Status of the Muon g-2

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Outline of Talk:

① **Lepton magnetic moments: basics**

② **The Anomalous Magnetic Moment of the Muon**

③ **Standard Model Prediction for a_μ**

④ $a_\mu^{\rm had}$ **and theory of the Pion form factor**

⑤ $a_\mu^{\rm had}$ **via the Adler function**

⑥ **Outlook**

Topics covered by talk of Simon Eidelman:

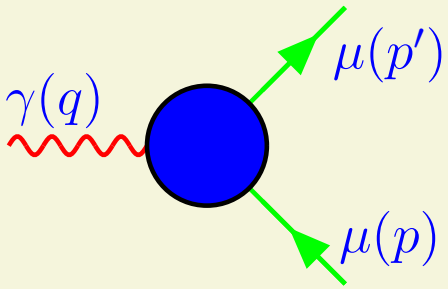
❑ e^+e^- **– Cross Sections and** τ **– Decay Spectral Functions**

❑ **Evaluation of** $a_\mu^{\rm had}$

① Lepton magnetic moments: basics

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s} \;\; ; \quad g_\mu = 2\,(1+a_\mu)$$

Dirac: $g_\mu = 2$ **,** a_μ **muon anomaly**



$$= (-ie)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu}F_2(q^2)\right]u(p)$$

$$F_1(0) = 1 \;\; ; \quad F_2(0) = a_\mu$$

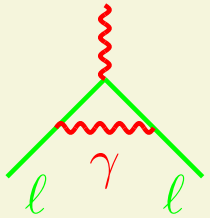
a_μ **responsible for the Larmor precession**

directly proportional at magic energy $\sim$ 3.1 GeV

CERN, BNL g-2 experiments

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu\vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right]^{E\sim 3.1\,\mathbf{GeV}}_{\text{at "magic }\gamma\text{"}} \simeq \frac{e}{m}\left[a_\mu\vec{B}\right]$$

a_ℓ: **dimensionless (just a number), must vanish at tree level in any renormalizable theory $\longrightarrow$ finite prediction , testing helicity flip, most precisely measured electroweak observables.** **Lowest order QED/SM prediction: a_ℓ universal**



$$a_e = a_\mu = a_\tau = \frac{\alpha}{2\pi}$$

Schwinger 48

a_e: **Presently most precise determination of $\alpha = e^2/4\pi$**

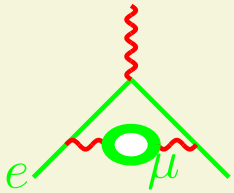
a_μ: **Enhanced sensitivity to New Physics, severe constraints to physics beyond the SM**

$$a_\ell \sim \left(\frac{m_\ell}{m_{\rm NP}}\right)^2 \Rightarrow \left(\frac{m_\mu}{m_e}\right)^2$$

$a_\mu \sim$ **40000 times more sensitive than** a_e

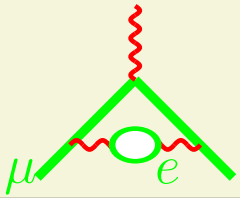
Even so a_μ is 143$\times$ less precise than a_e its still 280$\times$ more sensitive to NP

Mass effects: **– HEAVY internal masses decouples**



$$= \left[\frac{1}{45}\left(\frac{m_e}{m_\mu}\right)^2 + O\left(\frac{m_e^4}{m_\mu^4}\ln\frac{m_\mu}{m_e}\right)\right]\left(\frac{\alpha}{\pi}\right)^2$$

– LIGHT internal masses give rise to log's of mass ratios



$$= \left[\frac{1}{3}\ln\frac{m_\mu}{m_e} - \frac{25}{36} + O\left(\frac{m_e}{m_\mu}\right)\right]\left(\frac{\alpha}{\pi}\right)^2$$

Universal contributions

- 2-loop diagrams [7] with common fermion lines

$$a_\ell^{(4)} = \left[\frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2}\ln 2 + \frac{3}{4}\zeta(3)\right]\left(\frac{\alpha}{\pi}\right)^2$$

Peterman 57, Sommerfield 57

- 3-loop diagrams [72] with common fermion lines

$$\begin{aligned} a_\ell^{(6)} \quad = \quad & \left[\frac{28259}{5184} + \frac{17101}{810}\pi^2 - \frac{298}{9}\pi^2\ln 2 + \frac{139}{18}\zeta(3) + \frac{100}{3}\left\{\mathrm{Li}_4(\frac{1}{2}) + \frac{1}{24}\ln^4 2 - \frac{1}{24}\pi^2\ln^2 2\right\}\right. \\ & \left. -\frac{239}{2160}\pi^4 + \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5)\right]\left(\frac{\alpha}{\pi}\right)^3 \end{aligned}$$

Laporta & Remiddi 96

- 4-loop diagrams [891] with common fermion lines numerically

Kinoshita 99, Kinoshita & Nio 02

$$a_\ell^{\text{universal}} = 0.5\left(\frac{\alpha}{\pi}\right) - 0.3284789655\ldots\left(\frac{\alpha}{\pi}\right)^2 + 1.18124145\ldots\left(\frac{\alpha}{\pi}\right)^3 - \boxed{1.7502(384)}\left(\frac{\alpha}{\pi}\right)^4$$

Kinoshita & Nio 02 coefficient of $\left(\frac{\alpha}{\pi}\right)^4$ changed by -0.24

Corrections due to internal μ- and τ-loops

$$a_e = a_e^{\rm uni} + a_e(\mu) + a_e(\tau)$$

$$a_e(\mu) = \underbrace{5.197\times 10^{-7}}_{\mu\ \ \rm VP}\left(\frac{\alpha}{\pi}\right)^2 + (\underbrace{-2.1768\times 10^{-5}}_{\mu\ \ \rm ho\ VP} + \underbrace{1.4394\times 10^{-5}}_{\mu\ \ \rm LBL}) \times \left(\frac{\alpha}{\pi}\right)^3$$

$$a_e(\tau) = \underbrace{1.838\times 10^{-9}}_{\tau\ \ \rm VP}\left(\frac{\alpha}{\pi}\right)^2$$

Hadronic corrections: $a_e^{\rm had} = 1.67(3)\times 10^{-12}$

Weak corrections: $a_e^{\rm weak} = 0.03\times 10^{-12}$

$\Rightarrow$ a_e **is excellent observable for extracting** $\alpha_{\rm QED}$ **!**

Most precise $\alpha_{\rm em}$ from $a_e \equiv (g_e - 2)/2$

Compare extraordinary precise measurements of electron anomalous magnetic moment

$$a_e \equiv (g_e - 2)/2$$

$$a_{e^-}^{\rm exp} = 0.001\,159\,652\,188\,4(43), \quad a_{e^+}^{\rm exp} = 0.001\,159\,652\,187\,9(43)$$

Dehmelt et al. 1987

with the prediction

$$a_e^{\rm SM} = \frac{\alpha}{2\pi} - 0.328\,478\,444\,00\left(\frac{\alpha}{\pi}\right)^2$$
$$+1.181\,234\,017\left(\frac{\alpha}{\pi}\right)^3 - 1.7502(384)\left(\frac{\alpha}{\pi}\right)^4 \ ^*$$
$$+1.66(3)\times 10^{-12}$$ **(hadronic & electroweak loops)**

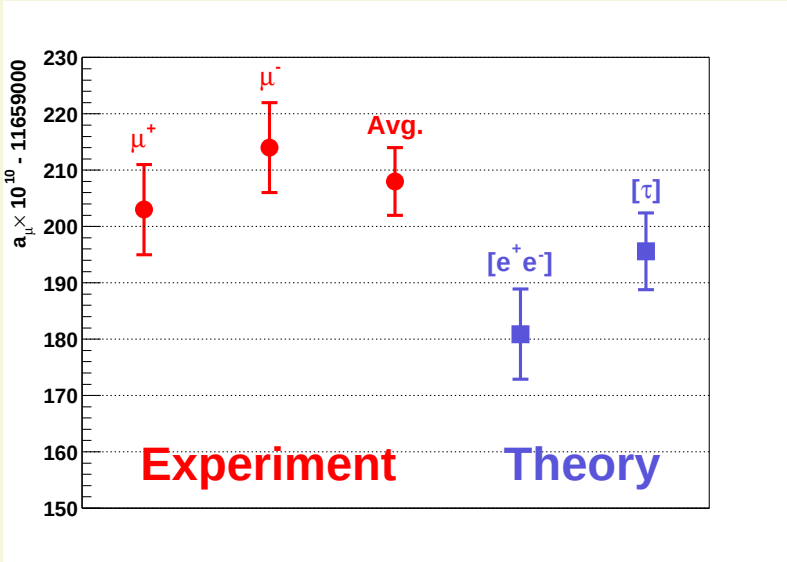
$$\Downarrow$$

currently provides the best determination of the fine structure constant

$$\alpha^{-1}(a_e) = 137.035\,998\,75(52)$$ **(3.8 ppb)**

* **Kinoshita, Nio 2003** $[-0.24]\times(\frac{\alpha}{\pi})^4$ **update announced**

② The Anomalous Magnetic Moment of the Muon



2.7 σ (e^+e^-) [1.4 σ (τ)]

BNL - E821

Muon (g-2) Collaboration

hep-ex/0401008

	experiment	0.5ppm	=	6×10^{-10}	[5×10^{-10} stat, 4×10^{-10} syst]
Uncertainties:	theory	0.7ppm	=	8×10^{-10}	(e^+e^-) [0.6ppm = 7×10^{-10} (τ)]
	BNL design	0.35ppm	=	4×10^{-10}	

New physics sensitivity: (example) $\Delta a_\mu^{\rm SUSY}/a_\mu \simeq 1.25{\rm ppm}\left(\frac{100{\rm GeV}}{\tilde m}\right)^2 \tan\beta$

2-loops may amount $\sim$ 1.0 σ

for allowed parameter space

$\tilde m$ lightest SUSY particle

Heinemeyer, Stöckinger, Weiglein

hep-ph/0312264

$\tan\beta = \frac{v_1}{v_2}, v_i =< H_i > \;\; ; \;\; i = 1, 2$

SUSY requires two Higgs fields

③ Standard Model Prediction for a_μ

❶ QED Contribution

The QED contribution to a_μ has been computed (or estimated) through 5 loops

$$
\begin{aligned}
a_\mu^{\rm QED} &= \frac{\alpha}{2\pi} + 0.765\,857\,376(27)\left(\frac{\alpha}{\pi}\right)^2 \\
&\quad +24.050\,508\,98(44)\left(\frac{\alpha}{\pi}\right)^3 \\
&\quad +126.07(41)\left(\frac{\alpha}{\pi}\right)^4 \\
&\quad +930(170)\left(\frac{\alpha}{\pi}\right)^5 .
\end{aligned}
$$

Growing coefficients in the α/π expansion reflect the presence of large $\ln\frac{m_\mu}{m_e} \simeq 5.3$ terms coming from electron loops. Employing the value of α from a_e leads to

$$a_\mu^{\rm QED} = 116\,584\,705.7(2.9)\times 10^{-11}$$ **(old)**

The current uncertainty is well below the $\pm 40\times 10^{-11}$ ultimate experimental error anticipated from E821 and should, therefore, play no essential role in the confrontation between theory and experiment.

Not quite: Corrections due to internal e- and τ-loops updated

$$a_\mu = a_e^{\rm uni} + a_\mu(m_\mu/m_e) + a_\mu(m_\mu/m_\tau) + a_\mu(m_\mu/m_e, m_\mu/m_\tau)$$

$$\begin{aligned}
a_\mu(m_\mu/m_e) &= 1.094\,258\,282\,8\,(98)\left(\frac{\alpha}{\pi}\right)^2 + 22.868\,379\,36\,(23)\left(\frac{\alpha}{\pi}\right)^3 \\
&+ \boxed{132.682\,3\,(72)}\left(\frac{\alpha}{\pi}\right)^4 \\
a_\mu(m_\mu/m_\tau) &= 7.8059\,(25)\times 10^{-5}\left(\frac{\alpha}{\pi}\right)^2 + 36.054\,(21)\times 10^{-5}\left(\frac{\alpha}{\pi}\right)^3 \\
&+ \boxed{127.50\,(41)}\left(\frac{\alpha}{\pi}\right)^4 \\
a_\mu(m_\mu/m_e, m_\mu/m_\tau) &= 52.763\,(17)\times 10^{-5}\left(\frac{\alpha}{\pi}\right)^3 \\
&+ \boxed{0.037\,594\,(83)}\left(\frac{\alpha}{\pi}\right)^4
\end{aligned}$$

with $\alpha^{-1}(\text{a.i.}) = 137.036\,000\,3\,(10)$ **[7.4 ppb]**

$$\boxed{a_\mu^{\rm QED} = 116\,584\,719.35\ \underbrace{(0.03)}_{\alpha^4}\underbrace{(1.15)}_{\alpha^5}\underbrace{(0.85)}_{\alpha_{\rm inp}}\times 10^{-11}}$$

shift by $\boxed{+13.7\times 10^{-11}}$

Kinoshita, Nio 04

# of loops	$a_\mu^{\rm QED} \times 10^{11}$
1	116140972.87 (0.44)
2	413217.60 (0.02)
3	30141.90 (0.00)
4	367.01 (1.19)
5	6.29 (1.15)
tot	116584705.66 (2.80)

❶ 1 diagram — Schwinger 1948

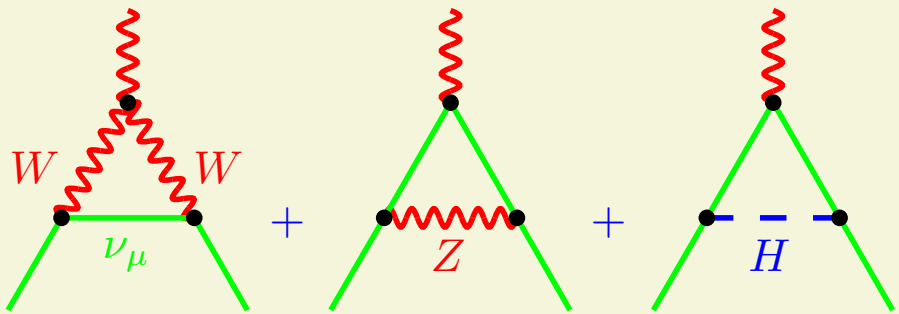
❷ 7 diagrams — Peterman 1957, Sommerfield 1957

❸ 72 diagrams — Lautrup, Peterman, de Rafael 1974, Laporta, Remiddi 1996

❹ about 1000 diagrams — Kinoshita 1999, Kinoshita, Nio 2004

❺ RG estimate — Czarnecki, Marciano 00

❷ Weak Contributions

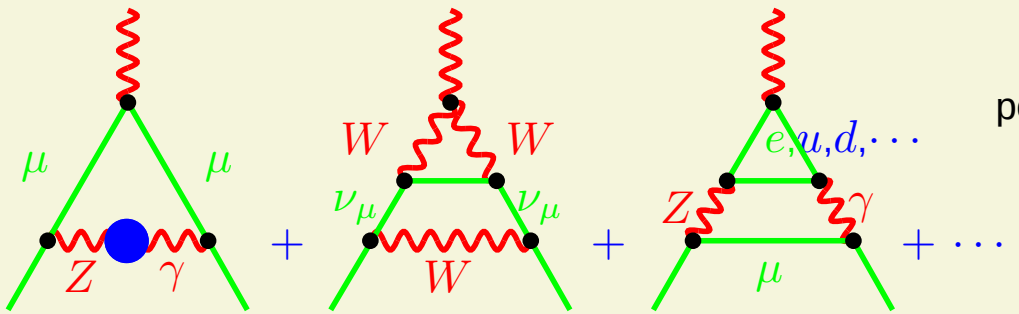


$$a_\mu^{\mathrm{weak}(1)} = (195 \pm 0) \times 10^{-11}$$

Brodsky, Sullivan 67, ...,

Bardeen, Gastmans, Lautrup 72

Higgs contribution tiny!



$$a_\mu^{\mathrm{weak}(2)} = -(44 \pm 4) \times 10^{-11}$$

Kukhto et al 92

potentially large terms $\sim G_F m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_\mu}$

Peris, Perrottet, de Rafael 95

quark-lepton (triangle anomaly) partial

Czarnecki, Krause, Marciano 96 full result

Most recent evaluations: improved hadronic part (beyond QPM)

$$a_\mu^{\mathrm{weak}} = (152 \pm 1[\mathrm{had}] \pm ?) \times 10^{-11}$$ **(Knecht, Peris, Perrottet, de Rafael 02)**

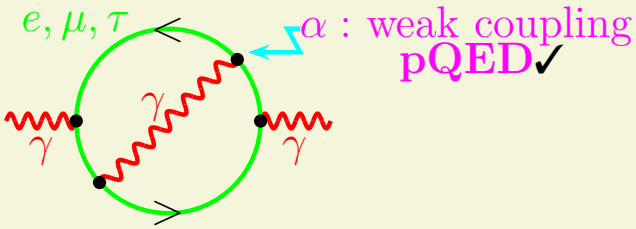
$$a_\mu^{\mathrm{weak}} = (154 \pm 1[\mathrm{had}] \pm 2[\mathrm{m_H}, \mathrm{m_t}, 3-\mathrm{loop}]) \times 10^{-11}$$ **(Czarnecki, Marciano, Vainshtein 02)**

❸ Hadronic Contributions

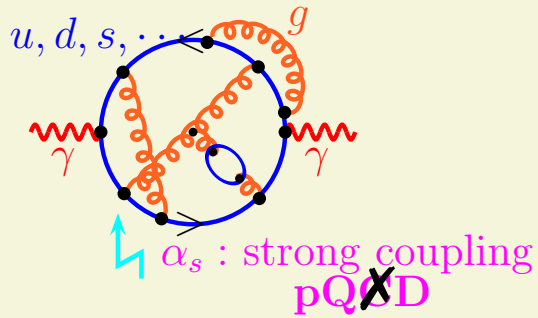
General problem in electroweak precision physics:

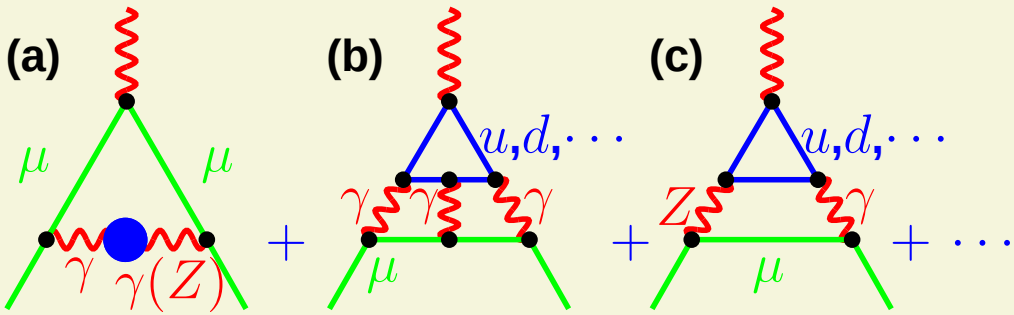
contributions from hadrons (quark loops) at low energy scales

Leptons



Quarks





(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$

(b) Hadronic light-by-light scattering $O(\alpha^3)$

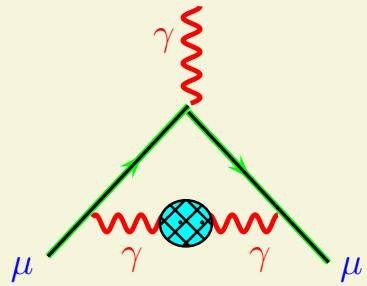
(c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$

Light quark loops $\longrightarrow$ Hadronic "blob"

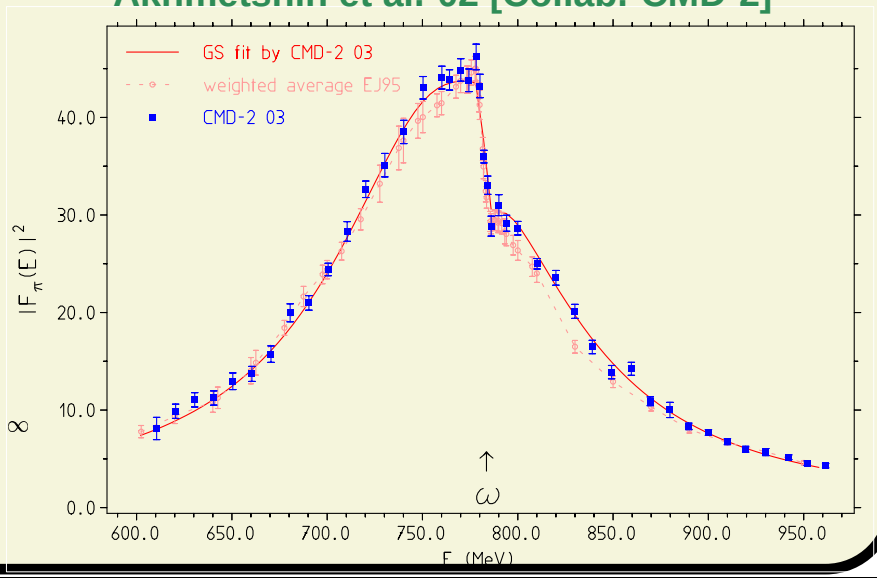
❑ Evaluation of $a_\mu^{\rm had}$

Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via dispersion integral:

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E_{\rm cut}^2} ds \, \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int_{E_{\rm cut}^2}^{\infty} ds \, \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right)$$



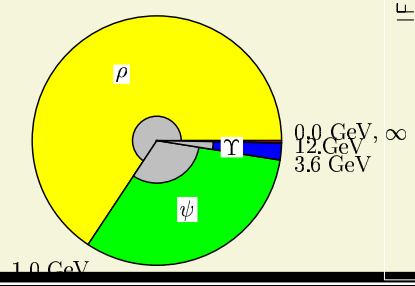
Data:
Akhmetshin et al. 02 [Collab. CMD-2]



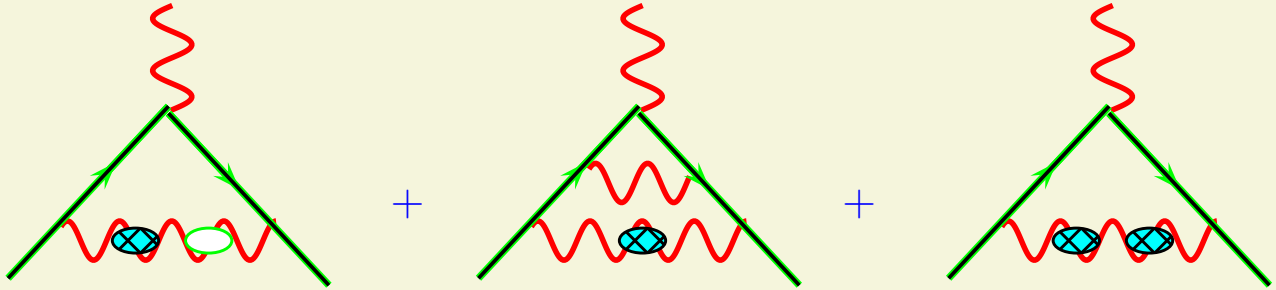
- **Experimental error implies theoretical uncertainty!**
- **Low energy contributions enhanced: $\sim 67\%$ of error on $a_\mu^{\rm had}$ comes from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$**

$$a_\mu^{\rm had(1)} = (694.8 \pm 8.6)\ 10^{-10}$$

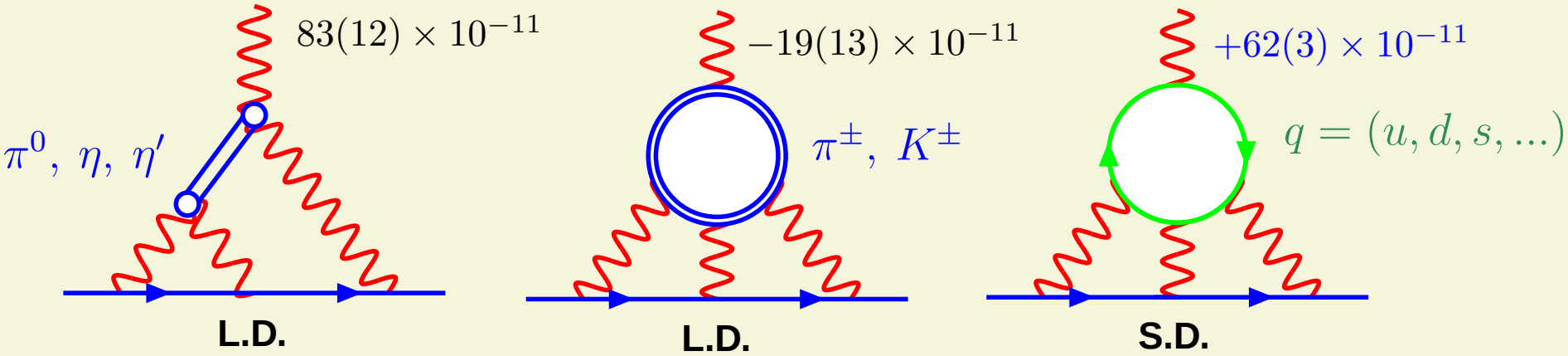
e^+e^-–data based



- **Higher order hadronic contributions** $a_\mu^{\mathrm{had}(2)} = -(100 \pm 6) \times 10^{-11}$ **(Krause 96)**
  $a_\mu^{\mathrm{had}(2)} = -(\ 98 \pm 1) \times 10^{-11}$ **(Hagiwara et al. 03)**

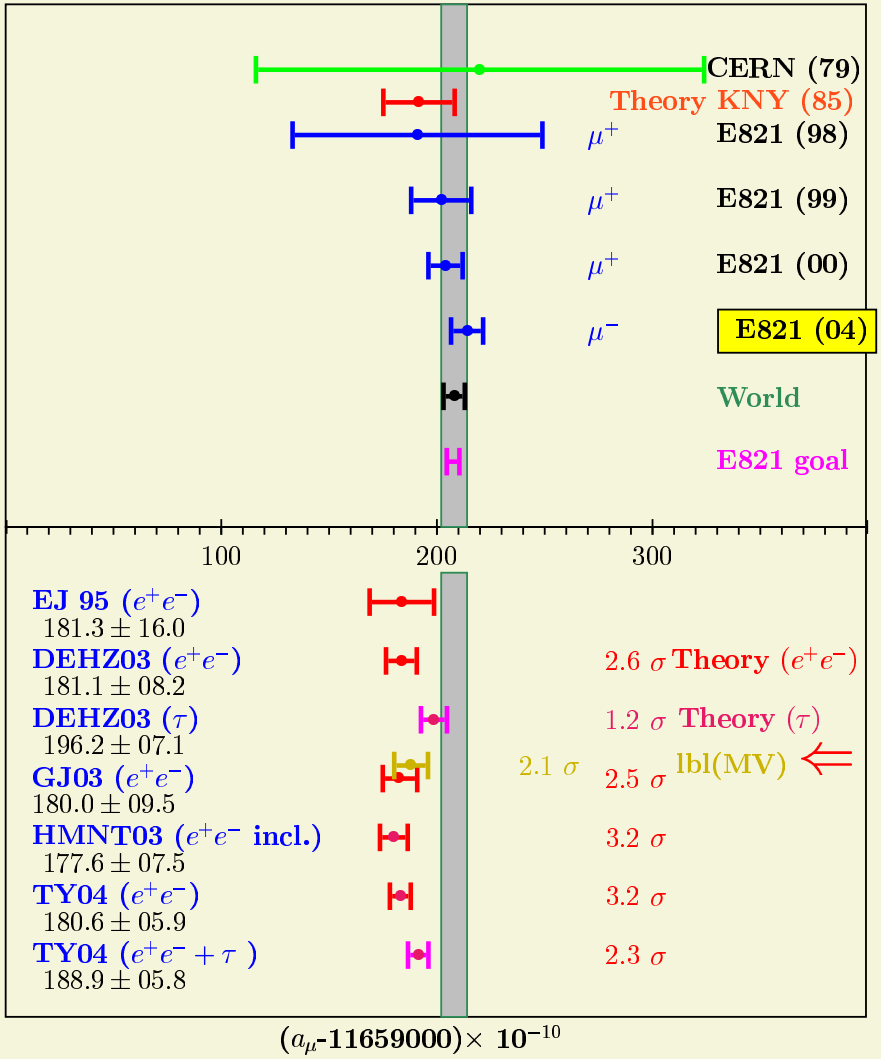


- **Hadronic light–by–light scattering** $a_\mu^{\mathrm{lbl}} = (\ 80 \pm 40)\ \times 10^{-11}$ **(Nyffeler 02)**
  $a_\mu^{\mathrm{lbl}} = (136 \pm 25)\ \times 10^{-11}$ **(Melnikov & Vainshtein 03)** **shift by** $\boxed{+56 \times 10^{-11}}$



Low energy effective theory: e.g. ENJL

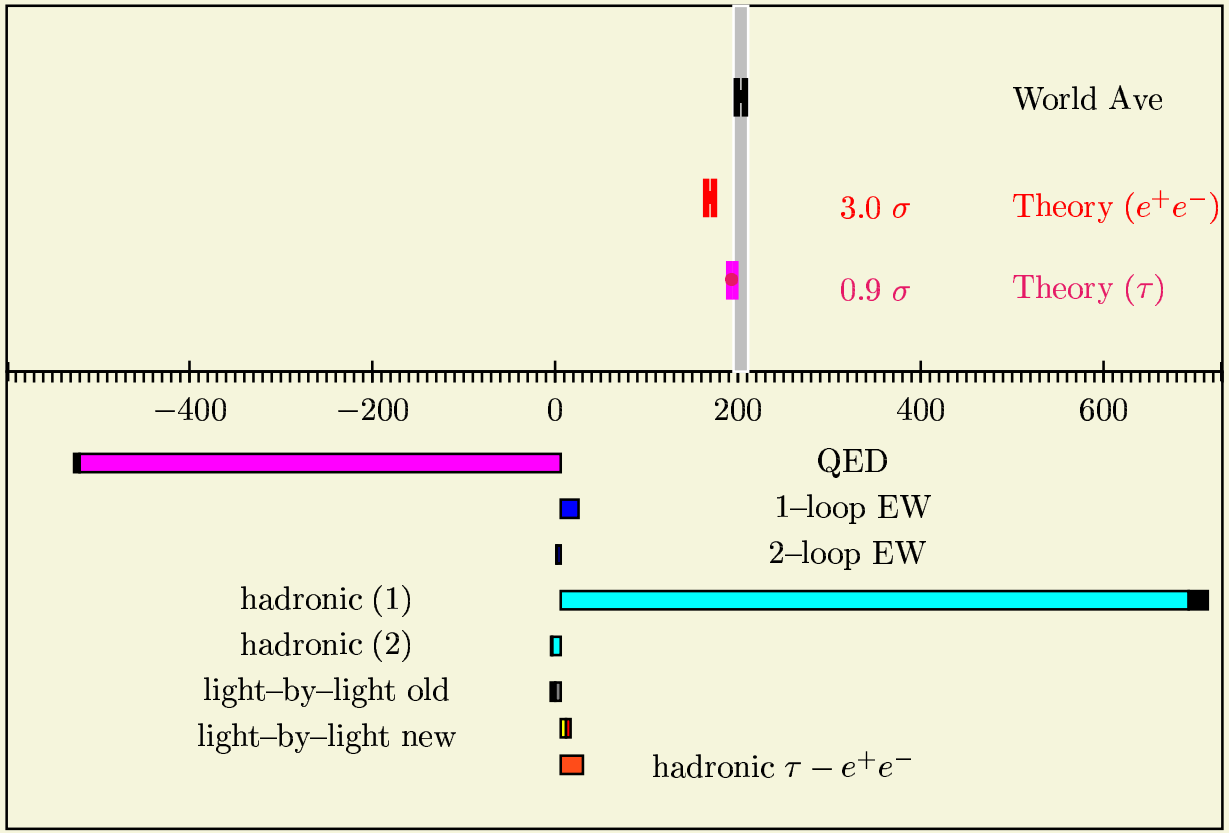
a_μ Summary: experiment vs. theory



Given theory results only differ by $a_\mu^{\mathrm{had}(1)}$!

a_μ: type and size of contributions

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm had(1)} + a_\mu^{\rm had(2)} + a_\mu^{\rm weak(1)} + a_\mu^{\rm weak(2)} + a_\mu^{\rm lbl} \; (+a_\mu^{\rm new\ physics})$$



All kind of physics meets !

④ $a_\mu^{\rm had}$ based on theory of the Pion form factor

Electromagnetic Form Factor constraint by analyticity, unitarity and chiral limit: [see also (Trocóniz and Yndurain 01 and others)]

$$F(s) = \exp\Delta(s) \times G_\omega(s) \times G(s)$$

- Omnès factor (cut due to 2π intermediate states)

  In elastic region curvature in $F(s)$ generated by these states is determined by P-wave phase shift $\delta(s)$ of $\pi\pi$ scattering

$$\Delta(s) = \frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{dx\delta(x)}{x(x-s)}$$

  using more accurate phase shifts relying on the Roy equation analysis Colangelo, Gasser, Leutwyler 01

  Behavior of $\delta(s)$ in region below matching point $E_0 = 0.8$ GeV controlled by 3 parameters: 2 S-wave scattering length $a_0^0,\ a_0^2$ and $\phi \equiv \delta(E_0)$ phase at matching point. We treat ϕ as a free parameter and rely on very accurate predictions for $a_0^0,\ a_0^2$ from chiral perturbation theory.

- $\rho-\omega$–mixing contribution

$$G_\omega(s) = 1 + \varepsilon\frac{s}{s_\omega - s} + \ldots \qquad s_\omega = (M_\omega - \frac{1}{2}i\Gamma_\omega)^2$$

  In fact: in order to get it real in space-like region, we replace it by dispersion integral with proper behavior at threshold (is inessential numerically for our purpose). $G_\omega(s)$ is (a) fully determined by $\varepsilon,\ M_\omega$ and Γ_ω and (b) in the experimental range $|G_\omega(s)|$ is very close to magnitude of the pole approximation

- Low energy singularities generated by states with 2 or 3 pions are accounted for by the first two factors of the "master equation" above. the function $G(s)$ represents the smooth background that contains the curvature generated by the remaining singularities. The $4\,\pi$ channel opens at $s = 16\,M_\pi^2$ but phase space strongly suppresses the strength of the corresponding branch point singularity - a significant inelasticity only manifests itself for $s > s_{\rm in} = (M_\omega + M_\pi)^2$. Conformal mapping:

$$z = \frac{\sqrt{s_{\rm in} - s_1} - \sqrt{s_{\rm in} - s}}{\sqrt{s_{\rm in} - s_1} + \sqrt{s_{\rm in} - s}}$$

maps the plane cut along $s > s_{\rm in}$ onto the unit disk in the z–plane. It contains a free parameter s_1 - the value of s which gets maps into the origin. We find that if s_1 is taken in the vicinity of M_ρ^2, then the fit becomes rather insensitive to the details of the parametrization. In the following we set $s_1 = 0.6$ GeV^2. We approximate $G(s)$ by a second degree polynomial in z:

$$G(s) = 1 + c_1\,(z - z_0) + c_2\,(z^2 - z_0^2)$$

where z_0 is the image of $s = 0$.

- charge $G(0) = 1$ normalization: z_0–terms
- no unphysical root singularity: $c_2 = -\frac{1}{2}c_1$

one free parameter: c_1

freedom: choice of s_1, degree of polynomial, $\Rightarrow$ model uncertainty

work in progress Caprini, Colangelo, Jegerlehner, Leutwyler

P	χ^2/d.o.f.	$\chi^2_{\rm CMD2/NA7}$	$10^{10}a_\rho$	$10^{10}a_{2M_K}$	$\langle r^2\rangle$(fm^2)
0	84.9/83	43.6 / 43.7	420.1 ± 2.1	489.5 ± 2.2	0.4254 ± 0.0020
5	78.4/82	35.9 / 42.6	423.8 ± 2.6	494.1 ± 2.7	0.4300 ± 0.0024
6	78.1/81	36.0 / 42.2	424.4 ± 2.8	494.7 ± 2.9	0.4339 ± 0.0051
7	73.5/80	31.7 / 42.2	423.4 ± 2.9	493.2 ± 3.0	0.4350 ± 0.0051
8	73.5/79	31.6 / 42.2	423.5 ± 5.7	493.4 ± 7.4	0.4347 ± 0.0052

Numerical results for fits to CMD-2 and (spacelike) NA7 data. The errors given are purely statistical.

To be compared with: 429.02 ± 4.95 (stat) from trapezoidal rule. Gain factor of 2 in precision in stat error!

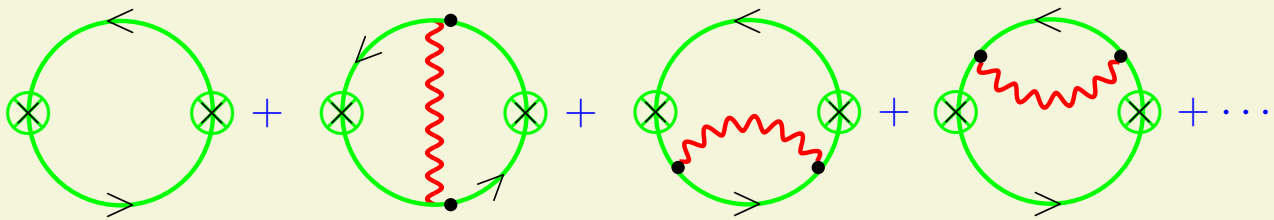
Note on new KLOE result:

my old value:	694.75 (5.15) (6.83) [8.56]	
subtract cmd2:	389.36 (2.75) (2.59) [3.78]	extended to KLOE range
	305.39 (4.35) (6.32) [7.67]	
KLOE:	388.75 (0.52) (5.05) [5.08]	KLOE range: 591.6-969.5 MeV
add weighted	389.24 (1.14) (2.39) [2.65]	
my new value	694.63 (4.50) (6.76) [8.12]	

⑤ $a_\mu^{\rm had}$ via the Adler function

Controlling pQCD via the Adler function

❶ pQCD calculations of vacuum polarization amplitudes



up to 4–loops massless — Groshny, Kataev, Larin 91

up to 3–loops massive — Chetyrkin, Kühn et al. 97

up to 2–loops massive BF–MOM RG — F. J., Tarasov 98

❷ use old idea: testing non–perturbative effects with help of the Adler function

Eidelman, F. J., Kataev, Veretin 98

$$D(-s) \;\doteq\; \frac{3\pi}{\alpha}\, s\, \frac{d}{ds}\Delta\alpha_{\rm had}(s) = -\left(12\pi^2\right)\; s\, \frac{d\Pi'_\gamma(s)}{ds}$$

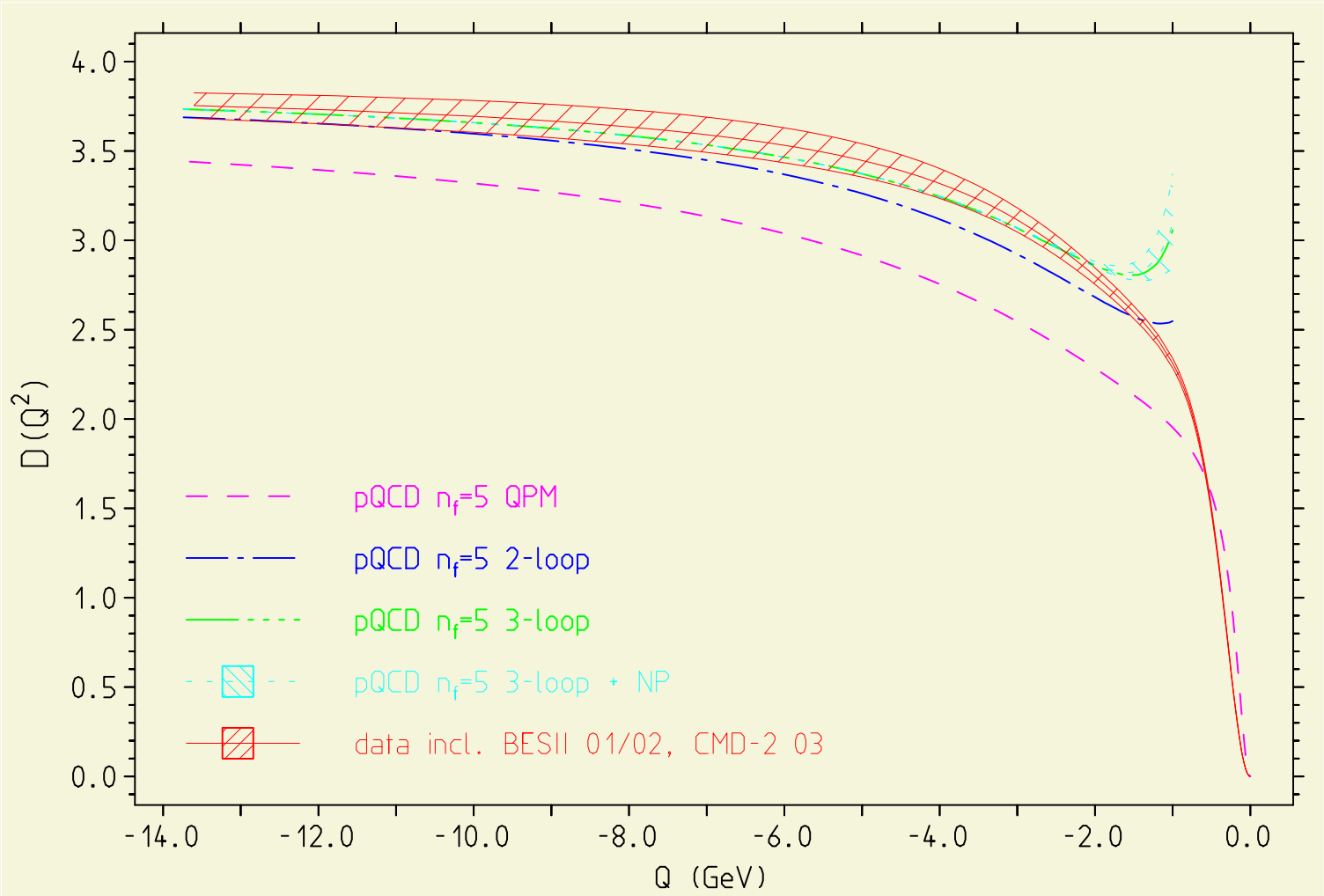
$$\Rightarrow \qquad D(Q^2) \;=\; Q^2 \int\limits_{4m_\pi^2}^{\infty} ds\, \frac{R(s)}{(s+Q^2)^2}$$

pQCD $\leftrightarrow R(s)$	pQCD $\leftrightarrow D(Q^2)$
very difficult to obtain in theory	smooth simple function in Euclidean region

Conservative conclusion:

- **time-like approach: pQCD works well in "perturbative windows"**
  3.00 - 3.73 GeV, 5.00 - 10.52 GeV and 11.50 - ∞
  (Kühn,Steinhauser)
- **space-like approach: pQCD works well for $Q^2 = -q^2 > 2.5$ GeV (see plot)**
  (EJKV 98/04)

"Experimental" Adler–function versus theory (pQCD + NP)



(Eidelman, F.J., Kataev, Veretin 98, FJ 03 update (BES, CMD-2))

$\Rightarrow$ **pQCD works well to predict $D(Q^2)$ down to $s_0 = (2.5\,\textbf{GeV})^2$**

(not down to m_τ ! however);

use this to calculate

$$\Delta\alpha_{\rm had}(-Q^2) \sim \frac{\alpha}{3\pi}\int dQ'^2 \frac{D(Q'^2)}{Q'^2}$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = \left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-s_0)\right]^{\rm pQCD} + \Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data}$$

and obtain, for $s_0 = (2.5\,\textbf{GeV})^2$: **(FJ 98/03)**

$$\Delta\alpha^{(5)}_{\rm had}(-s_0)^{\rm data} = 0.007417 \pm 0.000086$$

$$\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.027613 \pm 0.000086 \pm 0.000149[0.000149]$$

The second error comes from the variation of the pQCD parameters. In square brackets the error if we assume the uncertainties from different parameters to be uncorrelated. The uncertainties coming from individual parameters are listed in the following table (masses are the pole masses):

parameter	range	pQCD uncertainty	total error
α_s	0.117 ... 0.123	0.000051	0.000155
m_c	1.550 ... 1.750	0.000087	0.000170
m_b	4.600 ... 4.800	0.000011	0.000146
m_t	170.0 ... 180.0	0.000000	0.000146
all correlated		0.000149	0.000209
all uncorrelated		0.000101	0.000178

The largest uncertainty is due to the poor knowledge of the charm mass. I have taken errors to be 100% correlated. The uncorrelated error is also given in the table.

$$\Rightarrow \quad \delta\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) = 0.00015$$

New values: pQCD/SR (moment method) and Lattice QCD

Ref	$\alpha_s(M_Z)$	$\Lambda_{\overline{\rm MS}}^{N_f=0}$ [MeV]	$m_s(m_s)$ [MeV]	$m_c(m_c)$ [GeV]	$m_b(m_b)$ [GeV]
PDG	0.118(3)	-	-	-	-
Steinhauser	$0.124^{+0.011}_{-0.014}$	-	-	1.304(27)	4.191(51)
Rolf	-	238(19)[Q]	97(4)[Q]	1.301(34)[Q]	4.12(7)(4)[Q]

Lattice: waiting for full QCD results [unquenched], continuum limit performed

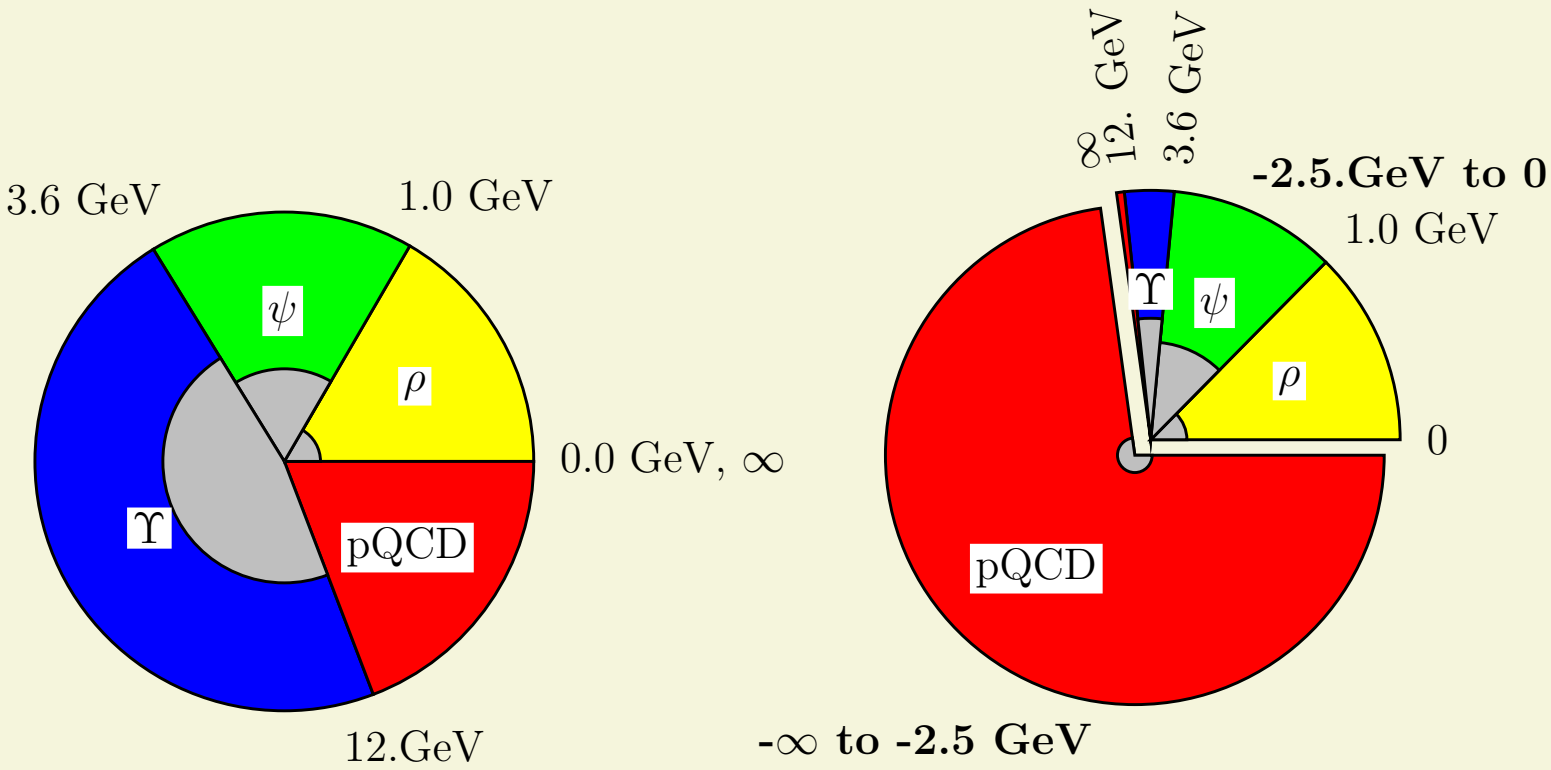
The virtues of this analysis are obvious:

- **no problems with physical threshold and resonances**
- **pQCD is used only where we can check it to work (Euclidean, $Q^2 \gtrsim$ 2.5 GeV).**
- **no manipulation of data, no assumptions about global or local duality.**
- **non–perturbative "remainder" $\Delta\alpha^{(5)}_{\rm had}(-s_0)$ is mainly sensitive to low energy data !!!**

Distribution of hadronic contributions to $\Delta\alpha^{\rm had}$

e^+e^- data based approach:

Based on available data up to 12 GeV (FJ 03)



Comparison of the distribution of contributions and errors (shaded areas scaled up by 10) in the standard (left) and the Adler function based approach (right), respectively.

$a_\mu^{\rm had}$ via the Adler function

t–channel or Euclidean field theory approach

non–perturbative regime directly accessible by lattice QCD !

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int_0^\infty \frac{ds}{s}\int_0^1 dx\,\frac{x^2\,(1-x)}{x^2+(1-x)s/m_\mu^2}\,e^2\frac{1}{\pi}{\rm Im}\Pi'_{\gamma\,{\rm had}}$$

interchange of integrations yields:

$$a_\mu^{\rm had} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha^{\rm had}\left(-Q^2(x)\right) \qquad (1)$$

where $Q^2(x) \equiv \frac{x^2}{1-x}m_\mu^2$ is the space like square momentum transfer or

$$x = \frac{Q^2}{2m_\mu^2}\left(\sqrt{1+\frac{4m_\mu^2}{Q^2}}-1\right)$$

By writing $(1-x) = -\frac{1}{2}\frac{d}{dx}(1-x)^2$ and performing a partial integration we obtain

$$a_\mu^{\rm had} = \frac{\alpha}{\pi} m_\mu^2 \int_0^1 dx\, x\,(2-x)\,\left(D(Q^2(x))/Q^2(x)\right) \qquad (2)$$

the number of integrations is reduced by one.

The evaluation in both forms provides a good stability test of the numerical integrations involved.

Utilizing the 2001 compilation of the e^+e^-–data we obtain the following results: (FJ01)

$a_\mu^{\rm had}$	method
699.47 (10.43)	e^+e^--data ,time-like, pQCD above 11 GeV
697.40 (10.44)	Eq. (1)
697.40 (10.47)	Eq. (1) or (2) pQCD above 2.5 GeV

Error dominated by data in range 1.4 to 2 GeV! and/or QCD parameters. For $a_\mu^{\rm had}$ no improvement yet via Adler function.

Interesting for the future [also in conjunction with latticeQCD].

⑥ Outlook

Summary:

- **the new a_{μ^-} measurements is in very good agreement with the a_{μ^+} value in accordance with CPT and the combined result with an error of 60×10^{-11} not so far from the original goal of 40×10^{-11}.**
- **the increasing precision required and triggered a lot of new efforts to understand and/or settle the observed "discrepancy"**
- **still a big challenge on both the experimental side (e^+e^- cross sections) and on the theory side (also Lattice QCD could become an important tool (Blum 03, Schierholz et al. 03).**
- **with unprecedented precision of the g-2 of the Muon experiment new obstacles showed up in many places which require more efforts on the theory side.**
- **remember situation similar to the one with CERN experiment 25 years ago (error in theory, leptonic lbl)**

- e^+e^--data more directly to what we need in the evaluation of $a_\mu^{\rm had}$
- use of τ-data obsolete at present (Ghozzi&Jegerlehner, Davier, Höcker); requires more reliable estimates of isospin breaking effects (theory???)
- a conservative estimate is

$$|a_\mu^{\rm exp} - a_\mu^{\rm SM}| = (28 \pm 11) \times 10^{-10}$$

Still an exciting situation, some room for new physics, serious challenges for theory and experiment

- Still need better e^+e^--data: KLOE first final result (Aug 04) by the radiative return method
  similar results expected from the B–factories
  ongoing upgrades: Novosibirsk, Beijing
- τ–charm factory
  able to perform an energy scan between 2 and 3.6 GeV
  would satisfy requirements of future precision experiments g-2, GigaZ,...

- **Last but not least: need further progress in theory**
  - ✗ **more careful study of isospin breaking in τ–data vs. e^+e^-–data**
  - ✗ **constraints to $F_\pi(s)$ from χPT, unitarity and analyticity below the ρ**
  - ✗ **still a theoretical challenge:**
    the hadronic light–by–light scattering contribution !
  - ✗ **Concerning τ–data vs. e^+e^-–data discrepancy: need more careful check of radiative corrections which have been applied !**
  - ✗ **Further progress in radiative corrections calculations needed for the processes involved in R–measurements**
  - ✗ **Further progress in determination of QCD parameters crucial for improving pQCD part**

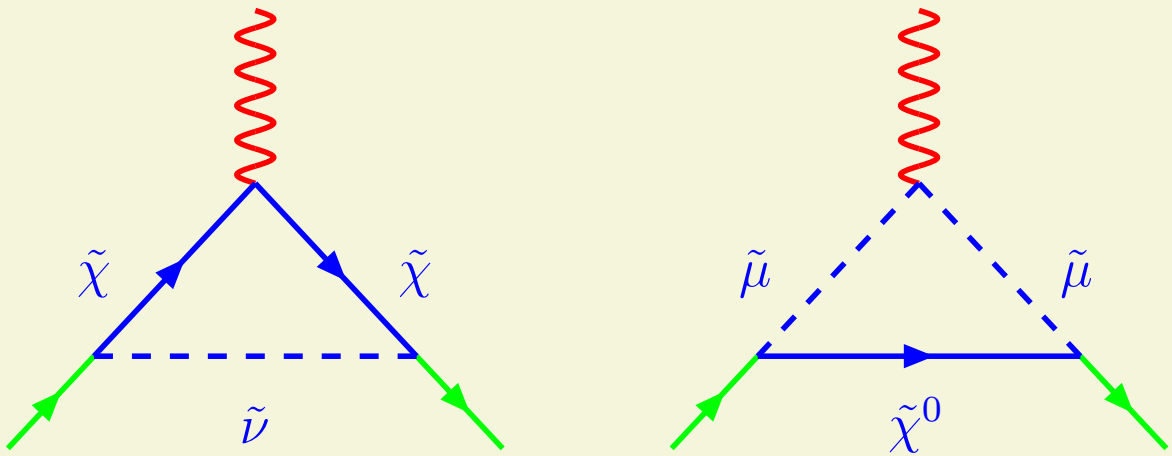
Big experimental challenge: attempt cross-section measurements at 1% level up to J/ψ[Υ] !!! crucial for $\alpha_{\rm QED}(M_Z)$ at GigaZ. Plans for new $g-2$ experiment!

The race for precision goes on!!!

❼ New Physics ?

① SUSY

The supersymmetric contributions to a_μ stem from smuon–neutralino and sneutrino-chargino loops:



Some contributions are enhanced by $\tan\beta \equiv \frac{v_2}{v_1}$ which may be large (in some cases of order $m_t/m_b \approx 40$). Then

$$\left|a_\mu^{\rm SUSY}\right| \simeq \frac{\alpha(M_Z)}{8\pi\sin^2\theta_W}\,\frac{m_\mu^2}{\widetilde{m}^2}\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{\widetilde{m}}{m_\mu}\right)$$

$\widetilde{m} = m_{\rm SUSY}$ **typical SUSY loop mass**

In large $\tan\beta$ regime

$$\left|a_\mu^{\rm SUSY}\right| \simeq 130\times 10^{-11}\left(\frac{100\ {\rm GeV}}{\widetilde{m}}\right)^2 \tan\beta$$

$a_\mu^{\rm SUSY}$ **generally has the same sign as the μ-parameter in SUSY models. For positive sgn(μ):**

$$\tan\beta\left(\frac{100\ \textbf{GeV}}{\widetilde{m}}\right)^2 \simeq 3.3\pm 1.3$$

or

$$\widetilde{m}\simeq (55\ \textbf{GeV})\sqrt{\tan\beta}$$

Negative μ models give the opposite sign contribution to a_μ and are strongly disfavored.

For $\tan\beta$ in the range $4\sim 40$, assuming $\widetilde{m}>100$ GeV:

$$\widetilde{m}\simeq 100-450\ \textbf{GeV}$$

precisely the range where SUSY particles are often expected.

Note: **Ferrara, Remiddi 1974**

$$a_\mu^{\rm tot} = a_\mu^{\rm SM} + a_\mu^{\rm SUSY} = 0$$

in SUSY limit

unbroken SUSY: $a_\mu^{\rm SUSY} < 0$

broken SUSY: either sign

$(g-2)$ **requires:** $a_\mu^{\rm SUSY} > 0$ **in addition to** $a_\mu^{\rm SM} > 0$

② Anomalous W couplings

Anomalous W boson couplings:

Magnetic dipole moment $\mu_W = \frac{e}{2m_W}(1+\kappa+\lambda)$

Electric quadrupole moment $Q_W = -\frac{e}{2m_W}(\kappa-\lambda)$

$\kappa = 1$ **and** $\lambda = 0$ **in the SM** **Contribution to a_μ:**

$$a_\mu(\kappa,\lambda) \simeq \frac{G_\mu m_\mu^2}{4\sqrt{2}\pi^2}\left[(\kappa-1)\ln\frac{\Lambda^2}{m_W^2} - \frac{1}{3}\lambda\right]$$

Λ high momentum cutoff required to give a finite result.

For $\Lambda \simeq 1$ TeV the BNL constraint yields

$$\kappa - 1 = 0.37 \pm 0.14 \qquad \textbf{(BNL 01/04)}$$

Is already ruled out by $e^+e^- \to W^+W^-$ data at LEP II

$$\kappa - 1 = 0.04 \pm 0.08 \qquad \textbf{(LEP II)}$$

③ New Physics: general structure

For a variety of models typically

$$|a_\mu(\textbf{New Physics})| \simeq m_\mu^2/M^2 \quad \cdot \quad \underbrace{\frac{\alpha}{\pi}}_{\text{if loop induced}}$$

Current constraint suggests (very roughly)

$$\boxed{M \simeq 1-2\ \textbf{TeV}}$$

Of course, for a specific model, one must check that the sign of the induced $a_\mu^{\rm NP}$ is in accord with experiment (i.e. it should be positive).

Since Feb. 8, 2001 (BNL):	**$\sim$700**
PRL86 (2001) 488 citations	**new physics**
PRL89 (2002) 240	**explanations**
PRL92 (2004) 047	**presented on the hep archives**

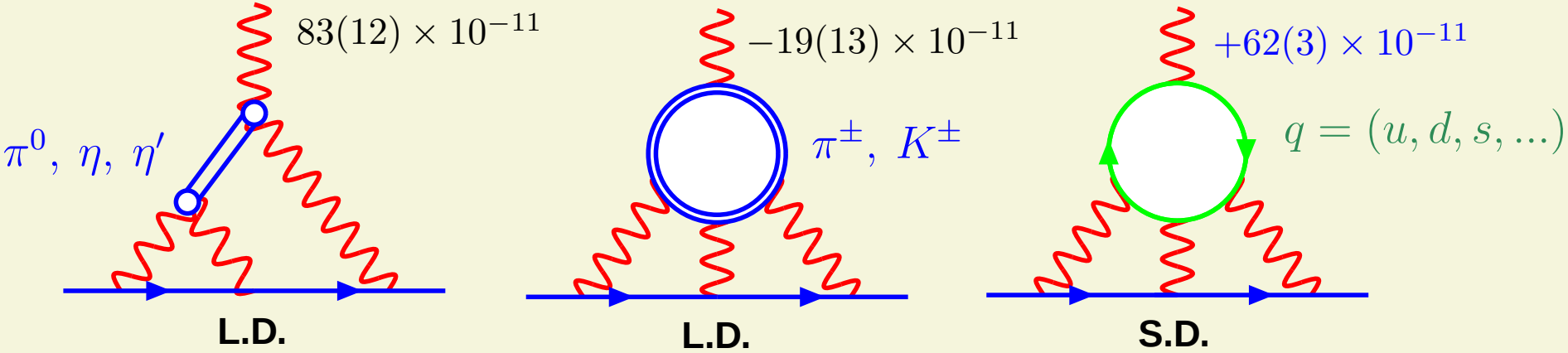
Maybe we really see a first hint of physics beyond the SM!

(deviation persisted since 2001)

Light-by-light-scattering contribution

Melnikov-Vainshtein approach vs EJLN/HGS approach:

- **Hadronic light–by–light scattering** $a_\mu^{\rm lbl} = (\ 80 \pm 40)\ \times 10^{-11}$ **(Nyffeler 02)**
  $a_\mu^{\rm lbl} = (136 \pm 25)\ \times 10^{-11}$ **(Melnikov & Vainshtein 03)** **shift by** $\boxed{+56 \times 10^{-11}}$



Low energy effective theory: e.g. ENJL

MV and KN utilize the same model LMD+V form factor:

$$F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{4\pi^2 F_\pi^2}{N_c}\ \frac{q_1^2 q_2^2 (q_1^2 + q_2^2) - h_2 q_1^2 q_2^2 + h_5 (q_1^2 + q_2^2) + (N_c M_1^4 M_2^4 / 4\pi^2 F_\pi^2)}{(q_1^2 + M_1^2)(q_1^2 + M_2^2)(q_2^2 + M_1^2)(q_2^2 + M_2^2)},$$

where $M_1 = 769\,\mathrm{MeV}$, $M_2 = 1465\,\mathrm{MeV}$, $h_5 = 6.93\,\mathrm{GeV}^4$.

with two modifications:

- **form factor: undressed soft photon (non-renormalization of ABJ)**
- $h_2 = 0 \pm 20$ **GeV**^{2} **(KN) vs.** $h_2 = -10$ **GeV**^{2} **(MV) fixed by twist 4 in OPE** $(1/q^4)$

	π^0, η, η'[π^0]	a_1[f_1, f_1^*]	$\pi^\pm$	pQCD/QPM	tot
HK	83(06)	1.7 [a_1	-4.5(8.5)	10(11)	90(15)
BPP	85(13)	-4(3) [$a_1 + f_0$]	-19(5)	21(3)	83(32)
KN	83(12)			80(40)	
MV	114.5[76.5]	22[7]	0	0	136(25)