Cosmological Nucleosynthesis

❖ In the discussion of the thermal history of the early universe form temperatures $10^{11}\ ^\circ\mathrm{K}$ down to about $10^{6}\ ^\circ\mathrm{K}$ we were neglecting the tiny contribution of relict nucleons which were not completely annihilated in particle-antiparticle annihilation.

❖ Nucleons, the building blocks of atomic nuclei, were formed abundantly in the QCD phase transition out of the quark-gluon plasma at temperatures $T \sim 150$ MeV corresponding to a temperature $T \sim 2 \times 10^{12}\ ^\circ\mathrm{K}$ and cosmic time $t \sim 20\mu\mathrm{s}$.

❖ The QCD phase transition sets in after the annihilation of the heavier quarks (t, b, c) during the annihilation of s quarks. During the QCD phase transition color neutral $SU(3)_c$ triplet states form, the baryons, as well as anti-triplets representing the antibaryons together with quark–antiquark singlets, the mesons. The lightest hadrons are the nucleons (N), proton (p) and neutron (n), and their antiparticles (spin 1/2 fermions), which when formed are non-relativistic (essentially at rest) and the lightest mesons, the pions $\pi^\pm$ and π^0 (spin 0 bosons).

❖ Baryon–antibaryon annihilation immediately sets in during their formation. Here the mystery of the baryon asymmetry plays the key role for the future evolution, namely, the annihilation turned out to be incomplete with a relict

$$\eta = n_N(t_0)/n_\gamma(t_0) \sim 10^{-10} - 10^{-9}$$

of surviving baryons. The only particles left in large numbers at this time were pions, muons, electrons, neutrinos and photons.

After baryon–antibaryon annihilation, for $T < 10$ MeV, the total number of baryons $n_B = n_N - n_{\bar{N}} = n_p - n_{\bar{p}} + n_n - n_{\bar{n}}$, in practice $n_B = n_p + n_n$ since now $n_{\bar{N}} \ll n_N$, remains conserved due to baryon number conservation. Baryons are free protons, free neutrons or protons and neutrons bound in nuclei. While nuclear strong interaction forces bind nucleons in nuclei weak interactions convert neutron and protons into each other.

❒ As we will see, about one quarter of all nucleons form nuclei ($A > 1$), with about an equal amount of neutrons and protons, while approximately three quarters remain free protons.

❒ Later, when $T < 10$ keV, after e^+e^-–annihilation, we also will have $n_{e^+} \ll n_{e^-}$ and $n_{e^-} = n_p$, where n_p represents all protons, the free ones as well as the ones bound in nuclei (presumed charge neutrality of the universe).

❒ Radiation and matter (electron, protons and nuclei) remain in thermal equilibrium as long as there are lots of free electrons. After e^+e^-–annihilation, atoms in particular hydrogen may form

$$\boxed{e^- + p \rightleftharpoons H + \gamma}$$

and essentially, when $T_\gamma < 13.6\ \mathrm{eV}$, the ionization energy of hydrogen, electrons will have been trapped by protons and nuclei ions (recombination). The universe becomes transparent and photons decouple from matter. More details on that later.

We are now discussing the evolution of the relict densities of baryons and the formation of light nuclei.

Relevant basic processes:

$$\boxed{n+\nu \leftrightharpoons p+e^- \;,\quad n+e^+ \leftrightharpoons p+\bar{\nu} \;,\quad n \leftrightharpoons p+e^- +\bar{\nu}}$$

- $\nu \equiv \nu_e$, other neutrinos inactive here.
- $k_\mathrm{B}\, T \ll m_N \to$ nucleons at rest (non-relativistic)
- Q–value: $Q = m_n - m_p \simeq 1.293$ MeV

$$\begin{array}{ll} Q = E_e - E_\nu & n+\nu \leftrightharpoons p+e^- \\ Q = E_\nu - E_e & n+e^+ \leftrightharpoons p+\bar{\nu} \\ Q = E_\nu + E_e & n \leftrightharpoons p+e^- +\bar{\nu} \end{array}$$

Calculation of rate:

❒ cross-section: weak 2–body reaction $n+e \to p+\bar{\nu}$

$$\boxed{\sigma = \frac{2\pi^2\hbar^3\, A\, E_\nu^2}{\mathrm{v}_e} \;;\quad A = \frac{G_F^2\left(1+3\,g_A^2\right)\,\cos^2\theta_C}{2\pi^3\,\hbar}}$$

$G_F = 1.16637(1) \times 10^{-5}$ GeV^{-2} Fermi constant
$g_A = 1.257$ nucleon axial vector coupling of β-decay
$\cos\theta_C = 0.9745(6)$, Θ_C Cabibbo mixing angle
v_e the electron velocity

Electrons and neutrinos are relativistic $\rightarrow$ have to take into account Fermi distribution (Pauli-Principle):

- number density of electrons (with momentum between p_e and $p_e + \mathrm{d}p_e$)

$$\boxed{\frac{4\pi\, p_e^2\, \mathrm{d}p_e}{(2\pi\hbar)^3} \left\{\exp\left(E_e/k_{\mathrm{B}}T\right) + 1\right\}^{-1}}$$

- fraction of unfilled antineutrino levels:

$$\boxed{1-\{\exp\left(E_\nu/k_{\mathrm{B}}T_\nu\right)+1\}^{-1}=\{\exp\left(-E_\nu/k_{\mathrm{B}}T_\nu\right)+1\}^{-1}}$$

such that the total rate reads

$$\boxed{\lambda(n+e^+\to p+\bar\nu)=A\int\limits_0^\infty E_\nu^2\,p_e^2\,\mathrm{d}p_e\,\left[\exp\left(E_e/k_{\mathrm{B}}T\right)+1\right]^{-1}\times\left[\exp\left(-E_\nu/k_{\mathrm{B}}T_\nu\right)+1\right]^{-1}}$$

changing variable to $q\equiv -E_\nu=-Q-E_e$ yields an integral over q

- from $q=-\infty$ to $q=-Q-m_e$ for $n+e^+\to p+\bar\nu$
- from $q=-Q+m_e$ to $q=0$ for $n\to p+e^-+\bar\nu$
- from $q=0$ to $q=+\infty$ for $n+\nu\to p+e^-$

while the integrand is unchanged.

The total rate of disappearance of neutrons then is given by

$$\boxed{\lambda(n \to p) = \fint_{-\infty}^{+\infty} \sqrt{1 - \frac{m_e^2}{(Q+q)^2}} \frac{(Q+q)^2\, q^2\, \mathrm{d}q}{\left(1+\mathrm{e}^{q/k_\mathrm{B}T_\nu}\right)\left(1+\mathrm{e}^{-(Q+q)/k_\mathrm{B}T}\right)}}$$

with a gap from $q = -Q - m_e$ to $q = -Q + m_e$ in the integration range (unphysical region).

Similarly, one calculates the total rate of production of neutrons to be

$$\boxed{\lambda(p \to n) = \fint_{-\infty}^{+\infty} \sqrt{1 - \frac{m_e^2}{(Q+q)^2}} \frac{(Q+q)^2\, q^2\, \mathrm{d}q}{\left(1+\mathrm{e}^{-q/k_\mathrm{B}T_\nu}\right)\left(1+\mathrm{e}^{(Q+q)/k_\mathrm{B}T}\right)}}$$

again, with a gap from $q = -Q - m_e$ to $q = -Q + m_e$ in the integration range.

This allows us to calculate the fraction of neutrons among the nucleons, by solving the differential equation

$$\boxed{\frac{\mathrm{d}X_n}{\mathrm{d}t} = -\lambda(n \to p)\, X_n + \lambda(p \to n)\,(1 - X_n)}$$

with appropriate initial conditions.

Initial Condition

One can check (Exercise) that for fixed time–independent temperature $T = T_\nu$ the two rates $\lambda(p \to n)$ and $\lambda(n \to p)$ have a ratio

$$\boxed{\frac{\lambda(p\to n)}{\lambda(n\to p)} = \exp\left(-Q/k_{\mathrm{B}}T\right) \text{ for } T = T_\nu}$$

which would imply a time–independent solution

$$\boxed{\frac{X_n}{X_p} = \frac{X_n}{1-X_n} = \exp\left(-Q/k_{\mathrm{B}}T\right) \ ,}$$

the non-relativistic equilibrium distribution. Thus what keeps this ratio away from equilibrium is the inequality of T and T_ν and the fact that they depend on time. When baryons are produced during the QCD–phase transition we have $k_{\mathrm{B}}T \gg Q$ and we can calculate the integrals by setting $T = T_\nu$ and $Q, m_e = 0$, such that

$$\begin{aligned} \lambda(n \to p) = \lambda(p \to n) \quad &= \quad A \int\limits_{-\infty}^{+\infty} \frac{q^4\,\mathrm{d}q}{\left(1+\mathrm{e}^{q/k_{\mathrm{B}}T}\right)\left(1+\mathrm{e}^{-q/k_{\mathrm{B}}T}\right)} \\ &= \quad \frac{7}{15}\pi^4 A\ (k_{\mathrm{B}}T)^5 = 0.400\ \mathrm{sec}^{-1}\left(\frac{T}{10^{10}\ {}^\circ\mathrm{K}}\right)^5 \end{aligned}$$

which is > 1 for $T > 1.22 \times 10^{10}\ {}^\circ\mathrm{K}$.

Thermal equilibrium means

$$\boxed{\frac{\mathrm{d}X_n}{\mathrm{d}t} = 0 \Leftrightarrow X_n = \frac{\lambda(p\to n)}{\lambda(p\to n)+\lambda(n\to p)} \Leftrightarrow X_n = \frac{1}{\left(1+\exp(Q/k_\mathrm{B}T)\right)}}$$

Crucially important is that the equilibrium value is established at high enough temperatures when the electron chemical potential is zero (still large numbers of electrons and positrons) such that no assumption about the initial proton/neutron ratio is necessary!

This is expected to be satisfied for $T > 3\times 10^{10}\ {}^\circ\mathrm{K}$ and we may solve our exact equation for X_n with this boundary condition. The solution can be obtained numerically; see the Table

$T(°\mathrm{K})$	$t(\mathrm{sec})$	X_n
1×10^{12}	0.0001	0.4962
3×10^{11}	0.0011	0.4875
1×10^{11}	0.0099	0.4626
3×10^{10}	0.1106	0.3798
1×10^{10}	1.008	0.2386
3×10^{9}	12.67	0.1654
1.3×10^{9}	91.09	0.1458
1.2×10^{9}	110.2	0.1425
1.1×10^{9}	135.1	0.1385
1×10^{9}	168.1	0.1333
9×10^{8}	2.12.7	0.1268
8×10^{8}	274.3	0.1182
7×10^{8}	362.6	0.1070
6×10^{8}	496.3	0.0919
3×10^{8}	1980	0.0172
1×10^{8}	17780	3.07×10^{-10}

Given the rate λ, the maximum number of collisions can be found by multiplying the rate with the Hubble time $H = 1/2t$. Earlier, we calculated

$$t = \sqrt{\frac{3}{16\pi G a g*}}\frac{1}{T^2} + \mathrm{const} = 0.994\,\mathrm{sec}\left(\frac{10^{10}\ {}^\circ\mathrm{K}}{T}\right)^2 + \mathrm{const}$$

such that, with $g* = 43/4$, $\lambda/H \simeq 0.8 \times \left(\frac{T}{10^{10}\ {}^\circ\mathrm{K}}\right)^3$ which is > 1 for $T > 1.1 \times 10^{10}\ {}^\circ\mathrm{K}$. Because of the T^3 behavior the rate drops fast and equilibrium is lost quickly between $3 \times 10^{10}\ {}^\circ\mathrm{K}$ and $10^{10}\ {}^\circ\mathrm{K}$. Little later all but neutron β–decay plays a role in proton– neutron transmutation, also due to the fact that electron and positron densities rapidly decrease because of e^+e^-–annihilation. Then the decrease of X_n just follows neutron decay.

Exercise: Show that $X_n \to 0.1609\ \exp\left(-t/\tau_n\right)$ with $\tau_n = 885.7$ sec is the neutron lifetime.

The above would be the story if nucleons would not get bound into nuclei where also neutrons get stable.

Nuclei if in Equilibrium

We are interested here in what happens after temperatures have dropped way below the proton mass $T \ll m_p$ such that all nucleons and nuclei are non-relativistic. In thermal equilibrium we then have (see Exercise at end of previous lecture)

$$\boxed{n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \mathrm{e}^{\frac{\mu_i - m_i}{T}}}$$

for the number densities of proton ($i = p$), neutron ($i = n$) or a nuclei of type $i = (A_i - Z_i)\, n + Z_i\, p$.

If nuclear reaction rates for the formation of a nucleus $(A_i - Z_i)\, n + Z_i\, p \leftrightharpoons i$ from protons and neutrons would be sufficiently high to maintain chemical equilibrium, we would have

$$\boxed{\mu_i = (A_i - Z_i)\, \mu_n + Z_i\, \mu_p}$$

for the chemical potentials. Since for free nucleons $g_i = 2$ for $i = p, n$ we have

$$\boxed{n_i = g_i \, A_i^{\frac{3}{2}} 2^{-A_i} \left(\frac{2\pi}{m_N T}\right)^{\frac{3}{2}(A_i-1)} \; n_p^{Z_i} \, n_n^{A_i - Z_i} \, \mathrm{e}^{B_i/T}}$$

where

$$\boxed{B_i = Z_i \, m_p + (A_i - Z_i) \, m_n - m_i}$$

is the binding energy of the nucleus. Besides in exponents, we use the nucleon mass m_N as an approximation $m_p = m_n = m_i/A_i = m_N$.

Nuclear binding energies of light nuclei

${}^{A_i}Z_i$	B_i	g_i	T_i for $\Omega_B h^2 \simeq 0.2$
${}^1H$	13.6 eV	(atomic binding energy)	
${}^2H$	2.22 MeV	3	0.75×10^9 °K
${}^3H$	8.48 MeV	2	1.4×10^9 °K
${}^3He$	7.72 MeV	2	1.3×10^9 °K
${}^4He$	28.3 MeV	1	3.3×10^9 °K
${}^7Li$	39.24 MeV	4	2.3×10^9 °K
${}^7Be$	37.60 MeV	4	2.2×10^9 °K
${}^{12}C$	92.2 MeV	1	2.9×10^9 °K

Total baryon number density now is:

$$\boxed{n_B = \sum_i n_i}$$

Its present value is usually encoded in the ratio

$$\boxed{\eta = \frac{N_B(T_0)}{n_\gamma(T_0)}\ ; \quad n_\gamma(T) = \frac{2\,\zeta(3)}{\pi^2}\,T^3\ ; \quad T_0 = 2.725\ °\mathrm{K}}$$

Baryon number conservation and the scaling property $n(t) \propto S(t)^{-3}$ allow us to calculate n_B at temperature T. We may also use the fact that the entropy is constant $S(t)^3\, s(t) = \mathrm{const}$ with $s(t) = \frac{1}{T(t)}\,(\rho(t) + p(t)) = g_{*s}(T)\,\frac{2\pi^2}{45}\,T^3(t)$ or $g_{*s}(T)\,S^3(t)\,T^3(t) = \mathrm{const}$:

$$\boxed{n_B = \eta\,\frac{2\zeta(3)}{\pi^2}\,\frac{g_{*s}(T)}{g_{*s}(T_0)}\,T^3}$$

as long as baryon number is conserved and the universe expands adiabatically. For $T \ll m_e$ we have the simpler relation $n_B = \eta\,n_\gamma = \eta\,\frac{2\zeta(3)}{\pi^2}\,T^3$.

In order to get ride of the unknown chemical potentials one considers $\frac{n_i}{n_p^{Z_i}\,n_n^{A_i-Z_i}}$ and represents the equilibrium distributions in terms of dimensionless ratios

$$X_i \equiv n_i/n_N\;,\quad X_p \equiv n_p/n_N\;,\quad X_n \equiv n_n/n_N$$

where n_N is the number density of all nucleons. Then

$$\boxed{X_i = \frac{g_i}{2}\,X_p^{Z_i}\,X_n^{A_i-Z_i}\,A_i^{\frac{3}{2}}\varepsilon^{(A_i-1)}\,\mathrm{e}^{B_i/T}}$$

where

$$\boxed{\varepsilon \equiv \frac{1}{2}\, n_N \mathrm{h}^3\, (2\pi m_N T)^{-\frac{3}{2}} = 2.96 \times 10^{-11} \left(\frac{S(t)}{10^{10}\, S(t_0)}\right)^{-3} \left(\frac{T}{10^{10}\ {}^\circ\mathrm{K}}\right)^{-3/2} \Omega_B\, h^2}$$

with $n_N = 3\Omega_B\, H_0^2\, (S/S_0)^3/8\pi G\, m_N$ the total number density of nucleons.

In the period of interest after e^+e^-–annihilation $T \propto 1/S$ such that

$$\boxed{\varepsilon = 1.46 \times 10^{-12} \left(\frac{T}{10^{10}\ {}^\circ\mathrm{K}}\right)^{+3/2} \Omega_B\, h^2}$$

Thus ε is very small such that in equilibrium the nuclear species i are essentially absent until the binding term takes over for sufficiently small temperatures:

$$\boxed{T_i \simeq \frac{B_i}{(A_i - 1)\, |\ln \varepsilon|}\ .}$$

where the T_i are given in the Table above. The dependence on $\Omega_B h^2$ is weak. Heavier elements have T_i's similar to He^4 as they have similar binding energy per nucleon.

If thermal and chemical equilibrium would have been maintained from temperatures of about $3 \times 10^{10}\ {}^\circ\mathrm{K}$ down to $10^{9}\ {}^\circ\mathrm{K}$ then during this time He^4 and the heavier elements would have been built first followed by the lighter ones He^3

and H^3, which would decay later into He^3 and finally H^2. This is not what has happened! The reason: densities were too low for any other than two–body reactions to take place!

The Chain of 2–body processes

1) $p + n \to d + \gamma$
2) $d + d \to H^3 + p\,,\ He^3 + n$
3) $d + H^3 \to He^4 + n\ ;\ \ d + He^3 \to He^4 + p$
4) slower processes involving γ's

- Step 1) no problem, rate of deuterium production per free neutron

$$\begin{aligned}\lambda_d &= 4.55 \times 10^{-20}\,\mathrm{cm}^3/\mathrm{sec} \times n_p = 511\,\mathrm{sec}^{-1}\left(\frac{S\,(t)}{10^{-9} S_0}\right)^{-3} X_p \Omega_B h^2 \\ &= 2.52 \times 10^4\,\mathrm{sec}^{-1}\left(\frac{T}{10^{10}\ {}^\circ\mathrm{K}}\right) X_p\,\Omega_B h^2\end{aligned}$$

In this period, after e^+e^-–annihilation we have $g* = 3.363$ and the time vs. temperature relation (last lecture) yields

$$t = \sqrt{\frac{3}{16\pi G a g*}}\frac{1}{T^2} + \mathrm{const} = 1.78\,\mathrm{sec}\left(\frac{10^{10}\ {}^\circ\mathrm{K}}{T}\right)^2 + \mathrm{const}$$

so

$$\boxed{\lambda_d\, t \simeq 4.5\times 10^4\left(\frac{T}{10^{10}}\right) X_p\, \Omega_B h^2}$$

which is substantially larger than unity until well after the temperature drops below $10^9\ {}^\circ\mathrm{K}$. Thus in this period the deuterium abundance is close to its equilibrium value

$$\boxed{X_d = 3\sqrt{2} X_p X_n\, \varepsilon\, \exp\left(\frac{B_d}{k_\mathrm{B} T}\right)}$$

Now the trouble: $T_d \simeq 0.7\times 10^9\ {}^\circ\mathrm{K}$ is low because B_d is small, hence deuteron remains rare until long after He^4 would have been abundant in thermal equilibrium $\Rightarrow$ processes 2) $\Rightarrow$ stops nucleosynthesis (processes $p + d \to He^3 + \gamma$ and $n + d \to H^3 + \gamma$ have too low rates to help much)!

❒ But when T drops below T_d the still present neutrons rapidly assembled in the most deeply bound light element He^4! However, must overcome Coulomb barrier! Does not proceed via 2) but

$$\boxed{\begin{array}{l} p + n \to Deuterium + n \to Tritium + p \to He^4 \\ \text{or} \\ p + n \to Deuterium + p \to He^3 + n \to He^4 \end{array}}$$

❒ Now per proper volume: n_{He} helium, n_H hydrogen nuclei: mass $4n_{He} + n_H$, hence fractional abundance by weight of helium $Y = 4n_{He}/(4n_{He} + n_H)$. Number of protons and neutrons $n_p = 2n_{He} + n_H$ and $n_n = 2n_{He}$. Fraction of nucleons which are neutrons $X_n = 2n_{He}/(4n_{He} + n_H)$. Hence $Y = 2X_n$!

❒ formation of heavier nuclei blocked: no stable nuclei of atomic weight 5 and 8.

In stars formation of unstable Be^8 in $He^4 + He^4$ collisions followed by resonant capture of another He^4 to form excited state of C^{12}

❐ When $T \sim T_d$ the exothermic reactions 2) start to build up heavier elements: (cross sections $\propto 1/\mathrm{v}$). They are

$$\begin{aligned} \langle \sigma(d + d \to H^3 + p)\,\mathrm{v}\rangle &\simeq 1.8 \times 10^{-17}\ \mathrm{cm}^3/\mathrm{sec}\ , \\ \langle \sigma(d + d \to He^3 + n)\,\mathrm{v}\rangle &\simeq 1.6 \times 10^{-17}\ \mathrm{cm}^3/\mathrm{sec}\ , \end{aligned}$$

and for the total rate one obtains

$$\begin{aligned} \lambda_{A^3} &= \left[\langle \sigma(d + d \to H^3 + p)\,\mathrm{v}\rangle + \langle \sigma(d + d \to He^3 + n)\,\mathrm{v}\rangle\right] X_d\, n_N \\ &= 1.9 \times 10^7\,\mathrm{sec}^{-1} \left(\frac{T}{10^{10}\ {}^\circ\mathrm{K}}\right)^3 X_d\, \Omega_B h^2 \end{aligned}$$

This compares to the expansion rate after e^+e^-–annihilation:

$H = 1/2t = 0.28 \left(\frac{T}{10^{10}\ {}^\circ\mathrm{K}}\right)^2\ \mathrm{sec}^{-1}$

for T in the region $10^9\ {}^\circ\mathrm{K}$ we have $\lambda_{A^3} = H$ at

$X_d \simeq 1.2 \times 10^{-7}/\Omega_B h^2$ thus $X_d \simeq 0.6 \times 10^{-5}$ for $\Omega_B h^2 \simeq 0.02$

This value is reached in thermal equilibrium at T about $10^9\ {}^\circ\mathrm{K}$ and depends weakly on $\Omega_B h^2$.

What it means: nucleosynthesis began around $10^9\ {}^\circ\mathrm{K}$

and not at $0.75 \times 10^9\ {}^\circ\mathrm{K}$. From table: $t = 168\ \mathrm{sec}$ where $X_n = 0.1609 \times \exp(-168/885)$ such that (p =primordial)

$$\boxed{Y_p \simeq 2 \times 0.1609 \times \exp(-168/885) \simeq 0.27}$$

Note:

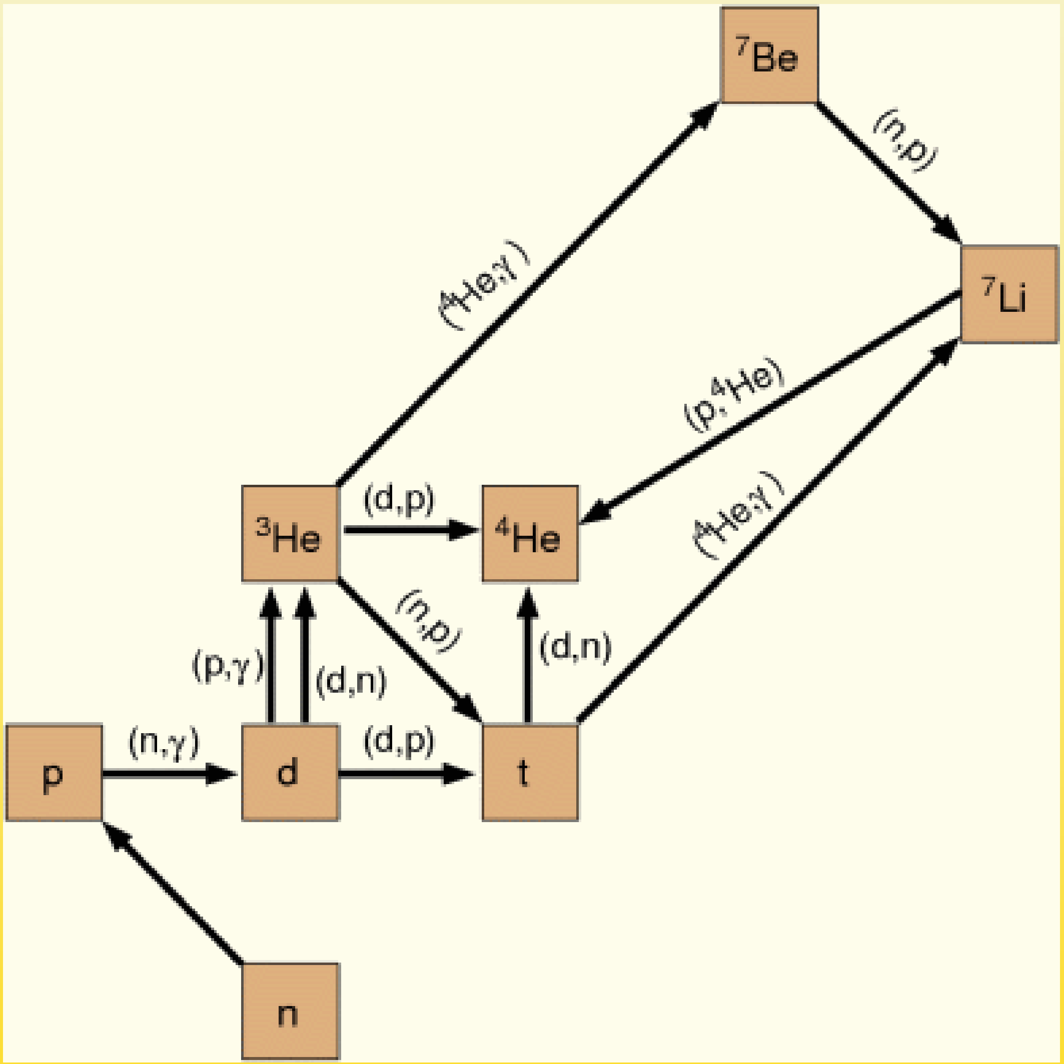
- the larger the nucleon density the higher the temperature nucleosynthesis began

- the less time there was for neutrons to decay before nucleosynthesis

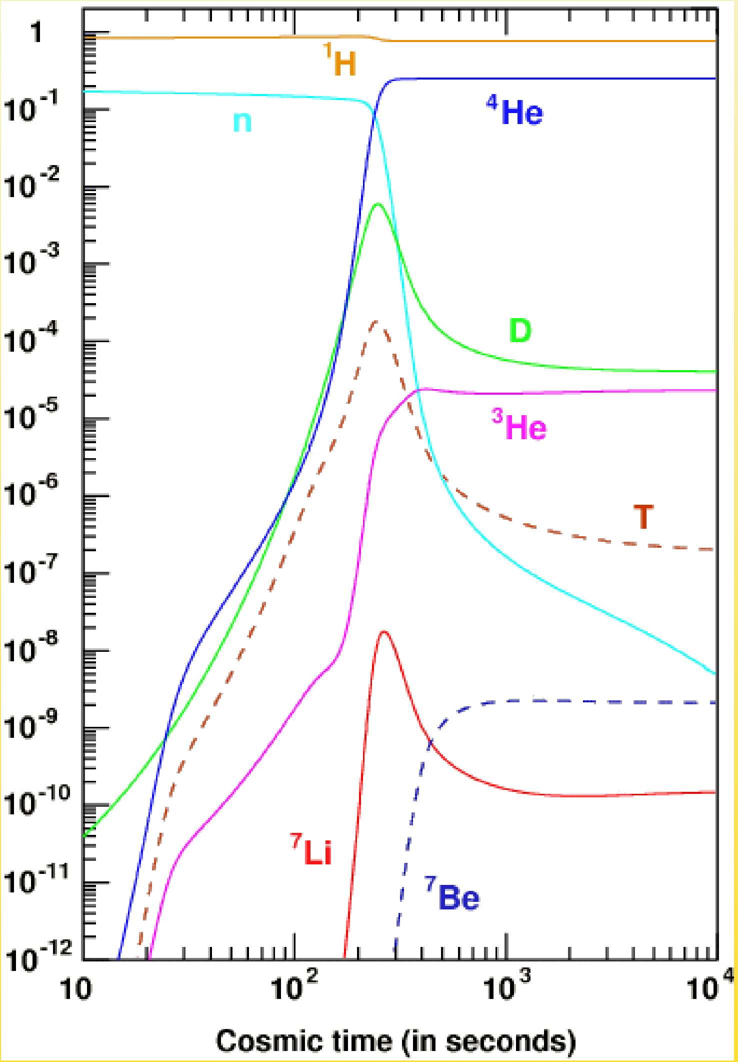
- the higher the He^4 abundance

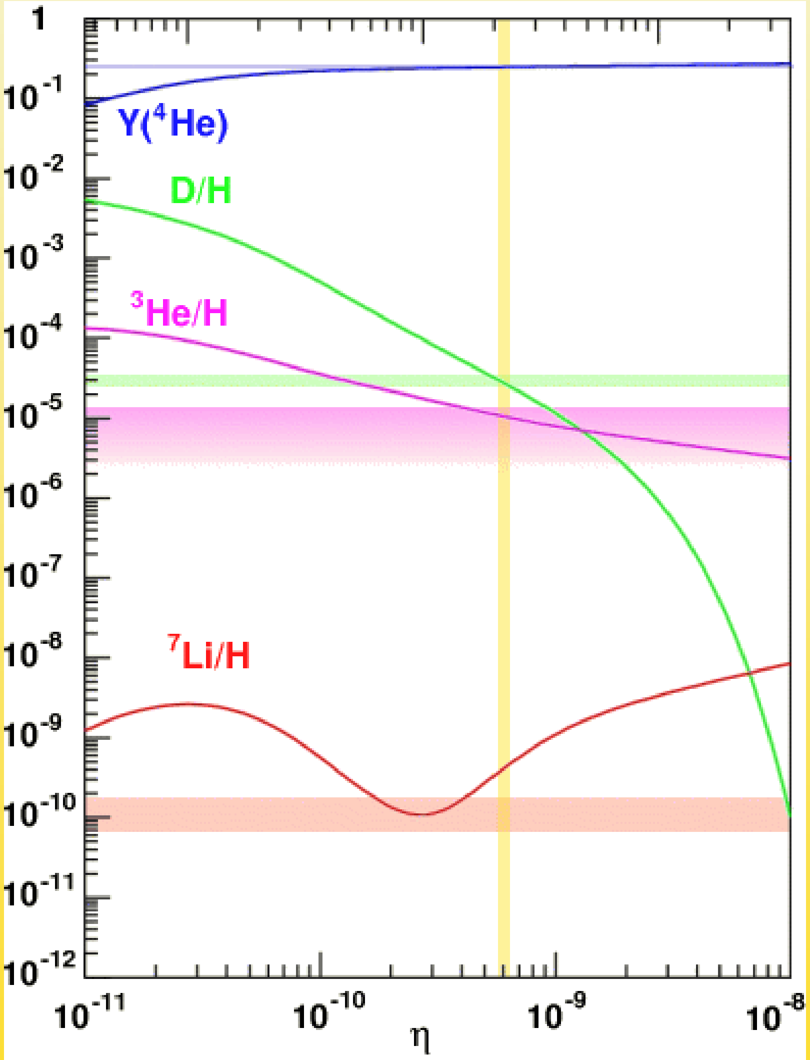
Results of modern state of the art calculations:

$$\boxed{\Omega_B h^2 = 3.65\,\eta \times 10^7 \;\; [\Omega_B h^2 = 0.02 \quad \Leftrightarrow \quad \eta = 5.5 \times 10^{-10}}$$



Big Bang nucleosynthesis: reaction chains





Big Bang nucleosynthesis: predicted abundances

Data on He^4/H ratio

He^4/H ratio: spectroscopy of so called HII regions (ionized gas containing low abundances of "metals" [elements other than hydrogen and helium], typically in blue compact galaxies) ✲⋙

$$\boxed{Y_p = 0.2477 \pm 0.0029 \text{ corresponding to } \eta = (5.813 \pm 1.81) \times 10^{-10}}$$

- The best Y_p determination available, that by Peimbert et al. (2007a), is in agreement with the D_p determination and with the WMAP observations under the assumption of SBBN. The errors in the Y_p determination are still large and there is room for non-standard physics. ["S"BBN="Standard" Big Bang Nucleosynthesis]

- To improve the accuracy of the Y_p value derived under the assumption of SBBN and the $\Omega_B h^2$ derived from the background radiation, a new determination of the neutron lifetime is needed to sort out the difference between the τ_n obtained by Arzumanov et al. (2000) and the τ_n obtained by Serebrov et al. (2005, 2008):

COSMOLOGICAL PREDICTIONS BASED ON SBBN AND OBSERVATIONS

Table 1. FOR τ_n = 885.7 ± 0.8 s

Input	Y_p	D_p	η_{10}	$\Omega_B h^2$
Y_p	0.2477 ± 0.0029^a	$2.93\pm^{2.53}_{1.06}{}^b$	5.625 ± 1.81^b	0.02054 ± 0.00661^b
D_p	0.2479 ± 0.0007^b	2.82 ± 0.28^a	5.764 ± 0.360^b	0.02104 ± 0.00132^b
WMAP	0.2487 ± 0.0006^b	2.49 ± 0.13^b	6.226 ± 0.170^b	0.02273 ± 0.00062^a

^{a} Observed value. ^{b} Predicted value. References: τ_n Arzumanov et al. (2000); Y_p Peimbert et al. (2007a); D_p O Meara et al.(2006); WMAP Dunkley et al. (2009).

Table 2. FOR τ_n = 878.5 ± 0.8 s

Input	Y_p	D_p	η_{10}	$\Omega_B h^2$
Y_p	0.2477 ± 0.0029^a	$2.22\pm^{1.46}_{0.71}{}^b$	6.688 ± 1.81^b	0.02442 ± 0.00661^b
D_p	0.2462 ± 0.0007^b	2.82 ± 0.28^a	5.764 ± 0.360^b	0.02104 ± 0.00132^b
WMAP	0.2470 ± 0.0006^b	2.49 ± 0.13^b	6.226 ± 0.170^b	0.02273 ± 0.00062^a

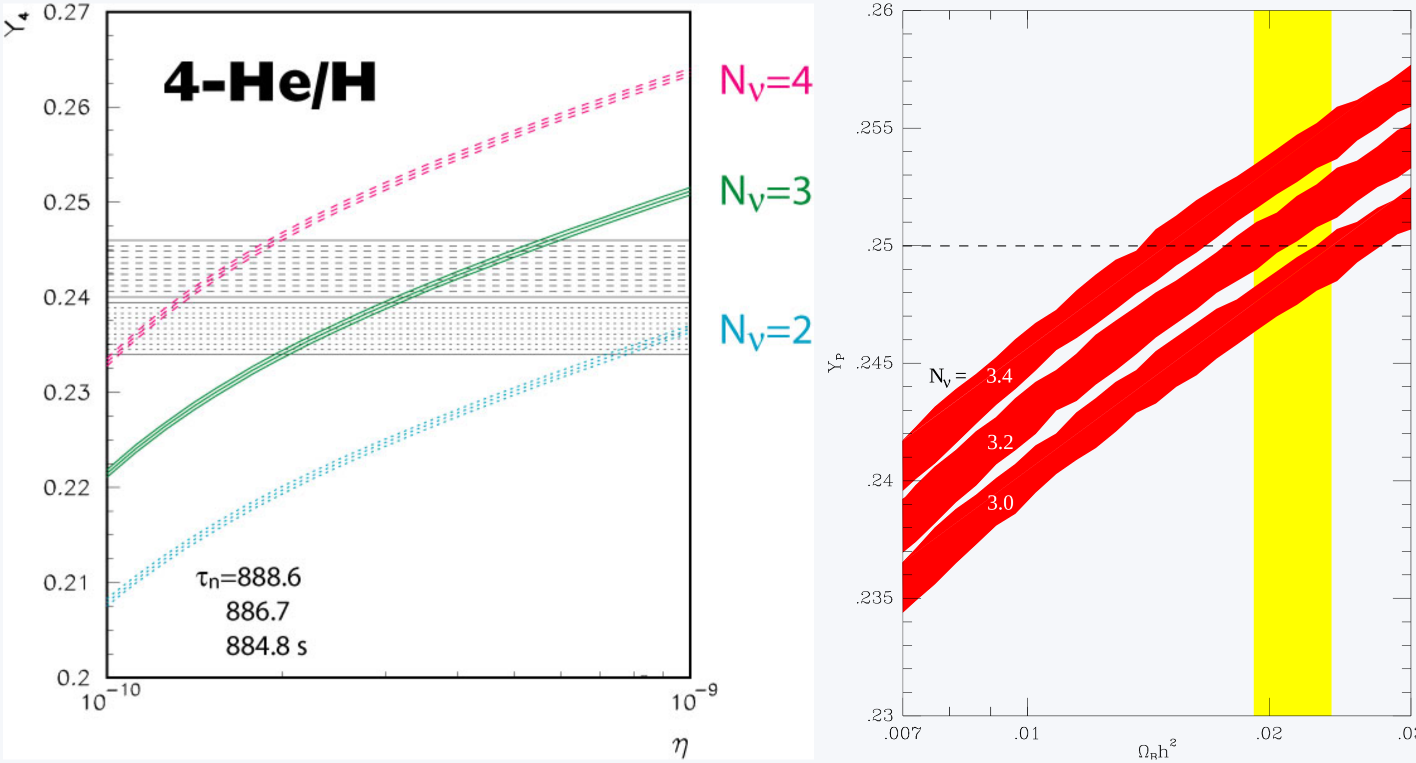
^{a} Observed value. ^{b} Predicted value. References: same as in Table 1 with the exception of τ_n that comes from Serebrov et al. (2005, 2008).

Characteristics of Y_p:

❖ The figure shows that Y_p is not very sensitive to η, and therefor not a very good observable to constrain η.

❖ The helium rate is a good and important constraint on the expansion rate which yields $T(t)$ in which $g*(T)$ plays a crucial role. One test here is the number of light neutrinos N_ν. With $N_\nu = 4$ we would have $g*(T) = 3.813$ instead of $g*(T) = 3.363$, which would shorten the time as $t \to t \times \sqrt{g*(N_n u = 3)/g*(N_\nu = 4)} = t \times 0.94$. This would lead to a higher helium abundance at the temperature $10^9\ ^\circ\mathrm{K}$ when nucleosynthesis begins.

Note: crucial interrelation between particle physics and cosmology/astrophysics: number of neutrinos, neutron lifetime!



^4He production for $N_\nu = 3.0, 3.2, 3.4$. The vertical band indicates the baryon density consistent with $(D/H)_p = (2.7 \pm 0.6) \times 10^{-5}$ and the horizontal line indicates a primeval ^4He abundance of 25%. The widths of the curves indicate the 2σ theoretical uncertainty [David N. Schramm and Michael S. Turner 1997].

Other light elements:

- H^2, H^3, He^3, Li^7, and Be^7 are produced in BBN.
- The unstable nuclei H^3 decayed later by β^+–decay to He^3, and Be^7 by e^-–capture to Li^7.
- The formation of He^4 from free neutrons at $T \sim 10^9\ ^\circ\mathrm{K}$ of course was not 100% efficient, and free neutrons still were available to build other light nuclei via the two–body reactions of deuterium formation and deuterium fusion

Data on D/H ratio

The measurement of the deuterium abundance is complicated because of the small binding energy, which makes it easy to destroy it in stars. Modern techniques are based on observation of spectra from quasi-stellar objects.

D/H ratio: High-resolution spectra reveal the presence of D in high-redshift, low-metallicity quasar absorption systems via its isotope-shifted Lyman-α absorption. It is believed that there are no astrophysical sources of deuterium. Averaging the seven most precise observations of deuterium in quasar absorption systems gives

$$\boxed{D/H = (2.82 \pm 0.12) \times 10^{-5} \text{ corresponding to } \eta = (5.813 \pm 1.81) \times 10^{-10}}$$

✲⋙, ✲⋙

Characteristics of D/H:

❖ The figure shows that D/H is very sensitive to η, and therefore it is a very good observable to constrain η. The weak point is that its hard to be measured reliably.

❖ The deuterium ratio is determined by $X_d = 3\sqrt{2} X_p X_n \,\varepsilon\, \exp\left(\frac{B_d}{k_{\rm B} T}\right)$ (derived above). The higher the baryon density, the more complete is the incorporation of neutrons into ${}^4He$, and hence the smaller the abundance of deuterium. As we found above, at the start of BBN $X_d \sim 0.6 \times 10^{-5}$ for $\Omega_B h^2 = 0.02$. First X_d continued to rise for some time as the temperature dropped and the exponential term increased. But then X_d decreased again because X_n decreased due to the incorporation of free neutrons into deuterium and the conversion of deuterium to ${}^3H$ and ${}^3He$, which convert to ${}^4He$. The final result of a detailed analysis is not far from the deuterium fraction X_d we found at $T = 10^9\ {}^\circ\mathrm{K}$ where we obtained $X_d \simeq 1.2 \times 10^{-7}\ /\Omega_B h^2$.

Data on He^3/H ratio

Data were obtained from galactic HII regions and from planetary nebulae. Problem: He^3 is produced and destroyed in stars, and it is not clear whether the measured abundance is primordial. Quoted bound is

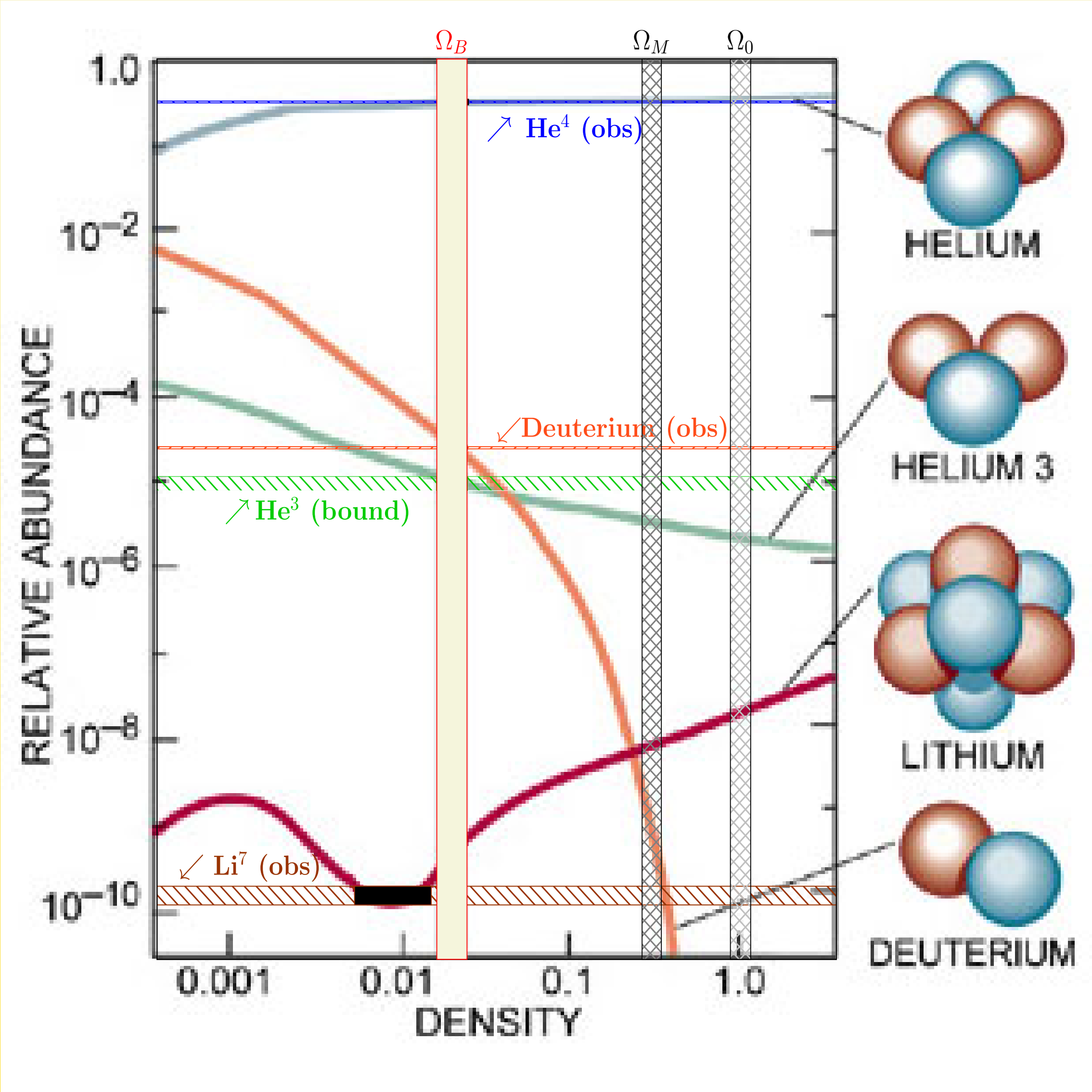
$$\boxed{He^3/H < (1.1 \pm 0.2) \times 10^{-5}}$$

Data on Li^7/H ratio

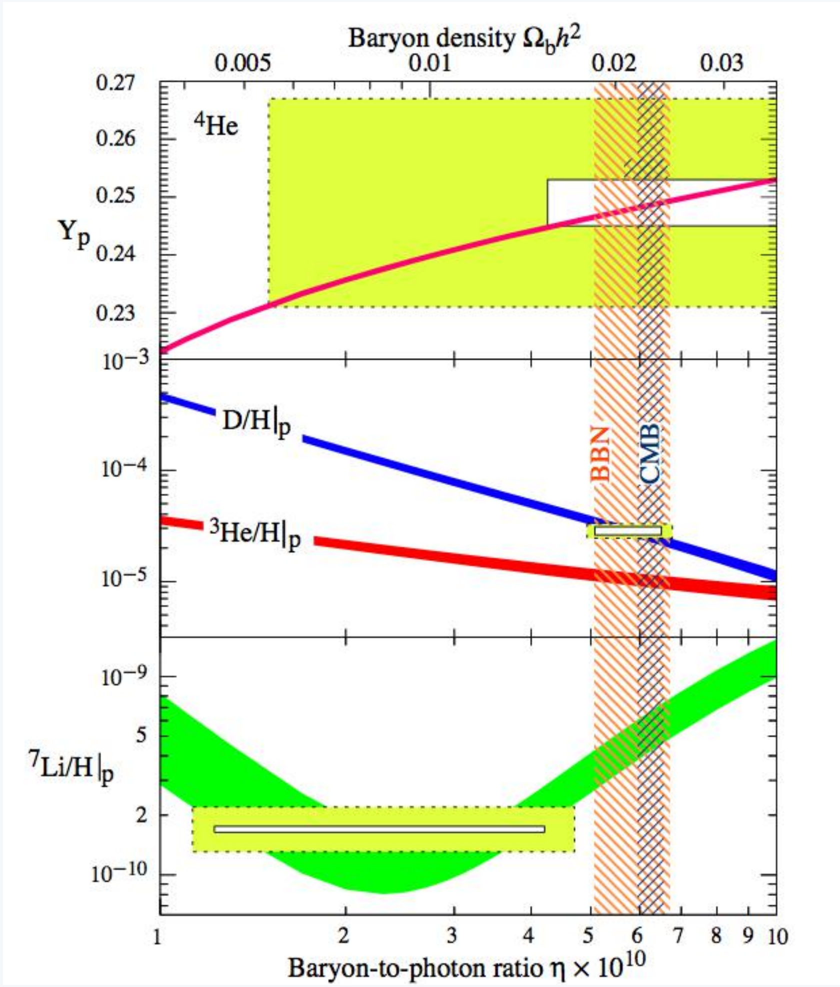
Is interesting because of its more complicated dependence on $\Omega_B h^2$. Li^7 is formed in two ways, by $H^3 + He^4 \to Li^7 + \gamma$ and $He^3 + He^4 \to Be^7 + \gamma$, the latter followed much later by $e^- + Be^7 \to \nu + Li^7$. At intermediate baryon densities Li^7 is destroyed by $p + Li^7 \to He^4 + He^4$, which leads to the dip at $\eta \sim 2.5 \times 10^{-10}$.

$$\boxed{Li^7/H = (1.7 \pm 0.06 \pm 0.44) \times 10^{-10}}$$

Theory vs. Data: the BBN concordance range



Status Particle Data Group: (note different scales for different ratios)



Predicted abundances of ^{4}He, D, ^{3}He, and ^{7}Li (bands show 95% CL range). Boxes show observed values (smaller boxes: $\pm 2\sigma$ statistical errors; larger boxes: $\pm 2\sigma$ statistical and systematic errors). Narrow vertical band: CMB measure of the cosmic baryon density; wider band: BBN concordance range (both at 95% CL).

For other summaries of BBN: ✲⋙ ✲⋙ ✲⋙ ✲⋙ or ✲⋙

Conclusions:

❖ $\sim$ 25% ^4He is a key piece of evidence for the Big Bang model, at $\sim$ 10% precision.

❖ D is the "baryometer"; best estimates of D/H imply $\Omega_B h^2 \approx 0.02$.

❖ ^3He and ^7Li give two additional, low precision (factor $\sim$ 10) tests. Problem here:
both elements can be produced and destroyed in stars.

BBN concordance provides a baryon content

$$\boxed{0.019 \leq \Omega_B h^2 \leq 0.024 \text{ at } (95\% \text{ CL})}$$

The key result is the understanding of the baryonic matter budget ($\sim$ 75% Hydrogen, $\sim$ 25% Helium plus well understood tiny contributions of other light elements, much less than 1% of all other elements combined). Note that $\Omega_B \ll \Omega_0$ far from being able to close the universe. Furthermore, the cosmic density of (optically) luminous matter is $\Omega_{\text{lum}} \simeq 0.0024\, h^{-1}$, hence $\Omega_{\text{lum}} \ll \Omega_B$: most baryons are optically dark, probably in the form of a diffuse intergalactic medium, brown dwarfs, planets etc. Finally, given that $\Omega_M \sim 0.3$, it appears that most matter in the Universe is not only dark, but also is non-baryonic.

$\Omega_B \simeq 0.02$ obtained from BBN is in good agreement with the independent determination from CMB [$\Omega_B \simeq 0.0227$]. As is well known Ω_B is less than 10% of $\Omega_M \simeq 0.3$ [CMD gives $\Omega_M \simeq 0.133$ only], which includes any form of cold dark matter. The latter is still way below the critical density $\Omega_0 = 1$, at the limit of closing the universe and presumed by theory (inflation).

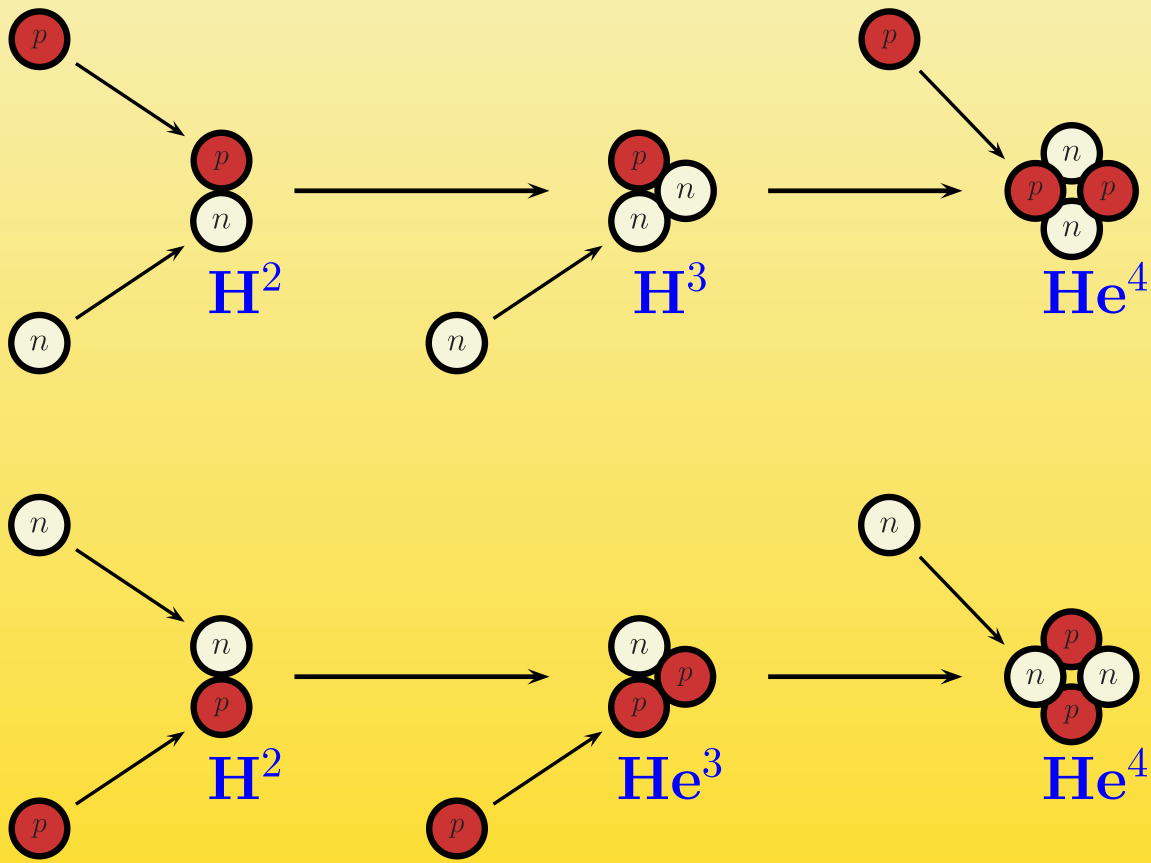
Actual cosmological parameters (PDG 2010):

Parameter	Symbol	Value
Hubble parameter	h	0.72 ± 0.03
Total matter density	Ω_M	$\Omega_M h^2 = 0.133 \pm 0.006$
Baryon density	Ω_B	$\Omega_B h^2 = 0.0227 \pm 0.0006$
Cosmological constant	Ω_Λ	0.74 ± 0.03
Radiation density	Ω_r	$\Omega_r h^2 = 2.47 \times 10^{\ 5}$
Neutrino density	$\Omega_\nu h^2 = \frac{\sum m_\nu}{93\ \mathrm{eV}}$	$\Omega_\nu \sim 0.001$

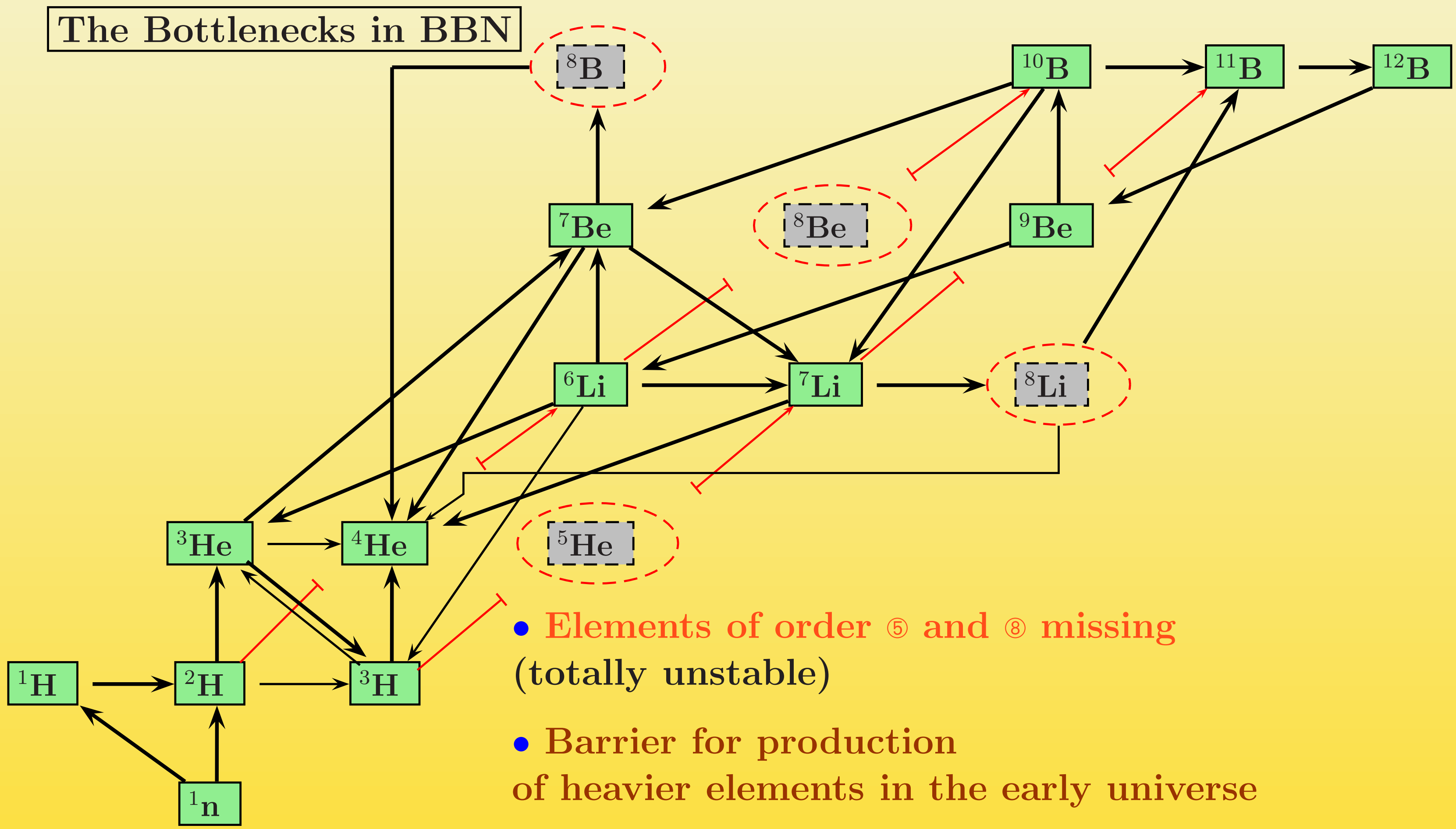
Summary and Outlook

As the universe expands, it cools. Free *neutrons* and *protons* are less stable than *helium* nuclei, and the *protons* and *neutrons* have a strong tendency to form *helium*–4. However, forming *helium*–4 requires the intermediate step of forming *deuterium*. At the time at which nucleosynthesis occurs, the temperature is high enough for the mean energy per particle to be greater than the binding energy of *deuterium*; therefore any *deuterium* that is formed is immediately destroyed (a situation known as the deuterium bottleneck). Hence, the formation of *helium*–4 is delayed until the universe becomes cool enough to form *deuterium* (at about $T = 0.1$ MeV), when there is a sudden burst of element formation. Shortly thereafter, at twenty minutes after the Big Bang, the universe becomes too cool for any nuclear fusion to occur. At this point, the elemental abundances are fixed, and only change as some of the radioactive products of BBN (such as *tritium*) decay.

$\boxed{\text{Nucleosynthesis}}$ is predominantly He^4 fusion



As the universe cools down for a short time protons and neutron can fuse to form light atomic nuclei

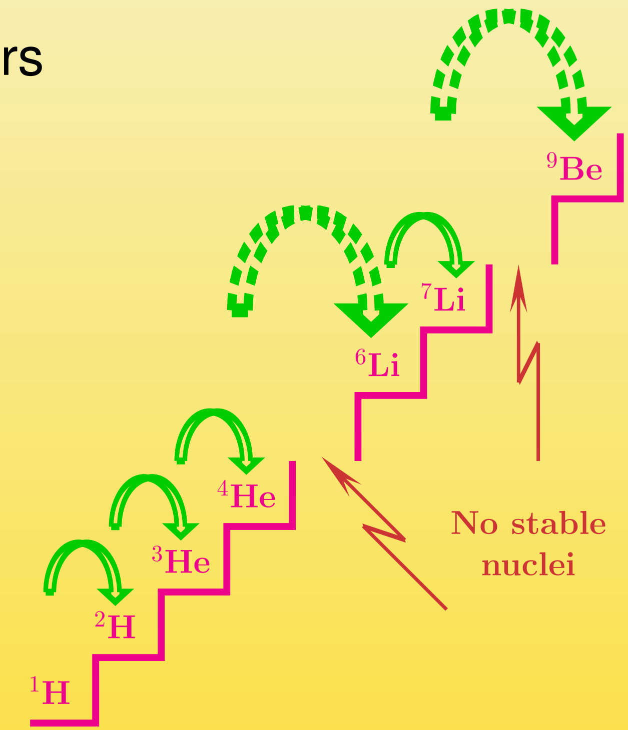


- Elements of order ⑤ and ⑧ missing (totally unstable)
- Barrier for production of heavier elements in the early universe
- heavy elements must have been produced in stars (extreme temperature and pressure)

For element fusion in stars see "Neutrino Physics meets Astrophysics" in Lecture 3 ✲⋙

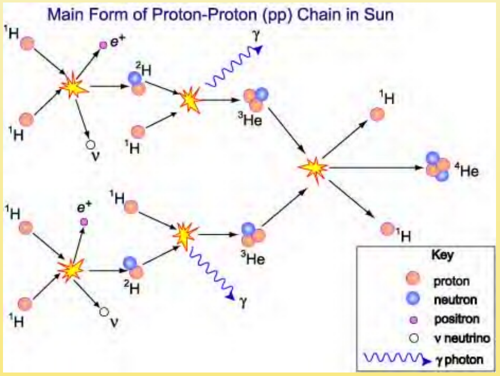
Primordial versus Stellar Nucleosynthesis

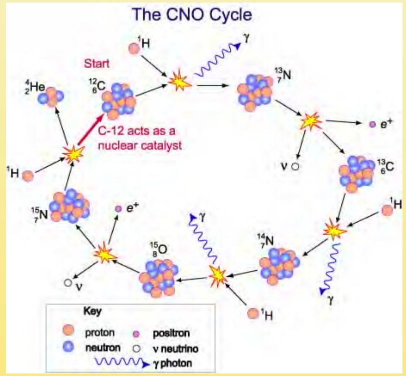
- Timescale
  - Stellar Nucleosynthesis (SN): billions of years
  - Primordial Nucleosynthesis (PN): minutes
- Temperature evolution
  - SN: slow increase over time
  - PN: rapid cooling
- Density
  - SN: 100 gr/cm^3
  - PN: 10^{-5} gr/cm^3 (like air)
- Photon to baryon ratio
  - SN: less than a single photon per baryon
  - PN: billions of photons to every baryon

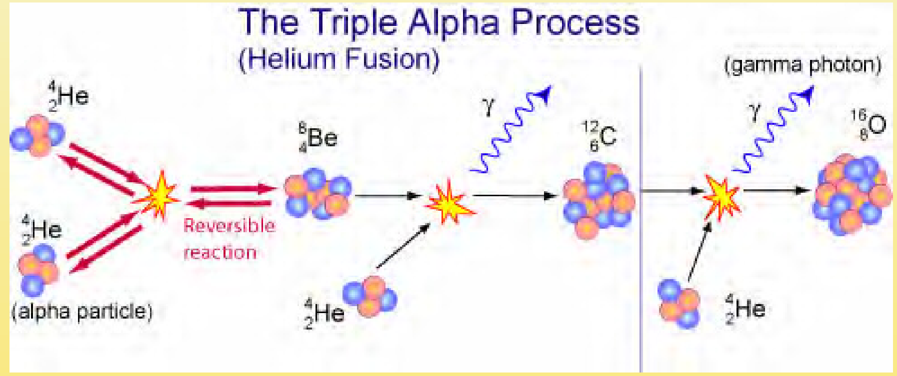


The lack of stable elements of masses 5 and 8 makes it very difficult for BBN to progress beyond Lithium and even Helium.
Need conditions found in stars for formation of the heavy elements.

Nucleosynthesis by Neutron Capture





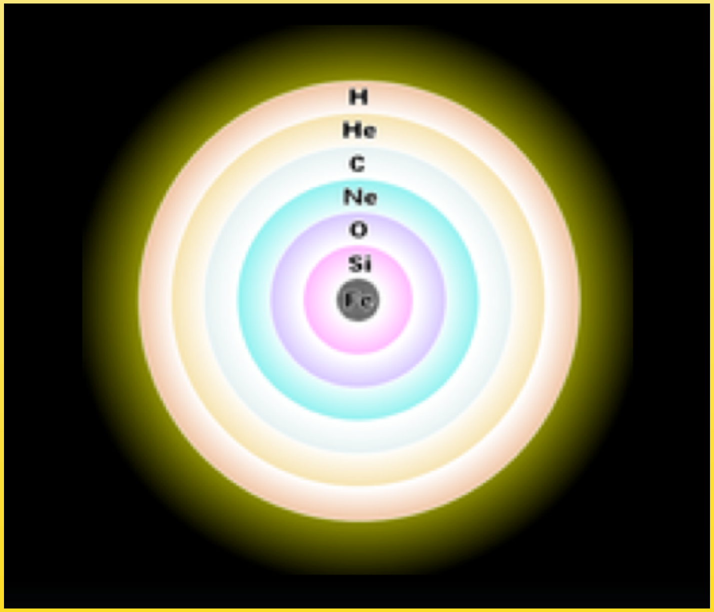


Stars like the sun use the proton-proton chain (main branch). Stars heavier than the sun (say 20 to 120 times the mass of the sun) use carbon-12 as a catalyst (CNO-cycle). Very heavy stars use the triple-alpha process forming $^{12}\mathrm{C}_6$ from 3 Helium-4 nuclei (via intermediate Beryllium-8). Formation of elements beyond iron ($^{56}\mathrm{Fe}_{26}$) is possible only by neutron capture to produce isotopes.

Unstable isotopes decay into new elements. Example:

$^{110}\mathrm{Cd}_{48} + ^{1}n_0 \rightarrow ^{111}\mathrm{Cd}_{48}$	neutron capture
$^{111}\mathrm{Cd}_{48} + ^{1}n_0 \rightarrow ^{112}\mathrm{Cd}_{48}$	stable isotope
$^{112}\mathrm{Cd}_{48} + ^{1}n_0 \rightarrow ^{113}\mathrm{Cd}_{48}$	stable isotope
$^{113}\mathrm{Cd}_{48} + ^{1}n_0 \rightarrow ^{114}\mathrm{Cd}_{48}$	stable isotope
$^{114}\mathrm{Cd}_{48} + ^{1}n_0 \rightarrow ^{115}\mathrm{Cd}_{48}$	$\boxed{\text{unstable}}$ isotope
$^{115}\mathrm{Cd}_{48} \rightarrow ^{115}\mathrm{In}_{49} + e^- + \bar{\nu}_e$	radioactive decay

Shell structure of a mass rich star:



Nucleosynthesis is rather well understood, the main problems seem to be the unambiguous determination of the element abundances (in particular ^{3}He and Lithium). Nevertheless, results show a remarkable pattern for the abundances and the existence of a concordance range is highly non-trivial, as it covers almost 10 orders of magnitude in the element densities. Even more remarkable is the agreement with data obtained from the analysis of the CMB, which is completely independent. The CMB will be discussed below.

Summary: “Cosmic Concordance” via BBN

- Primordial nucleosynthesis
  - explains observed light element abundances if the density of normal matter (baryons) in the universe is in the range $(3.2 - 4.5) \times 10^{-31}$ gr/cm^3 or 0.23 hydrogen atoms per cubic meter
- Precise observational test
  - independent measurements of the abundances of the four lightest stable elements lead to consistent constraints on the density of normal matter
  - provides confidence that primordial or Big Bang nucleosynthesis provides the correct explanation of the formation of the light elements

With BBN we have quited the pure particle physics era and entered the domain of theoretically more involved nuclear physics. At this stage only the simplest nuclei come into play, this allows us to make rather unambiguous predictions. The trouble with clear cut experimental tests follows where elements produced in primordial nucleosynthesis later also were produced in stars which makes the interpretation of observed abundances difficult.

Exercise pages

Exercise: check (numerically) that for fixed time–independent temperature $T = T_\nu$ the two rates $\lambda(p \to n)$ and $\lambda(n \to p)$ have a ratio $\frac{\lambda(p\to n)}{\lambda(n\to p)} = \exp\left(-Q/k_\mathrm{B}T\right)$ for $T = T_\nu$.

Exercise: show that $X_n \to 0.1609\ \exp\left(-t/\tau_n\right)$, with $\tau_n = 885.7$ sec is the neutron lifetime, fits the tabulated values of X_n for not too small times.

Exercise: at some time in the expansion of the universe photons were in thermal equilibrium with non-relativistic electrons (matter in general). Photons then followed the Planck black-body radiation equilibrium distribution with equilibrium temperature $T_\gamma = T_e$. As expansion continues, assume (for simplicity) that at some fixed time t_L (time of last-scattering) photon stop to interact with matter, because photon energy falls below the interaction threshold energy. Show that, up to a change in temperature, the form of the distribution remains unchanged by the expansion of the universe even though photons get out of equilibrium with matter. How does the temperature change?

Previous ⋘| , next |⋙ lecture.