Mathematische Grundlagen
Übungsblatt 9 – Lösungen

1. (a) \( x = g(y) = \frac{y^2 - 1}{2y} = \frac{1}{2} \left( y - \frac{1}{y} \right) \)

   (b) \( f'(x) = 1 + \frac{x}{\sqrt{x^2 + 1}} \)
   \( g'(y) = \frac{1}{2} \left( 1 + \frac{1}{y^2} \right) \)

   (c) \( x^2 + 1 = \frac{(y^2 - 1)^2 + 4y^2}{4y^2} = \frac{(y^2 + 1)^2}{4y^2} \)
   \[ \Rightarrow \quad f'(x) = 1 + \frac{2xy}{y^2 + 1} = 1 + \frac{y^2 - 1}{y^2 + 1} = \frac{2y^2}{y^2 + 1} = \frac{1}{g'(y)} \]

2. 
\[
\begin{align*}
\frac{\partial f}{\partial r} &= \cos \varphi e^r \cos \varphi \\
\frac{\partial f}{\partial \varphi} &= -r \sin \varphi e^r \cos \varphi \\
\frac{\partial^2 f}{\partial r^2} &= \cos^2 \varphi e^r \cos \varphi \\
\frac{\partial^2 f}{\partial \varphi^2} &= (-r \cos \varphi + r^2 \sin^2 \varphi) e^r \cos \varphi \\
\frac{\partial}{\partial \varphi} \frac{\partial f}{\partial r} &= (-\sin \varphi - r \sin \varphi \cos \varphi) e^r \cos \varphi = \frac{\partial}{\partial r} \frac{\partial f}{\partial \varphi}
\end{align*}
\]

3. (a) \(-\frac{1}{a} \cos at\)
   (b) \(2\sqrt{1 + x^2}\)
   (c) \(-\log |1 - t^2|\)
   (d) \(\frac{x^3}{3} \log x - \frac{x^3}{9}\)

4. \(\heartsuit\)
   (a) Integrand ist ungerade, Intervall ist symmetrisch
   (b) Integrand ist ungerade bzgl. \( \phi = \pi/2 \)