Open problems in g-2 and related topics.

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(\(g - 2\))_\mu

Outline of Talk:

1. **Hadronic Light-by-Light**
2. **Isospin breaking**: \(\tau\ vs. e^+ e^-\)
3. **CMD-2 vs. KLOE**
4. **Final state radiation of hadrons**
5. **4-loop QED**
6. **Outlook**
1) The \((g - 2)_\mu\) problem:  
(Experiment (BNL 2004))
\[
a_\mu^{\pm} = 11659208(6) \times 10^{-10}
\]

(Theory)
\[
a_\mu^{\pm} = 11659182.7(7.3) \times 10^{-10}
\]
\[
\alpha_{\mu}^{\text{Exp}} - \alpha_{\mu}^{\text{The}} = 25.3 \pm 9.4 \times 10^{-10} \quad 2.7\sigma
\]

Note:
\[
\delta \alpha_{\mu}^{\text{Exp}} = 6.0 \times 10^{-10} 
\delta \alpha_{\mu}^{\text{HVP}} = 6.4 \times 10^{-10}
\]

2) The \(\alpha(M_Z)\) problem: input for electroweak precision physics
\[
\frac{\delta \alpha}{\alpha} \sim 3.6 \times 10^{-9}
\]
\[
\frac{\delta G_\mu}{G_\mu} \sim 8.6 \times 10^{-6}
\]
\[
\frac{\delta M_Z}{M_Z} \sim 2.4 \times 10^{-5}
\]
\[
\frac{\delta \alpha(M_Z)}{\alpha(M_Z)} \sim 1.6 \div 6.8 \times 10^{-4} \quad \text{(present)}
\]
\[
\frac{\delta \alpha(M_Z)}{\alpha(M_Z)} \sim 5.3 \times 10^{-5} \quad \text{(ILC requirement)}
\]
Hadronic Light-by-Light scattering contribution

Melnikov-Vainshtein improvement of EJLN/HGS approach:

- Hadronic light–by–light scattering $a_{\mu}^{\text{lbl}} = (80 \pm 40) \times 10^{-11}$ (Knecht & Nyffeler 02)
- $a_{\mu}^{\text{lbl}} = (136 \pm 25) \times 10^{-11}$ (Melnikov & Vainshtein 03)

(Kinoshita et al., Bijnens et al.)

Low energy effective theory: e.g. ENJL

MV and KN utilize the same model LMD+V form factor:
\[ F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{4\pi^2 F_\pi^2}{N_c} \frac{q_1^2 q_2^2 (q_1^2 + q_2^2) - h_2 q_1^2 q_2^2 + h_5 (q_1^2 + q_2^2) + (N_c M_1^4 M_2^4/4\pi^2 F_\pi^2)}{(q_1^2 + M_1^2)(q_1^2 + M_2^2)(q_2^2 + M_1^2)(q_2^2 + M_2^2)}, \]

where \( M_1 = 769 \text{ MeV}, M_2 = 1465 \text{ MeV}, h_5 = 6.93 \text{ GeV}^4. \)

with two modifications:

- form factor: undressed soft photon (non-renormalization of ABJ)
- \( h_2 = 0 \pm 20 \text{ GeV}^2 \) (KN) vs. \( h_2 = -10 \text{ GeV}^2 \) (MV) fixed by twist 4 in OPE \((1/q^4)\)

<table>
<thead>
<tr>
<th>( \pi^0, \eta, \eta'[\pi^0] )</th>
<th>( a_1[f_1, f_1^*] )</th>
<th>( \pi^\pm )</th>
<th>pQCD/QPM</th>
<th>tot</th>
</tr>
</thead>
<tbody>
<tr>
<td>HK</td>
<td>83(06)</td>
<td>1.7 ([a_1])</td>
<td>-4.5(8.5)</td>
<td>10(11)</td>
</tr>
<tr>
<td>BPP</td>
<td>85(13)</td>
<td>-4(3) ([a_1 + f_0])</td>
<td>-19(5)</td>
<td>21(3)</td>
</tr>
<tr>
<td>KN</td>
<td>83(12)</td>
<td></td>
<td></td>
<td>80(40)</td>
</tr>
<tr>
<td>MV</td>
<td>114.5[76.5]</td>
<td>22[7]</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
The iso-vector part of \( \sigma(e^+e^- \rightarrow \text{hadrons}) \) may be calculated by an iso-spin rotation from \( \tau^- \)-decay spectra (to the extend that CVC is valid).

\[ \tau^- \rightarrow X^- \nu_{\tau} \quad \Longleftrightarrow \quad e^+e^- \rightarrow X^0 \]

\( X^- \) and \( X^0 \) are hadronic states related by iso-spin rotation.

The \( e^+e^- \) cross-section is then given by

\[ \sigma^{I=1}_{e^+e^- \rightarrow X^0} = \frac{4\pi\alpha^2}{s} v_{1,X^-}, \quad \sqrt{s} \leq M_{\tau} \]

in terms of the \( \tau \) spectral function \( v_1 \).
All kind of isospin breaking effects have to be taken into account !!!

(V. Cirigliano, G. Ecker and H. Neufeld)

After known isospin corrections:

Experimental problems?
Comparison of $\tau$–data:

$\tau$–data may be not so easy; DELPHI, L3 could not measure $\tau$ spectral–functions; ALEPH vs. OPAL no good agreement.
S. Ghozzi, F. J

Fit data by same Gounaris-Sakurai formula: Only parameters differ
in first place mass and width of $\rho$ ! Two parameter fit (crude)

ALEP vs. CMD-2: $\Delta m_\rho = 2.7 \pm 0.8$ and $\Delta \Gamma_\rho = 1.3 \pm 1.0$ (S. Ghozzi, F. J.)

$[\Delta m_\rho = 3.1 \pm 0.9$ and $\Delta \Gamma_\rho = 2.3 \pm 1.6]$ (M. Davier (Pisa))

Problem with theory: usual argument

$\Delta m_\rho^2 = \Delta m_\pi^2$

from a sum rule yields $m_{\rho^-} - m_{\rho^0} = \frac{1}{2} \frac{\Delta m_\pi^2}{m_{\rho^0}} \sim 0.82 \text{MeV}!$. Too large by factor 2! width??

A. Höcker at ICHEP Beijing August 2004:

An empirical isospin-breaking correction of the $\rho$ resonance lineshape (mass and width) improves but does not restore the agreement between the two data sets. It is a consequence of this confirmation that, until the CVC puzzle is solved, only $e^+ e^-$ data should be used for the evaluation of the dispersion integral. Doing so, and including the KLOE data, we find that the Standard Model prediction of $a_\mu$ differs from the experimental value by 2.7 standard deviations.

W. Morse, BNL:

Is the source of isospin breaking a charged Higgs exchange?
Discusses how a charged Higgs propagator would modify the form factor in $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$ decays.
Isospin breaking in $\tau$--decays via charged Higgs exchange

Typically in 2HDM: Large one-loop RC (M. Krawczyk, D. Temes) in leptonic $\tau$--decays see talk by M. Krawczyk

Large isospin violating $H^-$ exchange in hadronic $\tau$--decays (W. Morse)?

$$R = \frac{|\Psi_W + \Psi_H|^2 - |\Psi_W|^2}{|\Psi_W|^2}$$

Correctly normalized effect by far too small!
(g − 2)_\mu

\section*{3 KLOE vs. CMD-2 Hadronic Cross Sections}

\textbf{Radiative Return @ DAΦNE:} KLOE-Measurement of the hadronic cross section \(\sigma(e^+e^- \rightarrow \pi^+\pi^-)\) below 1 GeV

\textbf{Radiator-Function H(s) from Phokhara-Generator}

\[ M^2_{\pi\pi} \frac{d\sigma_{\pi\pi}}{dM^2_{\pi\pi}} = \sigma_{\pi\pi}(s) \times H(s) \]

\textbf{KLOE Measurement: Pion Formfactor}

\begin{align*}
|F_{\pi}(s)|^2 & \quad \text{CMD-2} \\
& \quad \text{KLOE}
\end{align*}
Note on new KLOE result: see talk by S. Müller

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>extended to KLOE range</th>
</tr>
</thead>
<tbody>
<tr>
<td>my old value:</td>
<td>694.75 (5.15) (6.83) [8.56]</td>
<td></td>
</tr>
<tr>
<td>subtract cmd2:</td>
<td>389.36 (2.75) (2.59) [3.78]</td>
<td></td>
</tr>
<tr>
<td>KLOE:</td>
<td>388.75 (0.52) (5.05) [5.08]</td>
<td></td>
</tr>
<tr>
<td>add weighted</td>
<td>389.24 (1.14) (2.39) [2.65]</td>
<td></td>
</tr>
<tr>
<td>my new value</td>
<td>694.63 (4.50) (6.76) [8.12]</td>
<td></td>
</tr>
</tbody>
</table>

Theory of Pion FF: unitarity, analyticity and \( \chi \)PT (G. Colangelo (Pisa))

<table>
<thead>
<tr>
<th>( P )</th>
<th>( \chi^2 / \text{d.o.f.} )</th>
<th>( \chi^2_{\text{CMD2/NA7}} )</th>
<th>( 10^{10} a_\rho )</th>
<th>( 10^{10} a_{2M_K} )</th>
<th>( \langle r^2 \rangle (\text{fm}^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>84.9/83</td>
<td>43.6 / 43.7</td>
<td>420.1 ± 2.1</td>
<td>489.5 ± 2.2</td>
<td>0.4254 ± 0.0020</td>
</tr>
<tr>
<td>5</td>
<td>78.4/82</td>
<td>35.9 / 42.6</td>
<td>423.8 ± 2.6</td>
<td>494.1 ± 2.7</td>
<td>0.4300 ± 0.0024</td>
</tr>
<tr>
<td>6</td>
<td>78.1/81</td>
<td>36.0 / 42.2</td>
<td>424.4 ± 2.8</td>
<td>494.7 ± 2.9</td>
<td>0.4339 ± 0.0051</td>
</tr>
<tr>
<td>7</td>
<td>73.5/80</td>
<td>31.7 / 42.2</td>
<td>423.4 ± 2.9</td>
<td>493.2 ± 3.0</td>
<td>0.4350 ± 0.0051</td>
</tr>
<tr>
<td>8</td>
<td>73.5/79</td>
<td>31.6 / 42.2</td>
<td>423.5 ± 5.7</td>
<td>493.4 ± 7.4</td>
<td>0.4347 ± 0.0052</td>
</tr>
</tbody>
</table>

Numerical results for fits to CMD-2 and (spacelike) NA7 data. The errors given are purely statistical.

To be compared with: \( 429.02 ± 4.95 \) (stat) from trapezoidal rule. Gain factor of 2 in precision in stat error!
Final state radiation of hadrons

Remainder: Need 1pi “blob” in dispersion integrals: VP undressed cross-section

\[ |F_\pi^{(0)}(s)|^2 = |F_\pi(s)|^2 (\alpha/\alpha(s))^2 \]

VP effects in physical quantities must include photonic corrections to the hadronic 1pi blob:

\[ \text{had} + \text{had} + \cdots \]

Add theoretical prediction for FS radiation (including full photon phase space):

\[ |F_\pi^{(\gamma)}(s)|^2 = |F_\pi^{(0)}(s)|^2 \left( 1 + \eta(s) \frac{\alpha}{\pi} \right) \]

to order $O(\alpha)$, where $\eta(s)$ is a known correction factor (Schwinger 1989). The
corresponding $O(\alpha)$ contribution to the anomalous magnetic moment of the muon is

$$\delta \gamma a^{\text{had}}_\mu = (38.6 \pm 1.0) \times 10^{-11}$$

Final–state radiation in electron–positron annihilation into a pion pair
(S. Dubinsky, A. Korchin, N. Merenkov, G. Pancheri, O. Shekhovtsova) ⇒ talk by O. Shekhovtsova
Recent: Corrections due to internal $\epsilon$- and $\tau$-loops updated

\[ a_\mu = a_\epsilon^{\text{uni}} + a_\mu (m_\mu/m_e) + a_\mu (m_\mu/m_\tau) + a_\mu (m_\mu/m_e, m_\mu/m_\tau) \]

\[ a_\mu (m_\mu/m_e) = 1.094 \, 258 \, 282 \, 8 \, (98) \left( \frac{\alpha}{\pi} \right)^2 + 22.868 \, 379 \, 36 \, (23) \left( \frac{\alpha}{\pi} \right)^3 + 132.682 \, 3 \, (72) \left( \frac{\alpha}{\pi} \right)^4 \]

\[ a_\mu (m_\mu/m_\tau) = 7.8059 \, (25) \times 10^{-5} \left( \frac{\alpha}{\pi} \right)^2 + 36.054 \, (21) \times 10^{-5} \left( \frac{\alpha}{\pi} \right)^3 + 127.50 \, (41) \left( \frac{\alpha}{\pi} \right)^4 \]

\[ a_\mu (m_\mu/m_e, m_\mu/m_\tau) = 52.763 \, (17) \times 10^{-5} \left( \frac{\alpha}{\pi} \right)^3 + 0.037 \, 594 \, (83) \left( \frac{\alpha}{\pi} \right)^4 \]

with $\alpha^{-1}(\text{a.i.}) = 137.036 \, 000 \, 3 \, (10) \, [7.4 \text{ ppb}]$

\[ a_\mu^{\text{QED}} = 116 \, 584 \, 719.35 \times 10^{-11} \left( \frac{\alpha}{\pi} \right)^4 \left( \frac{\alpha}{\pi} \right)^5 \alpha_{\text{inp}} \]

shift by $+13.7 \times 10^{-11}$
Most problematic region now 1.4-2.0 GeV

Contributions: \(\rho\) 72\% rel. error 2.1 \% contribution to final error 1.3\%

Region 1.4-2.0 GeV: 20\% rel. error contribution to final error 1\%! 
In progress:

- R from ISR:
  - KLOE collect and analyze more data for $\pi\pi$
  - BABAR in progress

- R from scan:
  - R from CLEO will resolve Mark I vs. CB “discrepancy” check soon, run at 7.0, 7.4, 8.4, 9.4 10.0 and 10.3 GeV

- future plans:
  - BEPCII/BESIII: run at 2.2, 2.6, 3.0 GeV at precision 5.5, 3.4, 3.4 % ([7.6, 7.0, 5.6] % now)
  - VEPP-2000: 0.4-2.0 GeV [2005-2010] factor of 10 in Lumi, CMD-2,SND factor of 2 in precision ... 1.4 - 2 GeV in progress

For precision physics at an ILC, at least a $\tau$–charm factory is required 1% hadronic cross section measurements up to 3 GeV mandatory!
Previously unaccounted contributions ?:

\[ e^+ e^- \rightarrow \sigma \gamma, \ f_0 \gamma \ \text{to} \ a_{\mu}^{\text{had}} \ \text{(data very poor)} \]

\[ \Rightarrow \delta a_{\mu}^S = 1.0(0.6)[13.0(11.0)] \times 10^{-10}, \ \text{i.e.,} \ \sim +0.1[1] \sigma \]

(Narison 03, Dubnikova et al) TH based[PDG based] \( \delta a_{\mu}(0.6 - 2.0 \text{GeV}) < 0.7 \times 10^{-10} \)
form \( \pi \pi \gamma, \pi \eta \gamma \) which include decay products from \( \pi^0 \gamma, \sigma \gamma, f \gamma, a_1 \gamma \) (Eidelman 03)

(A. Dubničková et al.)

\[
\begin{align*}
a_{\mu}(\pi^0 \gamma) & = 17.2 \times 10^{-11} \\
a_{\mu}(\eta \gamma) & = 2.2 \times 10^{-11} \\
a_{\mu}(\eta' \gamma) & = 1.5 \times 10^{-11} \\
a_{\mu}(\sigma \gamma) & = 12.5 \times 10^{-11} \\
a_{\mu}(a^0 \gamma) & = 0.9 \times 10^{-11}
\end{align*}
\]

Controversial!
Theory:

- Radiative corrections for
  1) Radiative return calculations: continuing progress ⇒ Talks by J. Kühn and H. Czyz
  2) Bhabha (small/wide) angle
      progress towards full two-loop: Dixon et al., Penin, Czakon et al.
      two-loop virtual + soft complete including collinear mass logs (Penin)
      two additional hard real photons exist (Jadach et al.)
      missing: one-loop + additional hard real photon I think in 1 year from now
      complete $O(\alpha^2)$ result available. However: at $O(\alpha)$ still 0.5% disagreement
      between existing calculations! Talk by F. Nguyen
  3) $\mu^+\mu^-$ for normalization/cross check/event separation

Check: in all calculations presently aimed precision requires e.g. check in your program
proper treatment of vacuum polarization (VP) subtraction: now need time–like $\alpha(s)$ in
s–channels: Bhabha: t–channel $\alpha(t)$, s–channel $\alpha(s)$
Challenge for the future:

- \((g - 2)_\mu\) new experiment under discussion, up to factor 10 improvement
- \(\alpha(M_Z)\) for ILC factor 5 improvement required!

The show will go on!