LHC case. The \((m_0, m_{1/2})\) plane for \(\mu > 0\) for (a) \(\tan \beta = 10\) and (b) \(\tan \beta = 40\) in the CMSSM scenario. Left: after LHC run I. The Higgs discovery has changed it all. Plot courtesy of K. Olive

SUSY-GUT scenarios ruled out. If SUSY, it requires large mass splittings in SUSY sector, particularly between shadrons and sleptons

Excluded are light states with “normal size” couplings. Dark photon or axion-like particles still candidates, but almost ruled out as an explanation for the muon \(g - 2\) gap.
see NA48/2 Coll. arXiv:1504.00607  
BaBar Coll. arXiv:1702.03327

Still possible something from low scales, axion like states etc ... not seen at LHC

Or is it unaccounted for real photon radiation effects?  
Do experiments measure what theory calculates?

At the present/future level of precision $a_{\mu}$ depends on all physics incorporated in the SM: electromagnetic, weak, and strong interaction effects and beyond that all possible new physics we are hunting for.
here we are and hope to go:

Past and future $g - 2$ experiments testing various contributions.

New Physics $\equiv$ deviation $\left( a_\mu^{\text{exp}} - a_\mu^{\text{the}} \right) / a_\mu^{\text{exp}}$.

Limiting theory precision: hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL)

*** digging deeper and deeper ***
Prospects

- a “New Physics” interpretation of the persisting $3$ to $4\,\sigma$ requires relatively strongly coupled states in the range below about $250\,\text{GeV}$.

- Search bounds from LEP, Tevatron and specifically from the LHC already have ruled out a variety of Beyond the Standard Model (BSM) scenarios, so much hat standard motivations of SUSY/GUT extensions seem to fall in disgrace.

- There is no doubt that performing doable improvements on both the theory and the experimental side allows one to substantially sharpen (or diminish) the apparent gap between theory and experiment.

In any case $a_\mu$ constrains BSM scenarios distinctively and at the same time challenges a better understanding of the SM prediction.

The big challenge: two complementary experiments: Fermilab with ultra hot muons and J-PARC with ultra cold muons (very different radiation) to come
Provided deviation is real and theory and needed cross section data improves the same as the muon $g - 2$ experiments $3\sigma \rightarrow 9\sigma$ possible?!

**Key:** more/better data and/or progress in non-perturbative QCD

For muon $g - 2$:

- main obstacle: hadronic light-by-light [data, lattice QCD, RLA]
- progress in evaluating HVP: more data (BaBar, Belle, VEPP 2000, BESIII,...), lattice QCD in reach (recent progress Jansen et al, Wittig et al, Blum et al)

  in both cases lattice QCD will be the answer one day,

- also low energy effective *Resonance Lagrangian* and *Dispersion Relation* approach need be further developed for what concerns HLbL.

- For future improvements one desperately needs more information from $\gamma\gamma \rightarrow \text{hadrons}$ in order to have better constraints on modeling of the relevant
hadronic amplitudes. The goal is to exploit possible new experimental constraints from $\gamma\gamma \rightarrow \text{hadrons}$ and crossed channels if possible.

mostly single-tag events: KLOE, KEDR (taggers), BaBar, Belle, BES III; Dalitz-decays: $\rho, \omega, \phi \rightarrow \pi^0(\eta)e^+e^-$ Novosibirsk, NA60, JLab, Mainz, Bonn, Jülich, BES would be interesting, but some buried in the background.
the dispersive approach to HLbL (Colangelo et al 2014) is able to allow for real progress since contributions which were treated so far as separate contributions will be treated “rolled into one” (as entirety). Note: 19 independent amplitudes contribute to $g - 2$ vs. data (HVP 1 amplitude vs data)

Last but not least: do theoreticians calculate what experiments measure? (form-factor vs cross-section)

$$a_\mu^\text{the} = a_\mu^\text{SM virtual} \left[= F_2(0)\right] + \Delta a_\mu^\text{SM real soft } \gamma \quad \left[\text{dep. on exp. setup}\right] + \Delta a_\mu^\text{NP}$$

Fermilab vs J-PARC

A lot remains to be done! while a new $a_\mu^\text{exp}$ is approaching!

Thanks you for your attention!
For the electron anomaly the hadronic and weak contributions read

\[
\begin{align*}
    a_{e}^{\text{had}(1)} & = (184.90 \pm 1.08) \times 10^{-14} \quad \text{(LO)} \\
    a_{e}^{\text{had}(2)} & = (-22.13 \pm 0.12) \times 10^{-14} \quad \text{(NLO)} \\
    a_{e}^{\text{had}(3)} & = (2.80 \pm 0.02) \times 10^{-14} \quad \text{(NNLO)} \\
    a_{e}^{\text{had, LbL}} & = (3.7 \pm 0.5) \times 10^{-14} \quad \text{(HLbL)} \\
    a_{e}^{\text{weak}} & = (3.053 \pm 0.002[m_H, m_t] + 0.023[\text{had}]) \times 10^{-14} \quad \text{(LO+NLO)}.
\end{align*}
\]

The QED prediction of \(a_e\) including the recent results Aoyama et al 12, Laporta 17 is given by

\[
a_{e}^{\text{QED}} = \frac{\alpha}{2\pi} - 0.32847844400254(33) \left( \frac{\alpha}{\pi} \right)^2 + 1.181234016816(11) \left( \frac{\alpha}{\pi} \right)^3 - 1.91134(182) \left( \frac{\alpha}{\pi} \right)^4 + 6.676(192) \left( \frac{\alpha}{\pi} \right)^5.
\]
The new quasi–analytic $O(\alpha^4)$ result by Laporta 17 is certainly a milestone in consolidating the QED part $a_e^{\text{QED}}$. For extracting $\alpha_{\text{QED}}$ the SM prediction

$$a_e^{\text{SM}} = a_e^{\text{QED}} + 1.723(12) \times 10^{-12}\text{(hadronic & weak)}$$

is to be confronted with $a_e^{\text{exp}} = 1159\,652\,180.73(28)$ from experiment Aoyama et al 12 as an input. I obtain

$$\alpha^{-1}(a_e) = 137.035\,999\,1550(331)(0)(27)(14)[333].$$

Using $\alpha$ from atomic interferometry, specifically $\alpha(h/M_{\text{Rb11}})[0.66 \text{ ppb}]

$$\alpha^{-1} = 137.035999037(91),$$

the prediction of $a_e$. 
\[
\begin{array}{ll}
\text{universal} & = 1159\ 652\ 177.28(77)(0)(4) \times 10^{-12}, \\
\mu\text{–loops} & = 2.738(0) \times 10^{-12}, \\
\tau\text{–loops} & = 0.009(0) \times 10^{-12}, \\
\text{hadronic} & = 1.693(13) \times 10^{-12}, \\
\text{weak} & = 0.030(0) \times 10^{-12}, \\
\text{from SM theory} & = 1159\ 652\ 181.73(77) \times 10^{-12},
\end{array}
\]

which confronts \(a_e^{\text{exp}}\). Thus

\[
a_e^{\text{exp}} - a_e^{\text{the}} = -1.00(0.82) \times 10^{-12},
\]

theory and experiment are in excellent agreement. We know that the sensitivity to new physics is reduced by \((m_\mu/m_e)^2 \cdot \delta a_e^{\text{exp}} / \delta a_\mu^{\text{exp}} \approx 19\) relative to \(a_\mu\). Nevertheless, one has to keep in mind that \(a_e\) is suffering less from hadronic uncertainties and thus may provide a safer test. One should also keep in mind that experiments determining \(a_e\) on the one hand and \(a_\mu\) on the other hand are very different with
different systematics. While $a_e$ is determined in a ultra cold environment $a_{\mu}$ has been determined with ultra relativistic (magic $\gamma$) muons so far. Presently, the $a_e$ prediction is limited by the, by a factor $\delta \alpha(\text{Rb11})/\delta \alpha(a_e) \approx 5.3$ less precise, $\alpha$ available. Combining all uncertainties $a_{\mu}$ is about a factor 43 more sensitive to new physics at present.

New evaluation of universal QED part $A_1^{(10)} = 6.675(192)$,

- $a_e(\text{HV08}) = 1159652180.73(28) \times 10^{-12}$,
- $a_e(\text{QED : mass – dependent}) = 2.7475719(13) \times 10^{-12}$,
- $a_e(\text{Weak}) = 0.03053(23) \times 10^{-12}$,
- $a_e(\text{Hadron}) = 1.6927(120) \times 10^{-12}$,

Mohr, Newell, and Taylor, Rev. Mod. Phys. 88, 035009 (2016) update

$\alpha^{-1}(\text{Rb : 2016}) = 137.035998995(85)$, $a_e(\text{HV08}) - a_e(\text{theory}) = (-1.30 \pm 0.77) \times 10^{12}$

$\alpha^{-1}(a_a : 2017) = 137.0359991491(15)(14)(330)$,
$\alpha^{-1}(a_e : 2017) \alpha^{-1}(\text{Rb : 2016}) = (0.155 \pm 0.091) \times 10^{-6}.$
Status and sensitivity of the $a_e$ experiments testing various contributions. The error is dominated by the uncertainty of $\alpha(Rb11)$ from atomic interferometry. “New Physics” = deviation $(a_e^{\text{exp}} - a_e^{\text{the}})/a_e^{\text{exp}}$, i.e. is absent. The blue band illustrates the improvement by the Harvard experiment. Note the very different sensitivities to non-QED contributions in comparison with $a_\mu$. 