The beginning of all sciences
is the astonishment that things are
the way they are

Aristoteles

How the Higgs can explain inflation, dark energy and the cosmological constant

A pedestrian introduction to a new view on the SM of particle physics

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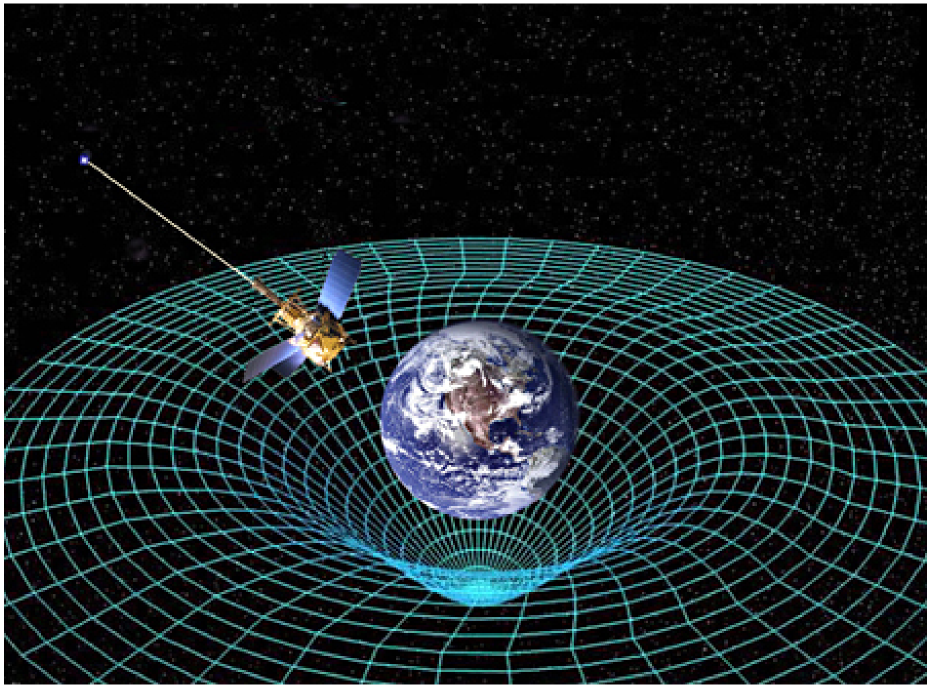
Gravitation and Einstein's General Relativity Theory

Gravitation $\Leftrightarrow$ all masses and even massless particle attract each other
Einstein's General Relativity Theory (GRT): masses (energy density) determine the geometry of space-time (Riemannian Geometry)



Mass tells space how to curve – curved space tells bodies how to move
$\Rightarrow$Einstein's equation $G_{\mu\nu} = \kappa T_{\mu\nu}$!

Cosmology, Cosmological Constant and Dark Energy

- Cosmology shaped by Einstein gravity +
  - ❒ Weyl's postulate (radiation and matter as ideal fluids)
  - ❒ Cosmological principle (isotropy of space implying homogeneity)
  - ⇒fix the form of the metric and of the energy-momentum tensor:

1. The metric (3-spaces of constant curvature $k = \pm 1, 0$)

$$\boxed{\mathrm{d}s^2 = (c\mathrm{d}t)^2 - a^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\,\mathrm{d}\Omega^2\right)}$$

where in the comoving frame $\mathrm{d}s = c\,\mathrm{d}t$ with t the cosmic time

2. The energy-momentum tensor

$$\boxed{T^{\mu\nu} = (\rho(t) + p(t))\,(t)\,u^\mu u^\nu - p(t)\,g^{\mu\nu}\ ;\quad u^\mu \doteq \frac{\mathrm{d}x^\mu}{\mathrm{d}s}}$$

Need $\rho(t)$ energy density and $p(t)$ pressure to get $a(t)$ radius of the universe

Einstein [CC $\Lambda = 0$]: curved geometry ↔ matter [empty space ↔ flat space]

3. Special form energy-momentum tensor $p(t) = -\rho(t)$??? "Dark Energy" only

$$\boxed{T^{\mu\nu} = \rho(t)\, g^{\mu\nu}}$$

Peculiar dark energy equation of motion: $w = p/\rho = -1$ no known physical system exhibits such strange behavior [anti-gravity]!

WHAT IS DARK ENERGY? Well, the simple answer is that we don't know.

First introduced by Einstein as "Cosmological Constant" (CC) as part of the geometry, [where empty space appears curved,] in order to get stationary universe.

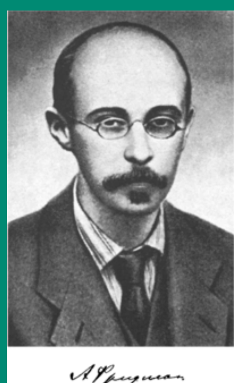
$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu}$ = κ $T_{\mu\nu}$

Einstein Tensor $\Leftrightarrow$ geometry of space-time

Gravitational interaction strength $\kappa = \frac{8\pi G_N}{3c^2}$

Energy-Momentum Tensor $\Leftrightarrow$ deriving from the Lagrangian of the SM

Cosmological solution: universe as a fluid of galaxies $\Rightarrow$Friedmann-Equations:



$$3\,\frac{\dot{a}^2+kc^2}{c^2a^2} - \Lambda = \kappa\,\rho$$

$$-\frac{2\,\ddot{a}a+\dot{a}^2+kc^2}{c^2a^2} + \Lambda = \kappa\,p$$

$a(t)$ Robertson-Walker radius of the universe

Λ Cosmological Constant

- universe must be expanding, **Big Bang**, and has finite age t
- Hubble's law [galaxies: $velocity_{\mathrm{recession}} = H\,Distance$], H Hubble constant
- temperature, energy density, pressure huge at begin, decreasing with time

$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R \quad = \quad \kappa \quad (T_{\mu\nu} + \rho_\Lambda\, g_{\mu\nu}) \quad = \kappa\, T^{\mathrm{tot}}_{\mu\nu} \;\; ; \; \rho_\Lambda = \Lambda/\kappa$

Einstein Tensor $\Leftrightarrow$ geometry of space-time

Gravitational interaction strength $\kappa = \frac{8\pi G_N}{3c^2}$

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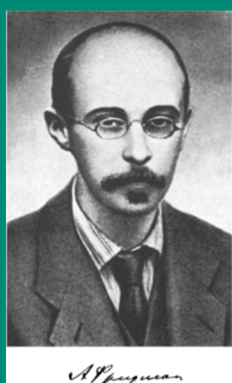


$$3\,\frac{\dot{a}^2 + kc^2}{c^2a^2} = \kappa\,(\rho + \rho_\Lambda)$$

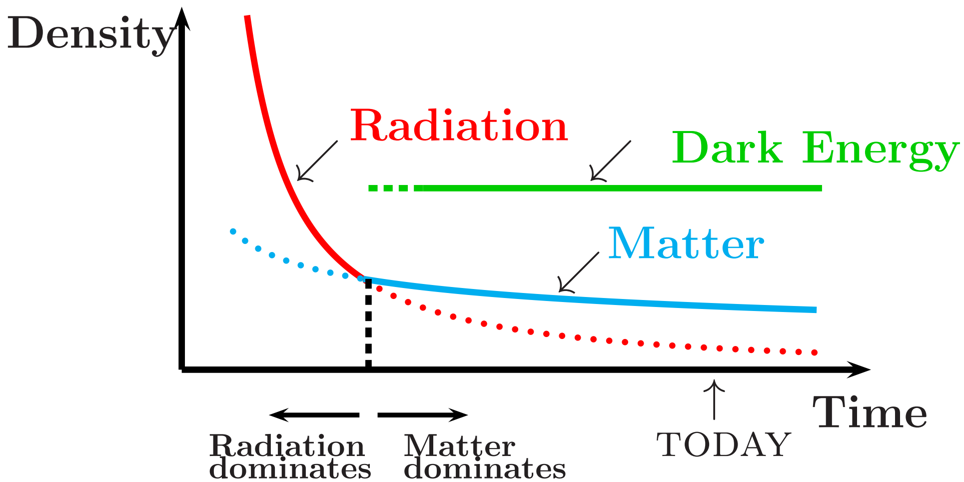
$$-\,\frac{2\,\ddot{a}a + \dot{a}^2 + kc^2}{c^2a^2} = \kappa\,(p + p_\Lambda)$$

$a(t)$ Robertson-Walker radius of the universe

$p_\Lambda = -\rho_\Lambda$ Dark Energy

- universe must be expanding, **Big Bang**, and has finite age t
- Hubble's law [galaxies: $velocity_{\mathrm{recession}} = H\, Distance$], H Hubble constant
- temperature, energy density, pressure huge at begin, decreasing with time

Sketch of the Evolution of the Universe



At Start ➽ **a Light-Flash!**
- high density
- high temperature

$\Rightarrow$ radiation dominates!

Late Times ➽ Dark Energy only
all other stuff dilutes into nothing!
unless universe recontracts $[k = +1]$

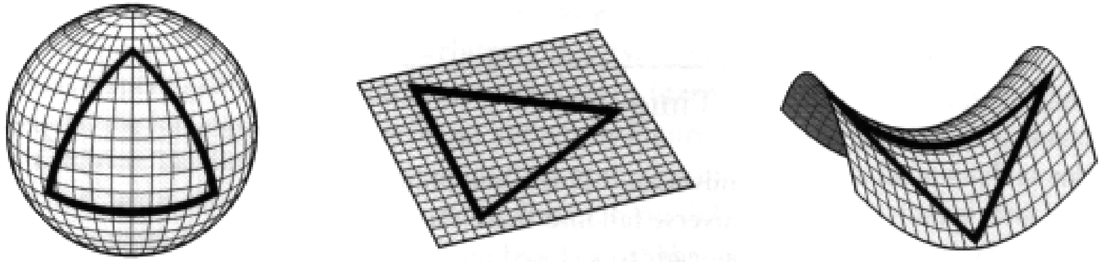
Forms of energy:

❒ radiation: photons, highly relativistic particles $p_{\rm rad} = \rho_{\rm rad}/3$

❒ normal and dark matter (non-relativistic, dilute) $p_{\rm matter} \simeq 0\,,\ \ \rho_{\rm matter} > 0$

❒ dark energy (cosmological constant) $p_{\rm vac} = -\rho_{\rm vac} < 0$

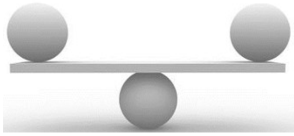
Note: Radiation $\rho_{\rm rad} \propto 1/a(t)^4$, Matter $\rho_{\rm mat} \propto 1/a(t)^3$, Dark Energy $\rho_\Lambda \propto a(t)^0$



Curvature: closed $k = 1$ $[\Omega_0 > 1]$, flat $k = 0$ $[\Omega_0 = 1]$ and open $k = -1$ $[\Omega_0 < 1]$

Interesting fact: flat space geometry $\Leftrightarrow$ specific critical density, “very unstable”

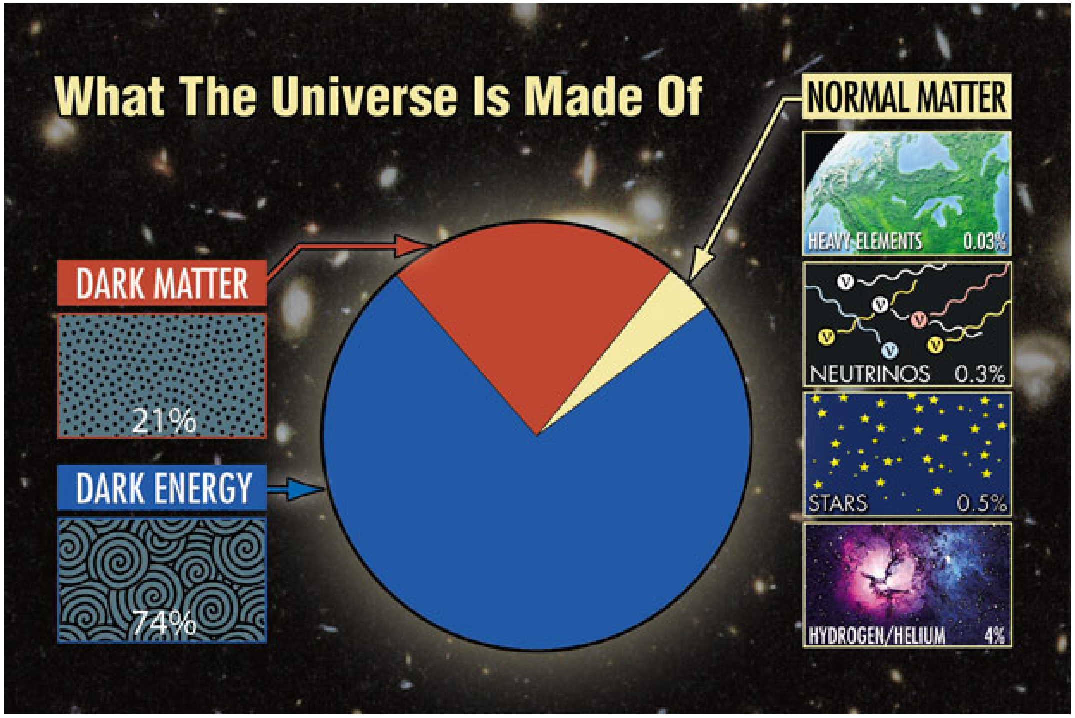
$$\boxed{\rho_{0,\mathrm{crit}} = \rho_{\mathrm{EdS}} = \frac{3H_0^2}{8\pi G_N} = 1.878 \times 10^{-29}\, h^2\, \mathrm{gr/cm}^3,}$$



where H_0 is the present Hubble constant, and h its value in units of 100 km s^{-1} Mpc^{-1}. Ω expresses the energy density in units of $\rho_{0,\mathrm{crit}}$. Thus the present density ρ_0 is represented by

$$\Omega_0 = \rho_0/\rho_{0,\mathrm{crit}}$$

❒ findings from Cosmic Microwave Background (COBE, WMAP, PLANCK)

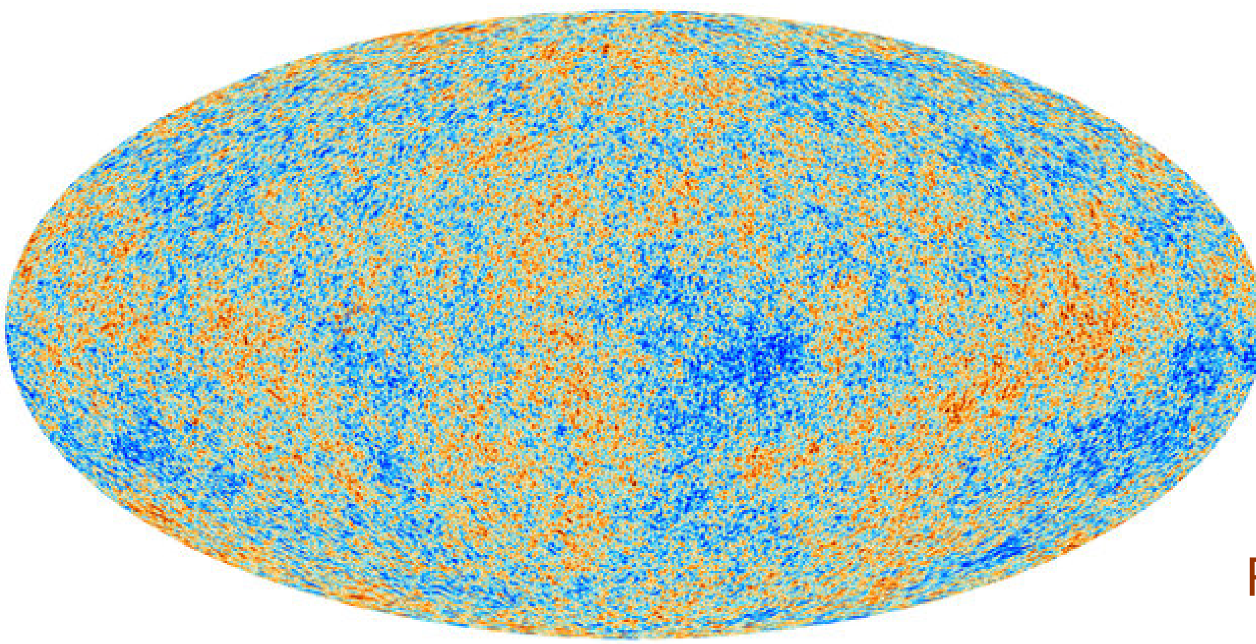


❒ the universe is flat! $\boxed{\Omega_0 \approx 1}$. How to get this for any $k = \pm 1, 0$? ⇒inflation

The Cosmic Microwave Background

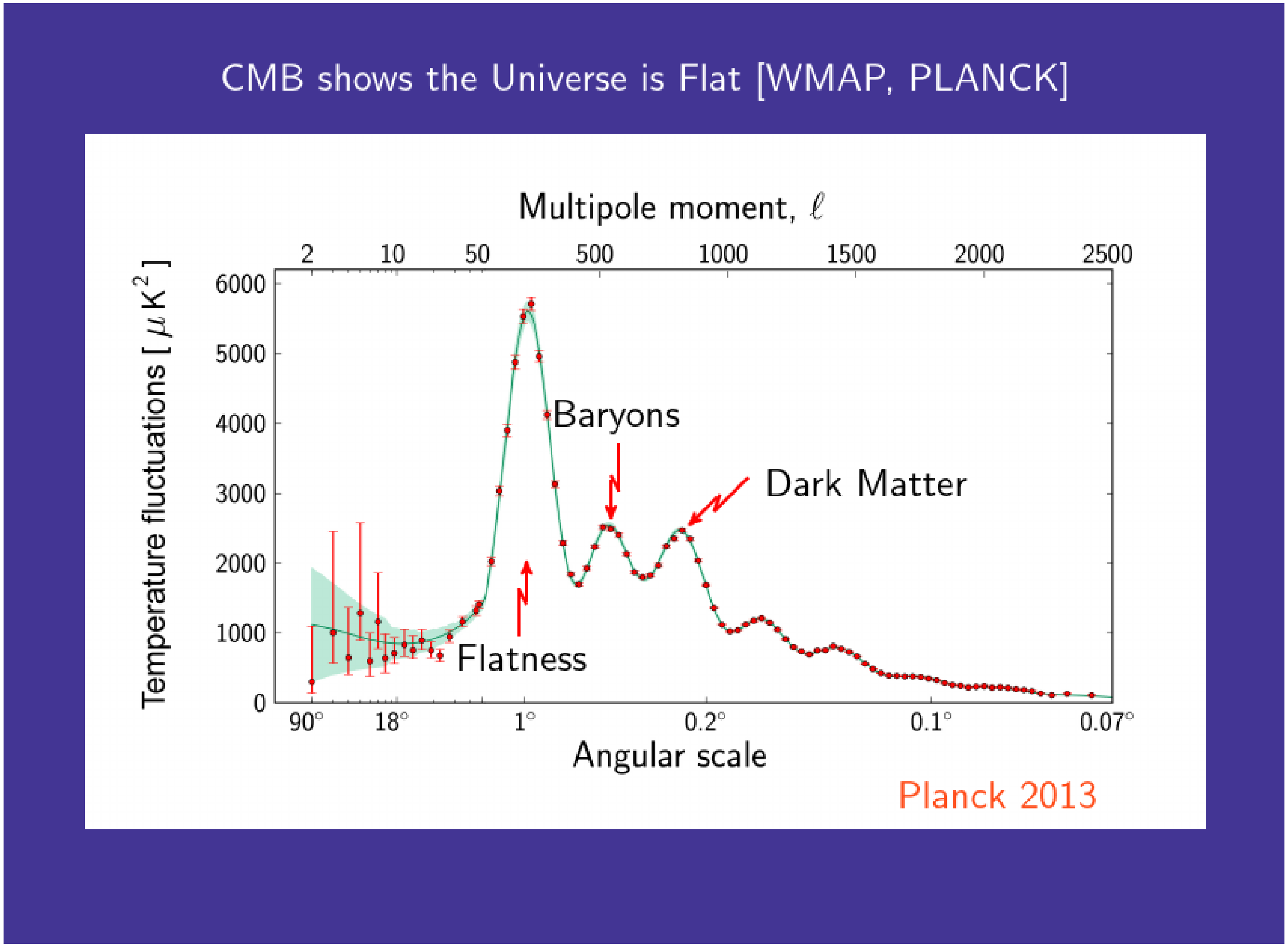
Cosmic black-body radiation of $3\ ^\circ\mathrm{K}$ Penzias, Wilson 1965, NP 1978

❒ The CMB fluctuation pattern: imprinted on the sky when the universe was just 380 000 years (after B.B.) old. Photons red-shifted by the expansion until the cannot ionize atoms (Hydrogen) any longer (snapshot of surface of last scattering). Smoot, Mather, NP 2006



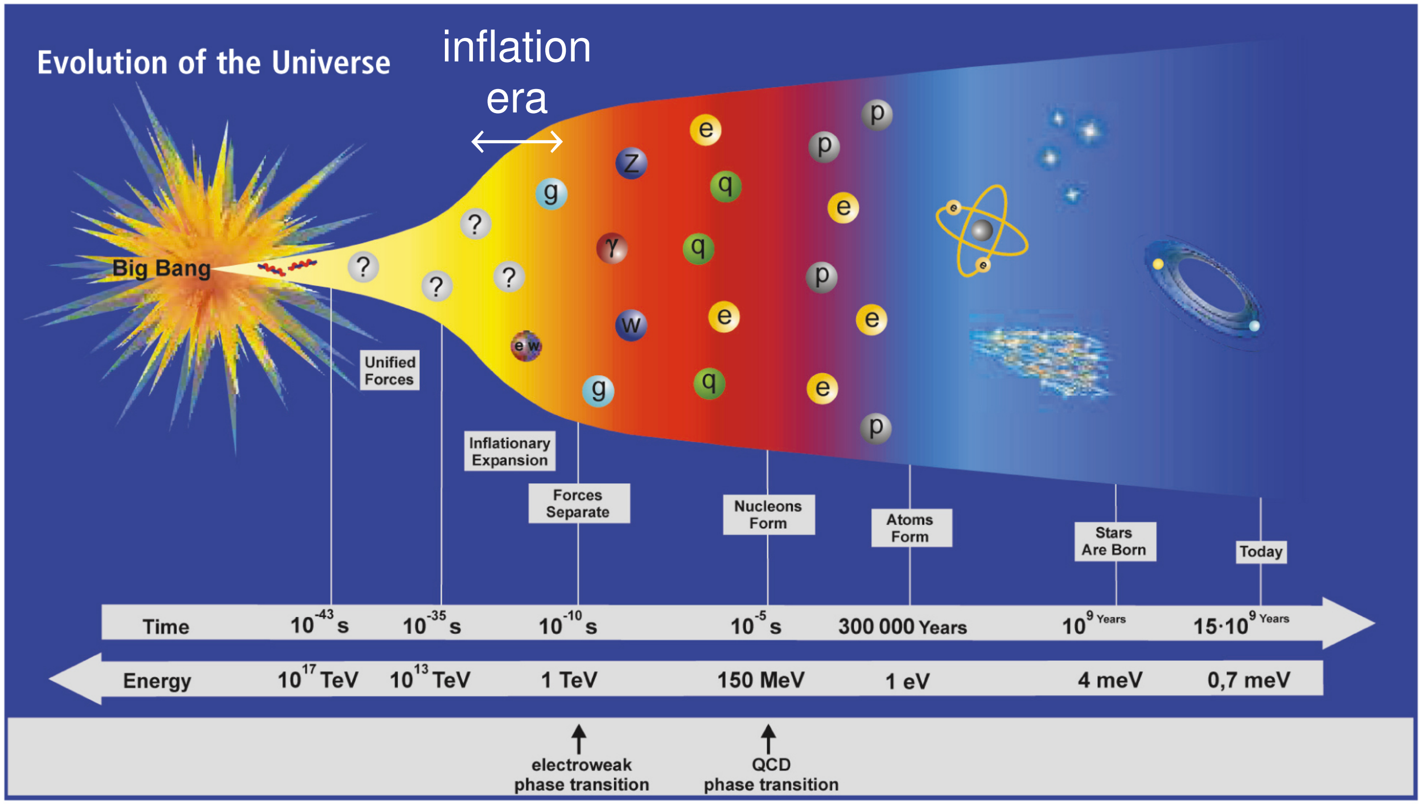
PLANCK 2013

❐ The power spectrum of CMB noise: (the acoustic peaks)



Inflation

Need inflation! universe must blow up exponentially for a very short period, such that we see it to be flat! [switch on anti-gravity for very short period of time]



Solves:

- Flatness problem i.e. why $\Omega \approx 1$ (although unstable) ?
- Horizon problem finite age t of universe, finite speed of light c: $D_{\rm Hor} = c\,t$ what we can see at most?

  CMB sky much larger [$d_{t_{\rm CMB}} \simeq 4 \cdot 10^7\ \ell{\rm y}$] than causally connected patch [$D_{\rm CMB} \simeq 4 \cdot 10^5\ \ell{\rm y}$] at $t_{\rm CMB}$ (380 000 yrs), but no such spot shadow seen!

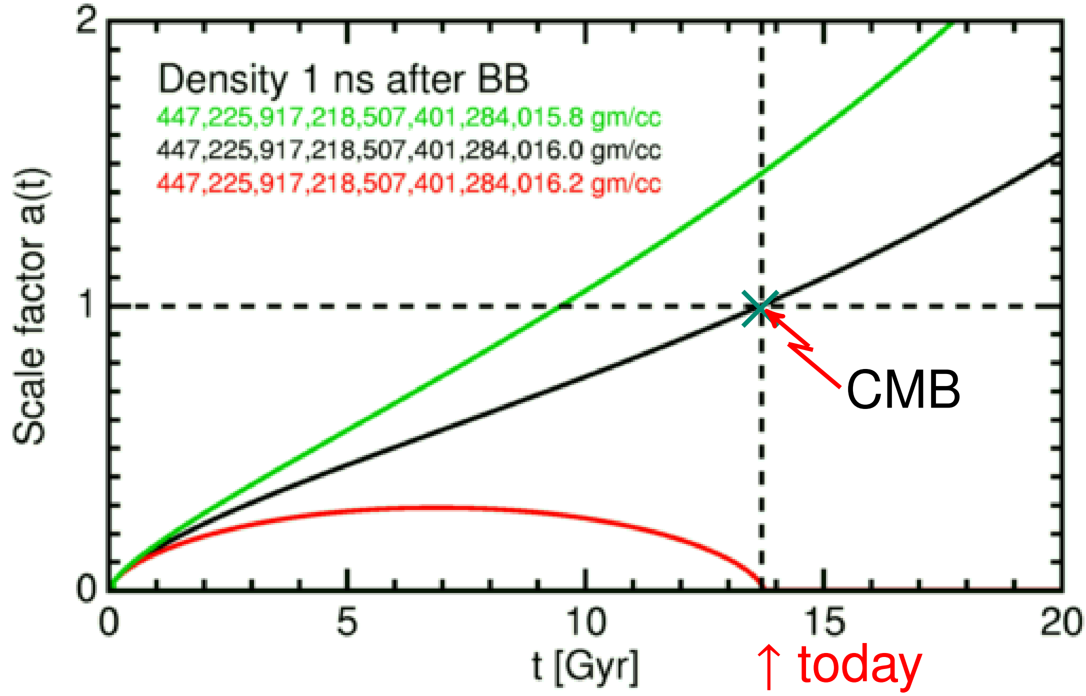
More general: what does it mean homogeneous or isotropic for causally disconnected parts of the universe? Initial value problem required initial data on space-like plane. Data on space-like plane are causally uncorrelated!

- Problem of fluctuations magnitude, various components (dark matter, baryons, photons, neutrinos) related: same fractional perturbations
  $\Rightarrow$Planck lenght $\ell_{\rm Pl}$ sized quantum fluctuations at Planck time?

As we will see: - $\Omega = 1$ unstable only if not sufficient dark energy!
- dark energy is provided by SM Higgs via $\kappa\, T_{\mu\nu}$
- no extra cosmological constant $+\Lambda\, g_{\mu\nu}$ supplementing $G_{\mu\nu}$
- i.e. all is standard GRT + SM (with minimal UV completion)

$$T^{\rm tot}_{\mu\nu} = T^{\rm SM}_{\mu\nu}$$

Flatness problem: observed today: (COBE, WMAP, PLANCK) $\Omega_{\rm tot} = 1.02 \pm 0.02$



Flat space unstable against perturbations (if Ω_Λ absent): shown here initial data agreeing to 24 digits! CMB data say we are living in flat space!

$$\frac{|\Omega_{\rm tot}(t)-1|_{\rm Pl}}{|\Omega_{\rm tot}(t)-1|_0} = \frac{a^2(t_{\rm Pl})}{a_0^2} \simeq \frac{T_0^2}{T_{\rm Pl}^2} \sim \mathcal{O}(10^{60})$$

Inflation at Work

Flatness, Causality, primordial Fluctuations $\Rightarrow$Solution: Guth 1980

Inflate the universe

Add an "Inflation term" to the r.h.s of the Friedmann equation, which dominates the very early universe blowing it up such that it looks flat afterwards

Need scalar field $\phi(x)$ $\equiv$ "inflaton" : $\Rightarrow$inflation term $\boxed{\frac{8\pi}{3\,M_{\mathrm{Pl}}^2}\left(V(\phi)+\frac{1}{2}\,\dot{\phi}^2\right)}$

Means: switch on strong anti-gravitation for an instant [sounds crazy]

Inflation: $a(t) \propto \mathrm{e}^{Ht}\,;\; H = H(t) \equiv \dot{a}(t)/a(t)$ Hubble "constant", i.e. $\frac{\mathrm{d}a}{a} = H(t)\,\mathrm{d}t$

➠ $\| \; N \equiv \ln \frac{a_{\mathrm{end}}}{a_{\mathrm{initial}}} = H\,(t_e - t_i)$ automatic iff $\boxed{V(\phi) \gg \dot{\phi}^2}$! slow roll!

"flattenization" by inflation: curvature term $k/a^2(t) \sim k\,\exp(-2Ht) \to 0$ ($k = 0, \pm 1$ the normalized curvature)

SM Higgs as inflaton?

Energy-momentum tensor of SM $T_{\mu\nu} \hat{=} \Theta_{\mu\nu} = V(\phi)\, g_{\mu\nu} +$ derivative terms

$$\boxed{\rho_\phi = \frac{1}{2}\,\dot{\phi}^2 + V(\phi) \;\; ; \;\; p_\phi = \frac{1}{2}\,\dot{\phi}^2 - V(\phi)}$$

- Substitute energy density and pressure into Friedmann and fluid equation
- Expansion when potential term dominates

$$\boxed{\ddot{a} > 0 \Longleftrightarrow p < -\frac{\rho}{3} \Longleftrightarrow \dot{\phi}^2 < V(\phi)}$$

Equation of state: $w = \frac{p}{\rho} = \frac{\frac{1}{2}\,\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\,\dot{\phi}^2 + V(\phi)}$ is $\boxed{V(\phi) \gg \dot{\phi}^2}$?

- small kinetic energy ➠ $w \to -1$ is dark energy $p_\phi = -\rho_\phi < 0$!
  indeed Planck (2013) finds $w = -1.13^{+0.13}_{-0.10}$.

Friedmann equation: $H^2 = \frac{8\pi G_N}{3}\left[V(\phi) + \frac{1}{2}\,\dot{\phi}^2\right] \Rightarrow H^2 \simeq \frac{8\pi G_N}{3}\, V(\phi)$

Field equation: $\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) \Rightarrow 3H\dot{\phi} \simeq -V'(\phi)$, for $V(\phi) \approx \frac{m^2}{2}\,\phi^2$ harmonic oscillator with friction $\Rightarrow$Gaussian inflation (Planck 2013)

$$N \equiv \ln \frac{a(t_{\mathrm{end}})}{a(t_{\mathrm{initial}})} = \int_{t_i}^{t_e} H(t)\mathrm{d}t \simeq -\frac{8\pi}{M_{\mathrm{Pl}}^2} \int_{\phi_i}^{\phi_e} \frac{V}{V'} \mathrm{d}\phi$$

fixed entirely by scalar potential

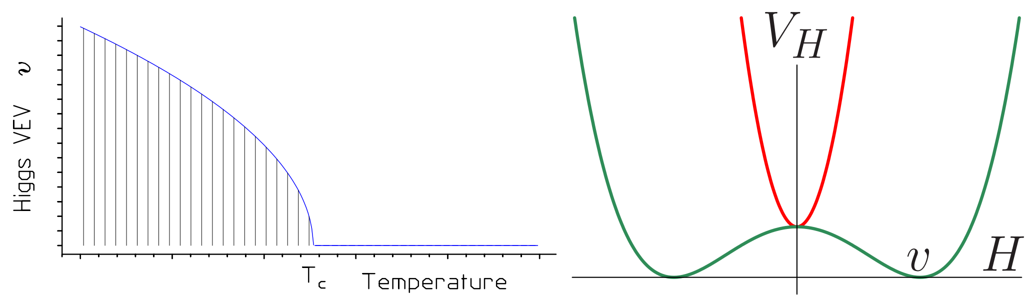
- need $N \gtrsim 60$, so called e-folds (CMB causal cone)

Key object of our interest: the Higgs potential

$$\boxed{V = \frac{m^2}{2} H^2 + \frac{\lambda}{24} H^4}$$

- Higgs mechanism

- when m^2 changes sign and λ stays positive $\Rightarrow$first order phase transition
- vacuum jumps from $v = 0$ to $v \neq 0$



Summary part I:

- Inflation is established by observation (Flatness, Primordial Fluctuation etc)
- SM Higgs particle is ideal candidate for the Inflaton and dark energy

Key questions:

- does SM Higgs potential satisfy slow roll condition?
- does the SM provide sufficient amount of inflation?

Key problem:

- renormalized SM Higgs potential established at low energy cannot trigger inflation!

Therefore: standard opinion Higgs cannot be the inflaton
(Guth 1980 originally suggested the Higgs to be the inflaton!)

Standard paradigm:

- renormalizability is fundamental principle, only renormalized SM is physical
- symmetries if broken are broken spontaneously
- the higher the energy the more symmetry (SUSY, GUT, Strings)
- hierarchy problem requires SUSY, extra dimensions, little Higgs, ETC, etc

Quantum Field Theory, Regularization and Renormalization

$\boxed{\text{Special relativity + quantum mechanics = relativistic QFT}}$

One crucial point: necessarily predicts infinities for non-free case! Loops!
⇒Regularization! well defined system requires cutoff!, e.g. lattice QCD, lattice SM underlying true system? defines theory beyond perturbation theory. After renormalization limit $\Lambda \gg E$ devoid of cutoff effects! i.e. UV cutoff effects renormalized away. Lattice SM as a representative of the SM universality class! True (real world) UV completion is unknown! $\boxed{\text{The Ether!}}$

- Infinities in Physics are the result of idealizations and show up as singularities in formalisms or models. A closer look usually reveals infinities to parametrize our ignorance or mark the limitations of our understanding or knowledge.

- "New" scenario of the Standard Model (SM) of elementary particles: ultraviolet singularities which plague the precise definition as well as concrete calculations in quantum field theories are associated with a physical cutoff, represented by the Planck length.

❒ Infinities are replaced by eventually very large but finite numbers, such huge effects may be needed in describing reality. Our example is huge dark energy triggering inflation of the early universe.

Limiting scales from the basic fundamental constants: $c, \hbar, G_N$

⇒Relativity and Quantum physics married with Gravity yield

Planck length: $\ell_{\rm Pl} = \sqrt{\frac{\hbar G_N}{c^3}} = 1.616252(81) \times 10^{-33}$ cm

Planck time: $t_{\rm Pl} = \ell_{\rm Pl}/c = 5.4 \times 10^{-44}$ sec

Planck (energy) scale: $M_{\rm Pl} = \sqrt{\frac{c\hbar}{G_N}} = 1.22 \times 10^{19}$ GeV

Planck temperature: $\frac{M_{\rm Pl} c^2}{k_{\rm B}} = \sqrt{\frac{\hbar c^5}{G_N k_{\rm B}^2}} = 1.416786(71) \times 10^{32}\ ^\circ{\rm K}$

- shortest distance $\ell_{\rm Pl}$ and beginning of time $t_{\rm Pl}$, $\boxed{t_{\rm Pl} < t}$
  the Planck epoch would have occurred instantly after the Big Bang!
- highest energy $E_{\rm Pl} = \Lambda_{\rm Pl} \equiv M_{\rm Pl}$ and temperature $T_{\rm Pl}$

Emergence Paradigm and UV completion (the LEESM)

The SM is a low energy effective theory of a unknown Planck medium [the "ether"], which exhibits the Planck energy as a physical cutoff: i.e. the SM emerges from a system shaped by gravitation

$$\boxed{\Lambda_{\rm Pl} = (G_N)^{-1/2} \simeq 1.22 \times 10^{19} \text{ GeV}}$$

G_N Newton's gravitational constant

- SM works up to Planck scale, means that in makes sense to consider the SM as the Planck medium seen from far away i.e. the SM is emergent at low energies. Expand in $E/\Lambda_{\rm Pl} \Rightarrow$ see renormalizable tail only.
- looking at shorter and shorter distances (higher energies) we can see the bare Planck system as it was evolving from the Big Bang! Energy Scan!
- the tool for accessing early cosmology is the RG solution of SM parameters: we can calculate the bare parameters from the renormalized ones determined at low (accelerator) energies.

❒ In the symmetric phase at very high energy we see the bare system:

the Higgs field is a collective field exhibiting an effective mass generated by radiative effects

$$m^2_{\rm bare} \approx \delta m^2 \text{ at } M_{\rm Pl}$$

eliminates fine-tuning problem at all scales!

Many examples in condensed matter systems, Coleman-Weinberg mechanism

❒ "free lunch" in Low Energy Effective SM (LEESM) scenario:

- renormalizability of long range tail automatic!
- so are all ingredients required by renormalizability:
- non-Abelian gauge symmetries, anomaly cancellation, fermion families etc
- last but not least the existence of the Higgs boson!

The low energy expansion at a glance

	dimension	operator	scaling behavior
hidden world	$\cdot$	∞–many	
	$\cdot$	irrelevant	
$\uparrow$	$\cdot$	operators	
no			
data	$d=6$	$(\Box\phi)^2, (\bar{\psi}\psi)^2, \cdots$	$(E/\Lambda_{\rm Pl})^2$
$\vert$	$d=5$	$\bar{\psi}\sigma^{\mu\nu}F_{\mu\nu}\psi, \cdots$	$(E/\Lambda_{\rm Pl})$
world as seen			tamed by symmetries
$\vert$	$d=4$	$(\partial\phi)^2, \phi^4, (F_{\mu\nu})^2, \cdots$	$\ln(E/\Lambda_{\rm Pl})$
experimental	$d=3$	$\phi^3, \bar{\psi}\psi$	$(\Lambda_{\rm Pl}/E)$
data	$d=2$	$\phi^2, (A_\mu)^2$	$(\Lambda_{\rm Pl}/E)^2$
$\downarrow$	$d=1$	ϕ	$(\Lambda_{\rm Pl}/E)^3$

Note: d=6 operators at LHC suppressed by $(E_{\rm LHC}/\Lambda_{\rm Pl})^2 \approx 10^{-30}$

$\Rightarrow$ require chiral symmetry, gauge symmetry, supersymmetry???

The Standard Model up to the Planck scale

The Cosmic Bridge

Universe is expanding: began in a very hot and dense state!

At Start a Light-Flash: **BIG BANG!**

Light quanta very energetic, all matter totally ionized, all nuclei disintegrated.

Elementary particles only!: $\gamma, e^+, e^-, p, \bar{p}, \cdots$

Early cosmology is Particle Physics!

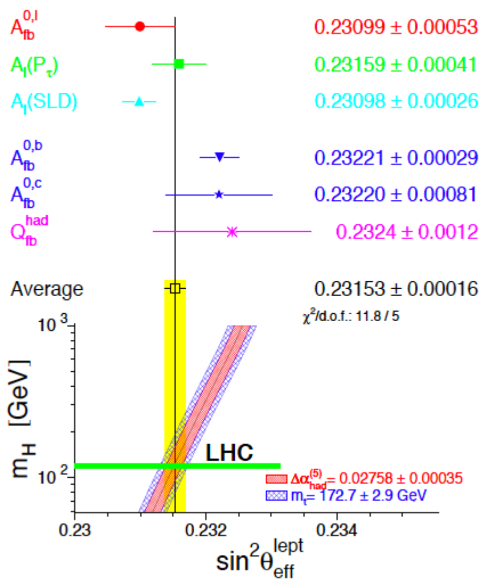
LEP type processes $e^+e^- \leftrightarrow \gamma^* \leftrightarrow X\bar{X}$ new forms of matter

Energy scale $\leftrightarrow$ Temperature $\leftrightarrow$ cosmic Time

$$E = 2\, M_X\, c^2 \Leftrightarrow T = E/k_B\ ^\circ\mathrm{K} \Leftrightarrow t = \frac{2.4}{\sqrt{g^*(T)}} \left(\frac{1MeV}{k_B T}\right)^2 \text{ sec. after B.B.}$$

✌ LHC ATLAS&CMS Higgs discovered ⇒ the SM completion

Higgs mass found by ATLAS and CMS agrees perfectly with the indirect bounds



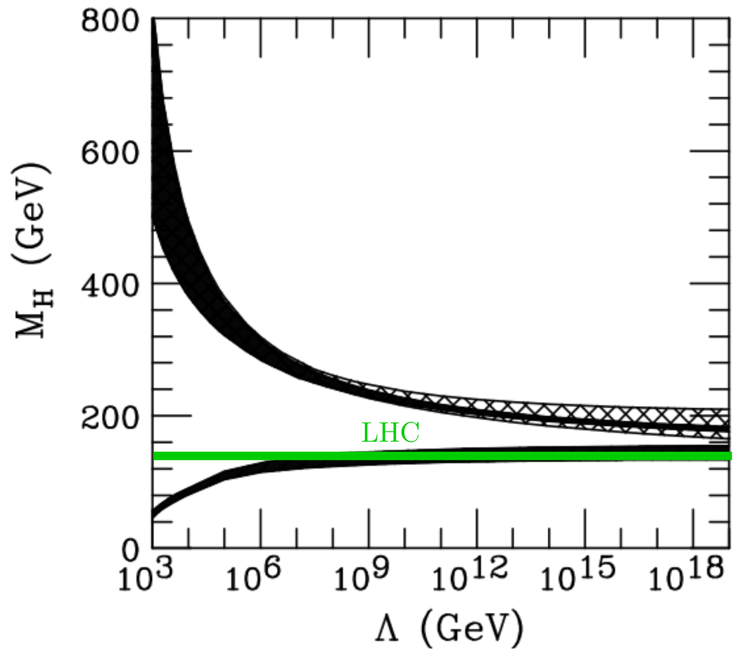
LEP 2005 +++ LHC 2012

Englert&Higgs Nobel Prize 2013

Higgs mass found in very special mass range 125.9 ± 0.4 GeV

Common Folklore: SM hierarchy problem requires a supersymmetric (SUSY) extension of the SM (no quadratic/quartic divergences) SUSY = infinity killer!

Do we really need new physics? Stability bound of Higgs potential in SM:



$$V = \frac{m^2}{2} H^2 + \frac{\lambda}{24} H^4$$

Riesselmann, Hambye 1996

$M_H < 180$ GeV

– first 2-loop analysis, knowing M_t –

SM Higgs remains perturbative up to scale $\Lambda_{\rm Pl}$ if it is light enough (upper bound=avoiding Landau pole) and Higgs potential remains stable ($\lambda > 0$) if Higgs mass is not too light [parameters used: $m_t = 175[150 - 200]$ GeV ; $\alpha_s = 0.118$]

SM – Fermions: 28 per family $\Rightarrow$ 3x28=84 ; Gauge-Bosons: 1+3+8=12 ; Scalars: 1 Higgs
Photon massless, gluons massless but confined

Before Higgs mechanism (triggering EW phase transition):

SM in symmetric phase : $W^\pm, Z$ and all fermions massless

Higgs "ghosts" $\phi^\pm, \phi^0$ physical, heavy degenerate with the Higgs!

At "low" energy [likely up to 10^{16} GeV]:

$$\boxed{V = \frac{m^2}{2} H^2 + \frac{\lambda}{24} H^4 \; ; \quad m^2 = -\mu^2 < 0}$$

SM in broken phase : H, $W^\pm, Z$ and all fermions massive [each mass requires separate new interaction via the Higgs: 2+12+1 decay channels];
3 Higgs "ghosts" $\phi^\pm, \phi^0$ disappear and transmute into longitudinal DOFs of $W^\pm, Z$

Basic parameters: gauge couplings $g' = g_1$, $g = g_2$, g_3, top quark Yukawa coupling y_t, Higgs self-coupling λ and Higgs VEV v, besides smaller Yukawas.
Note: $1/(\sqrt{2}v^2) = G_F$ is the Fermi constant! [$v = \left(\sqrt{2}G_F\right)^{-1/2}$]

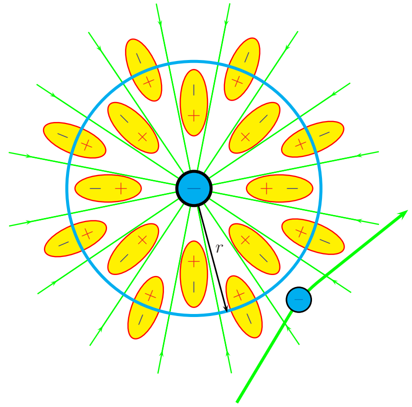
SSB $\Rightarrow$ mass $\propto$ interaction strength $\times$ Higgs VEV v

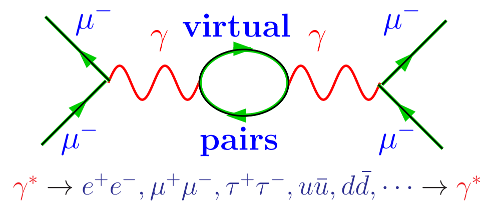
$$\begin{aligned} M_W^2 &= \tfrac{1}{4}\, g^2\, v^2\;; & M_Z^2 &= \tfrac{1}{4}\,(g^2+g'^2)\, v^2\;; \\ m_f^2 &= \tfrac{1}{2}\, y_f^2\, v^2\;; & M_H^2 &= \tfrac{1}{3}\,\lambda\, v^2 \end{aligned}$$

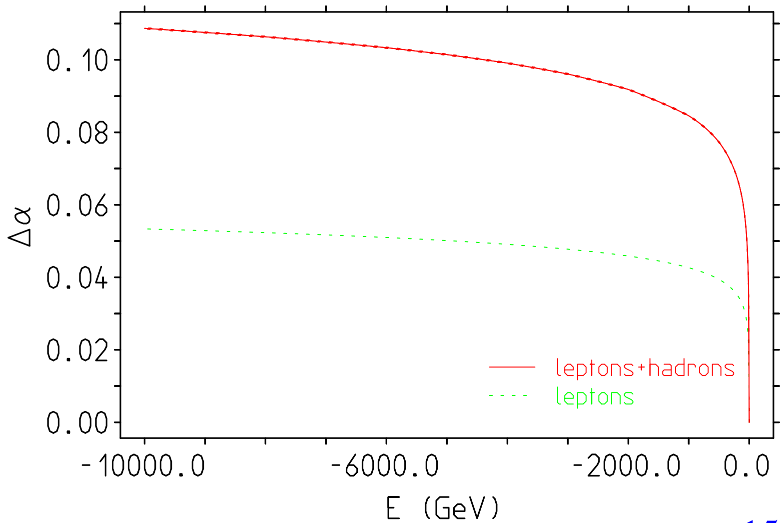
Effective parameters depend on renormalization scale μ [normalization reference energy!], scale at which ultraviolet (UV) singularities are subtracted

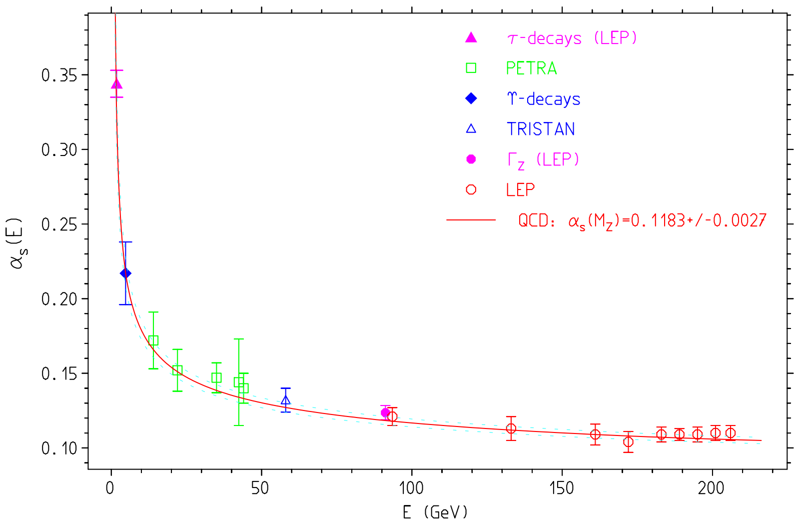
- running couplings change substantially with energy and hence as a function of time during evolution of the universe!
- high energy behavior governed by $\overline{\mathrm{MS}}$ Renormalization Group (RG) [$E \gg M_i$]
- key input matching conditions between $\overline{\mathrm{MS}}$ and physical parameters !
- running well established for electromagnetic α_{em} and strong coupling α_s: α_{em} screening, α_s anti-screening (Asymptotic Freedom)

The role of running couplings: $\alpha_{\rm em}$ screening, α_s anti-screening (Asymptotic Freedom)









$\uparrow\ t_{\rm LHC} \sim 1.66 \times 10^{-15}$ sec $\qquad \uparrow\ t_{\rm LEPI} \sim 2.58 \times 10^{-11}$ sec

$\mu \leftrightarrow$ energy scale $E = \sqrt{s} \leftrightarrow$ center of mass energy of a physical process

Asked questions:

- does SM physics extend up to the Planck scale?
- do we need new physics beyond the SM to understand the early universe?
- does the SM collapse if there is no new physics?

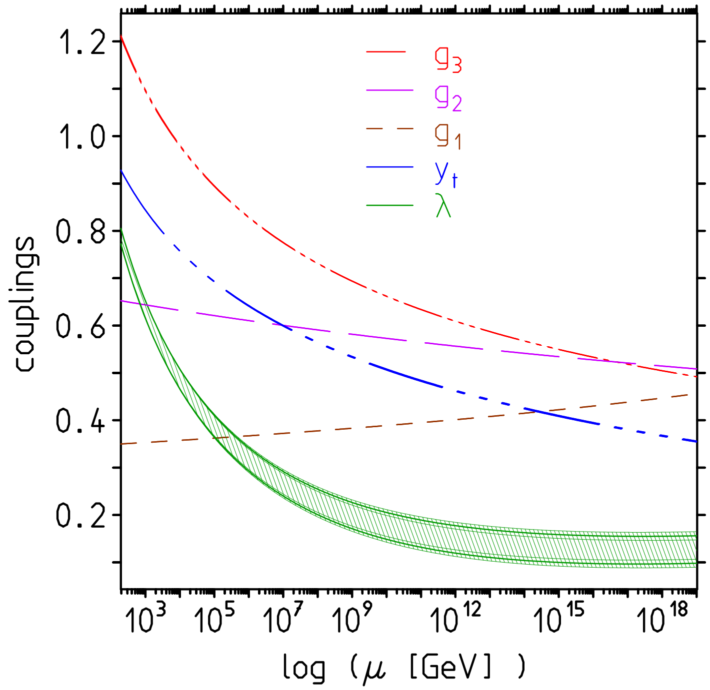
"collapse": Higgs potential gets unstable below the Planck scale; actually several groups claim to have proven vacuum stability break down!
Shaposhnikov et al, Degrassi et al, Maina, Hamada et al, ...

Scenario this talk: Higgs vacuum remains stable up and beyond the Planck scale
$\Rightarrow$seem to say we do not need new physics affecting the evolution of SM couplings to investigate properties of the early universe. In the focus:

- does Higgs self-coupling stay positive $\lambda > 0$ up to Λ_{Pl} ?
- the key question/problem concerns the size of the top Yukawa coupling y_t decides about stability of our world! — [$\lambda = 0$ would be essential singularity!]

Will be decided by:
- more precise input parameters
- better established EW matching conditions

The SM running parameters



The SM dimensionless couplings in the $\overline{\mathrm{MS}}$ scheme as a function of the renormalization scale for $M_H = 124 - 127$ GeV.

- perturbation expansion works up to the Planck scale!

  no Landau pole or other singularities $\Rightarrow$ Higgs potential remains stable!

- $U(1)_Y$ screening (IR free), $SU(2)_L$, $SU(3)_c$ antiscreening (UV free) [asymptotic freedom (AF)] – g_1, g_2, g_3

Right – as expected (standard wisdom)

- Top Yukawa y_t and Higgs λ: screening (IR free, like QED)

Wrong!!! – as part of SM transmutation from IR free to AF

- running top Yukawa – QCD takes over: IR free ⇒UV free

- running Higgs self-coupling – top Yukawa takes over: IR free ⇒UV free

- Higgs coupling decreases up to the zero of β_λ at $\mu_\lambda \sim 3.5 \times 10^{17}$ GeV, where it is small but still positive and then increases up to $\mu = \Lambda_{\mathrm{Pl}}$

The Higgs is special: before the symmetry is broken: all particles massless protected by gauge or chiral symmetry except the four Higgses. Two quantities affected: Higgs boson mass and Higgs vacuum energy

The SM's naturalness problems and fine-tuning problems

Issue broached by 't Hooft 1979 as a relationship between macroscopic phenomena which follow from microscopic physics (condensed matter inspired), i.e., bare versus renormalized quantities. Immediately the "hierarchy problem" has been dogmatized as a kind of fundamental principle.

Assume Planck scale $\Lambda_{\rm Pl} \simeq 1.22 \times 10^{19}$ GeV as a UV cutoff regularization:

❒ the Higgs mass: [note bare parameters parametrize the true Lagrangian]

$$m^2_{\rm Higgs,\ bare} = m^2_{\rm Higgs,\ ren} + \delta m^2 \; ; \quad \delta m^2 = \frac{\Lambda^2_{\rm Pl}}{(16\pi^2)}\, C(\mu)$$

coefficient typically $C = O(1)$. To keep the renormalized mass at the observed small value $m_{\rm ren} = O(100$ GeV$)$, $\Rightarrow m^2_{\rm bare}$ has to be tuned to compensate the huge term δm^2: about 35 digits must be adjusted in order to get the observed value.

Hierarchy Problem!

❒ the vacuum energy density $\langle V(\phi)\rangle$:

$$\rho_{\text{vac, bare}} = \rho_{\text{vac, ren}} + \delta\rho \; ; \quad \delta\rho = \frac{\Lambda_{\text{Pl}}^4}{(16\pi^2)^2}\, X(\mu)$$

SM predicts huge cosmological constant (CC) at Λ_{Pl}:

$$\rho_{\text{vac, bare}} \simeq V(0) + \Delta V(\phi) \sim 2.77\, \Lambda_{\text{Pl}}^4 \sim (1.57 \times 10^{19}\ \text{GeV})^4 \ \text{ vs. } \rho_{\text{vac}} = (0.002\ \text{eV})^4 \text{ today}$$

Cosmological Constant Problem!

Note: in symmetric phase the only trouble maker is the Higgs!

Note: naive arguments do not take into account that quantities compared refer to very different scales! $m^2_{\text{Higgs, bare}}$ short distance, $m^2_{\text{Higgs, ren}}$ long distance observables. Also: Λ as a regulator nobody forces you to take it to be Λ_{Pl}.

Need: UV-completion of SM: prototype lattice SM as true(r) system

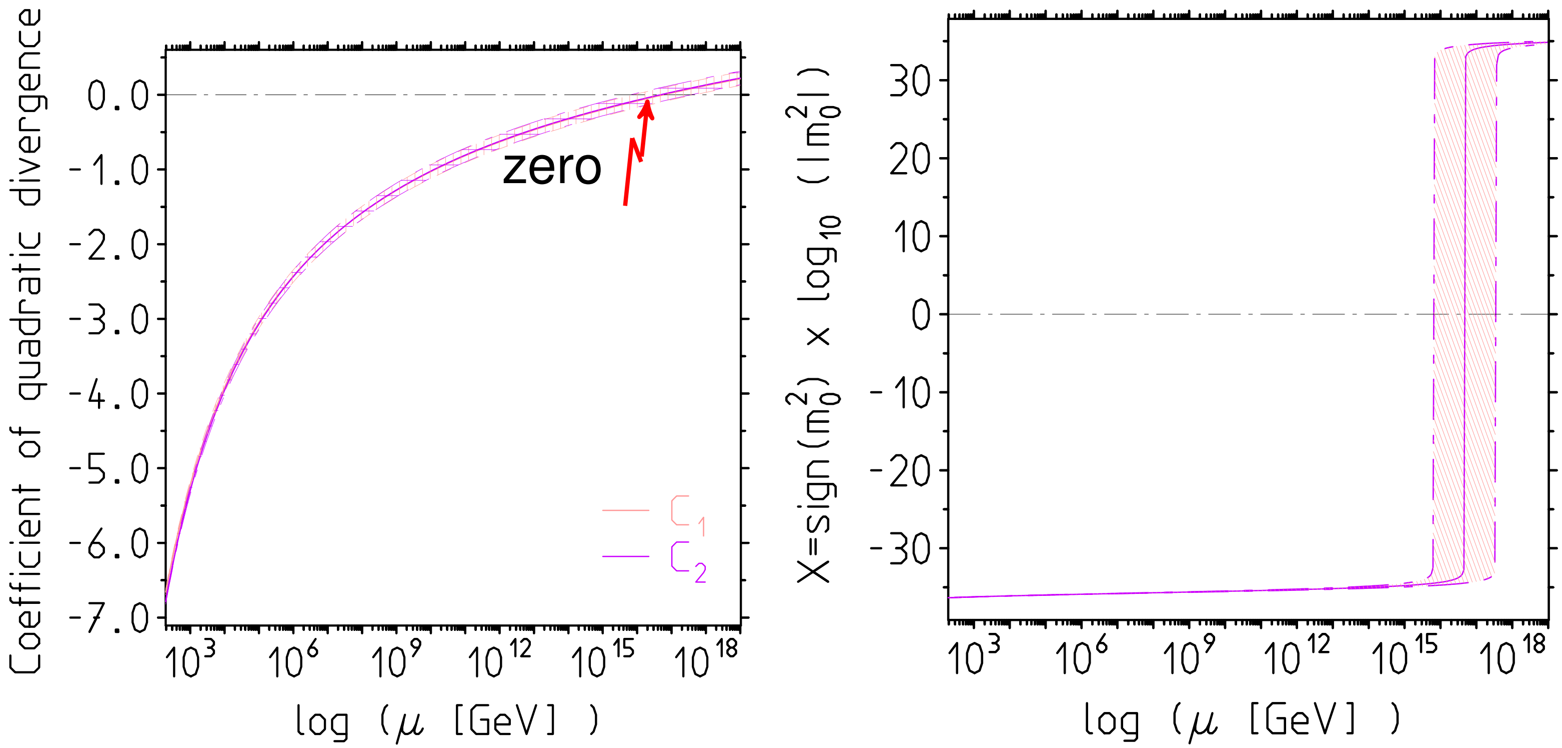
The Role of Quadratic Divergences in the SM

Veltman 1978 [NP 1999] modulo small lighter fermion contributions, one-loop coefficient function C_1 is given by

$$\delta m_H^2 = \frac{\Lambda_{\rm Pl}^2}{16\pi^2} C_1 \; ; \quad C_1 = \frac{6}{v^2}(M_H^2 + M_Z^2 + 2M_W^2 - 4M_t^2) = 2\,\lambda + \tfrac{3}{2}\,g'^2 + \tfrac{9}{2}\,g^2 - 12\,y_t^2$$

Key points:

- C_1 is universal and depends on dimensionless gauge, Yukawa and Higgs self-coupling only, the RGs of which are unambiguous. At two loops $C_2 \approx C_1$ numerically [Hamada et al 2013] stable under RCs!
- Couplings are running! $C_i = C_i(\mu)$
- the SM for the given running parameters makes a prediction for the bare effective mass parameter in the Higgs potential:



The Higgs phase transition in the SM [for $M_H = 125.9 \pm 0.4$ GeV].

$$\boxed{m^2_{\rm bare} = \text{sign}(m^2_{\rm bare}) \times 10^X}$$

Jump in vacuum energy: wrong sign and 50 orders of magnitude off $\Lambda_{\rm CMB}$!!!

$$\Delta V(\phi_0) = -\frac{m^2_{\rm eff} v^2}{8} = -\frac{\lambda v^4}{24} \sim -(176.0 \text{ GeV})^4$$

$\Rightarrow$one version of CC problem

- in the broken phase $m^2_{\text{bare}} = \frac{1}{2}\, m^2_{H\,\text{bare}}$, which is calculable!

- the coefficient $C_n(\mu)$ exhibits a zero, for $M_H = 126$ GeV at about $\mu_0 \sim 1.4 \times 10^{16}$ GeV, not far below $\mu = M_{\text{Planck}}$!!!

- at the zero of the coefficient function the counterterm $\delta m^2 = m^2_{\text{bare}} - m^2 = 0$ (m the $\overline{\text{MS}}$ mass) vanishes and the bare mass changes sign

- this represents a phase transition (PT), which triggers the Higgs mechanism as well as cosmic inflation as $\boxed{V(\phi) \gg \dot{\phi}^2}$

- at the transition point μ_0 we have $\boxed{v_{\text{bare}} = v(\mu_0^2)\ ; \quad m_{H\,\text{bare}} = m_H(\mu_0^2)}$, where $v(\mu^2)$ is the $\overline{\text{MS}}$ renormalized VEV

In any case at the zero of the coefficient function there is a phase transition, which corresponds to a restoration of the symmetry in the early universe.

Hot universe $\Rightarrow$finite temperature effects:

❒ finite temperature effective potential $V(\phi, T)$:

$\boxed{T \neq 0}$: $V(\phi, T) = \frac{1}{2}\left(g_T\, T^2 \; - \mu^2\right) \phi^2 + \frac{\lambda}{24}\, \phi^4 + \cdots$

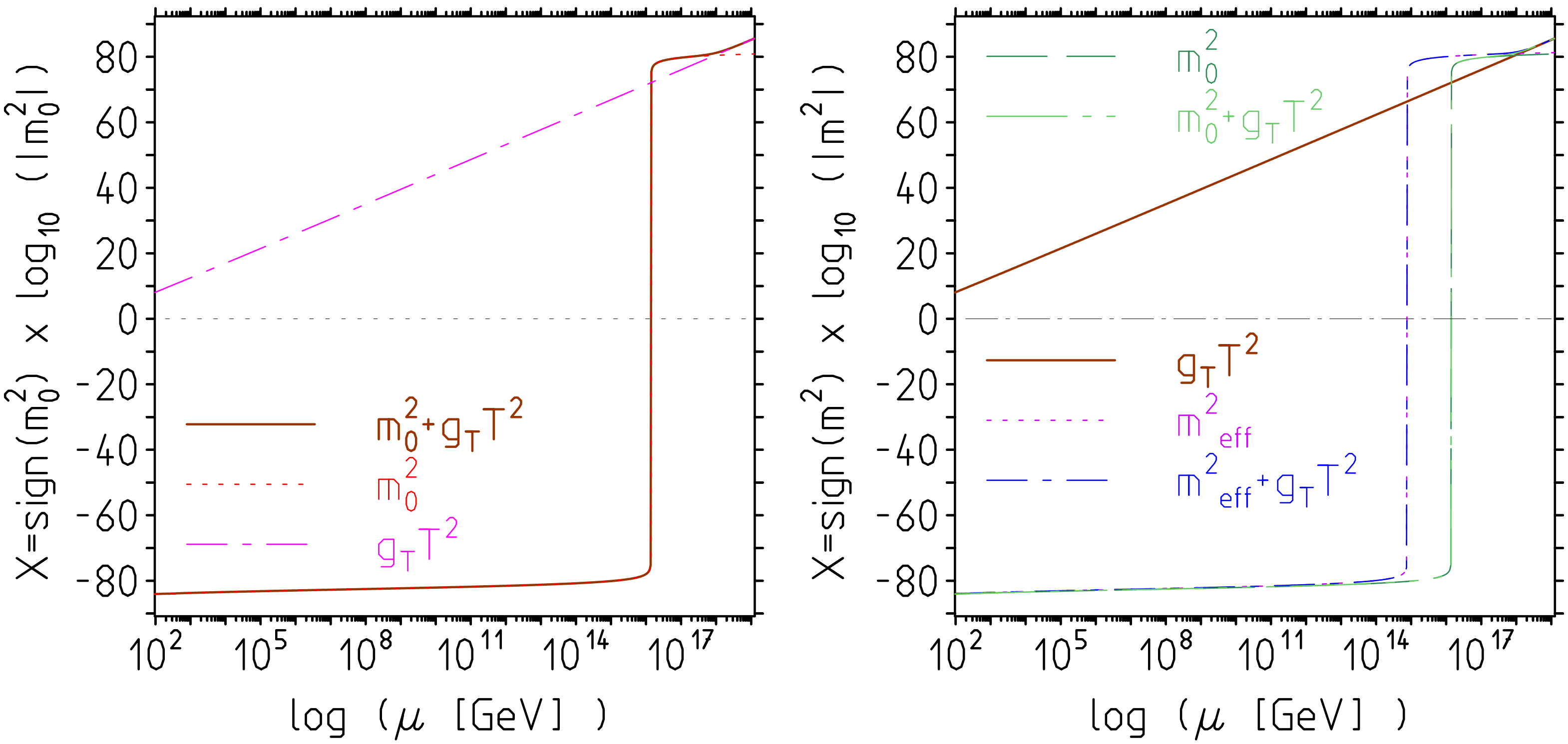
Usual assumption: Higgs is in the broken phase $\mu^2 > 0$ and $\mu \sim v$ at EW scale

EW phase transition is taking place when the universe is cooling down below the critical temperature $T_c = \sqrt{\mu^2/g_T}$.

My scenario: above PT at μ_0 SM in symmetric phase $-\mu^2 \to m^2 = (m_H^2 + \delta m_H^2)/2$

$$m^2 \sim \delta m^2 \simeq \frac{M_{\rm Pl}^2}{32\pi^2}\, C(\mu = M_{\rm Pl}) \simeq (0.0295\, M_{\rm Pl})^2 \;, \quad \text{or} \quad m^2(M_{\rm Pl})/M_{\rm Pl}^2 \approx 0.87 \times 10^{-3} \;.$$

In fact with our value of μ_0 almost no change of phase transition point by FT effects. True effective mass $m^2 \to m'^2$ from Wick ordered Lagrangian [$C \to C + \lambda$].



Effects on the phase transition by finite temperature and vacuum rearrangement

$$\mu_0 \approx 1.4 \times 10^{16}\ \text{GeV} \to \mu_0' \approx 7.7 \times 10^{14}\ \text{GeV}\,,$$

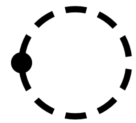
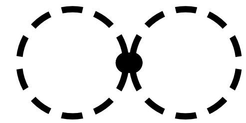
Up to shift in transition temperature PT is triggered by δm^2 and EW PT must be close by at about $\mu_0 \sim 10^{15}$ GeV not at EW scale $v \sim 246$ GeV!
Important for Baryogenesis!

The Cosmological Constant in the SM

- in symmetric phase $SU(2)$ is a symmetry: $\Phi \to -U(\omega)\Phi$ and $\Phi^+\Phi$ singlet;

$$\langle 0|\Phi^+\Phi|0\rangle = \tfrac{1}{2}\langle 0|H^2|0\rangle \equiv \tfrac{1}{2}\,\Xi\;; \quad \Xi = \frac{\Lambda_{\rm Pl}^2}{16\pi^2}\,.$$

just Higgs self-loops

$\langle H^2 \rangle =:$  ; $\langle H^4 \rangle = 3\left(\langle H^2 \rangle\right)^2 =:$ 

$\Rightarrow$ vacuum energy $V(0) = \langle V(\phi)\rangle = \frac{m^2}{2}\,\Xi + \frac{\lambda}{8}\,\Xi^2$; mass shift $m'^2 = m^2 + \frac{\lambda}{2}\,\Xi$

❐ for our values of the $\overline{\rm MS}$ input parameters $m^2 \to m'^2$

$\Rightarrow$ $$\mu_0 \approx 1.4\times 10^{16}\ \text{GeV} \to \mu_0' \approx 7.7\times 10^{14}\ \text{GeV}\,,$$

- potential of the fluctuation field $\Delta V(\phi)$.

$\Rightarrow$ quasi-constant vacuum density $V(0)$ representing $\boxed{\text{the cosmological constant}}$

$\Rightarrow$ $H \simeq \ell\,\sqrt{V(0)+\Delta V}$ at $M_{\rm Pl}$ we expect $\phi_0 = O(M_{\rm Pl})$ i.e. at start $\Delta V(\phi) \gg V(0)$

❒ fluctuation field eq. $3H\dot{\phi} \approx -(m'^2 + \frac{\lambda}{6}\,\phi^2)\,\phi$, ϕ decays exponentially, must have been very large in the early phase of inflation

● need $\phi_0 \approx 4.51 M_{\rm Pl}$, big enough to provide sufficient inflation. Note: this is the only free parameter in SM inflation, the Higgs field is not an observable in the renormalized low energy world (laboratory/accelerator physics).

Decay patterns:

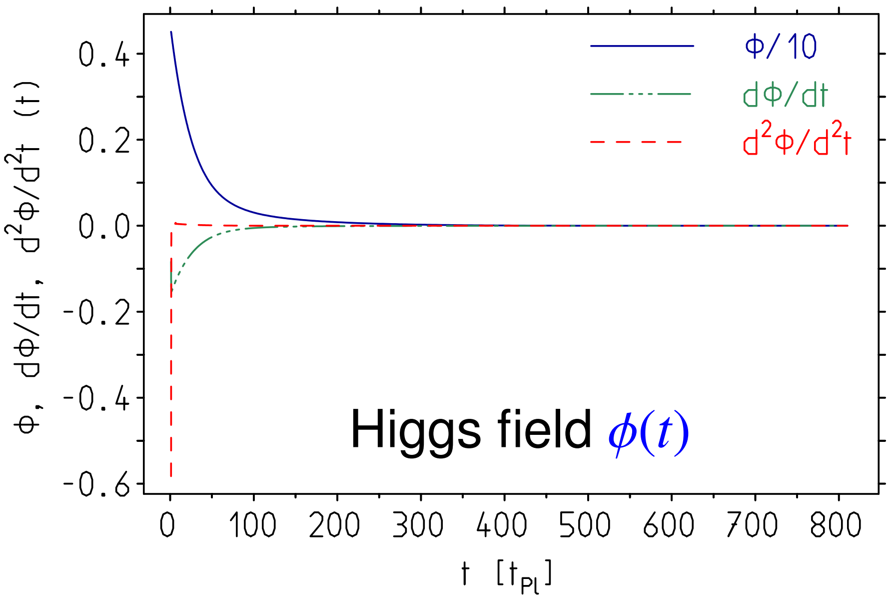
$$\phi(t) = \phi_0\,\exp\{-E_0\,(t-t_0)\}\,,\;\; E_0 \approx \frac{\sqrt{2\lambda}}{3\,\sqrt{3}}\,,\;\; \approx 4.3\times 10^{17}\ \text{GeV}\,,\;\; V_{\rm int} \gg V_{\rm mass}$$

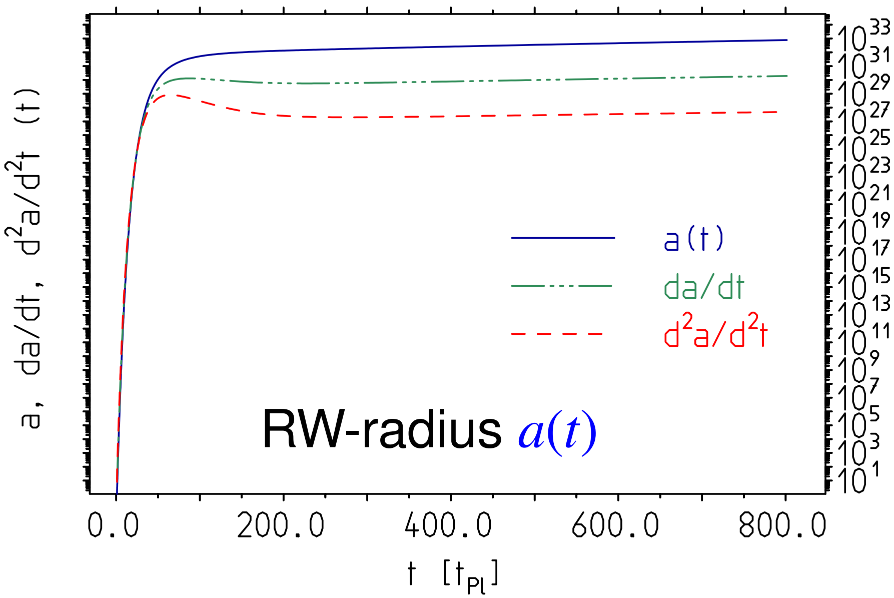
soon mass term dominates, in fact $V(0)$ and $V_{\rm mass}$ are comparable before $V(0)$ dominates and $H \approx \ell\,\sqrt{V(0)}$ and

$$\phi(t) = \phi_0\,\exp\{-E_0\,(t-t_0)\}\,,\;\; E \approx \frac{m^2}{3\ell\,\sqrt{V(0)}} \approx 6.6\times 10^{17}\ \text{GeV}\,,\;\; V_{\rm mass} \gg V_{\rm int}$$

Note: if no CC ($V(0) \approx 0$) as assumed usually

$$\phi(t) = \phi_0 - X_0\,(t-t_0)\,,\;\; X_0 \approx \frac{\sqrt{2}m}{3\ell} \approx 7.2\times 10^{35}\ \text{GeV}^2\,,\;\; V_{\rm mass} \gg V_{\rm int}$$





Note: the Hubble constant in our scenario, in the symmetric phase, during the radiation dominated era is given by (Stefan-Boltzmann law)

$$\boxed{H = \ell \sqrt{\rho_{\mathrm{rad}}} \simeq 1.66\ (k_B T)^2\ \sqrt{102.75}\ M_{\mathrm{Pl}}^{-1}}$$

such that at Planck time (SM predicted)

$$\boxed{H_i \simeq 16.83\ M_{\mathrm{Pl}}\ .}$$

i.e. trans-Planckian $\phi_0 \sim 5 M_{\mathrm{Pl}}$ is not unnatural!

How to get rid of the huge CC?

- $V(0)$ very weakly scale dependent (running couplings): how to get rid of?

Note total energy density as a function of time

$$\boxed{\rho(t) = \rho_{0,\mathrm{crit}} \left\{\Omega_\Lambda + \Omega_{0,\mathrm{k}} \left(a_0/a(t)\right)^2 + \Omega_{0,\mathrm{mat}} \left(a_0/a(t)\right)^3 + \Omega_{0,\mathrm{rad}} \left(a_0/a(t)\right)^4\right\}}$$

reflects a present-day snapshot. Cosmological constant is constant! Not quite!

- intriguing structure again: the effective CC counterterm has a zero, which again is a point where renormalized and bare quantities are in agreement:

$$\boxed{\rho_{\Lambda\,\mathrm{bare}} = \rho_{\Lambda\,\mathrm{ren}} + \frac{M_{\mathrm{Pl}}^4}{(16\pi^2)^2} X(\mu)}$$

with $X(\mu) \simeq 2C(\mu) + \lambda(\mu)$ which has a zero close to the zero of $C(\mu)$ when $2\,C(\mu) = -\lambda(\mu)$, which happens at

$$\mu_{\mathrm{CC}} \approx 3.1 \times 10^{15}\ \mathrm{GeV}$$

in between $\mu_0 \approx 1.4 \times 10^{16}$ GeV and $\mu_0' \approx 7.7 \times 10^{14}$ GeV .

Again we find a matching point between low energy and high energy world:

$$\boxed{\rho_{\Lambda\,\mathrm{bare}} = \rho_{\Lambda\,\mathrm{ren}}}$$

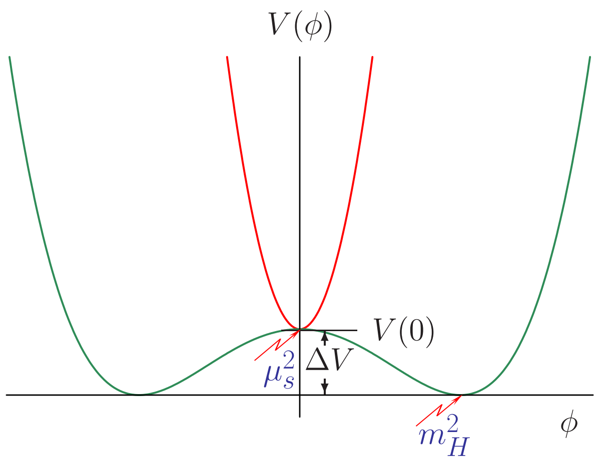
where memory of quartic Planck scale enhancement gets lost!

Has there been a cosmological constant problem?

Crucial point $X = 2C + \lambda = 5\,\lambda + 3\,g'^2 + 9\,g^2 - 24\,y_t^2$ acquires positive bosonic contribution and negative fermionic ones, with different scale dependence. X can change a lot (pass a zero), while individual couplings are weakly scale dependent $y_t(M_Z)/y_t(M_{\rm Pl}) \sim 2.7$ biggest, $g_1(M_Z)/g_1(M_{\rm Pl}) \sim 0.76$ smallest.

- SM predicts huge CC at $M_{\rm Pl}$: $\rho_\phi \simeq V(\phi) \sim 2.77\,M_{\rm Pl}^4 \sim \left(1.57 \times 10^{19}\ \text{GeV}\right)^4$ how to tame it?

At Higgs transition: $m'^2(\mu < \mu'_0) < 0$ vacuum rearrangement of Higgs potential



How can it be: $\boxed{V(0) + \Delta V \sim (0.002\ \text{eV})^4}$???
The zero $X(\mu_{\text{CC}}) = 0$ provides part of the answer as it makes $\rho_{\Lambda\,\text{bare}} = \rho_{\Lambda\,\text{ren}}$ to be identified with the observed value?
Seems to be naturally small, since Λ_{Pl}^4 term nullified at matching point.

Note: in principle, like the Higgs mass in the LEESM, also $\rho_{\Lambda\,\text{ren}}$ is expected to be a free parameter to be fixed by experiment.

Not quite! there is a big difference: inflation forces $\rho_{\rm tot}(t) \approx \rho_{0,\rm crit} = \mathrm{constant}$ after inflation era

$$\boxed{\Omega_{\rm tot} = \Omega_\Lambda + \Omega_{\rm mat} + \Omega_{\rm rad} = \Omega_\Lambda + \Omega_{0,\rm k}\,(a_0/a(t))^2 + \Omega_{0,\rm mat}\,(a_0/a(t))^3 + \Omega_{0,\rm rad}\,(a_0/a(t))^4 \approx 1}$$

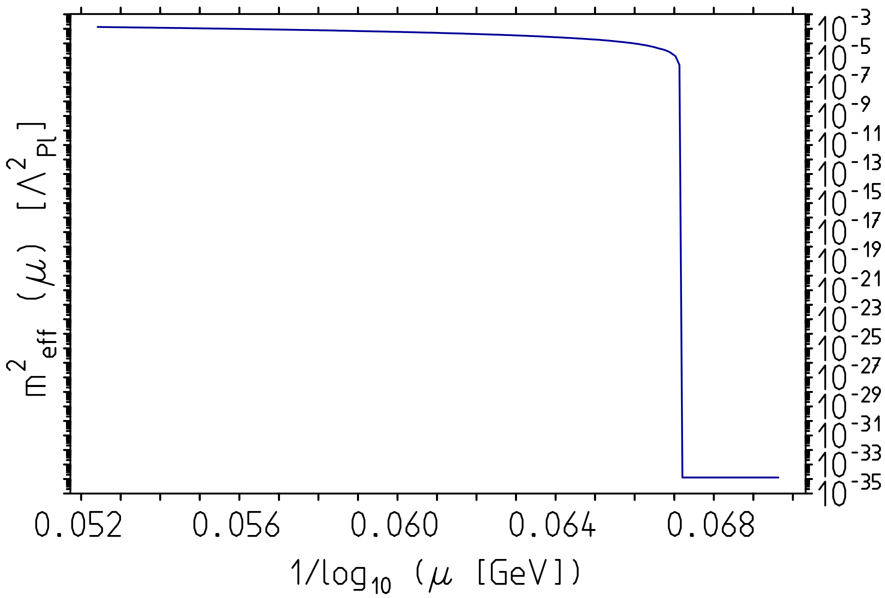
and since $1 > \Omega_{\rm mat},\ \Omega_{\rm rad} > 0$ actually Ω_Λ is fixed once we know dark matter, baryonic matter and the radiation density:

$$\boxed{\Omega_\Lambda = 1 - \Omega_{\rm mat} - \Omega_{\rm rad}}$$

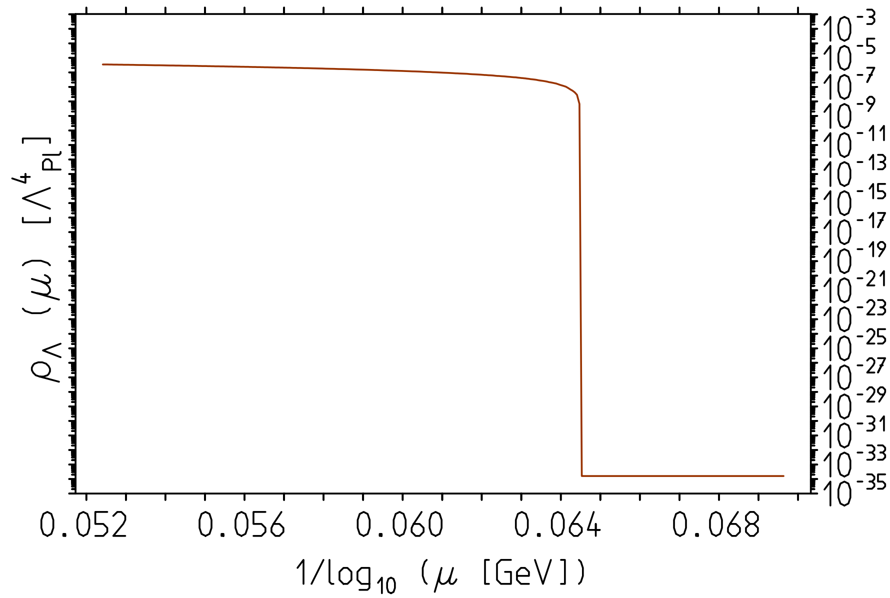
So, where is the miracle to have CC of the magnitude of the critical density of a flat universe? Also this then is a prediction of the LEESM!

Note that $\Omega_{\rm tot} = 1$ requires Ω_Λ to be a function of t, up to negligible terms,

$$\Omega_\Lambda \to \Omega_\Lambda(t) \approx 1 - (\Omega_{0,\rm dark\ mat} + \Omega_{0,\rm baryonic\ mat})\ (a_0/a(t))^3 \to 1\ ;\quad t \to \infty$$



effective Higgs mass square



effective dark energy density

in units of Λ_{Pl}, for $\mu < \mu_{\mathrm{CC}}$ we display $\rho_\Lambda[\mathrm{GeV}^4] \times 10^{13}$ as predicted by SM

$\rho_\Lambda = \mu_\Lambda^4$: $\mu_{0,\Lambda} = 0.002\ \mathrm{eV}$ today → approaching $\mu_{\infty,\Lambda} = 0.00216\ \mathrm{eV}$ with time

Remark: $\Omega_\Lambda(t)$ includes besides the large positive $V(0)$ also negative contributions from vacuum condensates, like $\Delta\Omega_{\mathrm{EW}}$ from the Higgs mechanism and $\Delta\Omega_{\mathrm{QCD}}$ from the chiral phase transition.

The Higgs Boson is the Inflaton!

- after electroweak PT, at the zeros of quadratic and quartic "divergences", memory of cutoff lost: renormalized low energy parameters match bare parameters

- in symmetric phase (early universe) bare effective mass and vacuum energy dramatically enhanced by quadratic and quartic cutoff effects

➠ slow-roll inflation condition $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$ satisfied

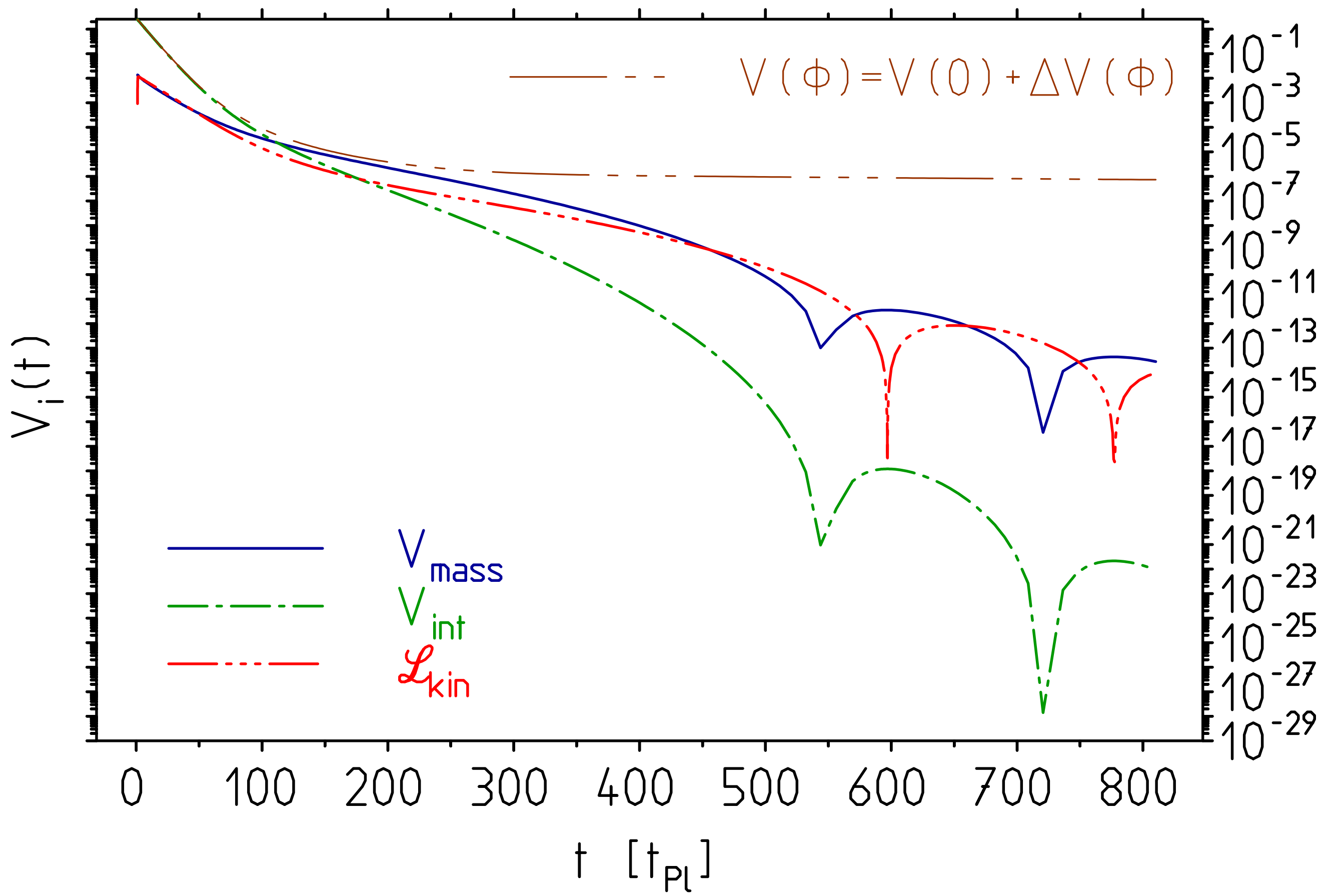
➠ Higgs potential provides huge dark energy in early universe which triggers inflation

The SM predicts dark energy and inflation!!!

dark energy and inflation are unavoidable consequences of the SM Higgs

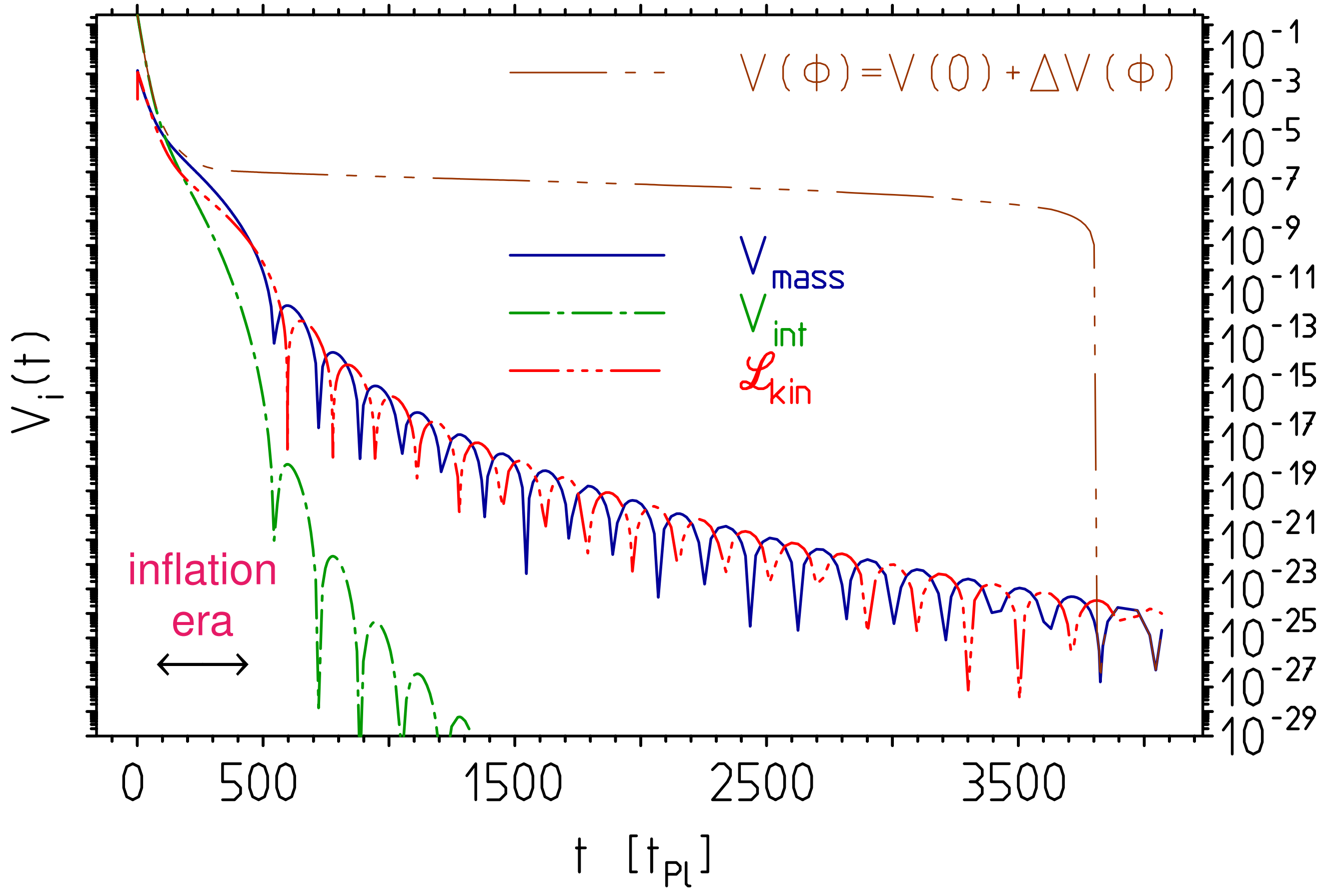
(provided new physics does not disturb it substantially)

The evolution of the universe before the EW phase transition:



Inflation epoch ($t \lesssim 450\, t_{\rm Pl}$): the mass-, interaction- and kinetic-term of the bare Lagrangian in units of $M_{\rm Pl}^4$ as a function of time.

The evolution of the universe before the EW phase transition:



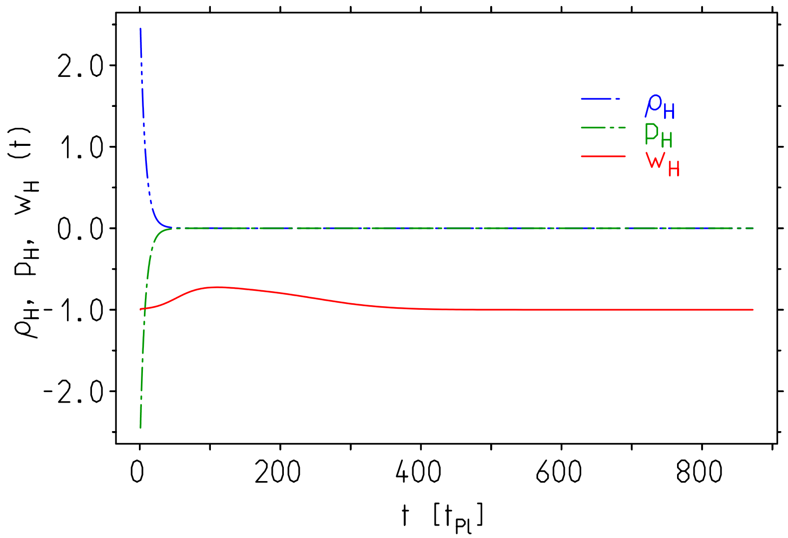
Evolution until symmetry breakdown and vanishing of the CC. After inflation quasi-free damped harmonic oscillator behavior (reheating phase).

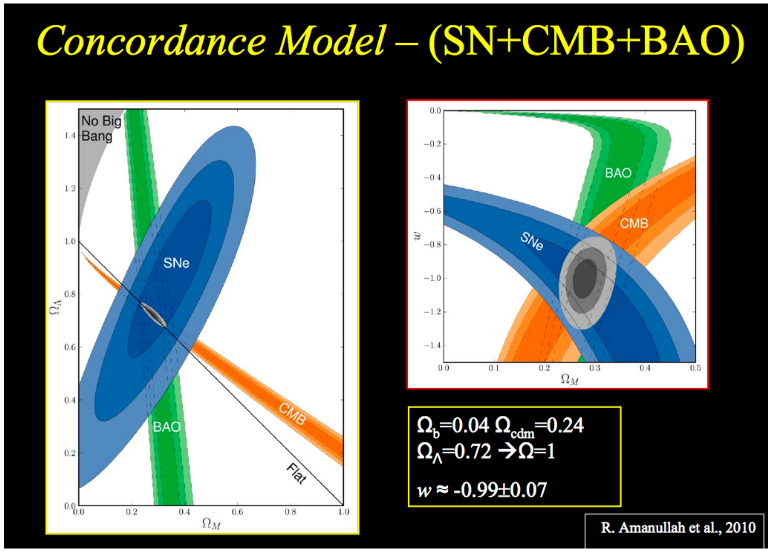
Comment on $w = -1$ or how dark is dark energy?

Note: SM prediction for Higgs equation of state is:

$$\boxed{w = p/\rho = (\tfrac{1}{2}\,\dot{\phi}^2 - V(\phi))/(\tfrac{1}{2}\,\dot{\phi}^2 + V(\phi)) \geq -1}$$

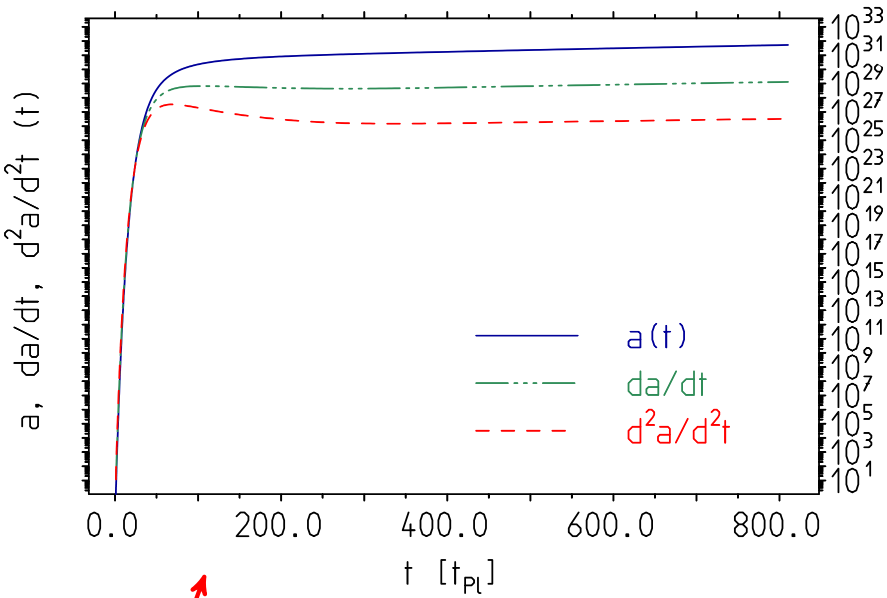
and not the ideal dark energy equation of state $w = -1$, which only holds if $V(\phi) \gg \dot{\phi}^2$. In fact $w = w(t)$ is a function of time and looks as



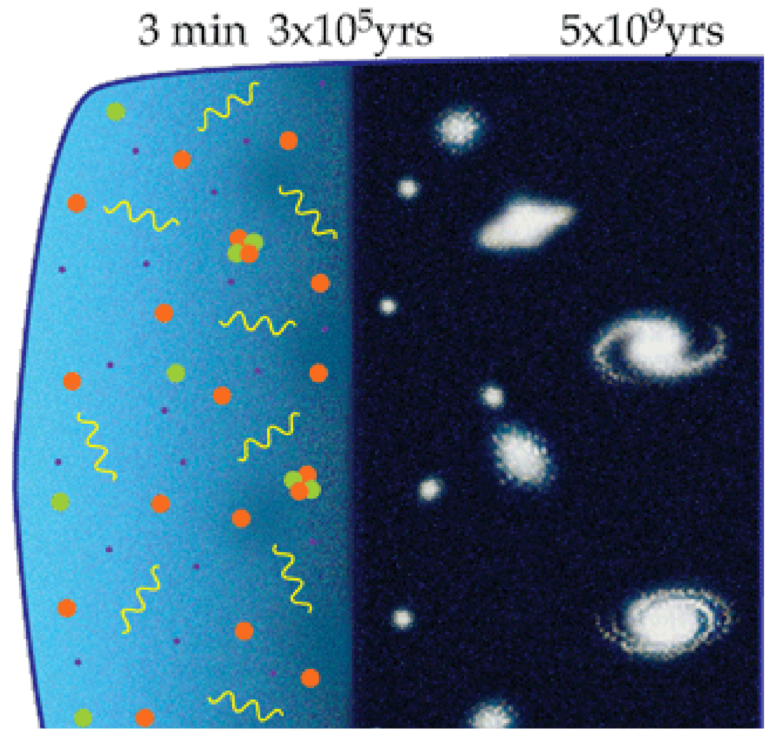


Higgs field density, pressure and equation of state. The Higgs provides dark energy beyond the inflation period which ends at about $t \simeq 450\, t_{\rm Pl}$.

The inflated expansion in the LEEJSM



SM prediction



Expansion before the Higgs transition: the FRW radius and its derivatives for $k = 1$ as a function of time, all in units of the Planck mass, i.e. for $M_{\rm Pl} = 1$. Here LEESM versus Artwork.

Crucial: minimal leading $\boxed{\text{UV completion}}$ by quadratic and quartic cut-off effects

Comment on Reheating and Baryogenesis

- inflation: exponential growth = exponential cooling

- reheating: pair created heavy states $X, \bar{X}$ in originally hot radiation dominated universe decay into lighter matter states which reheat the universe

- baryogenesis: X particles produce particles of different baryon-number B and/or different lepton-number L. $\not{B}$ by SM sphalerons or nearby dim 6 effective interactions

Sacharow condition for baryogenesis:

- $\not{B}$

- small $\not{B}$ is natural in LEESM scenario due to the close-by dimension 6 operators
  Weinberg 1979, Buchmüller, Wyler 1985,Grzadkowski et al 2010

❒ suppressed by $(E/\Lambda_{\rm Pl})^2$ in the low energy expansion. At the scale of the EW phase transition the Planck suppression factor is 1.3×10^{-6}.

❒ six possible four-fermion operators all $B - L$ conserving!

- $\cancel{C}$, $\cancel{CP}$, out of equilibrium

X is the Higgs! – "unknown" X particles now known very heavy Higgs in symmetric phase of SM: **Primordial Planck medium Higgses**

All relevant properties known: mass, width, branching fractions, CP violation properties!

Stages: ❒ $k_B T > m_X$ ⇒thermal equilibrium X production and X decay in balance

❒ $H \approx \Gamma_X$ and $k_B T < m_X$ ⇒X-production suppressed, out of equilibrium

- $H \to t\bar{t},\, b\bar{b},\, \cdots$ predominantly (largest Yukawa couplings)
- CP violating decays: $H^+ \to t\bar{d}$ [rate $\propto y_t y_d V_{td}$] $H^- \to b\bar{u}$ [rate $\propto y_b y_u V_{ub}$] and after EW phase transition: $t \to d e^+ \nu$ and $b \to u e^- \nu_e$ etc.
- Note: before Higgs mechanism bosonic triple couplings like HWW, HZZ are absent (induced by SSB after EW phase transition).
- Preheating absent! Reheating via $\phi \to f\bar{f}$ while all bosonic decays heavily suppressed (could obstruct reheating)!

Seems we are all descendants of four heavy Higgses via top-bottom stuff!

Baryogenesis most likely a “SM + dim 6 operators” effect!

Unlikely: $B+L$ violating instanton effects $\propto\ \exp\,[-\frac{8\pi^2}{g^2(\mu)} + \cdots] \approx \mathrm{e}^{-315.8}$ too small.

$\Rightarrow$observed baryon asymmetry $\eta_B \sim 10^{-10}$ cannot be a SM prediction, requires unknown B violating coupling. But order of magnitude should be “explainable”.

Conclusion

- The LHC made tremendous step forward in SM physics and cosmology: the discovery of the Higgs boson, which fills the vacuum of the universe first with dark energy and latter with the Higgs condensate, thereby giving mass to quarks leptons and the weak gauge bosons,
  but also drives inflation, reheating and all that

- Higgs not just the Higgs: its mass $M_H = 125.9 \pm 0.4$ GeV has a very peculiar value, which opens the narrow window to the Planck world!

- SM parameter space tailored such that strange exotic phenomena like inflation and likely also the continued accelerated expansion of the universe are a direct consequence of LEESM physics.

➠ ATLAS and CMS results may "revolutionize" particle physics in an unexpected way, namely showing that the SM has higher self-consistency (conspiracy) than expected and previous arguments for the existence of new physics may turn out not to be compelling

- SM as a low energy effective theory of some cutoff system at $M_{\rm Pl}$ consolidated; crucial point $M_{\rm Pl} >>>> \ldots$ from what we can see!
- the huge gap $E_{\rm lab} <<<< M_{\rm Pl}$ lets look particle physics to follow fundamental laws (following simple principles, QFT structure)
- change in paradigm:

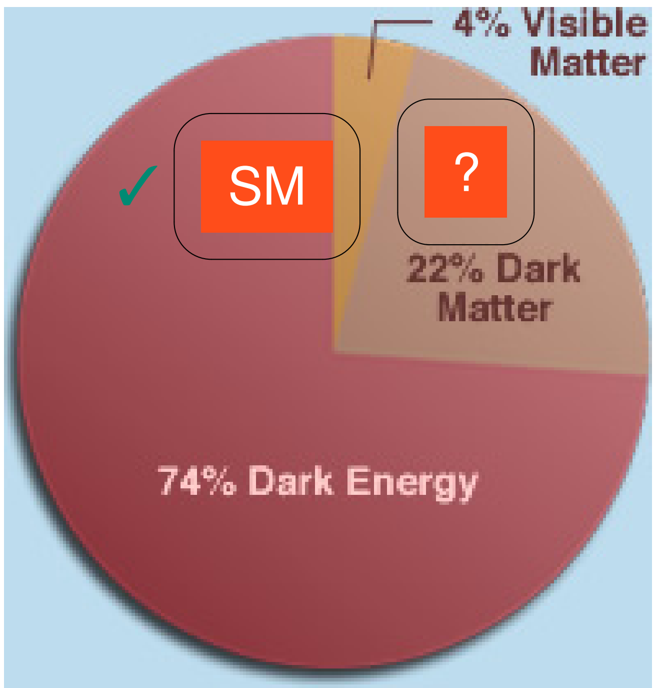
Natural scenario understands the SM as the “true world” seen from far away

$\Rightarrow$ Methodological approach known from investigating condensed matter systems. (QFT as long distance phenomenon, critical phenomena) Wilson 1971, NP 1982

- cut-offs in particle physics are important to understand early cosmology, i.e. inflation, reheating, baryogenesis and all that (see additional slides).

- the LEESM scenario, for the given now known parameters, the SM predicts dark energy and inflation, i.e. they are unavoidable

Last but not least: today's dark energy = relict Higgs vacuum energy?



WHAT IS DARK ENERGY?

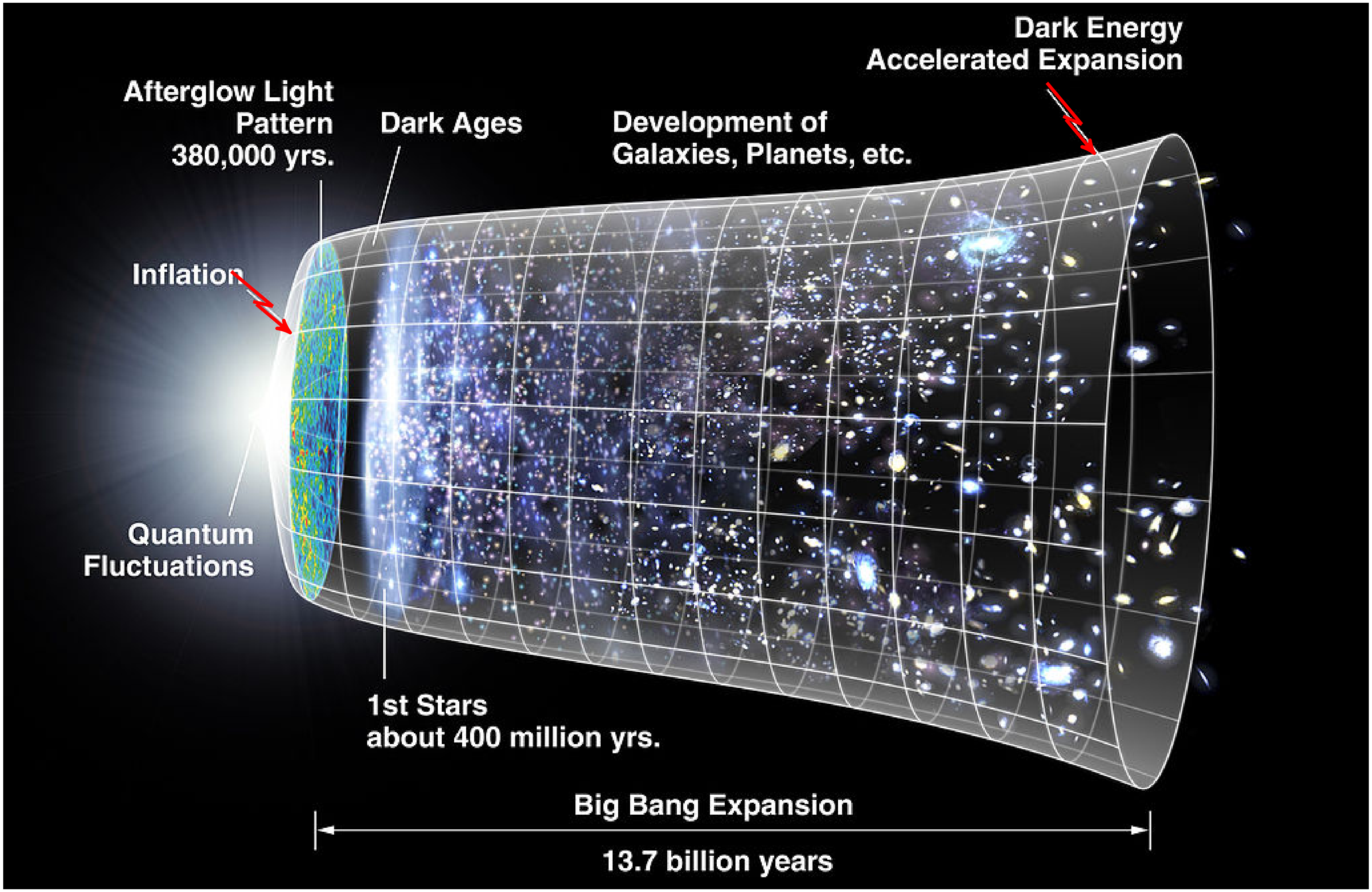
Well, the simple answer is that we don't know.

It seems to contradict many of our understandings about the way the universe works.

. . .

Something from Nothing?

It sounds rather strange that we have no firm idea about what makes up 74% of the universe.



the Higgs at work

- So what is “new”?
  Take hierarchy problem argument serious, SM should exhibit Higgs mass of Planck scale order (what is true in the symmetric phase),
  as well as vacuum energy of order $\Lambda_{\rm Pl}^4$, but do not try to eliminate them by imposing supersymmetry or what else, just take the SM regularized by the Planck cutoff as it is.

➠ inflation seems to be strong indication that quadratic and quartic cutoff enhancements are real, as predicted by LatticeSM for instance, i.e.

$\boxed{\text{Power divergences of local QFT are not the problem they are the solution!}}$

- New physics: still must exist
  ❶ cold dark matter
  ❷ axions as required by strong CP problem
  ❸ singlet neutrino puzzle (Majorana vs Dirac) and likely more $\cdots$,
  however, NP should not kill huge effects in quadratic and quartic cutoff sensitive terms and it should not deteriorate gross pattern of the running of the SM couplings.

Points in direction that high precision physics and astroparticle physics play a mayor role in disentangling corresponding puzzles.

Keep in mind: the Higgs mass miraculously turns out to have a value as it was expected form vacuum stability. It looks like a tricky conspiracy with other couplings to reach this “purpose”. If it misses to stabilize the vacuum, why is it so close to stability?

Why not simple although it may well be more complicated?

A lot yet to be understood!

Paths to Physics at the Planck Scale

M–THEORY(BRAIN WORLD)	Energy scale	E–THEORY(REAL WORLD)
candidate TOE		“chaotic” system
exhibits intrinsic cut-off	Planck scale	with intrinsic cut–off
↑	‖	
STRINGS	10^{19} GeV	
↑		↓
SUGRA	↓	QFT
↑		↓
SUSY–GUT	10^{16} GeV	
↑	↓	↓
SUSY	1 TeV	“??SM??”
↖	↓	↙
soft SB only	SM	SB soft at low/hard at high energies

top-down approach

bottom-up approach

symmetry low → → → symmetry high

?? symmetry ≡ blindness for details ??

the closer you look the more you can see when approaching the cut-off scale

Unless you accept the SM supplemented with a physical cutoff!

Thanks

Thanks for your attention!

References:

"The Standard model as a low-energy effective theory: what is triggering the Higgs mechanism?," Acta Phys. Polon. B **45** (2014) 1167 [arXiv:1304.7813 [hep-ph]].

"The hierarchy problem of the electroweak Standard Model revisited," arXiv:1305.6652 [hep-ph] also arXiv:1503.00809 [hep-ph]

"Higgs inflation and the cosmological constant," Acta Phys. Polon. B **45** (2014) 1215 [arXiv:1402.3738 [hep-ph]].

Krakow/Durham Lectures:
`http://www-com.physik.hu-berlin.de/~fjeger/SMcosmology.html`

see also: "The Vector Boson and Graviton Propagators in the Presence of Multipole Forces," Helv. Phys. Acta **51** (1978) 783.

Remarks for the Skeptic

How do our results depend on the true UV completion? In other words, how universal are the numbers we have presented?

In order to answer these questions we have to stress once more the extreme size of the cutoff [$M_{\rm Pl}$ >>>> ... from what we can see!], which lets look what we can explore to be ruled by fundamental principles like the Wightman axioms (the “Ten Commandments” of QFT) or extensions of them as they are imposed in deriving the renormalizable SM. In the LEESM approach many things are much more clear-cut than in condensed matter systems, where cutoffs are directly accessible to experiment and newer as far away and also lattice QCD simulations differ a lot, as cutoffs are always close-by, such that lattice artifacts affect results throughout before extrapolation to the continuum.

We also have to stress that taking actual numbers too serious is premature as long as even the existence of vacuum stability is in question. Detailed results evidently depend sensitively on accurate input values and on the perturbative approximations used for the renormalization group coefficients as well as for the matching relations needed to get the $\overline{\mathrm{MS}}$ input parameters in terms of the physical (on-shell) ones. After all we are extrapolating over 16 orders of magnitude in the energy scale.

The next question is how close to M_{Pl} can we trust our extrapolation? It is very important to note that above the EW scale [$v \sim 250$ GeV] perturbation theory seems to works the better the closer we are near the Planck cutoff, vacuum stability presupposed. As long as we are talking about the perturbative regime we can expand perturbative results in powers of E/Λ_{Pl} up to logarithms. Then we have full control over cutoff dependence to order $O((E/\Lambda_{\mathrm{Pl}})^2)$ (dim ≥ 6 operator corrections). Effects $O((E/\Lambda_{\mathrm{Pl}}))$ (dim 5 operators) only show up in special circumstances e.g. in scenarios related to generating neutrino masses and mixings and the sea-saw mechanism.

The true problem comes about when we approach the Planck scale, where the expansion in E/Λ_{Pl} completely breaks down. Especially, it does not make sense to talk about a tower of operators of increasing dimensions. This does not mean that everything gets out of control. If the "ether" would be something which can be modeled by a lattice SM, implemented similar to lattice QCD, one could still make useful predictions, which eventually could be tested in cosmological phenomena. In condensed matter physics it is well known that an effective Heisenberg Hamiltonian allows one to catch essential properties of the system, although the real structure cannot be expected to be reproduced in the details. Nevertheless it is possible to find out to what extent the description fits to reality.

It is well known that long range physics naturally emerges from underlying classical ststistical systems exhibitibg short-range exchange interactions (e.g. nearest-neighbour interactions on a lattice system) ($\cdots$ Wilson 1971). The Planck system besides such typical short-range interactions certainly exhibits a long-range gravitational potential, which develops multipole excitations showing up as spin 1, spin 2, etc modes at long distances (Jegerlehner 1978).

Obviously, too close to the Planck scale, predictions start to be sensitive to the UV completion and results get model dependent. However, this does not mean that predictions get completely obsolete. Such effects like the quadratic and quartic enhancements are persisting, as well as the running (screening or anti-screening effects) of couplings and their competition and conspiracy, which are manifest in the existence of the zeros of the enhanced terms, provided these zeros are not to close to the cutoff. Again, the perturbativeness, together with the fact that leading corrections to these results are by dim 6 operators, let us expect that result are reliable at the 10^{-4} level up to 10^{17} GeV, which is in the middle of the symmetric phase already. Once the phase transition has happened, the running is anyway weak and if cutoff effect are starting to play a role they cannot spoil the relevant qualitative features concerning triggering inflation, reheating and all that.

Lattice SM simulations in the appropriate parameter range of vacuum stability, keeping top quark Yukawa and Higgs self-energy couplings to behave asymptotically free, which requires to include simultaneously besides the Higgs system also the top Yukawa sector and QCD, could help to investigate such problems quantitatively. Experience from lattice QCD simulations may not directly be illustrative since usually the cutoff is rather close and a crucial difference is also the true non-perturbative nature of low energy QCD.

In any case, not to include the effects related to the relevant operators (dim < 4)

simply must give wrong results. Even substantial uncertainties, which certainly show up closer to the cutoff, in power-like behaved quantities seem to be an acceptable shortcoming in comparison to not taking into account the cutoff enhancements at all (as usually done).

In conclusion, these arguments strongly support the gross pattern of LEESM Higgs transition, inflation, reheating and all that.

Summary part II:

- with Higgs discovery: SM essentially complete, Higgs mass $M_H \simeq 126$ GeV very special for Higgs vacuum stability
- SM couplings are energy dependent, all but g' decrease towards $M_{\rm Pl}$, perturbation theory works well up to Planck scale.
- SM Higgs potential likely remains stable up to $M_{\rm Pl}$ (i.e. $\lambda(\mu) > 0$ for all $\mu < M_{\rm Pl}$)
- bare parameters are the true parameters at very high energy approaching $M_{\rm Pl}$, relevant for early universe
- bare parameters are calculable in SM as needed for early cosmology
- ❒ cutoff enhanced quantities: effective bare Higgs mass (quadratic $\propto \Lambda_{\rm Pl}^2$) as well as dark energy (quartic $\propto \Lambda_{\rm Pl}^4$)
  $\Rightarrow$provide inflation condition $V(\phi) \gg \frac{1}{2}\dot{\phi}^2$
- SM originally (at very high energies) in symmetric phase, all particles massless except for the four very heavy Higgses
- both the Higgs mass as well as the dark energy exhibit matching points where bare and renormalized values coincide, separates low energy form bare Planck regime responsible for inflation

- ❐ need trans-Planckian initial Higgs field $\phi_i = \phi(t_{\rm Pl}) \sim 5\, M_{\rm Pl}$ in order to get sufficient inflation $N \gtrsim 60$
- ❐ trans-Planckian fields do no harm: fast exponential decay of Higgs field
- ❐ after inflation in reheating phase: very heavy Higgses mainly decay into top–antitop pairs, which latter (after the EW phase transition) decay into normal baryonic matter
- ● except for ϕ_i all properties known: inflation and reheating are SM predictions within uncertainties of SM initial parameters and RG evolution approximations (presently 3-loops)
- ❐ EW phase transition in this scenario happens at much higher energy than anticipated so far and close by natural Baryon number violating dimension 6 operators likely trigger baryogenesis.

❐ SM inflation requires very precise input parameters and appropriate higher order corrections (precise knowledge of the SM itself) Presently: $\overline{\rm MS}$ RG to 3 loops (massless), matching conditions leading 2 loops (need full massive SM calculations, yet incomplete)

SM inflation vs added inflation scenarios

LEESM scenario is easy to rule out:

1. find any type of New Physics (NP) as motivated by the naive hierarchy problem argument. These are most SM extension scenarios (SUSY, Extra Dimensions, Little/st Higgs, ETC and what else), i.e. any physics affecting substantially the quadratic and quartic “divergences”.

2. find any type of “new heavy states with substantial couplings to SM spectrum” like 4th family, GUT, “light” heavy (far below $M_{\rm Pl}$) singlet Majorana neutrino etc, i.e. anything affecting the g', g, g_s, y_t and λ SM running coupling pattern.

3. confront precise SM inflation predictions with inflation pattern itself: large enough N_e, $w \approx -1$, spectral indices n_S, n_T, Gaussianity etc

The LEESM scenario is natural as it predicts a bulk of properties, which usually are assumed/ imposed as basic principles. All these are emergent properties!

Predicted as long range phenomenon:

- QFT structure,
- renormalizability and requirements needed for it:
  - non-Abelian gauge structure,
  - chiral symmetry,
  - anomaly cancellation and fermion family structure
  - the existence of the Higgs particle! (renormalizability)
- space-time dimensionality $D = 4$, no renormalizable non-trivial QFT in $D > 4$
- rotation invariance and Lorentz invariance (pseudo-rotations)
- analyticity, effective unitarity etc

All result are checkable through real calculations (mostly existent).

SM inflation is based on SM predictions, except for the Higgs field value ϕ_0, which is the only quantity relevant for inflation, which is not related to an observable low energy quantity.

All other inflation scenarios are set up “by hand”: the form of the potential as well as all parameters are tuned to reproduce the observed inflation pattern.

Example Minkowski-Zee-Shaposnikov et al so called non-minimal SM inflation

❶ Change Einstein Gravity by adding $G_{\mu\nu} \to G_{\mu\nu} + \xi\,(H^+H)\,R$ together with renormalized low energy SM $T_{\mu\nu}$ (no relevant operator enhancement)

❷ Choose ξ large enough to get sufficient inflation, need $\xi \sim 10^4$, entire inflation pattern essentially depends on ξ only (inflation “by hand”)

❸ assume quadratic and quartic SM divergences are absent (argued by dimensional regularization (DR) and $\overline{\mathrm{MS}}$ renormalization)

❹ assume SM to be in broken phase at Planck scale, which looks unnatural. Note: SSB is a low energy phenomenon, which assumes the symmetry to be restored at the short distance scale!)

All but convincing!

Note: DR and $\overline{\mathrm{MS}}$ renormalization are possible in perturbation theory only. There is no corresponding non-perturbative formulation (simulation on a lattice) or measuring prescription (experimental procedure). It is based on a finite part prescription (singulatities nullified by hand), which can only be used to calcculate quantities which do not exhibit any singularities at the end. The hierarchy problem cannot be addressed in the $\overline{\mathrm{MS}}$ scheme.

❐ My scenario: take Einstein Gravity serious (geometry, equivalence principle etc unaffected) together with true SM energy-momentum tensor, i.e. as effective at given scale, beyond $X_{\rm ren} = X_{\rm bare}$ matching point: true=bare as relevant near Planck energies. Need vacuum stability and Higgs phase transition below $M_{\rm Pl}$.

My evaluation of $\overline{\rm MS}$ parameters revealed Vacuum Stability

Although other evaluations of the matching conditions seem to favor the metastability of the electroweak vacuum within the experimental and theoretical uncertainties, one should not exclude the possibility that other experiments and improved matching conditions will be able to establish the absolute stability of Standard Model in the future.

Shaposhnikov et al. `arXiv:1412.3811` say about Vacuum Stability

Although the present experimental data are perfectly consistent with the absolute stability of Standard Model within the experimental and theoretical uncertainties, one should not exclude the possibility that other experiments will be able to establish the metastability of the electroweak vacuum in the future.

"Baryogenesis in the Annihilation Drama of Matter"

10^{-35} sec.

$X, \bar{X}$–Decay: $\Rightarrow$

10^{-30} sec.

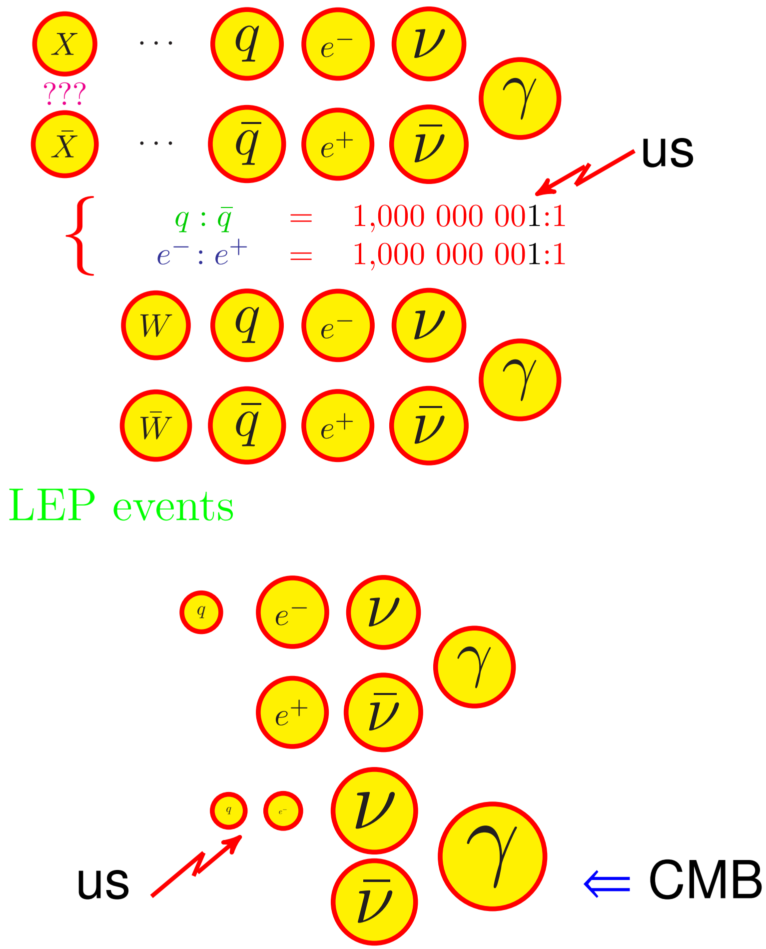
0.3×10^{-10} sec.

$q\bar{q} \to \gamma\gamma$:

10^{-4} sec.

$e^+e^- \to \gamma\gamma$:

1 sec.

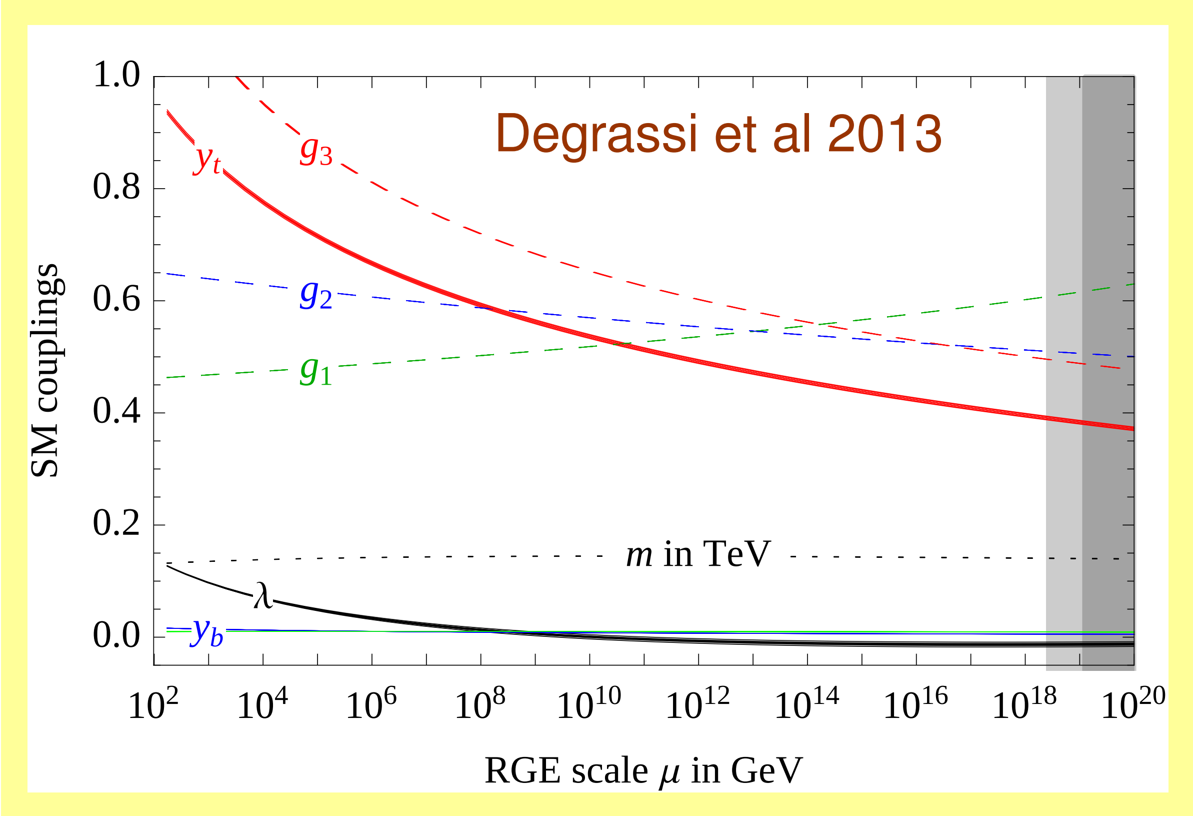


Is X the Higgs?

➠ Additional remarks ⇐

- Test of tricky conspiracy between SM couplings the new challenge
- Very delicate on initial values as we run over 16 orders of magnitude from the EW 250 GeV scale up to the **Planck scale!**
- Running couplings likely have dramatic impact on cosmology! The existence of the world in question?
- LHC and ILC will dramatically improve on Higgs self-coupling λ (Higgs factory) as well as on top Yukawa y_t ($t\bar{t}$ factory)
- for running $\alpha_{\rm em}$ and $\sin^2\Theta_{\rm eff}$ $\Leftrightarrow$ g_1 and g_2 need more information from low energy hadron production facilities, improving QCD predictions and EW radiative corrections! Lattice QCD will play key role for sure.

Comparison of SM coupling evolution



Renormalization of the SM gauge couplings $g_1 = \sqrt{5/3}g_Y, g_2,\ g_3$, of the top, bottom and τ couplings ($y_t,\ y_b,\ y_\tau$), of the Higgs quartic coupling λ and of the Higgs mass parameter m. We include two-loop thresholds at the weak scale and three-loop RG equations. The thickness indicates the $\pm 1\sigma$ uncertainties.

Comparison of $\overline{\mathrm{MS}}$ parameters at various scales: Running couplings for $M_H = 126$ GeV and $\mu_0 \simeq 1.4 \times 10^{16}$ GeV.

	my findings				Degrassi et al. 2013	
coupling \ scale	M_Z	M_t	μ_0	$M_{\rm Pl}$	M_t	$M_{\rm Pl}$
g_3	1.2200	1.1644	0.5271	0.4886	1.1644	0.4873
g_2	0.6530	0.6496	0.5249	0.5068	0.6483	0.5057
g_1	0.3497	0.3509	0.4333	0.4589	0.3587	0.4777
y_t	0.9347	0.9002	0.3872	0.3510	0.9399	0.3823
$\sqrt{\lambda}$	0.8983	0.8586	0.3732	0.3749	0.8733	i 0.1131
λ	0.8070	0.7373	0.1393	0.1405	0.7626	- 0.0128

Most groups find just unstable vacuum at about $\mu \sim 10^9$ GeV! [not independent, same $\overline{\mathrm{MS}}$ input]

Note: $\lambda = 0$ is an essential singularity and the theory cannot be extended beyond a possible zero of λ: remind $v = \sqrt{6m^2/\lambda}$!!! i.e. $v(\lambda) \to \infty$ as $\lambda \to 0$
besides the Higgs mass $m_H = \sqrt{2}\, m$ all masses $m_i \propto g_i\, v \to \infty$ different cosmology

What about the hierarchy problem?

- In the Higgs phase:

There is no hierarchy problem in the SM!

In the broken phase, characterized by the non-vanishing Higgs field vacuum expectation value (VEV) $v(\mu)$, all the masses are determined by the well known mass-coupling relations

$$
\begin{aligned}
m_W^2(\mu) &= \tfrac{1}{4}\, g^2(\mu)\, v^2(\mu)\ ; \quad m_Z^2(\mu) = \tfrac{1}{4}\,(g^2(\mu) + g'^2(\mu))\, v^2(\mu)\ ; \\
m_f^2(\mu) &= \tfrac{1}{2}\, y_f^2(\mu)\, v^2(\mu)\ ; \quad m_H^2(\mu) = \tfrac{1}{3}\, \lambda(\mu)\, v^2(\mu)\ .
\end{aligned}
$$

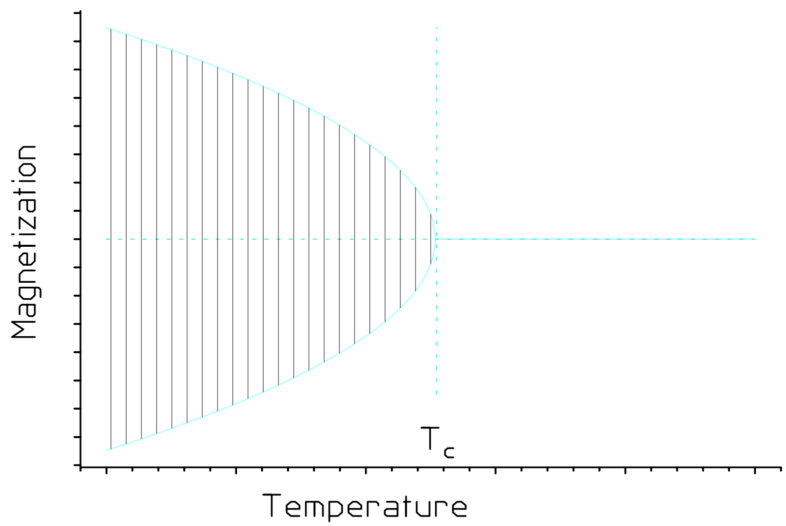
- Higgs mass cannot by much heavier than the other heavier particles!

- Extreme point of view: all particles have masses $O(M_{\rm Pl})$ i.e. $v = O(M_{\rm Pl})$.
  This would mean the symmetry is not recovered at the high scale,

notion of SSB obsolete! Of course this makes no sense.

● Higgs VEV v is an order parameter resulting form long range collective behavior,
can be as small as we like.

Prototype: magnetization in a ferromagnetic spin system



$M = M(T)$ and actually $M(T) \equiv 0$ for $T > T_c$ furthermore $M(T) \to 0$ as $T \overset{\to}{<} T_c$

- $v/M_{\rm Pl} \ll 1$ just means we are close to a 2$^{\rm nd}$ order phase transition point.

- In the symmetric phase at very high energy we see the bare system:

the Higgs field is a collective field exhibiting an effective mass generated by radiative effects

$$\boxed{m^2_{\rm bare} \approx \delta m^2 \text{ at } M_{\rm Pl}}$$

eliminates fine-tuning problem at all scales!

Many example in condensed matter systems.

In my view the hierarchy problem is a pseudo problem!

✶ What rules the β-functions:

Naively:

❒ $U(1)_Y$ screening (IR free), $SU(2)_L$, $SU(3)_c$ antiscreening (UV free) [asymptotic freedom (AF)]

Right – as expected

❒ Yukawa and Higgs: screening (IR free, like QED)

Wrong!!! – transmutation from IR free to AF

At the Z boson mass scale: $g_1 \simeq 0.350$, $g_2 \simeq 0.653$, $g_3 \simeq 1.220$, $y_t \simeq 0.935$ and $\lambda \simeq 0.796$

Leading (one-loop) β-functions at $\mu = M_Z$: [$c = \frac{1}{16\pi^2}$]

❖ gauge couplings:

$$\beta_1 = \frac{41}{6} g_1^3 c \simeq 0.00185 \; ; \;\; \beta_2 = -\frac{19}{6} g_2^2 c \simeq -0.00558 \; ; \;\; \beta_3 = -7 g_3^3 c \simeq -0.08045 \, ,$$

❖ top Yukawa coupling:

$$\begin{aligned}\beta_{y_t} &= (\frac{9}{2}\,y_t^3 - \frac{17}{12}\,g_1^2\,y_t - \frac{9}{4}\,g_2^2\,y_t - 8\,g_3^2\,y_t)\,c \\ &\simeq 0.02328 - 0.00103 - 0.00568 - 0.07046 \\ &\simeq -0.05389\end{aligned}$$

not only depends on y_t, but also on mixed terms with the gauge couplings g', g and g_3 which have a negative sign.

In fact the QCD correction is the leading contribution and determines the behavior. Notice the critical balance between the dominant strong and the top Yukawa couplings: QCD dominance requires $g_3 > \frac{3}{4}\,y_t$ in the gaugeless limit.

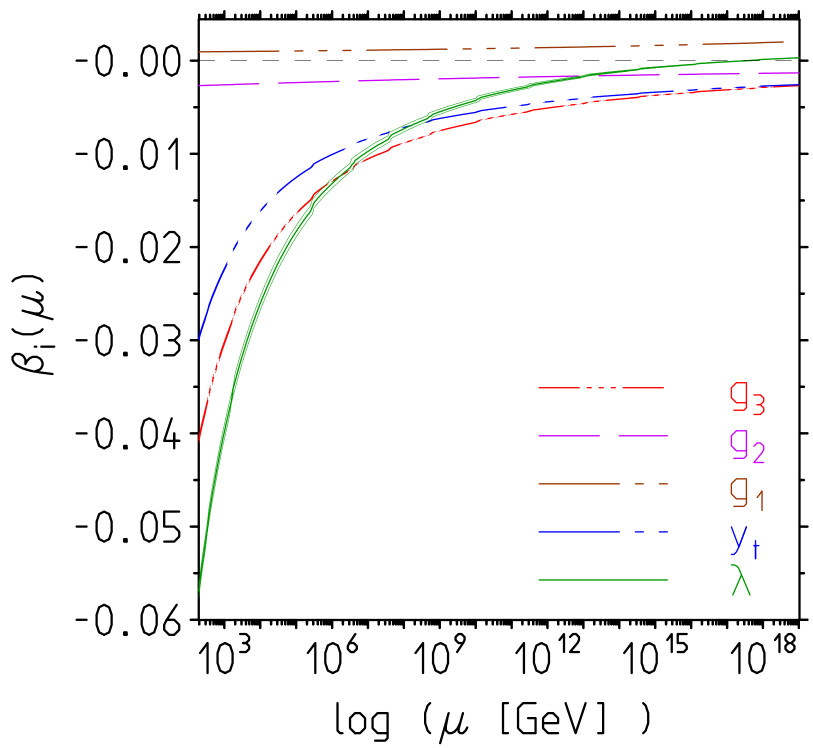
❖ the Higgs self-coupling

$$\begin{aligned}\beta_\lambda &= (4\,\lambda^2 - 3\,g_1^2\,\lambda - 9\,\lambda\,g_2^2 + 12\,y_t^2\,\lambda + \frac{9}{4}\,g_1^4 + \frac{9}{2}\,g_1^2\,g_2^2 + \frac{27}{4}\,g_2^4 - 36\,y_t^4)\,c \\ &\simeq 0.01606 - 0.00185 - 0.01935 + 0.05287 + 0.00021 + 0.00149 + 0.00777 - 0.17407 \\ &\simeq -0.11687\end{aligned}$$

dominated by y_t contribution and not by λ coupling itself. At leading order it is not subject to QCD corrections. Here, the y_t dominance condition reads $\lambda < \frac{3(\sqrt{5}-1)}{2}\, y_t^2$ in the gaugeless limit.

❒ running top Yukawa QCD takes over: IR free $\Rightarrow$UV free

❒ running Higgs self-coupling top Yukawa takes over: IR free $\Rightarrow$UV free



Including all known RG coefficients (EW up incl 3–loop, QCD up incl 4–loop)

- except from β_λ, which exhibits a zero at about $\mu_\lambda \sim 10^{17}$ GeV, all other β-functions do not exhibit a zero in the range from $\mu = M_Z$ to $\mu = M_{\rm Pl}$.

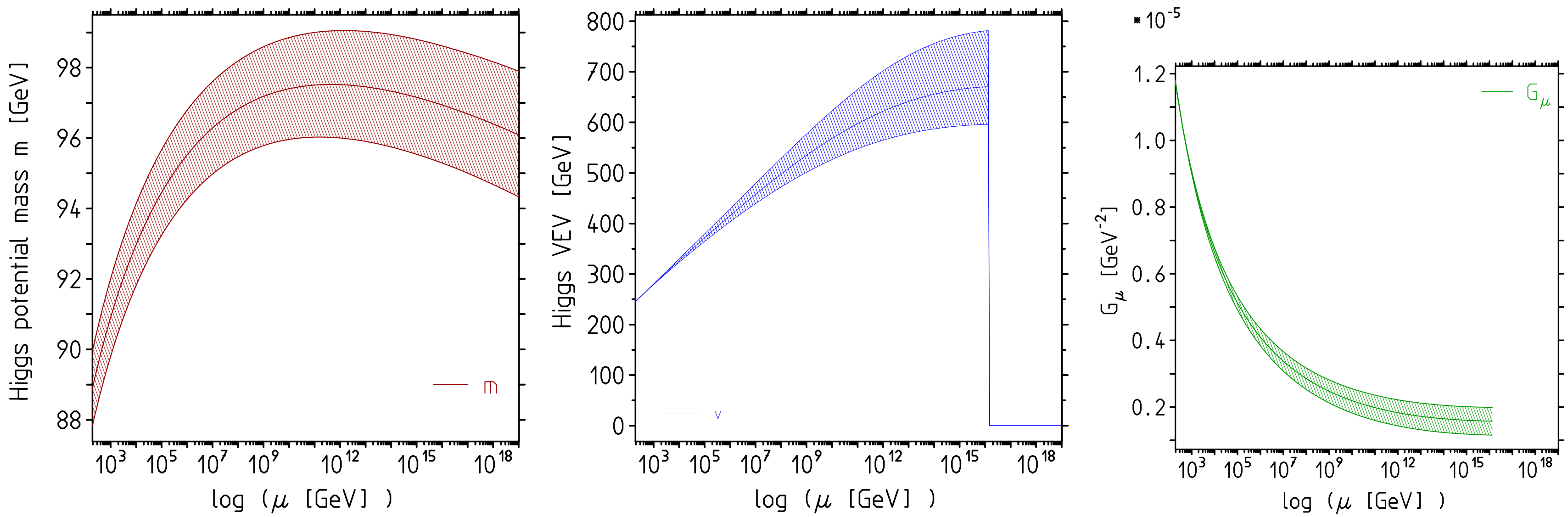
- so apart form the $U(1)_Y$ coupling g_1, which increases only moderately, all other couplings decrease and perturbation theory is in good condition.

- at $\mu = M_{\rm Pl}$ gauge couplings are all close to $g_i \sim 0.5$, while $y_t \sim 0.35$ and $\sqrt{\lambda} \sim 0.36$.

- effective masses moderately increase (largest for m_Z by factor 2.8): scale like

$$m(\kappa)/\kappa \text{ as } \kappa = \mu'/\mu \to \infty,$$

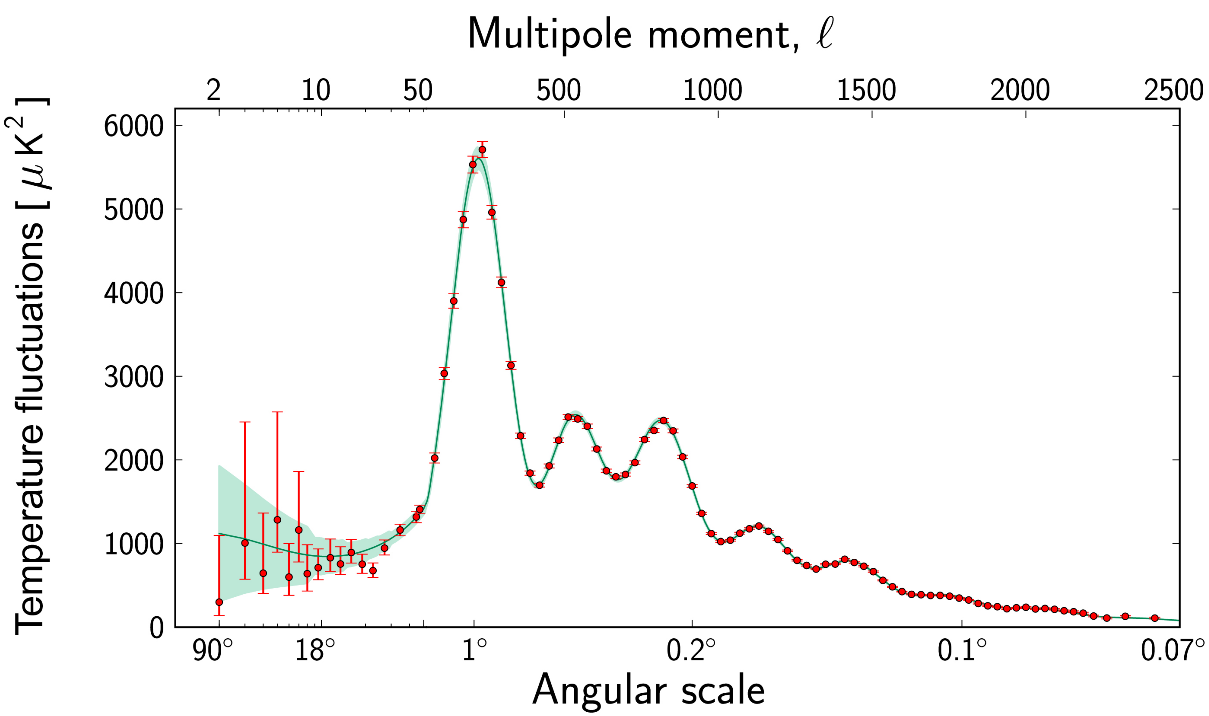
i.e. mass effect get irrelevant as expected at high energies.



Non-zero dimensional $\overline{\mathrm{MS}}$ running parameters: m, $v = \sqrt{6/\lambda}\, m$ and $G_F = 1/(\sqrt{2}\, v^2)$. Error bands include SM parameter uncertainties and a Higgs mass range 125.5 ± 1.5 GeV which essentially determines the widths of the bands. Note that v increases by a factor about 2.5 before it jumps to zero at the transition point.

Gaussianity of Inflation

❒ The PLANCK mission power spectrum:



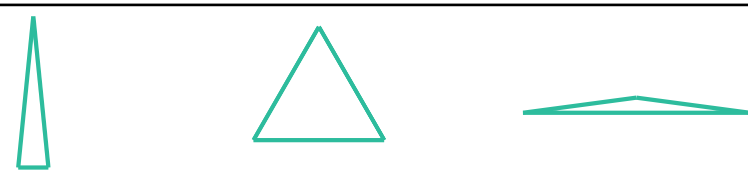
● a dominant mass term also looks to imply the inflaton to represent essentially a free field (Gaussian).

❑ Shapes in CMB

Gaussianity seems to be well supported by recent Planck mission constraints on non-Gaussianity: $\Phi(\vec{k})$ gravitational potential

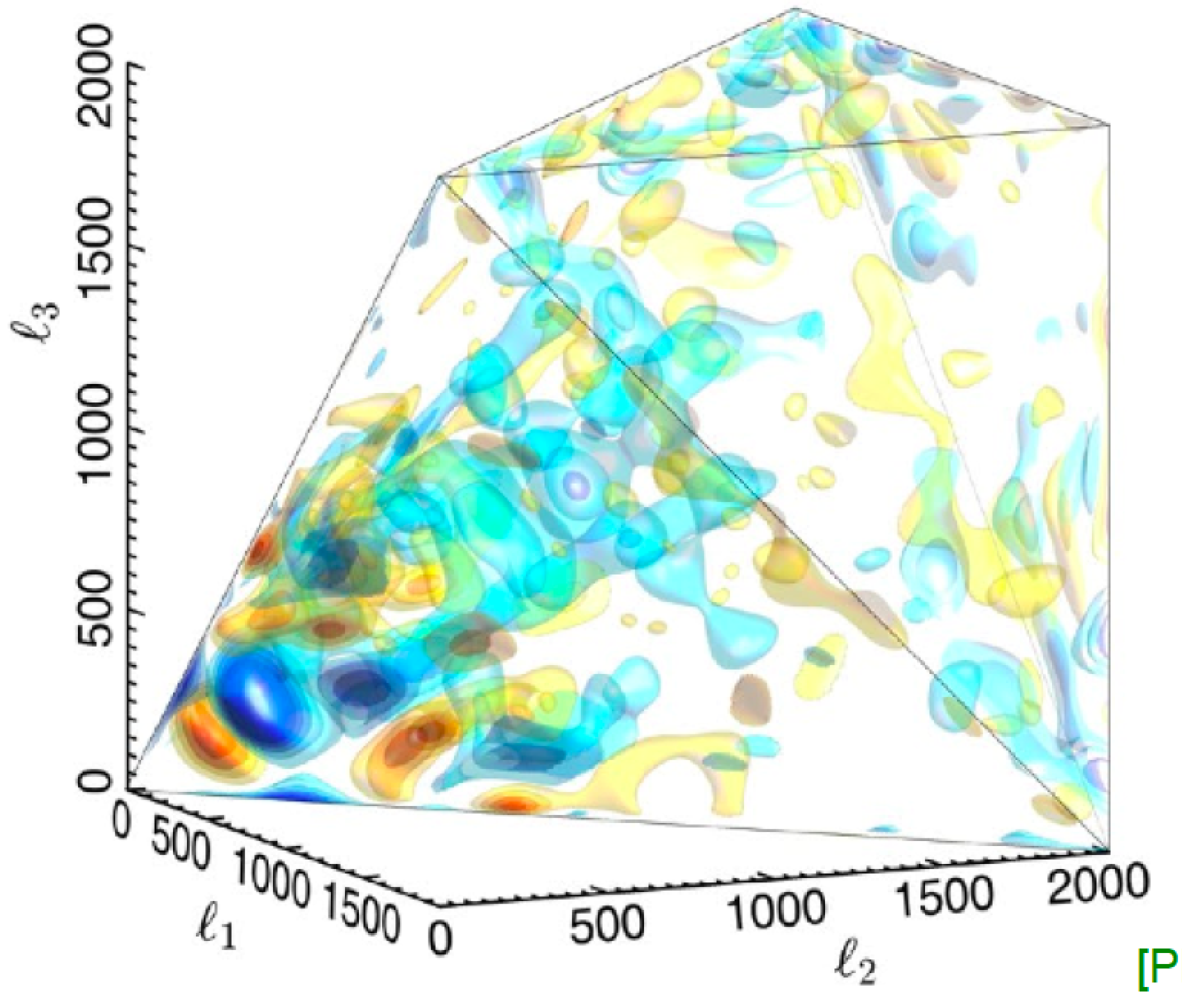
$$\underbrace{\langle \Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)\rangle}_{\text{three point correlation}} = (2\pi)^3 \underbrace{\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3)}_{\text{enforces triangular configuration}} \underbrace{f_{\mathrm{NL}}\,F(k_1,k_2,k_3)}_{\text{bispectrum}}$$

Three limiting cases



f_{NL}		
Local	Equilateral	Orthogonal
2.7 ± 5.8	-42 ± 75	-25 ± 39

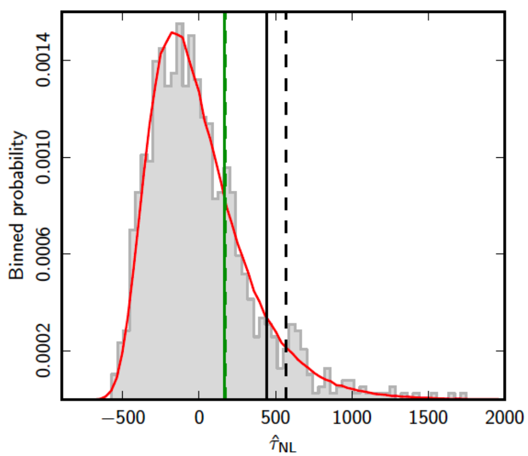
No evidence for non-Gaussianity



[Planck 2013]

Non-Gaussianity: CMB angular bispectrum

Planck data are consistent with Gaussian primordial fluctuations. There is no evidence for primordial Non Gaussian (NG) fluctuations in shapes (local, equilateral and orthogonal).



shape non-linearity parameters:

$$f_{NL}^{loc} = 2.7 \pm 5.8,\ f_{NL}^{eq} = -42 \pm 75,\ f_{NL}^{orth} = -25 \pm 39$$

(68% CL statistical)

- The scenario suggested by the present analysis is a Gaussian potential with small anharmonic perturbations, since $m^2_{\rm bare}$ is predicted to be large while $\lambda_{\rm bare}$ remains small. Also the bare kinetic term is logarithmically "unrenormalized" only.

- numbers depend sensibly on what $\lambda(M_H)$ and $y_t(M_t)$ are (LHC & future ILC!)

The SM renormalization group equations

References:

1-loop and 2-loop: Gross, Wilczek, Politzer 1973, Jones, Caswell 1974, Tarasov, Vladimirov 1977, Jones 1982, Fischler, Oliensis 1982, Machacek, Vaughn 1983/84/85, Luo, Xiao 2003

3–loop QCD: Tarasov, Vladimirov, Zharkov 1980, Larin, Vermaseren 1993

4–loop QCD: Ritbergen, Vermaseren, Larin 1997, Czakon 2005

2–loop QCD OS vs $\overline{\mathrm{MS}}$ mass: Gray, Broadhurst, Grafe, Schilcher 1990, Fleischer, F.J., Tarasov, Veretin 1999/2000

3–loop QCD OS vs $\overline{\mathrm{MS}}$ mass: Chetyrkin, Steinhauser 2000, Melnikov, Ritbergen 2000

$\beta_{g'}^{(3)}, \beta_{g}^{(3)}$:

Mihaila, Salomon, Steinhauser 2012, Bednyakov, Pikelner, Velizhanin 2012

$\beta_{y_t}^{(3)}, \beta_{\lambda}^{(3)}$:

Chetyrkin, Zoller 2012/2013, Bednyakov, Pikelner, Velizhanin 2012/2013

Matching conditions for $\overline{\mathrm{MS}}$ parameters in terms of physical parameters

References:

a) Higgs boson mass vs Higgs self-coupling:
The one-loop corrections give the dominant contribution in the matching relations
Fleischer, F.J. 1981, Sirlin, Zucchini 1986

Two-loop results are partially known F.J., Kalmykov, Veretin 2002/.../2004.
Completed recently: Kniehl, Pikelner, Veretin 2015

b) Top quark mass vs top Yukawa coupling:
The QCD corrections
Gray, Broadhurst, Grafe, Schilcher 1990; Fleischer, F.J., Tarasov, Veretin 1999; Chetyrkin, Steinhauser 1999/2000; Melnikov, Ritbergen 2000
Hempfling, Kniehl 1995 and F.J., Kalmykov 2003/2004
in the gaugeless-limit Martin 2005
more recent: F.J., Kalmykov, Kniehl 2012 Bezrukov et al 2012, Degrassi et al 2012

see `F. Jegerlehner, M. Y. Kalmykov, B. A. Kniehl, Phys. Lett. B` **722** `(2013) 123 [arXiv:1212.4319 [hep-ph]].`